Internet of Things - Exercises

Matteo Cesana

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Contents

Chapter 1

Exercises on Energy Consumption

Exercise–1 (*Exam of July 1, 2015*)

Nodes A, B and C in Figure 1.1 periodically collect and send temperature samples to the remote sink. The transmission phase is managed through a dynamic clustering approach which work as follows: two nodes send their samples to the clusterhead which then takes the average out of all the sample (two received + one obtained locally) and sends a single packet to the SINK. The clusterhead role is assigned in a round robin fashion starting from node A (node A, then B, then C, then A, etc.) (when clusterhead is C, B sends its message directly to C, and viceversa – not through A). Find the network lifetime (time to the first "death") with the following parameter set:

- energy required to operate the TX/RX circuitry $E_c = 6[\mu J/packet]$,
- energy required to support sufficient transmission output power $E_{tx}(d) = k \times d^2$ [nJ/packet], being k=120 [nJ/packet/m2]
- energy for taking the average of 3 samples E_p= 4 [μJ]
- initial energy budget E_b=128[μJ] for each node

Solution of Exercise–1

The energy consumed by the three nodes in one collection round is:

$$E_C = E_B = 2E_c + E_{tx}(5m) + E_{tx}(10m) + 2E_c + E_p + E_c + E_{tx}(\sqrt{125}m) = 64[\mu J]$$

$$E_A = 2E_c + 2E_{tx}(5m) + 2E_c + E_p + E_c + E_{tx}(10m) = 52[\mu J]$$

Two full rounds of data collection can be performed.

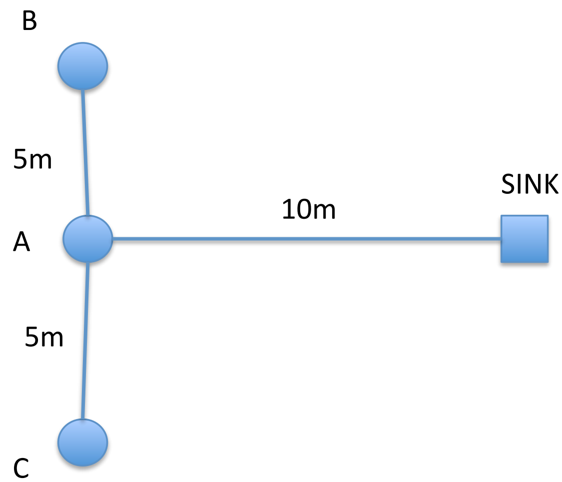


Figure 1.1: Reference topolgy

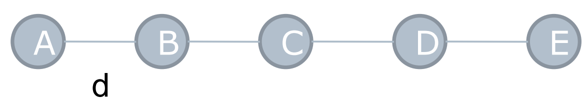


Figure 1.2: Reference topolgy

Exercise–2

A linear wireless sensor network is composed of 5 nodes. Each node is d=5 [m] away from its closest neighbor. Assuming that: (i) the energy required to operate the TX/RX circuitry, E_c=50 [nJ/bit], the energy required to support sufficient transmission output power E_{tx}(d) $= kd^2$ [nJ/bit], being k=1 [nJ/bit/m2], (iii) packets of b=2000 [bits], tell if its more "energy convenient" to use direct transmission (from A to E) or minimum per-transmission energy routing (A-B-C-D-E)

Solution of Exercise–2

The total consumed energy under direct transmission (A-E) is:

$$E_{direct} = E_c b + E_{tx}(4d)b + E_c b = 1[mJ]$$

The total consumed energy under minimum per-transmission energy routing (A-B-C-D-E) is:

$$E_{minenergy} = 4b(2E_c + E_{tx}(d)) = 1[mJ]$$

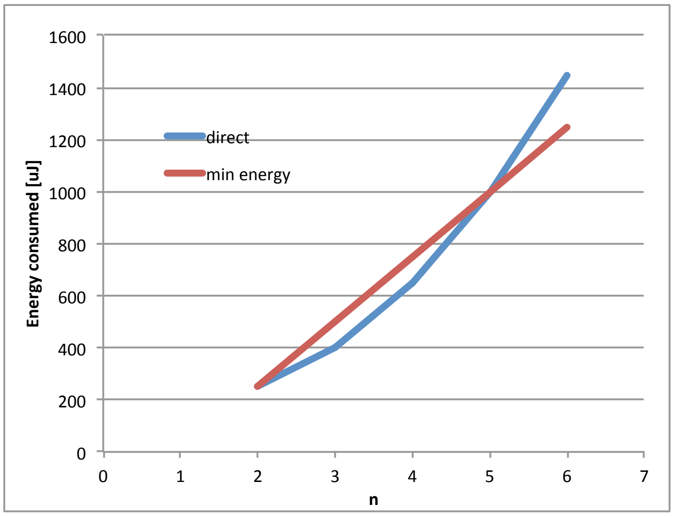


Figure 1.3: Energy consumption.

Exercise–3

Under the same network topology as Exercise 2, find out the number of sensor nodes for which the direct transmission consumes the same amount of energy as the min per-transmission energy routing.

Solution of Exercise–3

Calling n the total number of sensor nodes in the linear network topology, the expressions of the energy consumed in the two cases can be generalized as:

$$E_{direct} = 2E_c b + E_{tx}((n-1)d))b$$

$$E_{minenergy} = (n-1)b(2E_c + E_{tx}(d))$$

By setting up and solving in n the equality $E_{direct} = E_{minenergy}$, we get two solutions $n = 2$ and $n = 5$. The solution $n = 2$ corresponds to the case where we have only two nodes (one wireless link), and therefore the $min-energy$ and $direct$ routing policies coincide. Figure 1.3 reports the energy consumed in [mW] under the two routing policies. Expectedly, the consumed energy increases linearly in case of $min-energy$ routing, whereas it increases quadratically in case of $direct$ routing.

Exercise–4

A linear personal area network (PAN) (see Figure 1.4) is composed of 3 close-by motes and a distant Gateway. The communication is performed by electing a cluster head among the three motes which collects the traffic from the other motes and sends it remotely to the gateway. The energy required to operate the TX/RX circuitry, E_c=50 [nJ/bit]. The energy required to support sufficient transmission output power E_{tx}(d) = k d^2 [nJ/bit], being k=1 [nJ/bit/m2]. Packets to be exchanged are of b=2000 [bits]. The distance d is of 5 [m] and all the three nodes A, B and C are in transmission range.

1. Write the expression of the energy consumed for sending one packet per node to the gateway when using A, B and C as cluster heads respectively.
2. Find the values of D for which the best solution is to elect B as cluster-head.
3. Write the energy consumption under direct transmission (no cluster-head)

Solution of Exercise–4

Case 1: A is clusterhead

In this case node A needs to receive one packet from B, one packet from C and send the three packets (B's, C's and its own one) to the gateway. Thus, the total energy consumed by node A in this case can be written as:

$$E_A^1 = 2E_c b + 3(E_c b + E_{tx}(D + 2d)b)$$

The energy consumed by B and C in this very same case is:

$$E_B^1 = E_c b + E_{tx}(d)b$$

$$E_C^1 = E_c b + E_{tx}(2d)b$$

The total energy required to deliver the three packets to the central controller is:

$$E_{tot}^1 = E_A^1 + E_B^1 + E_C^1$$

Case 2: B is clusterhead

In this case node B needs to receive one packet from A, one packet from C and send the three packets (A's, C's and its own one) to the gateway. Thus, the total energy consumed by node B in this case can be written as:

$$E_B^2 = 2E_c b + 3(E_c b + E_{tx}(D + d)b)$$

Figure 1.4: Reference topology

The energy consumed by A and C in this very same case is:

$$E_A^2 = E_C^2 = E_c b + E_{tx}(d)b$$

The total energy required to deliver the three packets to the central controller is:

$$E_{tot}^2 = E_A^2 + E_B^2 + E_C^2$$

Case 3: C is clusterhead In this case node C needs to receive one packet from A, one packet from B and send the three packets (A's, B's and its own one) to the gateway. Thus, the total energy consumed by node C in this case can be written as:

$$E_C^3 = 2E_c b + 3(E_c b + E_{tx}(D)b)$$

The energy consumed by A and B in this very same case is:

$$E_B^3 = E_c b + E_{tx}(d)b$$

$$E_A^3 = E_c b + E_{tx}(2d)b$$

The total energy required to deliver the three packets to the central controller is:

$$E_{tot}^3 = E_A^3 + E_B^3 + E_C^3$$

We can safely say that $E^1 \geq E^3$ (think about it). Thus, we have to find the conditions in which $E^2 \leq E^3$.

The energy consumption is case the three nodes perform direct transmission to the gateway the total energy consumption would be:

$$E_{direct} = 3E_c b + E_{tx}(D + 2d)b + E_{tx}(d + D)b + E_{tx}(D)b$$

Note that we are not considering the energy for receiving the three packets by the gateway node.

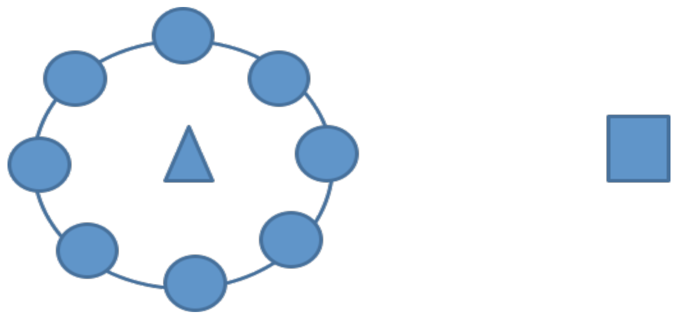


Figure 1.5: Reference topolgy

Exercise–5

The personal area network (PAN) in Figure 1.5 is used to gather periodical information on the average temperature of a given area. N motes are geared with temperature sensors. Consider the following two scenarios:

1. The sensors periodically report their measures to the PAN Coordinator which then relays each report to the information sink which performs the average operation.

2. The sensors periodically report their measures to the PAN Coordinator which performs the average operation and then sends one averaged sample to the information sink.

Assuming that:

- Sensors are d=5 meters away from the PAN Coordinator, The PAN Coordinator is D=10 meters from sink

- The packets containing the temperature samples and the averaged temperature are 127 [byte]

- the energy for operating TX/RX circuitry is E_c=50 [nJ/bit];

- the energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/bit], being k=1 [nJ/bit/m2]

- the energy required to perform the average operation is E_p=4 [μJ] for each temperature measure to be averaged

- the energy consumption for the averaging operation performed at the information sink is negligible

- Write the energy consumption to get one averaged temperature value at the sink when N=4 in the two aforementioned cases.
- Under the given parameters, is there any value of N beyond which performing the averaging operation at the PAN Coordinator is less energy-efficient than at the sink? If so, find out this value.

Solution of Exercise–5

In **Case 1**, the overall energy consumption can be written as:

$$E^1_{tot} = Nb(2E_c + E_{tx}(d)) + Nb(E_c + E_{tx}(D)),$$

where the first term accounts for the total energy consumed by all the sensor nodes to deliver 1 packet to the PAN coordinator and for the energy consumed by the PAN coordinator for receiving one packet from all the sensor nodes; the second term accounts for the energy consumed by the PAN coordinator to deliver all the received packets to the SINK. Note that in this case there's no energy consumed for performing the averaging operation across the temperature samples as this i performed by the SINK which is commonly expected to be attached to the mains.

In **Case 2**, the overall energy consumption can be written as:

$$E^2_{tot} = Nb(2E_c + E_{tx}(d)) + NE_p + b(E_c + E_{tx}(D)),$$

where the first term accounts for the total energy consumed by all the sensor nodes to deliver 1 packet to the PAN coordinator and for the energy consumed by the PAN coordinator for receiving one packet from all the sensor nodes; the second term accounts for the energy consumed by the PAN coordinator to perform the averaging operation across the N temperature samples received and the last term accounts for the energy consumed by the PAN coordinator for to deliver the single packet containing the averaged measure to the SINK.

If $N = 4$, it turns out that $E^1_{tot} = 1.117[mJ]$ and $E^2_{tot} = 676.4[\mu J]$

In order to verify if there exists any value of N for which Case 2 is less energy-efficient than Case 1, we have to solve (in N) the inequality $E^1_{tot} \leq E^2_{tot}$, that is,

$$Nb(2E_c+E_{tx}(d))+Nb(E_c+E_{tx}(D)) \leq Nb(2E_c+E_{tx}(d))+NE_p+b(E_c+E_{tx}(D))$$

After some maths, it turns out that $N \leq 1.01$, that is, the only case when **Case 2** is less energy efficient than **Case 1** corresponds to a topology with one single sensor node and the PAN coordinator. Note that this case is not much practical in the reference scenario since no averaging operation would be required with one single temperature sample.

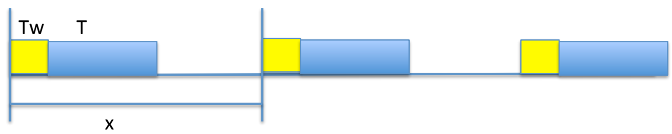


Figure 1.6: Reference case where mote switches on and off.

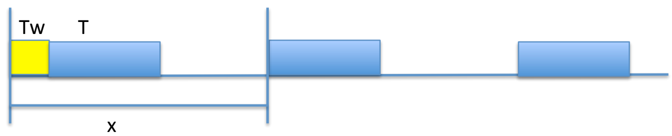


Figure 1.7: Reference case where mote activates at the beginning and then stays active.

Exercise–6 (*Exam of September 22, 2015*)

A sensor node generates a stream of 3 packets at a fixed rate of one packet every x [s]. The packets must be delivered to a sink node for further processing. The nominal data rate is R=250[kb/s] and the packets are of L=1000[bit]. The operating power level for TX circuitry is P_{tx} =100[mW]; the power emitted to the antenna is P_o=100[mW]; the power consumed while in idle and sleep states are P_{idle}=60[mW] and P_{sleep}=10[mW], respectively; In case the sensor goes to sleep, it needs a wake-up time of T_w=500 [μs]. Write the energy consumption for transmitting the 3 packets in the 2 cases where (1) the sensor node goes to sleep after each transmission and wakes up when the following packet is ready (2) the sensor node is always active (assume that in both cases the sensor node is asleep at the very beginning of the operations). Is there any value of x for which case (2) is more energy-efficient than case (1)?

Solution of Exercise–6

Figures 1.6 and 1.7 report the temporal evolution of the systems in the two cases where the mote switches on and off, and the mote stays active after first transmission, respectively.

Calling T the transmission time of one packet, with $T = \frac{L}{R} = 4[ms]$ The energy consumed in the two cases can be written as follows:

- **Case 1** - mote switches on and off:

$$E_1 = 3[T_w P_{tx} + (P_{tx} + P_o)T + P_{sleep}(x - T - T_w)]$$

- **Case 2** - mote stays active after first transmission

$$E_2 = P_{tx}T_w + (P_{tx} + P_o)T + P_{idle}x - P_{idle}T_w - P_{idle}T + 2[(P_{tx} + P_o)T + P_{idle}(x - T)]$$

We have to check for which values of x it is $E_2 \leq E_1$. By solving the inequality in x, we get: $x \leq 4.76$ [ms]

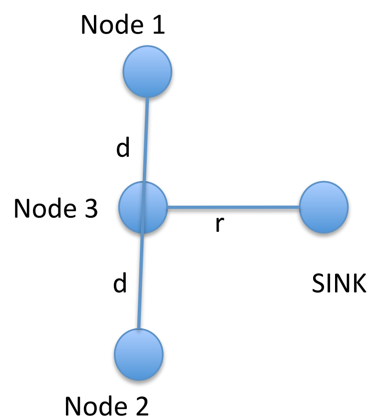


Figure 1.8: Reference topology for Ex. 7.

Exercise–7 (*Exam of September 22, 2014*)

A wireless sensor network is composed of three nodes and a sink. The three nodes mount temperature sensors and are set to deliver 1 temperature sample to the sink at a frequency of 1 [Hz]. The temperature sample is carried by packets with size L=128[byte]. Assume that: the energy for acquiring one temperature sample is E_s=40[μJ], the energy required to operate the TX/RX circuitry is E_c=50 [nJ/bit], the energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/bit], being k=1 [nJ/bit/m^2], the energy for processing one temperature sample E_p=20[μ J], d=5[m] and r=5[m].

Find out the energy consumed by each one of the three sensors in the two cases:

1. the sensors send the temperature sample directly to the sink,
2. sensor 1 and sensor 2 send their sample to sensor 3, sensor 3 performs the average of the three samples (from 1, 2 and its own) and sends a single packet to the sink (in this case, the energy consumed by sensor 3 for processing is $3E_p$)

Find out the network lifetime (time at which the first sensor node runs out of energy) in the two previous cases, if all the nodes have an initial energy budget E_b=100[mJ].

Solution of Exercise–7

The distance between sensor 1 and the SINK and between sensor 2 and the SINK is $d^* = \sqrt{(}50)$[m].

The energy consumption in **Case 1** is:

$$E_1 = E_2 = E_s + E_c L + E_{tx}(d^*)L = 142.4[\mu J]$$

$$E_3 = E_s + E_c L + E_{tx}(d)L = 116.8[\mu J]$$

The bottlenecks (energy-wise) are sensors 1 and 2, thus the lifetime of the network will be driven by the lifetime of these two nodes; with an energy budget of E_b= 100 [mJ], the maximum number of packet transmissions which can be performed before nodes 1 and 2 run out of energy is:

$$L_1 = \frac{E_b}{E_1} = \frac{100[mJ]}{142.4[\mu J]} \simeq 702[cycles]$$

The energy consumption in **Case 2** is:

$$E_1 = E_2 = E_s + E_c L + E_{tx}(d)L = 116.8[\mu J]$$

$$E_3 = E_s + 3E_p + E_c L + E_{tx}(r)L + 2E_c L = 279.2[\mu J]$$

The bottleneck (energy-wise) is sensor 3, thus the lifetime of the network will be driven by the lifetime of this node; with an energy budget of E_b= 100 [mJ], the maximum number of packet transmissions which can be performed before node 3 runs out of energy is:

$$L_2 = \frac{E_b}{E_3} = \frac{100[mJ]}{279.2[\mu J]} \simeq 358[cycles]$$

Figure 1.9: Reference topology for Ex. 8.

Exercise–8 (*Exam of June 30, 2014*)

A visual sensor network is composed of a camera node and a plain mote (see Figure 1.9). The camera node acquires an image of I=10 [Mbyte] which needs to be processed. The camera node sends a fraction of the image, xI, to Mote 1 for processing and processes locally the remaining part. In this case, the camera node first sends xI to Mote 1 and, upon completion of the transmission, starts processing the remaining part. Mote 1 starts processing its part as soon as it has received it.

Find out the value of x for which the camera node and the plain mote stop processing at the same time (initial time is the time the camera node sends out the first bit of xI to the plain mote). The capacity of the link camera-Mote 1 is C= 1[Mb/s] and the processing rates of the camera and Mote 1 are respectively, v_c= 100 [kb/s], v_1= 500[kb/s].

Find out the corresponding total energy consumption under the following parameters:

- Energy for transmitting/receiving: E_{tx}= E_{rx}=50 [nJ/bit] (including circuitry and transmission energy)
- Energy for processing E_{proc} = 100 [nJ/bit]

Solution of Exercise–8

The camera node and Mote 1 stop processing at the same time if the following holds (see Figure 1.10):

$$\frac{xI}{v_1} = \frac{(1-x)I}{v_c},$$

which leads to $x = \frac{v_1}{v1+vc}$=0.83.

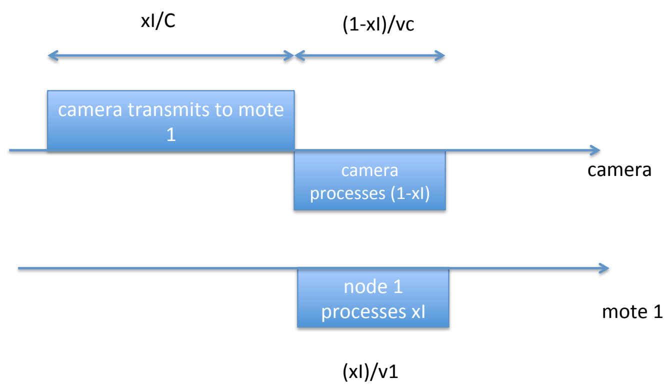


Figure 1.10: Temporal evolution for the reference system of Ex. 8.

The camera node consumes energy for transmitting xI [bits] and for processing $(1-x)I$ [bits]. In details,

$$E_{camera} = E_{tx}xI + E_{proc}(1-x)I = 3.32[Joule] + 1.36[Joule] = 4.68[Joule]$$

Mote 1 consumes energy for receiving xI bits and for processing xI bits:

$$E_{mote1} = xI(E_{rx} + E_{proc}) = 9.96[Joule]$$

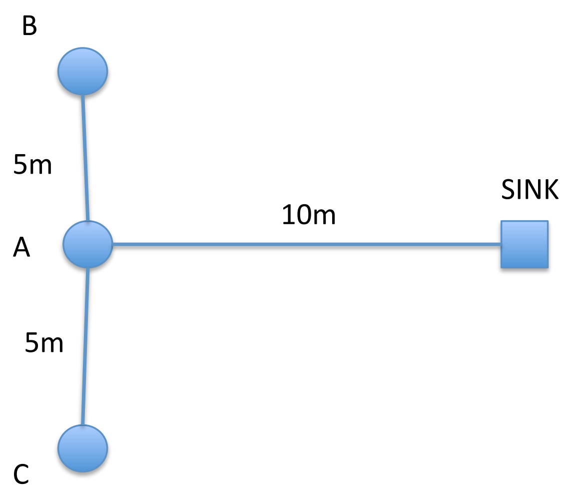


Figure 1.11: Reference topolgy

Exercise–9 (*Mock Exam, 2016*)

Nodes A, B and C in the figure periodically collect and send temperature samples to the remote sink. The transmission phase is managed through a clustering approach which work as follows: two nodes send their samples to the cluster head which then takes the average out of all the sample (two received + one obtained locally) and sends a single packet to the SINK.

Find the network lifetime (time to the first "death") in the three cases when cluster head is A, B and C respectively. Use the following parameters: energy required to operate the TX/RX circuitry E_c=6 [μJ/packet], energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/packet], being k=120 [nJ/packet/m^2], energy for taking the average of 3 samples E_p= 4 [μJ], initial energy budget E_b=150[μJ] for all the three nodes.

Solution of Exercise–9

In case mote A plays the cluster head, the energy consumed by the three motes is:

$$E_A = 2E_c + E_p + E_{tx}(10[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 12[\mu J] = 34[\mu J]$$

$$E_B = E_C = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

The energy bottleneck is mote A, and the corresponding lifetime is: $l = E_b/E_A \approx 4.4$ [cycles].

In case mote B plays the cluster head, the energy consumed by the three motes is:

$$E_B = 2E_c + E_p + E_{tx}(11.1[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 15[\mu J] = 37[\mu J]$$

$$E_A = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

$$E_C = E_c + E_{tx}(10[m]) = 6[\mu J] + 12[\mu J] = 18[\mu J]$$

The energy bottleneck is mote B, and the corresponding lifetime is: $l = E_b/E_B \approx 4.05$ [cycles].

In case mote C plays the cluster head, the energy consumed by mote A does not change with respect to the previous case, whereas the energy consumed by B and C are switched with respect to the previous case, that is:

$$E_C = 2E_c + E_p + E_{tx}(11.1[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 15[\mu J] = 37[\mu J]$$

$$E_A = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

$$E_B = E_c + E_{tx}(10[m]) = 6[\mu J] + 12[\mu J] = 18[\mu J]$$

The energy bottleneck is, in this case, mote C and the lifetime does not change with respect to the previous case, $l = E_b/E_C \approx 4.05$ [cycles].

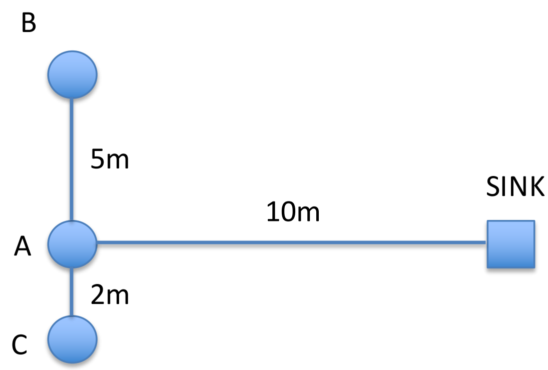


Figure 1.12: Reference topolgy

Exercise–10 (*July 1, 2016*)

Nodes A, B and C in the figure periodically collect and send temperature samples to the remote sink. The transmission phase is managed through a dynamic clustering approach which works as follows: two nodes send their samples to the cluster head which then takes the average out of all the sample (two received + one obtained locally) and sends a single packet to the SINK. The cluster head role is assigned in a round robin fashion starting from node A (node A, then B, then C, then A, etc.) (when cluster head is C, B sends its message directly to C, and viceversa – not through A).

Find the energy consumed by A, B and C in one round and the network lifetime (time to the first "death") with the following parameters: energy required to operate the TX/RX circuitry E_c=6 [μJ/packet], energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/packet], being k=120 [nJ/packet/m^2], energy for taking the average of 3 samples E_p= 4 [μJ], initial energy budget E_b=150[μJ] for all the three nodes.

Solution of Exercise–10

When mote A plays the cluster head, the energy consumed by the three motes is:

$$E_A = 2E_c + E_p + E_{tx}(10[m]) + E_c = 34[\mu J]$$

$$E_B = E_C = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

$$E_C = E_c + E_{tx}(2[m]) = 6[\mu J] + 0.48[\mu J] = 6.48[\mu J]$$

When it's Mote B turn:

$$E_B = 2E_c + E_p + E_{tx}(11.1[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 15[\mu J] = 37[\mu J]$$

$$E_A = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

$$E_C = E_c + E_{tx}(7[m]) = 6[\mu J] + 5.88[\mu J] = 11.88[\mu J]$$

When it's mote C's turn:

$$E_C = 2E_c + E_p + E_{tx}(\sqrt{104}[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 12.48[\mu J] = 34.48[\mu J]$$

$$E_A = E_c + E_{tx}(2[m]) = 6[\mu J] + 0.48[\mu J] = 6.48[\mu J]$$

$$E_B = E_c + E_{tx}(7[m]) = 6[\mu J] + 5.88[\mu J] = 11.88[\mu J]$$

The total energy consumed by A, B and C after one full round is:

$$E_A^{tot} = 49.48[\mu J]$$

$$E_B^{tot} = 57.88[\mu J]$$

$$E_C^{tot} = 52.84[\mu J]$$

The lifetime (measured in full rounds) is: $\frac{E_b}{E_B^{tot}} \approx 2.6$

Exercise–11 (*July 27, 2016*)

A sensor node generates a stream of 5 packets at a fixed rate of one packet every x [s]. The packets must be delivered to a sink node for further processing. The nominal data rate is R=250[kb/s] and the packets are of L=1000[bit]. The operating power level for TX circuitry is P_{tx} =10[mW]; the power emitted to the antenna is P_o=100[mW]; the power consumed while in idle and sleep states are P_{idle}=60[mW] and P_{sleep}=10[mW], respectively; In case the sensor goes to sleep, it needs a wake-up time of T_w=500 [μs]. Write the energy consumption when x=10[ms] for transmitting the 5 packets in the 2 cases where (1) the sensor node goes to sleep after each transmission and wakes up when the following packet is ready (2) the sensor node is always active (assume that in both cases the sensor node is asleep at the very beginning of the operations). Is there any value of x for which case (2) is more energy-efficient than case (1)?

Solution of Exercise–11

The packet transmission time, T, is: $T = \frac{L}{R} = 4[ms]$

Case 1 - node goes to sleep after transmission.

$$E_1 = 5(P_{tx}T_w+(P_{tx}+P_o)T)+5P_{sleep}(x-T-T_w) = 5(5\mu J+440[\mu J])+275[\mu J] = 2.5[mJ]$$

Case 2 - node always active

$$E_2 = T_wP_{tx}+5(P_{tx}+P_o)T+P_{idle}(5x-5T-T_w) = 5\mu J+5\times 440[\mu J]+1770[\mu J] = 3.975[mJ]$$

By solving the inequality $E_2(x) < E_1(x)$, we get $x < 3.98[ms]$, which is impossible given that the packet transmission time is $T = 4[ms]$.

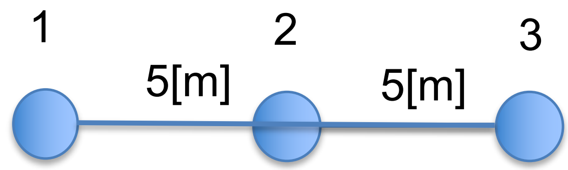


Figure 1.13: Reference topology for Ex. 12.

Exercise–12 (*September 2, 2016*)

Sensor node 1 and sensor node 2 are equipped with cameras and collect images with size I=12.8[kbyte]. The two sensors have to deliver the images to sensor 3 by using packets whose length is L=128[byte]. Assuming that: the energy required to operate the TX/RX circuitry is E_c=2 [uJ/packet], the energy required to support sufficient transmission output power $E_{tx}(d) = kd^2$ [nJ/packet], being k=120 [nJ/packet/m^2], find the total energy consumption (energy consumed by sensor 1, sensor 2 and sensor 3) to deliver one single image each in the following two cases:

- sensor 1 and sensor 2 send directly the images to the sink.
- sensor 1 sends the image to sensor 2, sensor 2 sends to sensor 3 its own image and a compressed version of sensor 1's image (compression ratio 0.1, that is, the compressed image has size .1 x I). In this case the energy required by sensor 2 to compress the image is E_p= 0.1 [μJ] for each packet of the original uncompressed image.

Solution of Exercise–12

The uncompressed image requires $N = I/L = 100$ packets to be delivered; the compressed image requires $0.1N/L = 10$ packets to be delivered.

In Case 1, the energy consumed by the three sensor nodes is:

$$E_1 = 100[E_c + E_{tx}(10[m])]$$

$$E_2 = 100[E_c + E_{tx}(5[m])]$$

$$E_3 = 200E_c$$

The total energy is, therefore, $E^1_{tot} = 400E_c{+}100E_{tx}(10[m]){+}100E_{tx}(5[m]) = 800[\mu J] + 1200[\mu J] + 300[\mu J] = 2.3[mJ]$

In Case 2, the energy consumed by the three sensor nodes is:

$$E_1 = 100[E_c + E_{tx}(5[m])]$$

$$E_2 = 110[E_c + E_{tx}(5[m])] + 100(E_p + E_c)$$

$$E_3 = 110E_c$$

The total energy is, therefore, $E^1_{tot} = 420E_c + 210E_{tx}(5[m]) + 100E_p = 840[\mu J] + 630[\mu J] + 10[\mu J] = 1.48[mJ]$

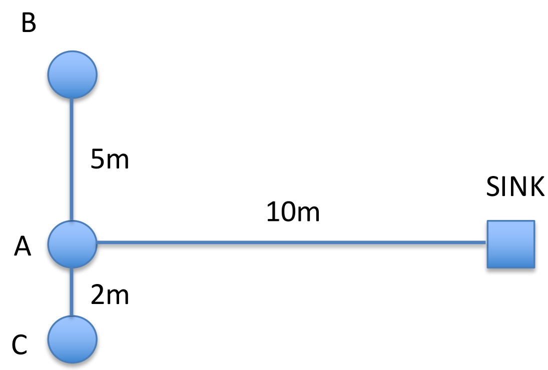


Figure 1.14: Reference topolgy

Exercise–13 (*September 22, 2016*)

Nodes A, B and C in the figure periodically collect and send temperature samples to the remote sink. The transmission phase is managed through a dynamic clustering approach which works as follows: two nodes send their samples to the cluster head which then takes the average out of all the sample (two received + one obtained locally) and sends a single packet to the SINK. The cluster head role is assigned repeating the following pattern A-A-B-B-C-C (when cluster head is C, B sends its message directly to C, and viceversa – not through A).

Find the energy consumed by A, B and C in one round and the network lifetime (time to the first "death") with the following parameters: energy required to operate the TX/RX circuitry E_c=6 [μJ/packet], energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/packet], being k=120 [nJ/packet/m^2], energy for taking the average of 3 samples E_p= 4 [μJ], initial energy budget E_b=150[μJ] for all the three nodes.

Solution of Exercise–13

When mote A plays the cluster head, the energy consumed by the three motes is:

$$E_A = 2E_c + E_p + E_{tx}(10[m]) + E_c = 34[\mu J]$$

$$E_B = E_C = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

$$E_C = E_c + E_{tx}(2[m]) = 6[\mu J] + 0.48[\mu J] = 6.48[\mu J]$$

When it's Mote B turn:

$$E_B = 2E_c + E_p + E_{tx}(11.1[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 15[\mu J] = 37[\mu J]$$

$$E_A = E_c + E_{tx}(5[m]) = 6[\mu J] + 3[\mu J] = 9[\mu J]$$

$$E_C = E_c + E_{tx}(7[m]) = 6[\mu J] + 5.88[\mu J] = 11.88[\mu J]$$

When it's mote C's turn:

$$E_C = 2E_c + E_p + E_{tx}(\sqrt{104}[m]) + E_c = 12[\mu J] + 4[\mu J] + 6[\mu J] + 12.48[\mu J] = 34.48[\mu J]$$

$$E_A = E_c + E_{tx}(2[m]) = 6[\mu J] + 0.48[\mu J] = 6.48[\mu J]$$

$$E_B = E_c + E_{tx}(7[m]) = 6[\mu J] + 5.88[\mu J] = 11.88[\mu J]$$

The total energy consumed by A, B and C after one full round is:

$$E_A^{tot} = 98.96[\mu J]$$

$$E_B^{tot} = 115.76[\mu J]$$

$$E_C^{tot} = 105.68[\mu J]$$

The lifetime (measured in full rounds) is: $\frac{E_b}{E_B^{tot}} \approx 1.2$

Exercise–14 (*Mock exam, January 20, 2017-Como*)

A wireless link between two sensor nodes is characterized by the following attenuation (in dB) $L = 70 + 32log(d)$, where d is the link length (expressed in km). Assuming that the receiver sensitivity is $P_{min} = -95$[dBm] , find the minimum required transmitted power which allows reception if the link length is d = 500[m].

Solution of Exercise–14

The received power is given by $P_r = P_t – L$. By setting $P_r \geq P_{min}$, we get $P_t – L > P_{min}$ which leads to $P_t \geq P_{min} + L$=-34,6 [dBm]

Chapter 2

Exercises on IEEE 802.15.4 Standard

Exercise–1 (*Exam of July 1, 2015*)

The two-hop personal area network (PAN) in Fiigure 2.1 is composed of 4 motes and a PAN Coordinator. The PAN works in beacon-enabled mode. Mote 1 and Mote 2 have statistical (non-deterministic) traffic towards the PAN coordinator characterized by the following probability distribution:

- P(required rate=75[bit/s])=0.5
- P(required rate=225 [bit/s])=0.1
- P(required rate=0 [bit/s])=0.4

Mote 3 and Mote 4 have deterministic traffic towards the PAN coordinator with a required rate of 450 [bit/s]. Assuming that: (i) the active part of the

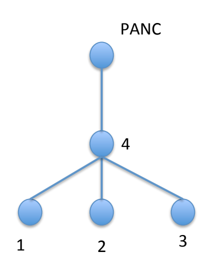


Figure 2.1: Ex. 1 Reference topolgy

Beacon Interval (BI) is composed of Collision Free Part only, (ii)the collision free part is divided in two: the first part is dedicated to the transmissions from Motes 1, 2 and 3 towards Mote 4, the second part is used by Mote 4 to deliver its own traffic and the relayed one to the PANC, (iii)the motes use b=128 [bit] packets to communicate with the PANC which fit exactly one slot in the CFP, and (iv) the nominal rate is R=250 [kb/s]

Solution of Exercise–1

The minimum rate required by all the four motes is 75 [bit/s]. The beacon interval can be dimensioned such that one slot in the CFP per beacon interval corresponds to 75 [bit/s]. Thus, BI = $\frac{128[bit]}{75[bit/s]}$ = 1.7 [s].

Mote 1 and 2 in the worst case require a bitrate of r_1=r_2=225 [kb/s] which corresponds to n_1=n_2=3 slots in the CFP.

Mote 3 requires a rate of r_3=450 [bit/s] which corresponds to n_3=6 slots in the CFP.

Mote 4 requires a rate of r_4=450 [bit/s] which corresponds to 6 slots for its own traffic and then it must be able to relay also the traffic from the other 3 motes. In total, mote 4 requires n_4=6+3+3+6=18 slots

The total number of slots in the CFP is thus: $N_{CFP} = n_1+n_2+n_3+n_4=$ 3 + 3 + 6 + 18 (Mote 4) = 30 slots.

The slot duration can be calculated as: $T_s = \frac{128[bit]}{250[kb/s]}$ = 512 [μs]

The duration of the CFP is $T_{CFP} = 30 \times T_s$ = 15.56 [ms]

The duration of the inactive part is: $T_{inactive} = BI - T_{cfp} - T_s$= 1.685 [s]

The duty cycle is $\eta = \frac{T_{active}}{BI}$ = 0.01

Exercise–2 (*Exam of July 28, 2015*)

A personal Area Network based on IEEE 802.15.4 beacon-enabled mode is deployed to collect temperature samples out of 1000 sensor nodes. Each sensor node collects one temperature sample every 5 minutes and has storage space to store one single sample (if a new sample is acquired and the previous one is still in the local memory, the previous sample is discarded and substituted by the new one).

Assuming that the nominal rate is R=250[kb/s], that the temperature samples fit exactly in packets of 50[byte], design the Beacon Interval structure (slot duration, BI duration, number of slots in the BI) which minimizes the duty cycle under the tight requirement that all the acquired samples get to the PAN coordinator (null sample loss). Do the same if the sensor nodes can store locally two temperature samples.

Solution of Exercise–2

To avoid sample loss, a temperature sample must be delivered to the PAN coordinator before a new sample is collected. Hence, each sensor node must be assigned a channel towards the PAN coordinator able to deliver at least one temperature sample every T=5 minutes (or even faster). This means that we have an upper bound on the beacon interval duration, that is, BI ¡= 5[minutes]. If we assume that one temperature sample fits exactly one slot, we have Ts=50[byte]/250[kb/s]=1.6[ms]. We need 1000 slots at least in the BI, thus T_{active}=1000 T_s = 1.6[s]. The corresponding duty cycle is minimum and is equal to $\eta = \frac{T_{active}}{T}$=0.005.

If each sensor node can store two samples locally, it means that it can wait 10 minutes before sending one temperature sample to the PANC. To avoid losses, though, each sensor node need to have two slots in the BI. In short, both the duration of the active part and the duration of the BI double, which leads to the same duty cycle calculated at the previous step.

Exercise–3 (*Exam of September 8, 2015*)

A Personal Area Network composed of 50 motes and a PAN Coordinator is operated according to the IEEE 802.15.4 beacon-enabled mode with the following parameters: (i) Active part composed of CFP only (no CAP) with slots of 128 [byte] packets; (ii) nominal data rate is R=250 [kbit/s]; (iii) 20 motes of Type 1 generate uplink traffic whose rate follows this with the following distribution: P(r_1=25[bit/s])=0.4 P(r_1=50[bit/s])=0.6; (iv) 30 motes of Type 2 generate uplink traffic whose rate follows the distribution: P(r_2=50[bit/s])=0.2 P(r_2=1[kbit/s])=0.8.

Define a consistent Beacon Interval structure including the number of slots in the CFP, the Beacon Interval duration, and the duty cycle. Define a consistent slot assignment in the CFP for all the devices in the network. Write the average energy consumption for motes of Type 1 and 2 in case all the motes are in radio range (let $E_{rx} = E_{tx}$=40[mJ] be the energy per slot for receiving/overhearing/transmitting, E_{idle}=20[mJ] and E_{sleep}=10[μJ] the energy per slot for being idle and sleeping, respectively).

Solution of Exercise–3

The minimum rate required by all the motes is 25 [bit/s]. The beacon interval can be dimensioned such that one slot in the CFP per beacon interval corresponds to 25 [bit/s].

Thus, BI = $\frac{128[byte]}{25[bit/s]}$ = 40.96 [s].

Mote of Type 1 and Type 2 in the worst case require a bitrate of r_1^{max} =50 [b/s] and r_2^{max} =1[kb/s], respectively, which corresponds to 2 slots and 40 slots in the CFP.

The total number of slots in the CFP is thus: $N_{CFP} = 2 \times 20(Motes of Type 1) + 40 \times 30(Motes of Type 2)$= 1240 slots.

The slot duration can be calculated as: T_s=128 [byte] /250 [kb/s] = 4.096 [ms].

The duration of the CFP is T_{CFP}=1240 × Ts = 5.079 [s].

The duration of the inactive part is: $T_{inactive} = BI - T_{CFP} - T_s$= 35 [s].

The duty cycle is $\eta = \frac{T_{active}}{BI}$= 0.12

Since the energy for receiving is the same as the energy for transmitting and overhearing and all the motes are in radio range, Motes of Type 1 and 2 have the same average energy consumption, that is:

$$E = 20 \times (0.6 \times 2E_{tx} + 0.4(E_{tx} + E_{idle})) + 30 \times (0.2(2E_{rx} + 38E_{idle}) + 0.8 \times 40 \times E_{rx}) + E_{rx} + E_{sleep}N_{sleep},$$

being N_{sleep} the number of equivalent slot in inactive part of the BI (approx. 8500).

Exercise–4 (*Exam of June 30, 2014*)

The two-hop personal area network (PAN) in Figure 2.1 is composed of 4 motes and a PAN Coordinator. The PAN works in beacon-enabled mode. Mote 1 and Mote 2 have statistical (non-deterministic) traffic towards the PAN coordinator characterized by the following probability distribution: P(required rate=50 [bit/s])=0.5, P(required rate=250 [bit/s])=0.25, P(required rate=0 [bit/s])=0.25. Mote 3 and Mote 4 have deterministic traffic towards the PAN coordinator with a required rate of 500 [bit/s].

Assuming that: (i) the active part of the Beacon Interval (BI) is composed of Collision Free Part only; (ii) the collision free part is divided in two: the first part is dedicated to the transmissions from Motes 1,2 and 3 towards Mote 4, the second part is used by Mote 4 to deliver its own traffic and the relayed one to the PANC; (iii) the motes use b=128 [bit] packets to communicate with the PANC which fit exactly one slot in the CFP; (iv)the nominal rate is R=250 [kb/s]; (v) Mote 4 is 10 [m] away from the PANC, find:

1. The duration of the single slot, the duration of Beacon Interval (BI), the duration of the CFP and the duration of the inactive part, a consistent slot assignment for the four motes, and the duty cycle

2. The energy consumption in a BI for Mote 4 if the energy required to operate the TX/RX circuitry is E_c=50 [nJ/bit], the energy required to support sufficient transmission output power $E_{tx}(d) = kd^2$ [nJ/bit], being k=1 [nJ/bit/m2], the energy of being idle in a slot is $E_{idle} = 20$ [μJ] and the energy for sleeping is $E_{sleep} = 5$ [nJ].

Solution of Exercise–4

The minimum rate required by all the four motes is r =50 [bit/s]. The beacon interval can be dimensioned such that one slot in the CFP per beacon interval corresponds to 50 [bit/s].

Thus, BI = $\frac{128[bit]}{50[bit/s]}$ = 2.56 [s].

Mote 1 and 2 in the worst case require a bitrate of r_1^{max}=250 [kb/s] which corresponds to N_1=5 slots in the CFP.

Mote 3 requires a rate of r_3^{max}=500 [bit/s] which corresponds to N_3=10 slots in the CFP.

Mote 4 requires a rate of r_4^{max}=500 [bit/s] which corresponds to 10 slots for its own traffic and then it must be able to relay also the traffic from the other 3 motes. In total, mote 4 requires N_4=10+5+5+10=30 slots.

The total number of slots in the CFP is thus: $N_{CFP} = 2\times N_1 + N_3 + N_4 =$ 50 slots.

The slot duration can be calculated as: $T_s = \frac{128[bit]}{250[kb/s]} = 512$ [μs].

The duration of the CFP is T_{CFP}=50 T_s = 25.6 [ms]

The duration of the inactive part is: $T_{inactive} = BI – T_{CFP} – T_s$= 2.56 [s] – 25.6 [ms] – 512 [us] = 2.533 [s]

The duty cycle is $\eta = \frac{T_{active}}{BI}$= 0.01

The energy consumption of Mote 4 is due to: (i) the traffic mote 4 receives from mote 1, 2 and 3, (ii) the traffic mote 4 sends to the PANC.

Mote 4 receives for 10 slots the traffic coming from Mote 3: 10 $E_c b$.

Mote 4 receives traffic form Mote 1 and Mote 2: $0.25 \times 5 \times E_{idle} + 0.25 \times 5 \times E_c \times b + 0.5 \times (4 \times E_{idle} + E_c \times b)$.

Mote 4 sends its own traffic to the PANC: $10(E_{tx} \times b + E_c \times b)$.

Mote 4 relays traffic from mote 3 to the PANC: $10(E_{tx} \times b + E_c \times b)$.

Mote 4 relays traffic from Mote 1 and 2 to the PANC: $0.25 \times 5 \times E_{idle} + 0.25 \times 5 \times (E_{tx} b + E_c b) + 0.5 \times (4 \times E_{idle} + E_{tx} \times b + E_c \times b)$

The overall energy consumption is:

$$\begin{aligned} E_{mote4} = \quad & 10E_c b + 2(0.255E_{idle} + 0.255E_c b + 0.5(4E_{idle} + E_c b)) + \\ & 20(E_{tx} b + E_c b) + 2(0.255E_{idle} + 0.255(E_{tx} b + E_c b) + \\ & +0.5(4E_{idle} + E_{tx} b + E_c b)) + E_c b + 4947E_{sleep} \end{aligned}$$

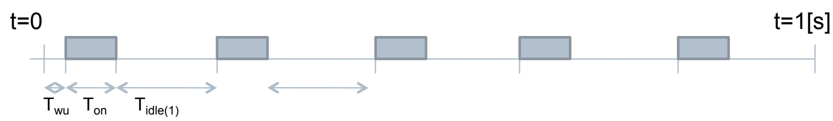


Figure 2.2: Temporal evolution in case the sensor node switches on at the beginning and then stays active.

Exercise–5

A IEEE 802.15.4 sensor switches on at time t=0 and start delivering samples (packets) to the PAN Coordinator at a constant rate of 5 [packet/s]. Each packet is 104 [byte] long and the channel bandwidth is 250 [kb/s]. Assuming the following parameters: (i) Consumed Power for the TX circuitry, P_{te}=5 [μW]; (ii) TX output power P_o=10 [μW], (iii) Wake up Time, T_{wakeUp}=500 [μs]; (iv) Sleep Power P_{sleep}=0.5 [μW]; (v) P_{idle}=8 [uW].

Find out the average power consumed by the sensor node in the following cases:

1. The node is always active (after switching on)

2. The node goes to sleep after each packet transmission

Solution of Exercise–5

In case the sensor node switches on at the beginning and then stays active (Figure 2.2), the consumed power in the first second of life of the sensor node can be calculated as follows: $P = 5(P_{te} + P_o)\ T_{on}+T_{wakeUp}P_{te} + T_{idle}P_{idle}$, being $T_{on}=\frac{104[byte]}{250[kb/s]}$=3.328[ms] and T_{idle}=1[s]-5 T_{on}-T_{wakeUp}=982.86[ms]. By substituting the values in the expression of the power consumption, we get P=8.11[μW].

In case the sensor node switches on and off continuously, the consumed power of the sensor node can be calculated as follows: $P = 5(P_{te}+P_o)\ T_{on}$+5 $T_{wakeUp}\times P_{te} + T_{sleep}P_{sleep}$, being T_{sleep}=1[s]-5 T_{on}-5T_{wakeUp}=980.86[ms]. By substituting the values in the expression of the power consumption, we get P=0.75[μW].

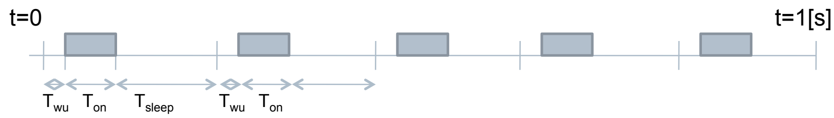


Figure 2.3: Temporal evolution in case the sensor node switches on an off continuously.

Exercise–6

An IEEE 802.15.4 mote wants to access the wireless channel. Assuming that the probability of finding the channel busy at each sensing event is p=0.05, find the probability, Q, that the mote is not able to access the channel within the first two backoff/sensing rounds.

Solution of Exercise–6

The backoff procedure starts with: NB=0, CW=2, BE=2.

At the first attempt, the accessing station waits for Rand(0, 2^BE-1) backoff periods and then senses the channel.

If the channel is sensed free for CW consecutive backoff periods, the station accesses the channel.

Otherwise, NB=1, CW=2, BE:=BE+1, and go back to 1.

Let's define the backoff success probability, P_{succ}, as the probability that the channel is sensed free in all the CW backoff periods. That is,

$$P_{succ} = (1-p)^2 = 0.9$$

The probability that the backoff procedure ends at the $i-th$ attempt is:

$$P_{succ}(i) = P_{succ}(1-P_{succ})^{(i-1)}$$

The require probability that the backoff procedure "fails" for the first two attempts is:

$$P^* = 1 - P_{succ}(1) - P_{succ}(2) = 0.01$$

Exercise–7

Refer to the CSMA procedure of the IEEE 802.15.4 standard. Assuming to know that procedure ends at the fourth sensing attempt, find out the average duration of the entire procedure (measured in backoff periods).

Solution of Exercise–7

The average number of backoff periods which are "wasted" at each backoff attempt is:

- 1st Attempt: $N^1_{wasted} = \frac{2^2-1}{2} = 1.5$
- 2nd attempt: $N^2_{wasted} = \frac{2^3-1}{2} = 3.5$
- 3rd attempt: $N^3_{wasted} = \frac{2^4-1}{2} = 7.5$
- 4th attempt: $N^4_{wasted} = \frac{2^5-1}{2} = 15.5$

At each failed attempt, the stations further wastes 1.5 backoff periods on average (it has to sense CW backoff periods)

At the last successful attempt, the stations has to wait for 2 periods.

Thus:

$$N_{wasted} = N^1_{wasted}+N^2_{wasted}+N^3_{wasted}+N^4_{wasted}+3\times1.5+2 = 34.5[backoff periods]$$

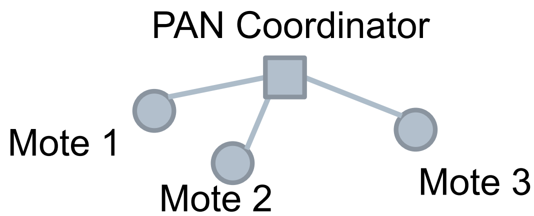


Figure 2.4: Reference network topology of 8.

Exercise–8

Consider the beacon-enabled Personal Area Network reported in Figure 2.4. Each mote is assigned a slot of 1 [ms] in the GTS which can carry one packet (the GTS has 3 slots in total). the Beacon Interval (BI) is 210 [ms]; the CAP is composed of 7 slots. The three motes have the following traffic characteristics: (i) Mote 1 generates uplink traffic according to a Poisson point process with parameter $\lambda_1 = 1$ [p/s]; (ii) Mote 2 generates uplink traffic according to a Poisson point process with parameter $\lambda_2 = 0.01$ [p/s]; Mote 3 generates uplink traffic according to a Poisson point process with parameter $\lambda_3 = 0.1$ [p/s]. The three motes and the PAN coordinator have similar hardware featuring the following energy consumption per unit slot: E_{sleep}=5[μJ], E_{idle}=4[mJ], E_{rx}=6[mJ], E_{tx}=7[mJ].

Find out the average consumed power by the three motes.

Solution of Exercise–8

The energy consumed within one BI by the three motes depends if they have/have not packets to be transmitted.

The probability that mote i has (has not) packets to be transmitted is the probability that at least 1 packet is (is not) generated by mote i within a BI, that is, $P_i = (1 - e^{-\lambda_i BI})$. By substituting the values of BI and λ_i into the formula, we get: $P_1 = 0.19$, $P_2 = 0.001$ and $P_3 = 0.02$.

The CAP is composed of 7 slots of 1[ms] and the CFP is composed of 3 slots of 1[ms]. Assuming that the transmission of one mote cannot be overheard by the other two, the average consumed power for mote i can be calculated as:

$$P_i^{consumed} = (9E_{idle} + E_{rx} + P_i E_{tx} + (1 - P_i)E_{idle} + N_{sleep}E_{sleep})/BI,$$

being N_{sleep} the number of slots in the BI in which the motes are sleeping which can be found by subtracting the duration of the active part duration (CAP+CFP+beacon slot) from the BI duration, that is, $N_{sleep} = \frac{210[ms]-10[ms]-1[ms]}{1[ms]} = 199$.

By substituting the values of the consumed energy per slot and P_i in the previous equation, one can obtain the values of average consumed power per mote.

$$P_1^{consumed} = [9 \times 4[mJ] + 6[mJ] + 0.19 \times 7[mJ] + 0.81 \times 4[mJ] + 199 \times 5[\mu J]]/210[ms] = 226.5[mW]$$
$$P_2^{consumed} = (9 \times 4[mJ] + 6[mJ] + 0.001 \times 7[mJ] + 0.999 \times 4[mJ] + 199 \times 5[\mu J])/210[ms] = 223.8[mW]$$
$$P_3^{consumed} = (9 \times 4[mJ] + 6[mJ] + 0.02 \times 7[mJ] + 0.98 \times 4[mJ] + 199 \times 5[\mu J])/210[ms] = 224[mW]$$

Assuming that the three motes are all in range, that is, they all can overhear the transmission of the other motes directed to the PAN coordinator, then the average power consumed by each mote can be calculated as:

$$\begin{aligned}
P_1^{consumed} &= (7 \times E_{idle} + E_{rx} + P_1 \times E_{tx} + (P_2 + P_3)E_{rx} + (3 - P_1 - P_2 - P_3) \times E_{idle} + 199 \times 5[\mu J])/210[ms] \\
&= 226.7[mW] \\
P_2^{consumed} &= (7 \times E_{idle} + E_{rx} + P_2 \times E_{tx} + (P_1 + P_3)E_{rx} + (3 - P_1 - P_2 - P_3) \times E_{idle} + 199 \times 5[\mu J])/210[ms] \\
&= 225.8[mW] \\
P_3^{consumed} &= (7 \times E_{idle} + E_{rx} + P_3 \times E_{tx} + (P_1 + P_2)E_{rx} + (3 - P_1 - P_2 - P_3) \times E_{idle} + 199 \times 5[\mu J])/210[ms] \\
&= 225.89[mW]
\end{aligned}$$

Exercise–9

Referring back to the previous exercise 8, find out the average consumed power by the PAN coordinator.

Solution of Exercise–9

The energy (power) consumed by the PAN coordinator depends on whether the PAN coordinator is actually receiving a packet in each single slot of the CFP, which, in turn, depends on whether the corresponding mote is actually transmitting an uplink packet.

In short, in each beacon interval the PAN coordinator "spends" energy: to transmit the beacon, to stay active/idle in the CAP, to receive packets from the motes (in case they are actually sending packets), and to sleep in the inactive part.

In formulas, the average power consumption can be written as:

$$P_{PANC} = E_{tx} + 7E_{idle} + (P_1 + P_2 + P_3)E_{rx} + (3 - P_1 - P_2 - P_3)E_{idle} + 199E_{sleep} = 230.55[mW]$$

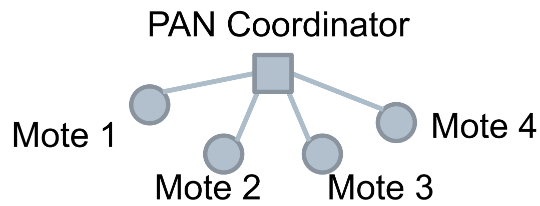


Figure 2.5: Reference network topology of 10.

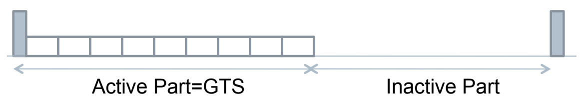


Figure 2.6: Reference beacon interval structure of 10.

Exercise–10

Consider the beacon-enabled Personal Area Network reported in Figure 2.5. Mote 1 and Mote 2 are equipped with temperature sensors that require a communication channel to the PANC of $r_1 = r_2$=500 [b/s]; Mote 3 requires a communication channel to the PANC of $r_3 = 1$ [kb/s]; Mote 4 requires a communication channel to the PANC of r_4 =10 [kb/s]. Assuming that: (i) the active part of the superframe is composed of GTS only; (ii)the motes use 127 [byte] packets to communicate with the PANC which fit exactly one slot in the GTS; (iii)each slot in the GTS lasts T_s=4.064 [ms], find:

- the superframe duration which allows to fulfill the traffic requirements
- The corresponding duty cycle of the PAN (or the PANC)
- How many additional motes with same requirements as Mote 3 could be added with the constraint of keeping the duty cycle below 20%

Solution of Exercise–10

Referring to Figure 2.6, the superframe is composed only of Guaranteed Time Slots (GTS). If one assigns one single slot per beacon interval to a specific mote, then the equivalent channel rate is: $r = \frac{127[byte]}{BI}$, being BI the beacon interval duration.

The lowest required rate to be supported is $r_1 = r_2 = 500[b/s]$; by imposing $r = r_1$, we get $BI = 2.032[s]$.

To define a consistent slot assignment, we must assign to each mote a number of slots in the beacon interval such that the equivalent data rate is at least as high as the maximum rate required by the specific mote. Since one slot per beacon interval is equivalent to a channel rate of $r = 500[b/s]$, then the required number of slots for mote i ($i \in \{1, 2, 3, 4\}$) is $N_i = \frac{r_i}{r}$, which leads to the following slot assignment: $N_1 = N_2 = 1$ slot, $N_3 = 2$ slots; $N_4 = 20$ slots.

Under this assignment, the GTS must be composed of $N_{GTS} = 24$ slots. The duration of the active part of the beacon interval is $T_{active} = 25T_s = 101.6$ [ms] (24 slots in the GTS and 1 slot for the beacon). By difference, the duration of the inactive part is $T_{inactive} = BI - T_{active} = 1.9304[s]$.

Finally, the duty cycle is $\eta = \frac{T_{active}}{BI} = 5\%$.

If the duty cycle can be as high as 20% (without modifying the BI duration), then the the duration of the active part can be increased to $T_{active} = 0.2BI = 406.4$ [ms], which corresponds to a total number of slots of $N_{active} = \frac{T_{active}}{T_s} = 100$.

The number of additional motes of type 3 can be calculated as: $n_3 = \lfloor (N_{active} - 25)\frac{r}{r_3} \rfloor$=37.

Exercise–11

A Personal Area Network (PAN) working in beacon-enabled mode is characterized by the following parameters: (i) activity cycle (duty cycle) η=10%; (ii)CAP duration: T_{cap}=50 [ms]; (iii) 10 slots in the CFP which carry L=200 [byte] packet and define data channels of r=1 [kbit/s].

Find out: (1) the duration of the superframe, T_{active}, (2) the duration of the beacon interval, (3)the duration of the CFP, T_{cfp}, (4)the nominal bit rate R, (5)the total capacity available in the CAP.

Solution of Exercise–11

Let's derive first the duration of the CFP. We know that 1 slot every beacon interval provides a data channel of r=1 [kb/s]. Thus, the duration of the beacon interval can be calculated as: BI=$\frac{L}{r} = \frac{200[byte]}{1[kbit/s]} = 1.6$ [s].

The duration of the active part can be derived by exploiting the information of the duty cycle. Namely, we can write:

$$T_{active} = \eta BI = 160[ms].$$

The duration of the single slot can be calculated as T_{slot}= $\frac{T_{active}-T_{cap}}{11} = 10[ms]$ (the denominator includes 10 slots of the CFP and one slot for the beacon). The Collision free Part (CFP) is: T_{cfp}=T_{active}- T_{cap}- T_{slot}= 100[ms].

The nominal bit rate is $R = L/T_{slot}$= 160[kb/s].

The CAP is composed of 5 "equivalent" slots; each equivalent slot provides a channel of 1 [kbit/s], thus the available capacity fo the CAP is 5 [kbit/s]

Exercise–12 (*Exam of July 28, 2015*)

A personal Area Network based on IEEE 802.15.4 beacon enabled mode is deployed to collect temperature samples out of 1000 sensor nodes. Each sensor node collects one temperature sample every 5 minutes and has storage space to store one single sample (if a new sample is acquired and the previous one is still in the local memory, the previous sample is discarded and substituted by the new one). Assuming that the nominal rate is R=250[kb/s], that the temperature samples fit exactly in packets of 50[byte], design the Beacon Interval structure (slot duration, BI duration, number of slots in the BI) which minimizes the duty cycle under the tight requirement that all the acquired samples get to the PAN coordinator (null sample loss).

Do the same if the sensor nodes can store locally two temperature samples.

Solution of Exercise–12

To avoid sample loss, a temperature sample must be delivered to the PAN coordinator before a new sample is collected. Hence, each sensor node must be assigned a channel towards the PAN coordinator able to deliver at least one temperature sample every 5 minutes (or even faster). This means that we have an upper bound on the beacon interval duration, that is, BI ¡= 5[minutes]. If we assume that one temperature sample fits exactly one slot, we have $T_s = \frac{50[byte]}{250[kb/s]}$=1.6[ms]. We need 1000 slots at least in the BI, thus T_{active}=1000 T_s = 1.6[s]. The corresponding duty cycle is minimum and is $\frac{1.6[s]}{5[minutes]}$=0.005.

If each sensor node can store two samples locally, it means that it can wait 10 minutes before sending one temperature sample to the PANC. In short, the duration of the active part remains the same but the BI duration doubles, which leads to half the duty cycle calculated at the previous step.

Exercise–13 (*Exam of July 1, 2015*)

A sensor node performs channel access according to the CSMA/CA scheme of the IEEE 802.15.4 standard. Assuming that the probability of finding the channel busy is p=0.05 at each backoff period, what is the probability that the sensor node does actually access the channel within the first two tries.

Solution of Exercise–13

The probability to find the channel free in two consecutive backoff periods is $P = (1-p)^2$=0.9.

The required probability is therefore: $P + (1-P)P$=0.9+0.09=0.99.

Exercise–14 (*Exam of September 22, 2014*)

A sensor network runs the IEEE 802.15.4 protocol and is composed by 4 sensor nodes directly connected to the PAN Coordinator. The Beacon Interval is composed of CFP slots only and each slot can carry a packet of size L=127[byte] and the nominal rate is R=250[kbit/s]. The sensor nodes have the following traffic requirements: (i) Sensor 1, Sensor 2: need a channel of r_1=100[bits/s], (ii) Sensor 3 and 4 need a channel of r_3=500[bits/s].

Find out a feasible structure for the BI indicating the CFP duration, the slot duration, the duty cycle and the number of slots assigned to each terminal. Assuming that the energy consumed in a slot for receiving/transmitting is E=40[μJ], the energy for overhearing other nodes' transmission is E_{ov}= 30[μJ], the energy for being idle is E_{idle}=10[μJ], and the energy for sleeping is E_{sleep}=1[nJ], find out the energy consumption for the three sensor node assuming that sensor 1 is in range of sensor 3 (and *viceversa*) and sensor 2 is in range of sensor 4 (and *viceversa*).

Solution of Exercise–14

Let's start be finding the slot duration, $T = \frac{L}{R} = 4.064$[ms]. The duration of the beacon interval can be found by assuming to assign one slot in the CFP for Sensor 1 and to Sensor 2. Under this assumption, the beacon interval duration must be set such that the data rate of the channel defined as "one slot per beacon interval" is higher or equal to r_1. In formulas, we can write:

$$BI = \frac{L}{r_1} = 10.16[s]$$

In this setting, the total number of slots needed in the CFP, N_{CFP}, must be $N_{CFP} = 12$ (1 slot for Sensor 1 and Sensor 2, and 5 slots fro Sensor 3 and Sensor 4).

The duty cycle of the network is then equal to:

$$\eta = \frac{T_{active}}{BI} = \frac{(N_{CFP}+1)T}{BI} = 5.210^{-3}$$

It is useful to find the equivalent number of slots in the inactive part of the beacon interval. The duration of the inactive part is:

$$T_{inactive} = BI - T_{active} = 10.16[s] - 52.8[ms] \simeq 2488$$

To find the energy consumption of the four sensors we have to observe that only sensor 1 and 3 and sensor 2 and 4 are in radio range. Thus, the energy consumed in one beacon interval by each one of the four sensors is:

$$E_1 = E_2 = E + E + 5E_{ov} + 6E_{idle} + 2488E_{sleep} \simeq 292.5[\mu J]$$

$$E_3 = E_4 = E + 5E + E_{ov} + 6E_{idle} + 2488E_{sleep} \simeq 332.5[\mu J]$$

Exercise–15 (*Exam of July 28, 2014*)

A personal area network (PAN) is operated according to the IEEE 802.15.4 beacon enabled mode. The Collision Access Part (CAP) of the superframe is composed of 20 slots. The inactive part of the beacon interval is composed of 1600 slots and the duty cycle is $\eta = 0.0202$. Each slot (of CAP and CFP) carries 127 [byte] packets. The nominal data rate is 250 [kb/s].

How many slots are available in the CFP? What is the rate of the channel defined as "one slot per beacon interval"?

Using the numbers found above, suppose that two motes (Mote 1 and Mote 2) are active in the PAN with the following traffic patterns:

- P(amount of data generated in one BI=508[byte])= 0.3
- P(amount of data generated in one BI =0 [byte])= 0.3
- P(amount of data generated in one BI=635 [byte])= 0.4

1. Define a consistent slot assignment in the CFP for the two motes (the motes do not use the CAP).
2. How many additional motes requiring a channel of 150 [bit/s] could be added to the network?
3. Find the average energy consumption of the two motes (assume that the two motes do not overhear the transmissions of each other). The total energy for receiving in a slot is E_{rx}=20 [μJ], the total energy to transmit in a slot is E_{tx}=30 [μJ], the energy of being idle in a slot is E_{idle}= 15 [μJ] and the energy for sleeping in a slot is E_{sleep}= 0.5[μJ]
4. What is the total capacity of the CAP?

Solution of Exercise–15

The total number of slots in the beacon interval is: $N_{BI} = N_{CAP} + N_{CFP} + 1 + N_{inactive}$.

The duty cycle is defined as: $\eta = \frac{N_{CAP}+N_{CFP}+1}{N_{BI}}$=0.0202. By solving this equation in N_{CFP} we obtain: $N_{CFP} \simeq 12$.

The slot duration is: $T_s = \frac{127[byte]}{250[kb/s]} = 4.064$[ms]. The duration of the beacon interval is: $T_{BI} = T_s N_{BI} \simeq$6.636[s].

If one slot is assigned to a Mote in one beacon interval, than the equivalent rate assigned to the Mote is: $r = \frac{127[byte]}{6.636[s]}$=153.10[bit/s].

In the worst traffic case,the two motes have generated 635[byte] which correspond to 635[byte]/127 [byte]=5 full-size slots. Thus, each one of the two motes must be assigned 5 slots in the CFP. After having assigned 5 slots to each one of the two motes, two slots (out of the available N_{CFP}=12) are still vacant.

Each slot in the beacon interval "corresponds" to an equivalent channel with rate 153.1[bit/s], thus two additional motes can be safely added.

Each one of the two motes spends energy for: (i) receiving the beacon in the beacon slot, (ii) being idle in the CAP, (iii)being idle in 7 slots of the CFP (5 belonging to the other mote and the two which are vacant), (iv) transmitting or being idle in its own 5 slots according to traffic pattern above. In short:

$$E = E_{rx}+20E_{idle}+1600E_{sleep}+7E_{idle}+0.3(4E_{tx}+E_{idle})+0.3\times5E_{idle}+0.4\times5E_{tx} = 1.225[mJ]$$

The CAP is composed of 20 slots each one "corresponding" to a channel of 153.1[bit/s]. Thus, the total capacity of the CAP is: r_{CAP}=3.062[kb/s].

Exercise–16 (*Exam of June 27, 2013*)

A personal area network (PAN) is composed of 4 motes and a PAN Coordinator. The PAN works in beacon-enabled mode. Mote 1 and Mote 2 require a deterministic communication channel to the PAN Coordinator of $r_1 = r_2$ =100 [bit/s]. Mote 3 requires a deterministic communication channel to the PAN Coordinator of r_3=600 [bit/s]. Mote 4 has statistical (non-deterministic) traffic towards the PAN coordinator characterized by the following probability distribution: P(r_4=50 [bit/s])= 0.2, P(rr_4=100 [bit/s])= 0.2, P(r_4=400 [bit/s])= 0.6.

Assume that: (i) the active part of the Beacon Interval (BI) is composed of Collision Free Part only; (ii) the motes use b=100 [byte] packets to communicate with the PANC which fit exactly one slot in the CFP; (iii) the nominal rate is R =250 [kb/s]; (iv) Mote 1 and Mote 2 are d_1=5[m] away from the PANC; (v) Mote 3 and 4 are d_2=10[m] away from the PANC; (vi)the 4 motes are out of reach one another.

Find:

1. The duration of the single slot, the duration of Beacon Interval (BI), the duration of the CFP and the duration of the inactive part, a consistent slot assignment for the four motes and the duty cycle.

2. The energy consumption in a BI for a mote of Type 3 if the energy required to operate the TX/RX circuitry is E_c=50 [nJ/bit], the energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/bit], being k=1 [nJ/bit/m2], the energy of being idle in a slot is E_{idle}= 20 [μJ] and the energy for sleeping is E_{sleep}= 5 [nJ].

3. Assuming that the energy budget for the four motes is E_{budget}=1 [J], what is the PAN lifetime (number of beacon interval after which the first sensor runs out of energy)?

Solution of Exercise–16

The minimum required rate is r_4=50 [bit/s] (Mote 4). Thus, the beacon interval duration can be found by imposing: $BI = \frac{b}{r_4} = \frac{100[byte]}{50[b/s]}$=16 [s].

Mote 1 and Mote 2 require twice the transmission rate of the lowest rate value required by Mote 4, thus they must be assigned two slots in the beacon interval.

Mote 3 requires 12 times larger rate than the lowest rate value required by Mote 4, thus it must be assigned 12 slots in the beacon interval.

Mote 4 requires, in the worst case, 8 slots per beacon interval.

The slot duration can be found as: $T_s = \frac{b}{R} = \frac{100[byte]}{250[kb/s]} = 3.2$ [ms].

The duration of the active part is the duration of a single slots multiplied by all the slots in the active part (1 slot for the beacon, 2 slots for Mote 1, 2 slots for Mote 2, 8 slots for Mote 4, 12 slots for Mote 3), thus:

$$T_{active} = 25T_s = 80[ms]$$

The duty cycle of the PAN is defined as $\eta = \frac{T_{active}}{BI} = 5 \times 10^{-3}$. It is also useful to find the number of slots in the inactive part, whihc can be expressed as: $N_{sleep} = \frac{BI - T_{active}}{T_s} \simeq 4975$.

Mote 3 cannot hear the other motes' transmissions, thus the energy consumed by Mote 3 can be expressed as:

$$\begin{aligned} E_3 = & \quad E_c b + 12(E_c b + E_{tx}(d_2)b) + 12E_{idle} + 4975E_{sleep} = \\ & 40[\mu J] + 12(40[\mu J] + 80[\mu J]) + 1220[\mu J] + 49755[nJ] = 1.744[mJ] \end{aligned}$$

The energy bottleneck is Mote 3 since it is at the highest distance to the PANC and has deterministic traffic which occupies the highest number of slots in the BI.

Thus, the PAN lifetime can be estimated as: $L = BI \frac{E_{budget}}{E_3} \simeq$2.5 [hours]

Exercise–17 (*Mock Exam 2016*)

A personal Area Network based on IEEE 802.15.4 beacon enabled mode (only CFP) is deployed to collect temperature samples out of 1000 sensor nodes. 500 sensor nodes need to collect and send up one temperature sample on average every minute, whereas the remaining 500 nodes need to collect and send up one temperature sample on average every 5 minutes. Assuming that the nominal rate is R=250[kb/s], that the temperature samples are L=50[byte] long and fit exactly in one slot of the CFP, design the Beacon Interval structure (slot duration, BI duration, number of slots in the BI) which minimizes the duty cycle under the requirement that all sensor nodes have the required average channel rate towards the sink/PAN coordinator.

Solution of Exercise–17

Let's dimension the Beacon Interval with respect to the sensor nodes with the "slowest" required channel rate. We can thus assign one slot in each BI to nodes of type 2 and safely set the BI=5 [minutes]=300[s].

Along the same lines, we need to assign 5 slots in each BI to motes of type 1.

The number o slots in the *active part* of the BI is: $N_{active} = 500 + 500 \times$ 5 $\cancel{1} + 1 = 3001$.

The slot duration is: $T_s = \frac{L}{R}$=1.6[ms].

The active part duration is: $T_{active} = N_{active} \times T_s$=4.8[s], and consequently:

$$T_{inactive} = BI - T_{active} = 295.2[s]$$

$$N_{inactive} = \frac{BI}{T_s} = 184500[slots]$$

$$\eta = \frac{T_{active}}{BI} = 1.6\%$$

Exercise–18 (*July 1, 2016*)

A personal area network (PAN) is composed of 4 motes and a PAN Coordinator. The PAN works in beacon-enabled mode. Mote 1 and Mote 2 have statistical (non-deterministic) traffic towards the PAN coordinator characterized by the following probability distribution: P($r_{1,2}$=75[bit/s])=0.5, P($r_{1,2}$=225 [bit/s])=0.1, P($r_{1,2}$=0 [bit/s])=0.4. Mote 3 and Mote 4 have deterministic traffic towards the PAN coordinator with a required rate, $r_{3,4}$ of 450 [bit/s]. The PAN coordinator has to deliver downlink traffic towards the four nodes according to the following pattern: traffic towards Mote 1 and Mote 2 P($r_{1,2}^{PANC}$=75[bit/s])=0.5, P($r_{1,2}^{PANC}$ =225 [bit/s])=0.1, P($r_{1,2}^{PANC}$ =0 [bit/s])=0.4; traffic towards Mote 3 and Mote 4 deterministic with required rate $r_{3,4}^{PANC}$ 225 [bit/s].

Assuming that: (i) the active part of the Beacon Interval (BI) is composed of Collision Free Part only; (ii) the motes and the PAN coordinator use b=128 [bit] packets for their transmissions which fit exactly one slot in the CFP, (iii) the nominal rate is 250 [kb/s], find the duration of the single slot, the duration of Beacon Interval (BI), the duration of the CFP, the duration of the inactive part, a consistent slot assignment for all the transmissions (UPLINK AND DOWNLINK), and corresponding the duty cycle. Assuming that the energy consumption parameters are the following ones, find the average energy consumption in a beacon interval for the PAN coordinator; energy for receiving a packet E_{rx} =4[μJ], energy for transmitting a packet E_{tx} =7[uJ], energy for being idle in a slot E_{idle}= 3[μJ], energy for sleeping in a slot E_{sleep}= 3[nJ].

Solution of Exercise–18

Let's dimension the Beacon Interval with respect to the sensor nodes with the "slowest" required channel rate, that is 75 [bit/s]. We have $75[bit/s] = \frac{b}{BI}$, and then $BI = \frac{128[bit]}{75[bit/s]} \approx 1.7[s]$..

In the worst cases (maximum required rate), the four motes need the following number of slots in each beacon interval:

$$N_{mote1} = N_{mote2} = 225[bit/s]/75[bit/s] = 3$$

$$N_{mote3} = N_{mote4} = 450[bit/s]/75[bit/s] = 6$$

Along the same lines, we need to assign bunch of slot to the PAN coordinator for the downlink traffic. Namely, in the worst cases, the PAN coordinator will need 3 slots for each one of the four motes.

The total number of slots in the Collision Free Part is then: $N_{CFP} = 3+3+6+6+3+3+3+3 = 30$

The slot duration is $T_s = \frac{b}{R} = 512[\mu s]$.

The total duration of the active part is: $T_{active} = T_s \times (N_{CFP} + 1) = 15.8[ms]$ and consequently:

$$T_{inactive} = BI - T_{active} = 1.684[s]$$

$$N_{inactive} = T_{inactive}/T_s \approx 3289$$

The duty cycle is:

$$\eta = \frac{T_{active}}{BI} = 0.9\%$$

The average energy consumed by the PAN coordinator can be written as:

$$\begin{aligned} E_{PANC} = E_{tx} + 3289E_{sleep} &+ 2 \times \overset{6}{\cancel{3}}E_{tx} + 2[0.4 \times 3E_{idle} + 0.1 \times 3E_{tx} + 0.5 \times (E_{tx} + 2E_{idle})] \\ &+2 \times \underset{3}{\cancel{6}}E_{rx} + 2[0.4 \times 3E_{idle} + 0.1 \times 3E_{rx} + 0.5 \times (E_{rx} + 2E_{idle})] \end{aligned}$$

Exercise–19 (*July 1, 2016*)

A sensor node performs channel access according to the CSMA/CA scheme of the IEEE 802.15.4 standard. Assuming that the probability of finding the channel busy is p=0.3 at each backoff period, find: (i) the probability that the sensor node does actually access the channel within the first two tries, (ii) the average time after which the sensor node does actually access the channel (assume infinite backoff attempts are allowed).

Solution of Exercise–19

The probability that the channel is sensed idle in two consecutive backoff periods is $P_{idle} = (1-p)(1-p) = 0.49$. The probability that the mote does access the channel within the first two tries is:

$$P = P_{idle} + (1 - P_{idle})P_{idle} \approx \cancel{0.4} \quad 0,7399$$

The average number of attempts before the sensor nodes accesses the channel is given by $\frac{1}{P} \approx 2.04$; this means that, on average, the sensor node finds the channel busy at the first try and idle at the second try. The average number of backoff periods (slots) to access the channel can be written as:

$$E[T] \approx 1.5 + \frac{2^2 - 1}{2} + 2 = 5.$$

Note that this is an approximation since the average number of required tries is not exactly equal to 2. To correct expression of $E[T]$ would be:

$$E[T] = 2P + \sum_{i=2}^{\infty}(1-P)^{i-1}P[2 + (i-1)1.5 + \sum_{k=2}^{i}\frac{2^k - 1}{2}]$$

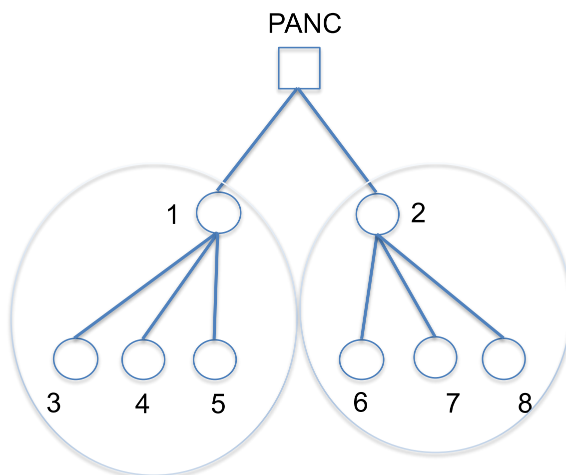


Figure 2.7: Reference topology for Ex. 20.

Exercise–20 (*July 27, 2016*)

The Personal Area Network in the figure is operated according to the IEEE 802.15.4 beacon enabled mode with the following parameters: (i) active part composed of CFP only (no CAP) with slots of 128 [byte] packets; (ii) nominal data rate is 250 [kbit/s]. The 8 motes in the figure are characterized by the following uplink traffic requirements:

- motes 1, 3, 5 and 7 generate uplink traffic whose rate has the following distribution: P(r=25[bit/s])=0.4 P(r=50[bit/s])=0.6
- motes 2, 4, 6 and 8 generate uplink traffic whose rate has the following distribution: P(r=50[bit/s])=0.2 P(r=1[kbit/s])=0.8

Motes 1 and 2, besides sending up to the PANC their own traffic, have to relay in each BI the traffic generated by their siblings nodes.

Define a consistent Beacon Interval structure including the number of slots in the CFP, the Beacon Interval duration, and the duty cycle. Define a consistent slot assignment in the CFP for all the devices in the network.

Solution of Exercise–20

The lowest required rate is 25 [bit/s]. Thus, the BI can be set as: $BI = \frac{128[byte]}{25[bit/s]} = 40.96[s]$.

Motes 1, 3, 5 and 7 require 2 slots in the BI for their own traffic. Motes 2, 4, 6 and 8 require 40 slots in the BI for their own traffic. Moreover, Mote

1 and Mote 2 also require extra slots for receiving and delivering traffic from their sibling nodes. Namely, Mote 1 requires extra 88 slots and Mote 2 requires extra 164 slots. To sum up, we have:

$$N_3 = N_5 = N_7 = 40$$

$$N_4 = N_6 = N_8 = 2$$

$$N_1 = 90$$

$$N_2 = 204$$

which leads to: $N_{CFP} = 40 \times 3 + 2 \times 3 + 90 + 204 = 4\cancel{22}$. 420

The slot duration is $T_s = \frac{128[byte]}{250[kbit/s]}$=4.096[ms].

The duration of the active and inactive parts and the duty cycle are, respectively:

$$T_{active} = (N_{CFP} + 1) \times T_s = 1.72[s]$$

$$T_{inactive} = BI - T_{active} = 39.231[s]$$

$$\eta = T_{active}/BI = 4.2\%$$

.

Exercise–21 (*September 2, 2016*)

A Personal Area Network is operated according to the IEEE 802.15.4 standard and is composed of 20 sensor nodes and a PAN Coordinator (only Collision Free Part, nominal data rate, R= 250[kbit/s], packet length, L=128[byte]). 15 sensor nodes (out of 20) require an equivalent rate of r [bit/s], whereas the remaining 5 sensor node require twice that rate (2r [bit/s]). Assuming that the network administrator sets a duty cycle of 1%, find the maximum allowed value for r, and the corresponding Beacon Interval duration (assume to assign one slot in the CFP to each mote requiring a rate of r. . .).

Solution of Exercise–21

The duty cycle is $\eta = \frac{N_{active}}{N_{total}} = 0.01$, where N_{active} and N_{total} are the number of slots in the active part and in the overall BI, respectively. If we assume that one slot is assigned to each sensor node requiring a rate of r, then we can write: $N_{active} = 15 \times 1 + 5 \times 2 + 1$= 26 (note that we have added here "1" corresponding to the beacon slot). We can then derive $N_{total} = N_{active}/\eta = 2600$.

The value of rate r is: $r = \frac{L}{N_{total}\frac{L}{R}} = 96.1[bit/s]$.

The duration of the BI is: BI $=\frac{L}{R}N_{total}$=10.64[s].

Exercise–22 (*September 22, 2016*)

A personal area network (PAN) is composed of 4 motes and a PAN Coordinator. The PAN works in beacon-enabled mode. Mote 1 and Mote 2 have statistical (non-deterministic) traffic towards the PAN coordinator characterized by the following probability distribution: P($r_{1,2}$=75[bit/s])=0.6, P($r_{1,2}$=225 [bit/s])=0.2, P($r_{1,2}$=0 [bit/s])=0.2. Mote 3 and Mote 4 have deterministic traffic towards the PAN coordinator with a required rate, $r_{3,4}$ of 525 [bit/s]. The PAN coordinator has to deliver downlink traffic towards the four nodes according to the following pattern: traffic towards Mote 1 and Mote 2 P($r_{1,2}^{PANC}$=75[bit/s])=0.5, P($r_{1,2}^{PANC}$ =225 [bit/s])=0.1, P($r_{1,2}^{PANC}$ =0 [bit/s])=0.4; traffic towards Mote 3 and Mote 4 deterministic with required rate $r_{3,4}^{PANC}$ 300 [bit/s].

Assuming that: (i) the active part of the Beacon Interval (BI) is composed of Collision Free Part only; (ii) the motes and the PAN coordinator use b=128 [bit] packets for their transmissions which fit exactly one slot in the CFP, (iii) the nominal rate is 250 [kb/s], find the duration of the single slot, the duration of Beacon Interval (BI), the duration of the CFP, the duration of the inactive part, a consistent slot assignment for all the transmissions (UPLINK AND DOWNLINK), and corresponding the duty cycle. Assuming that the energy consumption parameters are the following ones, find the average energy consumption in a beacon interval for the PAN coordinator; energy for receiving a packet E_{rx} =4[μJ], energy for transmitting a packet E_{tx} =7[uJ], energy for being idle in a slot E_{idle}= 3[μJ], energy for sleeping in a slot E_{sleep}= 3[nJ].

Solution of Exercise–22

Let's dimension the Beacon Interval with respect to the sensor nodes with the "slowest" required channel rate, that is 75 [bit/s]. We have $75[bit/s] = \frac{b}{BI}$, and then $BI = \frac{128[bit]}{75[bit/s]} \approx 1.7[s]$..

In the worst cases (maximum required rate), the four motes need the following number of slots in each beacon interval:

$$N_{mote1} = N_{mote2} = 225[bit/s]/75[bit/s] = 3$$

$$N_{mote3} = N_{mote4} = 525[bit/s]/75[bit/s] = 7$$

Along the same lines, we need to assign bunch of slot to the PAN coordinator for the downlink traffic. Namely, in the worst cases, the PAN coordinator will need 3 slots for Motes 1 and 2 and 4 slots for Motes 3 and 4.

The total number of slots in the Collision Free Part is then: $N_{CFP} = 3 + 3 + 7 + 7 + 3 + 3 + 4 + 4 = 34$

The slot duration is $T_s = \frac{b}{R} = 512[\mu s]$.

The total duration of the active part is: $T_{active} = T_s \times (N_{CFP} + 1) = 17.92[ms]$ and consequently:

$$T_{inactive} = BI - T_{active} = 1.682[s]$$

$$N_{inactive} = T_{inactive}/T_s \approx 3285$$

The duty cycle is:

$$\eta = \frac{T_{active}}{BI} = 0.9\%$$

The average energy consumed by the PAN coordinator can be written as:

$$\begin{aligned} E_{PANC} = E_{tx} + 3285E_{sleep} + 3E_{tx} + 4E_{tx} + 2[0.2 \times 3E_{idle} + 0.2 \times 3E_{rx} + 0.6 \times (E_{rx} + 2E_{idle})] \\ +2 \times 7E_{rx} + 2[0.4 \times 3E_{idle} + 0.1 \times 3E_{tx} + 0.5 \times (E_{tx} + 2E_{idle})] \end{aligned}$$

Exercise–23 (*September 22, 2016*)

A sensor node performs channel access according to the CSMA/CA scheme of the IEEE 802.15.4 standard. Assuming that the probability of finding the channel busy is p=0.1 at each backoff period, find: (i) the probability that the sensor node does actually access the channel within the first two tries, (ii) the average time after which the sensor node does actually access the channel (assume infinite backoff attempts are allowed).

Solution of Exercise–23

The probability that the channel is sensed idle in two consecutive backoff periods is $P_{idle} = (1-p)(1-p) = 0.81$. The probability that the mote does access the channel within the first two tries is:

$$P = P_{idle} + (1 - P_{idle})P_{idle} \approx 0.96$$

The average number of attempts before the sensor nodes accesses the channel is given by $\frac{1}{P_{idle}} \approx 1.23$; The average number of backoff periods (slots) to access the channel can be written as:

$$E[T] = 2P_{idle} + \sum_{i=2}^{\infty}(1 - P_{idle})^{i-1}P_{idle}[2 + (i-1)1.5 + \sum_{k=2}^{i}\frac{2^k - 1}{2}]$$

Exercise–24 (*February 6, 2017 - Como*)

A personal area network (PAN) is composed of 4 motes and a PAN Coordinator. The PAN works in beacon-enabled mode. Mote 1 and Mote 2 have statistical (non-deterministic) traffic towards the PAN coordinator characterized by the following probability distribution: P($r_{1,2}$=75[bit/s])=0.6, P($r_{1,2}$=225 [bit/s])=0.2, P($r_{1,2}$=0 [bit/s])=0.2. Motes 3 and 4 have deterministic traffic towards the PAN coordinator with a required rate, $r_{3,4}$ of 225 [bit/s]. The PAN coordinator has to deliver downlink traffic towards the four nodes according to the following pattern: traffic towards Mote 1 and Mote 2 P($r_{1,2}^{PANC}$=75[bit/s])=0.5, P($r_{1,2}^{PANC}$ =225 [bit/s])=0.1, P($r_{1,2}^{PANC}$ =0 [bit/s])=0.4; traffic towards Mote 3 and 4 deterministic with required rate $r_{3,4}^{PANC}$ 75 [bit/s]. Assuming that: (i) the active part of the Beacon Interval (BI) is composed of Collision Free Part only; (ii) the motes and the PAN coordinator use b=128 [bit] packets for their transmissions which fit exactly one slot in the CFP, (iii) the nominal rate is 250 [kb/s], find the duration of the single slot, the duration of Beacon Interval (BI), the duration of the CFP, the duration of the inactive part, a consistent slot assignment for all the transmissions (UPLINK AND DOWNLINK), and corresponding the duty cycle. Assuming that the energy consumption parameters are the following ones, find the average energy consumption in a beacon interval for the PAN coordinator; energy for receiving a packet E_{rx} =4[uJ], energy for transmitting a packet E_tx =7[uJ], energy for being idle in a slot E_{idle} = 3[uJ], energy for sleeping in a slot E_{sleep} = 3[nJ].

Solution of Exercise–24

The BI duration can be found by setting $BI = b/r_{min}$, being r_{min}=75[b/s], which leads to BI=1.706[s]. Uplink traffic (from motes to PANC) worst case:

- Mote 1: 3 slots,
- Mote 2: 3 slots,
- Mote 3: 3 slots,
- Mote 4: 3 slots

Downlink traffic (from PANC to motes) worst case:

- To Mote 1: 3 slots,
- To Mote 2: 3 slots;
- To Mote 3: 1 slots;

- To Mote 3: 1 slots

The total number of slots required in the CFP is therefore: N_{cfp}= 21

The duration of the slot is: $T_s = b/R$=512[μs]. The duration of the CFP is $T_{cfp} = N_{cfp}T_s$ = 10.76[ms] The duration of the inactive part is: BI-T_{cfp}=1.69[s] The duty cycle is η=T_{cfp}/BI=0.0063

The average energy consumption of the PANC is:

$$E = E_{tx}+2[0.2(3E_{idle})+0.2(3E_{rx})+0.6(E_{rx}+2E_{idle})]+2x3E_{rx}+2[0.43E_{idle}+0.5(E_{tx}+2E_{idle})+0.1(3E_{tx})$$

panc

Exercise–25 (*February 6, 2017 - Como*)

A sensor node runs the IEEE 802.15.4 Carrier Sensing Multiple Access (CSMA); what is the average backoff time after the third failed attempt to access the channel? (the node has tried to access the channel for 3 times and the channel was busy)

Solution of Exercise–25

After the third failed attempt, the sensor draws the random backoff time in the window [0, 2^4-1]. On average, it will have to wait 7.5 backoff periods.

Exercise–26 (*February 20, 2017 - Como*)

A personal area network (PAN) is operated according to the IEEE 802.15.4 beacon enabled mode. The Collision Access Part (CAP) is composed of 20 slots, the inactive part is composed of 2000 slots and the duty cycle is 2%; each slot (of CAP and CFP) carries 127 [byte] packets and the nominal data rate is 250 [kb/s]. Find (i) the total number of available slots in the CFP, (ii) the equivalent rate of the channel defined as "one slot per beacon interval". Using the numbers found in (i) and (ii), suppose now that two motes are active in the PAN with a required uplink rate, r, defined as follows P(r=1200 [bit/s])= 0.3, P(r=600 [bit/s])= 0.3, P(r=200 [bit/s])= 0.4. (iii) Define a consistent slot assignment in the CFP for the two motes (the motes do not use the CAP). (iv) how many additional motes requiring a channel of 120 [bit/s] could be added to the network?

Solution of Exercise–26

The slot duration is T_{slot}= 127[byte]/250[kb/s]=4.064[ms] Knowing that the duty cycle is 2%, we can write: 0.02=(N_{TOT}- $N_{inactive}$)/ N_{TOT}, being $N_{TOT}$and $N_{inactive}$the number of slots in the entire beacon interval and in the inactive part, respectively. $N_{inactive}$= 2000, (i) thus we can find N_{TOT}= 2041 (approximating to the closest larger integer).

It follows N_{CAP}=20, N_{CFP}=1 and BI = $T_{slot}N_{TOT}$= 8,29[s]. (ii) The equivalent rate of "one slot per beacon interval" is: r=127[byte]/8.29[s] = 122[bit/s]. (iii) each one of the two motes requires in the worst case a rate r=1200[bit/s], thus 10 slots must be assigned to each one of the motes. (iv) there's no additional space in the CFP for other motes.

Exercise–27 (*February 20, 2017 - Como*)

A personal area network (PAN) is composed of 10 motes and a PAN coordinator. The Beacon Interval duration is BI= 12[s]. Each mote is assigned a slot in the GTS within the Beacon Interval. Five motes are characterized by a traffic which is distributed as Poisson point process with parameter λ_1= 0.1 [packet/s] whereas the remaining five motes are characterized by a traffic distributed according to Poisson with parameter λ_2= 0.5 [packet/s]. The overall TX/RX energy (circuitry + transmission/reception) per slot is E_{tx}=50 [uJ]. The energy for being idle in a slot is E_{idle}=20 [uJ]. The sleeping energy is negligible. Write the expression of the average energy consumed by the PAN coordinator within one Beacon Interval

z

Solution of Exercise–27

The probability that a mote of type 1 and 2 has packets to be transmitted in a beacon interval is:

$$P_1 = 1 - e^{-\lambda_1 BI} = 0.69$$

$$P_2 = 1 - e^{-\lambda_1 BI} = 0.99$$

The average energy consumed by the PANC is:

$$E = E_{tx} + 5[(P_1 + P_2)E_{tx} + (2 - P_1 - P_2)E_{idle}]$$

Chapter 3

Exercises ZigBee, Routing and Application Layer

Exercise–1 (*Exam of July 1, 2015*)

The nodes in the Figure 3.1 exchange packets of L=128[byte] which are acknowledged with ACKs of the same size. The figure reports for each wireless link the corresponding link capacity C and *Expected Transmission Count (ETX)*. Find the *Expected Transmission Time (ETT)* metric for all the wireless links (assume negligible propagation delay); find the shortest path from node A to node B using the ETT as routing metric.

Solution of Exercise–1

The Round Trip Time (RTT) for each link can be calculated as $RTT = 2 \times \frac{L}{C}$ where C is the capacity of the specific link. The ETT can then be calculated as $ETT = ETX \times RTT$. The following Figure 3.2 reports the ETT values for all the links and the corresponding shortest path between A and B.

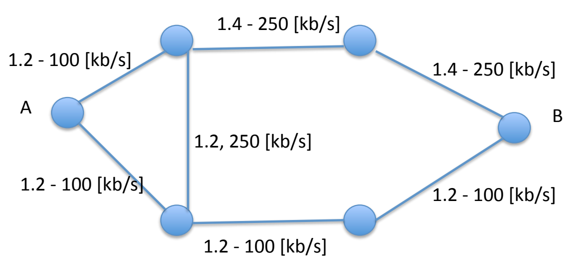


Figure 3.1: Ex. 1 Reference topolgy

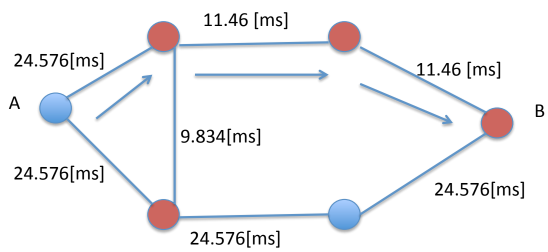


Figure 3.2: Ex. 1 Solution

Exercise–2

Given a network topology with the following characteristics: number of ZigBee routers per level, R_m=3; number of ZigBee end devices per level: D_m=2; number of levels in the cluster-tree topology, L_m=3; tell what is the number of addresses that the PAN Coordinator needs to assign.

Solution of Exercise–2

We can use the recursive formula seen in classes, that is, the number of addresses which need to be assigned to a ZigBee router at level $d \leq L_m$ in the tree is given by:

$$A(d) = \begin{cases} 1 + D_m + R_m, & \text{if } d = L_m - 1 \\ 1 + D_m + R_m A(d+1) & \text{if } 0 \leq d \leq L_m - 1 \end{cases}$$

The router devices at the second last level (level 2) have $R_m + D_m$ children, thus they will need $R_m + D_m + 1$=6 addresses.

The router devices at the third-last level (level 1) have R_m+D_m children, thus they will need $R_m + D_m + 1$ addresses for each one of their children router devices and D_m addresses for their children devices. Thus, they will need $D_m + 1 + R_m(R_m + D_m + 1)$=21 addresses.

The PAN Coordinator has R_m children router devices and D_m children simple devices. It will need $D_m + 1 + R_m(R_m + D_m + 1)$ addresses for each one of its children router devices and 1 address for each one of its simple children devices. Thus, $R_m[D_m + 1 + R_m(R_m + D_m + 1)] + D_m + 1$=66 addresses.

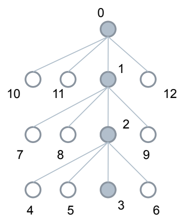


Figure 3.3: A feasible address assignment pattern for Ex. 3.

Exercise–3

A PAN Coordinator has 16 available short addresses to be assigned to ZigBee associated devices. Assuming that: the number of ZigBee routers per level in the tree is R_m=1, the number of ZigBee devices per level in the tree is D_m=3, find out the maximum tree depth and design an address distribution schedule.

Solution of Exercise–3

Each level in the tree has $R_m + D_m$=4 devices and thus requires 4 addresses. The total number of addresses which are available for devices (routers and end devices) is 15 (one address out of the original 16 ones is assigned by default to the PAN coordinator). Thus, the total number of ”full” (composed of 4 devices) levels which can be supported by the tree is: $L_m = \lfloor \frac{15}{4} \rfloor$=3. A feasible address assignment is reported in Figure 3.3.

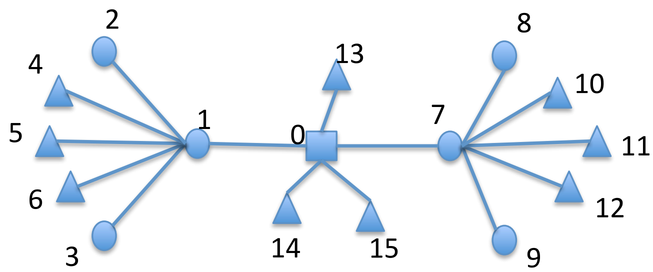


Figure 3.4: Network topology where the square represents the ZigBee coordinator, circles represent ZigBee routers and triangles represent ZigBee end devices.

Exercise–4

A multi-hop ZigBee personal area network (PAN) is characterized by the following topological parameters: number of ZigBee routers per tree level R_m=2, number of ZigBee end devices per tree level D_m=3, number of tree levels L_m=2 (The PAN coordinator plus two more levels). Plot a graph representing the network and assign network addresses to all the network devices according to the ZigBee address assignment rules.

Solution of Exercise–4

The network topology is reported in Figure 3.4

Exercise–5 (*Exam of June 30, 2014*)

A tree-based ZigBee network topology is used to collect traffic from a monitoring application. The topology has the following parameters: maximum number of ZigBee routers per tree-level R_m=2, maximum number of ZigBee end-devices per tree level D_m=2. Assuming that the first level of the tree (nodes directly connected to the PANC) share the same link capacity, C=100[kb/s], find out the maximum tree-depth, L_m, which allows not to exceed C under the following traffic conditions: (i) ZigBee end devices generate 2 [kb/s] of traffic, (ii) ZigBee routers, besides relaying simple nodes' traffic, generate additional 2 [kb/s] of traffic.

Solution of Exercise–5

The capacity of the link between the devices at level 1 and the PANC is C=100 [kb/s]. Each of the four devices requires 2 [kb/s] for its own traffic. Thus, the remaining capacity for the two zigBee routers at level 1 is 100[kb/s] – 8[kb/s] =92[kb/s].

Each one of the two routers can thus relay traffic up to 46 [kb/s].

In turn, the tree sub-branch rooted at each router at level 1 cannot exceed 46 [kb/s] of collected traffic. The four devices connected to the routers at level 1 require 2[kb/s] each. Thus, the remaining capacity for routers at level 2 is 46[kb/s] – 8[kb/s] = 38[kb/s]. Each one of the routers at level 2 can thus relay traffic from devices at level 3 up to 19 [kb/s]. To conclude, the maximum depth of the tree is L_m=3.

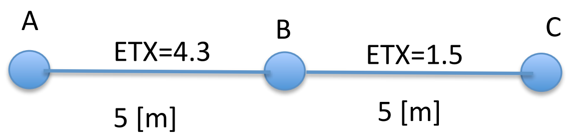


Figure 3.5: Network topology for Ex. 6.

Exercise–6 (*Exam of July 28, 2014*)

What is the overall average energy consumption for delivering one packet from node A to node C in the network? The figure shows the Expected Transmission Count (ETX) and the link length for each link. Assume that the A sends the packet to B which then forwards it to C. The energy required to operate the TX/RX circuitry is E_c=50 [nJ/bit], the energy required to support sufficient transmission output power E_{tx}(d)= k d^2 [nJ/bit], being k=1 [nJ/bit/m2]. Neglect overhearing.

Solution of Exercise–6

The expected transmission time (ETX) measures the average number of transmissions which are required to receive one packet successfully.

One single transmission/reception cycle across links A-B and C-D consumes:

$$E_{A-B} = E_{C-D} = E_c b + E_{tx}(5[m])b + E_c b$$

Let' focus on the A-B link. On average 4.3 transmission cycles are required to have the packet through. Similarly, for link C-D 1.5 cycles are required. Thus the (average) overall energy consumed end-to-end is:

$$E_{tot} = 4.3 E_{A-B} + 1.5 E_{C-D}$$

Figure 3.6: Network topology for Ex. 7.

Exercise–7

A routing protocol for WSNs leverages the expected transmission count (ETX) as a routing metric. To this end, two nodes, A and B, at the vertices of a link probe the quality of the link by sending out L=100 [byte] probing packets which are acknowledged (if correctly received) through explicit ACKs of L_{ACK}=10 [byte]. Assuming that the Bit Error Rate (BER) is p=0.005 in both directions, find out the ETX measure for the link (assume independent errors bit by bit).

Solution of Exercise–7

Since bits are independent, the probability of correct reception of a packet of L bits is: $P_{succ}^{packet} = (1-p)^L$. The packet error probability is: $P_{error}^{packet} = 1 - P$=0.98, and the error probability on the acknowledgement packets is, similarly, $P_{error}^{ACK} = 1 - (1-p)_{ACK}^{L}$=0.33.

The ETX for the link is the average number of packet retransmission (including the first transmission) to let the packet go through. The transmission success probability can be expressed as: $P_{succ} = P_{succ}^{packet} P_{succ}^{ACK}$.

The ETX is thus the average of a geometric probability distribution with parameter P_{succ}:

$$ETX = \sum_{k=0}^{\infty} (k+1) P_{succ} (1 - P_{succ})^k,$$

which means $ETX = \frac{1}{P_{succ}}$=74.6

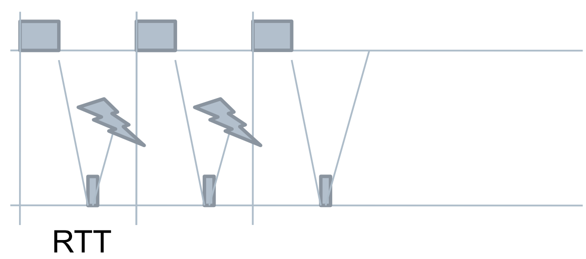


Figure 3.7: Round Trip Time for Ex. 8.

Exercise–8

Under the same parameters as the previous exercise 7, find out the expected transmission time (ETT) for the previous link, that is, the average amount of time taken by a transmission to go through.

Assume a link capacity (bidirectional) of R=250 [kb/s], null processing time and propagation delay t=10 [μs]. After a failed transmission, the retransmission is performed exactly after a round trip time. Comment on similarities/differences, pros/cons of ETX and ETT

Solution of Exercise–8

The round trip time for a successful transmission is defined as the time taken by the packet to be sent and by the corresponding ACK to be received back by the original transmitter. In formulas (see Figure 3.7):

$$RTT = T_{packet} + T_{ack} + 2t,$$

being $T_{packet} = \frac{L}{R} = \frac{100[byte]}{250[kb/s]} = 3.2[ms]$, and $T_{ack} = \frac{L_{ack}}{R} = 0.32[ms]$. Thus, $RTT = 3.54$[ms].

The Expected Transmission Time (ETT) is $ETT = RTTETX$= 261.9[ms].

Exercise–9 (*Exam of June 27, 2013*)

A wireless link is characterized by a bit error rate (probability) of p=0.001 and a nominal rate R=250 [kb/s]. What is the average transmission time for a packet of L=100 [byte]? (assume that a erred transmission is re-attempted straight away).

Solution of Exercise–9

The time for one transmission (successful or failed) is $T = \frac{L}{R} = \frac{100[byte]}{25[kb/s]} =$ 3.2 [ms].

The probability to successfully send a packet is $P = (1-p)^L$=0.449 (the assumption here is that the transmitted bit are independent one another, that is, the probability of failing one bit does not depend on the outcome of the previous and following bit transmissions).

The Expected Transmission Count (ETX), that is, the average number of transmissions to transmit a packet successfully is: ETX=$\frac{1}{P}$ =2.22.

The average transmission time, that is, the Expected Transmission Time (ETT) is: ETT=T 1/p = 7.12 [ms].

Exercise–10 (*Mock exam, 2016*)

A COAP client issues a CONFIRMABLE message. Assuming that the packet error rate is p=0.3 and the MAX_RETRANSMIT parameter is 3, find the probability that the CONFIRMABLE message in the end goes through. Now assume that MAX_RETRANSMIT=infinity and that the initial time out is set randomly in the interval [3s, 5s] and find the average time to the successful transmission of the CONFIRMABLE message.

Solution of Exercise–10

The probability that the CONFIRMABLE message goes through in the end is the probability that the sender does receive an acknowledge within the first three tries. That is, if p=0.3 is the probability not to receive an acknowledgement, we are looking for: $P_{success} = \sum_{i=1}^{3} P_{success}(try = i)$, being $P_{success}(try = i)$ is the probability to receive an ACK after try i. Thus we have:

$$P_{success} = 1 - p + p(1 - p) + p^2(1 - p) = 0.97$$

To find the average time to correctly transmit the CON message, we have to remember how COAP backoff procedure works: if the message is successful at the first try then COAP does not backoff; if the message "fails", then the first backoff is chosen randomly int the given interval (thus, the intial backoff time is on average t_0=4[s]) and the next access is re-attempted after the backoff is elapsed; at every following failed attempt the backoff is doubled. We can then write the expression of the average message delivery delay as:

$$E[T] = 0 \times (1 - p) + p(1 - p) \times t_0 + \sum_{i=3}^{\infty}[(1 - p)p^{i-1}(\sum_{k=2}^{i-1} 2^{k-1}t_0)] \approx 2.8[s]$$

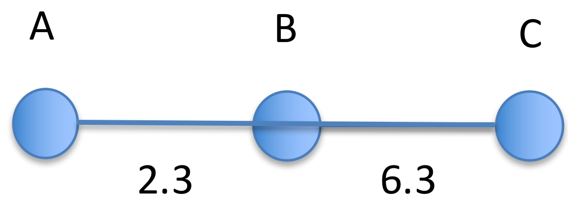


Figure 3.8: Reference topology for Ex. 11.

Exercise–11 (*July 1, 2016*)

Write the expression of the average energy consumed by all the sensor nodes for sending one packet from A to C through the two-hop path at the right. The numbers below each link express the ETX for the link (energy for operating the TX/RX circuitry for one packet E_c, energy for transmitting one packet E_{tx}).

Solution of Exercise–11

The total energy consumed across the two links for sending one single packet is: $E_{A-B} = E_{B-C} = E_c + E_{tx} + E_c$.

The total energy consumed is: $E_{tot} = ETX_{A-B} \times E_{A-B} + ETX_{B-C} \times E_{B-C}$.

Exercise–12 (*July 27, 2016*)

A COAP client issues a CONFIRMABLE message. Assuming that the packet error rate is p=0.2 and the MAX-RETRANSMIT parameter is 2, find the probability that the CONFIRMABLE message in the end goes through.

Solution of Exercise–12

The CONFIRMABLE message goes through within two attempts if it goes through at the first attempt (probability 1-p) or if it goes through at the second attempt (probability p(1-p)). The overall probability is: $P = (1 - p)(1 + p) = (1 - p^2) = 0.96$

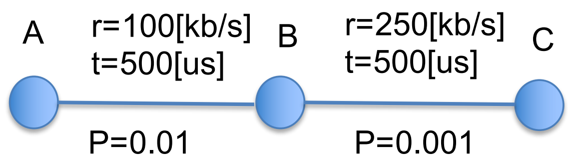


Figure 3.9: Reference topology for Ex. 13.

Exercise–13 (*July 27, 2016*)

Find out the Expected Transmission Time (ETT) for the two wireless links in the figure assuming packet size l=128[byte]. Each link is labeled with the corresponding packet error probability, p, the nominal data rate, r, and the propagation delay, t. Assume negligible size for the acknowledgements and a repetition time-out equal to the round trip time for the link.

Solution of Exercise–13

The total time for transmitting a packet through link i and receive the corresponding ACK is: $T_i = \frac{l}{r_i} + 2t$. Thus we have, T_1=11.24[ms] and T_2=5.096[ms].

The ETX for the two links is: $ETX_{AB} = \frac{1}{1-0.01} = 1.01$ and $ETX_{AB} = \frac{1}{1-0.001} = 1.001$.

The ETT for the two links is then: $ETT_{AB} = ETX_{AB}T_1$ and $ETT_{BC} = ETX_{BC}T_2$

Exercise–14 (*September 2, 2016*)

A wireless link is characterized by a packet error rate in the two directions (left-right, right-left) of 1% and 0.5% respectively. Assuming that the packet used for delivering information from left to right have size L=128[byte], the acknowledgements in the opposite direction have size A=8[byte], and the nominal rate in both directions is R=100[kbit/s], find the average transmission time to successfully send a packet and get the corresponding acknowledgement (negligible propagation delay).

Solution of Exercise–14

The probability to successfully send a packet and receive the corresponding ACK is: P=(1-0.01)(1-0.005)= 0.985. The average number of transmissions required to send a packet successfully is given by the ETX=1/P=1.015.

The time required to complete a successful transmission is:

$$RTT = L/R + A/R = 10.24[ms] + 0.64[ms] = 10.88[ms].$$

The expected transmission time is: ETT = $ETX \times RTT$ = 11.04[ms].

Exercise–15 (*September 22, 2016*)

Briefly describe COAP Observation mode

Solution of Exercise–15

See slides.

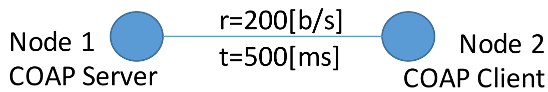


Figure 3.10: Ex. 16 Reference topology

Exercise–16 (*February 6, 2017-Como*)

Node 1 in the figure mounts a temperature sensor to collect temperature samples at an extremely high pace; node 1 runs a COAP server and the resource associated to temperature samples is /temp; Node 2 runs a COAP client and is interested in reading 20 consecutive temperature samples collected by node 1. The figure reports the nominal bit rate of the link between node 1 and node 2, r=200[bit/s] and the one-way propagation delay t=500[ms]. Assuming that COAP request messages have size L_{req}=8 [byte], COAP response messages have size L_{resp}=10 [byte], the energy consumed for transmitting/receiving a message (both request/response) E_{tx}=E_{rx}=8[μJ], find the total time for the COAP client to obtain the 20 temperature samples and the energy consumed by the COAP server in the two cases where COAP uses/doesn't use the Observation mode (assume that the size of request/response messages is the same in the two operation modes, assume perfect transmission, assume that notifications do not produce a client reply).

Solution of Exercise–16

$T_{req} = L_{req}/200$[b/s] =320[ms] $T_{resp} = L_{resp}/200$[b/s] = 400[ms]

No Observation mode

$$T = 20(T_{req} + T_{resp} + 2t) = 34.4[s]$$

$$E_{server} = 20(E_{tx} + E_{rx}) = 320[\mu J]$$

Observation mode

$$T = T_{req} + t + 20T_{resp} + t = 9.32[s]$$

$$E_{server} = 20E_{tx} + E_{rx} = 168[\mu J]$$

Exercise–17 (*February 20, 2017-Como*)

A CoAP transaction between a client and a server is operated through CONFIRMATION messages. Assuming that the packet error rate (on requests and responses) is p=0.01 (that is, 1% of probability to mistake a request message, 1% of probability to mistake a response message), find average total time to successfully retrieve a resource (assume infinite retrials are allowed).

Solution of Exercise–17

The transaction success probability is P=$(1-p)^2$; the total number of attempts to the first success is 1/P = 1.02. Knowing that the initial time out in COAP is (on average) 2[s], the total required time is approx. 2[s].

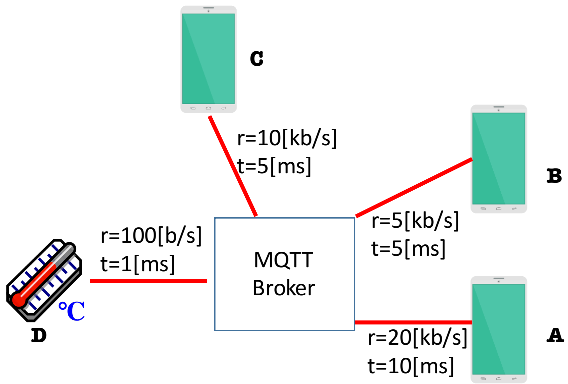


Figure 3.11: Ex. 18 Reference topology

Exercise–18 (*Mock Exam January 20, 2017-Como*)

Nodes A, B, C and D in the figure run MQTT protocol and they are all subscribed to topic1. Assuming that node D issues at time t=0 a PUBLISH message on topic1 find the time of arrival of such PUBLISH message at nodes A, B and C. Each link in the figure is characterized by the nominal data rate (r), and the propagation delay (t). The PUBLISH message has size L=40[byte].

Solution of Exercise–18

$$T_{D-broker} = L/100[b/s] + 1[ms] = 3.201[s]$$

$$T_{broker-A} = L/20[kb/s] + 10[ms] = 26[ms]$$

$$T_{broker-B} = L/5[kb/s] + 5[ms] = 69[ms]$$

$$T_{broker-C} = L/10[kb/s] + 5[ms] = 37[ms]$$

$$T_A = T_{D-broker} + T_{broker-A} = 3.227[s]$$

$$T_B = T_{D-broker} + T_{broker-B} = 3.27[s]$$

$$T_C = T_{D-broker} + T_{broker-C} = 3.238[s]$$

Exercise–19 (*Mock Exam January 20, 2017-Como*)

A CoAP client issues the following message to a CoAP server: [CON [0x6542], GET /cpu, Token 0x54]. Assuming that the CoAP server correctly receives the message, but the requested resource (/cpu) is not available at that very moment (will be available 10[s] later), show a possible follow up for the client-server transaction.

Solution of Exercise–19

Since the message is confirmable, the COAP server must answer straight away even if the requested resource is not available. A possible follow up for the COAP exchange is:

```
C                                                        S
---- CON [0x6542], GET /cpu, Token 0x54 ----------- >
< ----------- ACK[0x6542]-----------------------------------
          after some time
< --- CON [0x6767], 2.05 Content,  Token 0x54, "30%"
----------------ACK[0x6767]---------------------------------- >
```

Exercise–20 (*February 6, 2017-Como*)

A wireless link is characterized by a Bit Error Rate (BER), p=0.001. Assuming that transmissions on the link use packets of length L=128[byte] which are acknowledged with ACK with size L=8[byte], find an estimate for the Expected Transmission Count (ETX) of the link.

Solution of Exercise–20

The packet success probability is $P_s = (1-p)^{8(L+L_{ack})}$; the ETX $= 1/P_s = 2.97$

Exercise–21 (*February 20, 2017-Como*)

A ZigBee network is characterized by the following parameters: Lm=3 (three layers), Rm= 2 (number of zigbee routers), Dm = 2 (number of end devices). Tell how many addresses are needed to to support all the devices in the network and plot a consistent address assignment scheme.

Solution of Exercise–21

The routers at level 2 (second-last) need $1 + R_m + D_m = 5$ addresses.

The routers at level 1 need $1 + D_m + R_m(1 + R_m + D_m) = 13$ addresses.

The router at level 0 (PANC) needs $1 + D_m + R_m[1 + D_m + R_m(1 + R_m + D_m)] = 29$ addresses.

Exercise–22 (*Exercises March 23, 2023*)

MQTT-SN is an adaptation of the MQTT protocol for sensor networks. Publishers can use reduced-size publish messages using a short topic identifier against the usual topic path. This is achieved at the cost of a new registration phase (REGISTER + REGACK) to be done before publishing the messages for that topic. This phase is however not needed for subscribers, which can directly specify the topic path in the subscribe request. Consider a network having 20 publisher nodes, 20 subscriber nodes and a single central server. All the network entities are able of both communicating with MQTT-SN or CoAP. The subscribers show interest in the "/temperature" resource/topic in the server, while the publishers send messages to the same resource "/temperature". (a) Compute the total amount of traffic transferred in the network if the different publishers send 2 messages each. Distinguish the two cases where MQTT-SN and CoAP is used as protocol. (b) Tell how many number of messages sent by each publisher are needed to have MQTT-SN as the best choice (in terms of total traffic).
Assume that: (i) CoAP uses the pub/sub communication pattern (using the GET+observe). (ii) CoAP and MQTT-SN message sizes are those reported in Table 3.1. (iii) CoAP and MQTT-SN subscribers do not reply to the server notifications (MQTT-SN subscribe with qos=0, and CoAP uses NON notifications messages) (iv) Publish-side messages are acknowledged: CoAP publishers send CON messages, MQTT-SN publishers send QoS=1 messages. (v) All the clients are powered off at the beginning of the execution

Table 3.1: CoAP and MQTT-SN messages size (bytes)

CoAP		**MQTT-SN**	
GET + Observe	60	Subscribe Request	58
GET Response	55	Subscribe Ack	52
PUT Request	77	Publish Request	68
PUT Response	58	Publish Ack	51
Server-Sub Notification	71	Connect Request	54
		Connect Ack	47
		Register Request	59
		Register Ack	51

Solution of Exercise–22

Case 1: CoAP

The 20 subscribers need to subscribe using the GET request, and they will receive the corresponding response: $L_1 = 20 * (60 + 55) = 2300$B

The 20 publishers publish 2 messages each and receive the response: $L_2 = 20 * 2 * (77 + 58) = 5400$B

Finally, each publish message will produce 20 notifications to the subscribers, which is not acknowledged: $L_3 = 20 * 20 * 2 * 71 = 56800$B

Overall, the traffic using CoAP is the sum of the three values:L_{CoAP}=64500 B

Case 2: MQTT-SN

The 20 subscribers first need to connect (CONN+CONNACK) and subscribe (SUB+SUBACK): $L_1 = 20 * (54 + 47 + 58 + 52) = 4220$B

The 20 publishers first connect (CONN+CONNACK) and register the topic once (REG + REGACK), then send 2 messages each and receive the response (PUB+PUBACK): $L_2 = 20*(54+47+59+51+2*(68+51))$=8980B

Finally, each publish message is forwarded to 20 subscribers (not acknowledged): $L_3 = 20 * 20 * 2 * 68 = 54400$B

Overall, the traffic using CoAP is the sum of the three values:L_{MQTT_SN}=67600B

We can model the previous values in function of the number of messages m and we get: $L_{CoAP} = 2300 + 31100m$ and $L_{MQTT_SN} = 8440 + 29580m$

By posing $L_{CoAP} > L_{MQTT-SN}$ we get $m > 4.03$ meaning that we need at least 5 messages per publisher to have better results in MQTT-SN

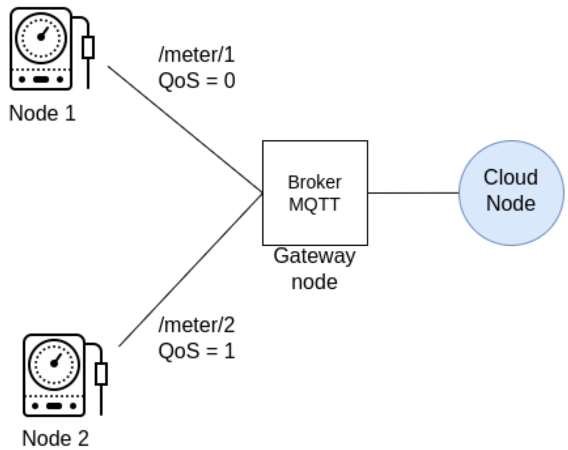


Figure 3.12: Ex. 23 Reference topology

Exercise–23 (*Exercises March 20, 2023*)

Suppose to have an IoT system that consists of four nodes: two gas meter sensor nodes, a gateway node that acts as an MQTT broker, and a cloud server node. The two gas meter sensor nodes collect the gas consumption data and publish it to the gateway node using MQTT once every hour with respectively QoS = 0 on topic /meter/1 and QoS = 1 on topic /meter/2. The gateway node receives the data and forwards it to the cloud server node using MQTT. The cloud server node is subscribed to topic /meter/# with CLEAN SESSION FLAG set to false and QoS=1. (a) Define the keep-alive value range x for which is more energy convenient to keep the connection always open rather than performing a connection and disconnection for each packet to publish for the gas meter 2. (b) Compute the number of messages lost after disconnection of the cloud server node for 1 day. Consider the following packet size in bytes: CONNECT 83, CONNACK 70, PUBLISH 85, PUBACK 70, DISCONNECT REQ 68, PING REQUEST 68, PING RESPONSE 68. Assume the nodes at time 0 are already powered on, the energy required to operate the TX/RX circuitry, $E_c = 50[nJ/bit]$, the energy required to support sufficient transmission output power $E_{tx} = 25[nJ/bit]$.

Solution of Exercise–23

Disconnect each time:

$$E_{CONN} = 83 \cdot 8 \cdot (E_c + E_{tx}) = 49,8[\mu J]$$

$$E_{CONNACK} = 70 \cdot 8 \cdot (E_c) = 28[\mu J]$$

$$E_{PUB} = 85 \cdot 8 \cdot (E_c + E_{tx}) = 51[\mu J]$$

$$E_{PUBACK} = 68 \cdot 8 \cdot (E_c) = 27,2[\mu J]$$

$$E_{DISCREQ} = 68 \cdot 8 \cdot (E_c + E_{tx}) = 40,8[\mu J]$$

$$\begin{aligned} E_{DISC} &= \\ &= E_{CONN} + E_{CONNACK} + E_{PUB} + E_{PUBACK} + E_{DISCREQ} \\ &= 196,8[\mu J/hour] \end{aligned}$$

Keep alive:

$$E_{PINGREQ} = 68 \cdot 8 \cdot (E_c + E_{tx}) = 40,8[\mu J]$$

$$E_{PINGRESP} = 68 \cdot 8 \cdot (E_c) = 27,2[\mu J]$$

$$E_{PUB} = 85 \cdot 8 \cdot (E_c + E_{tx}) = 51[\mu J]$$

$$E_{PUBACK} = 68 \cdot 8 \cdot (E_c) = 27,2[\mu J]$$

$$\begin{aligned} E_{KEEP} &= \\ &= E_{PUB} + E_{PUBACK} + (E_{PINGREQ} + E_{PINGRESP}) \cdot \frac{3600}{x} \\ &= 78,2 + 68 \cdot \frac{3600}{x}[\mu J/hour] \end{aligned}$$

$$\begin{aligned} E_{KEEP} < E_{DISC} \Rightarrow \\ &\Rightarrow 78,2 + 68 \cdot \frac{3600}{x}[\mu J/hour] < 196,8[\mu J/hour] \\ &\Rightarrow 2064[s] < x < 65535[s] \end{aligned}$$

Messages Lost: The cleansession flag is used to specify whether a client's session state should be maintained by the broker in the event of a disconnect. When the cleansession flag is set to true (or not set at all), the broker will discard any information about the client's session (e.g., subscriptions, queued messages) when the client disconnects. When the cleansession flag is set to false, the broker will retain this information for the client's next session. When a message is sent with a QoS greater than one, the broker will store the message and attempt to deliver it to any current or future subscribers with a matching subscription. Therefore the packets lost on one day are the ones coming from gas meter node 1 since it has QoS = 0

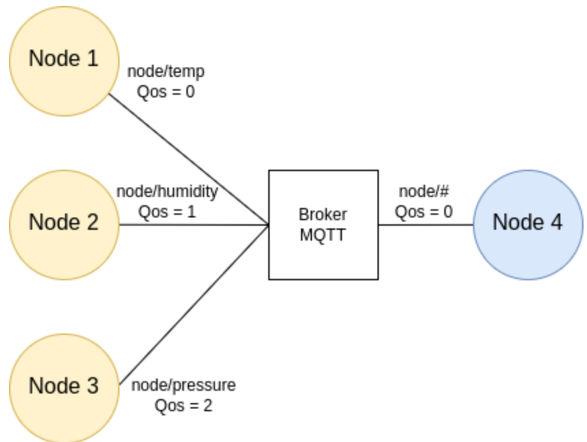


Figure 3.13: Ex. 24 Reference topology

Exercise–24 (*Exercises March 20, 2023*)

Consider an IoT network consisting of three MQTT nodes, labelled Node 1, Node 2, and Node 3, and one MQTT subscriber node, labelled Node 4. The nodes are connected over a network with the probability of successfully sending the packet from client to broker of P_{Req}, while the probability to successfully receive the packet from the broker is always 1 ($P_{Resp} = 1$).
Node 1 publishes messages on topic "node/temp" with QoS level 0. Node 2 publishes messages on topic "node/humidity" with QoS level 1. Node 3 publishes messages on topic "node/pressure" with QoS level 2. Node 4 subscribes to "node/# with QoS level 0. Assume all clients are already powered on and connected with the MQTT broker at the beginning of the execution, and that each message is: $L_{PUB} = 85[bytes]$, $L_{PUBACK} = 68[bytes]$, $L_{PUBREC} = 70[bytes]$, $L_{PUBREL} = 70[bytes]$, $L_{PUBCOMP} = 70[bytes]$. (a) compute the average time required to complete a publisher-broker interaction for each of the three nodes, assuming a nominal bit rate of $r = 200[bit/s]$ and a one-way propagation delay of $t = 500[ms]$ in case $P_{Req} = 1$. (b) compute the average time required by node 2 to complete the transmission with $P_{Req} = 0.8$. To simplify the calculations, consider: (i) The maximum number of attempts is set to 3. (ii) There is no implemented backoff mechanism. If no acknowledgment is received within a 30-second time window ($RTO = 30[s]$), the message is resent.

Solution of Exercise–24

Question a:

$$T_{Node1} = t + \frac{L_{PUB} \cdot 8}{r} = 0.5[s] + \frac{85 \cdot 8[bit]}{200[bit/s]} = 3,9[s]$$

$$\begin{aligned} T_{Node2} &= \frac{(L_{PUB} + L_{PUBACK}) \cdot 8}{r} + 2t \\ &= 2t + T_{PUB} + T_{PUBACK} \\ &= 1[s] + \frac{(85+68) \cdot 8[bit]}{200[bit/s]} \\ &= 1[s] + 3,4[s] + 2,72[s] = 7,12[s] \end{aligned}$$

$$\begin{aligned} T_{Node3} &= \frac{(L_{PUB} + L_{PUBREC} + L_{PUBREL} + L_{PUBCOMP}) \cdot 8}{r} + 4t \\ &= \frac{(85+70+70+70) \cdot 8[bit]}{200[bit/s]} + 2[s] = 13,8[s] \end{aligned}$$

Question b:

$$T_{PUB} = \frac{L_{PUB} \cdot 8}{r} \quad T_{PUBACK} = \frac{L_{PUBACK} \cdot 8}{r}$$

$$\begin{aligned} T_{Node2} = & P_{Req} \cdot (2t + T_{PUB} + T_{PUBACK}) + \\ & + (1 - P_{Req}) \cdot P_{Req} \cdot (T_{PUB} + RTO + (t + T_{PUB}) + (t + T_{PUBACK})) + \\ & + (1 - P_{Req})^2 \cdot P_{Req} \cdot (2(T_{PUB} + RTO) + (t + T_{PUB}) + (t + T_{PUBACK})) + \\ & + (1 - P_{Req})^3 \cdot 3(RTO + T_{PUB}) = \\ & = 5,7 + 6,6 + 2,4 + 0,8 = 15,5[s] \end{aligned}$$

Chapter 4

Exercises on Radio Frequency Identification

Exercise–1 (*Exam of September 8, 2015*)

An RFID system is composed of 3 tags and uses a Dynamic Frame ALOHA access protocol. Assume that the initial frame size is r=4 and that after the first frame no tags have been resolved. What is the backlog predicted by Schoute's estimate? Find out the probability that the resolved tags after the second frame is equal to i, with $i = 0, 1, 2, 3$ if the second frame length is set to Schoute's estimate.

Solution of Exercise–1

After the first frame no tags have been resolved, thus all the 3 tags have chosen the very same slot for transmitting. According to Schoute's estimate, the backlog is equal to the number of collided slots, c, multiplied by 2.39, all rounded up to the closest integer; in formulas,

$$\hat{n} = round(2.39c)$$

In our case, only one collided slot is observed, $c = 1$, so Schoute's estimate predicts a backlog $\hat{n}$=2 tags.

The length of the second frame is then set to r=2 slots.

The requested probability values can be derived as follows:

- $P(S = 3)$=0, each one of the three tags has to choose one out of two slots, thus it is impossible that two tags are successfully resolved;
- $P(S = 2)$=0, each one of the three tags has to choose one out of two slots, thus it is impossible that thress tags are successfully resolved;

- $P(S = 0) = 2(\frac{1}{2})^3$, no tag is resolved if all the three tags choose the same slot;
- $P(S = 1) = 1 - P(S = 0) = \frac{3}{4}$

Exercise–2 (*Exam of July 28, 2015 and June 27, 2013*)

A RFID collision arbitration system is based on multi-frame dynamic frame ALOHA. Find out the efficiency of the collision arbitration process if the initial number o tags is N=3 and the initial frame size is r=2.

Solution of Exercise–2

The efficiency is defined as $\eta = \frac{N}{L_N}$, where L_N is the average resolution time with a initial tag population of N tags. L_3 can be calculated by applying the recursive formula studied during classes. Namely,

$$L_3 = 2 + P(S=0)L_3 + P(S=1)L_2 + P(S=2)L_1,$$

where $P(S=2) = 0$, $P(S=0) = \frac{1}{4}$ and $P(S=1) = \frac{3}{4}$.

Similarly, L_2 can be written as:

$$L_2 = 2 + P(S=0)L_2 + P(S=1)L_1,$$

where $P(S=0) = \frac{1}{2}$, $P(S=1) = 0$. Solving for L_2 and substituting the obtained valued back in the formula of L_3 we get $L_3 = \frac{20}{3}$.

The efficiency is finally $\eta = \frac{9}{20}$.

Exercise–3 (*Exam of July 1, 2015*)

A Dynamic Frame ALOHA system is used to arbitrate 4 tags. What is the average throughput after the first two frames of the arbitration process knowing that the respective frame lengths are r_1=2, r_2=2?

Solution of Exercise–3

After the first frame either 0 or 1 tag are resolved. The probability to solve one tag after the first frame is $2\binom{4}{3}(\frac{1}{2})^4 = \frac{1}{2}$. If one tag is resolved after the first frame, 3 tags are left; either 1 or 0 tags can be resolved in this case with probability $\frac{3}{4}$ and $\frac{1}{4}$ respectively. If zero tags are resolved after the first frame, the initial 4 tags are left, thus with probability $\frac{1}{2}$ one tag is resolved after the second frame.

In summary, the average number of tags solved after 2 frames under the given conditions is:

$$E[S] = \frac{1}{2}(1 + \frac{3}{4}) + \frac{1}{2}\frac{1}{2}1 = \frac{9}{8}$$

Exercise–4 (*Exam of June 30, 2014*)

A Dynamic Frame ALOHA system is used to arbitrate 5 tags. What is the average throughput after the first two frames of the arbitration process knowing that the respective frame lengths are r_1=2, r_2=3?

Solution of Exercise–4

After the first frame either 0 or 1 tag are resolved. The probability that 1 tag is resolved after the first frame is: $P(S_1 = 1) = 5(\frac{1}{2})^5 2 = 0.3125$. The probability that 0 tags are resolved after the first frame is: $P(S_1 = 0) = 1 - P_1(S = 1)$= 0.6875.

If 1 tag is resolved after the first frame, there are 4 remaining tags to be resolved. The average throughput of Frame ALOHA on a single frame (see slide 32 of RFID slide set) is given by:

$$E[S] = n(1 - \frac{1}{r})^{n-1}, \tag{4.1}$$

being n the current backlog (number of tags) and r the frame size (in number of slots). In our case, it is: $n = 4$ and $r = r_2 = 3$. Substituting these two values in the formula, we get: $E[S_2/S_1 = 1] = 4(1 - \frac{1}{3})^3$=1.185.

If 0 tags were resolved after the first frame, the initial 5 tags still need to be resolved. We can then apply the same formula in Eq. 4.1 with the following numbers: n=5, $r = r_2$=3, thus getting: $E[S_2/S_1 = 0] = 5(1 - \frac{1}{3})^4$=0.98.

In summary, the average number of tags solved after 2 frames under the given conditions is:

$$E[S_2] = E[S_2/S_1 = 0]P(S_1 = 0) + (E[S_2/S_1 = 1] + 1)P(S_1 = 1) = 1.35$$

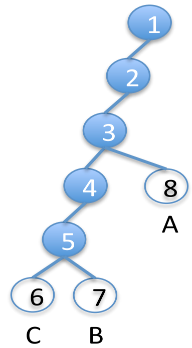


Figure 4.1: Network topology for Ex. 5.

Exercise–5 (*Exam of July 28, 2014*)

The figure represents a possible outcome of the execution of a binary tree algorithm with three tags A, B and C to be resolved. For each node in the tree, the figure reports the slot number and the outcome of the transmission (success or collision, empty circles are successful ones). What is the efficiency of this outcome? What is the probability for this outcome to happen?

Solution of Exercise–5

The efficiency is defined as the ration between the number of tags and the required slots, that is, $\eta = \frac{3}{8}$.

The probability for the reference realization can be calculated reasoning as follows: all the three tags collide at slot 1; after slot 1 all the three tags randomly draw the next value of their counter to be zero (re-attempt the access at slot 2), which happens with probability $(\frac{1}{2})^3$; after the collision at slot 2, all the three tags still draw zero as next value of their counter (re-attempt access at slot 3), which happens with probability $(\frac{1}{2})^3$; after collision at slot 3, tags B and C draw zero as counter value, whereas tag A draws one which still happens with probability $(\frac{1}{2})^3$; at slot 4, tags B and C collide and they draw zero as next counter value (re-attempt at slot 5) which happens with probability $(\frac{1}{2})^2$ (note that tag A does not take part in the random choice); after slot 5, tags B and C respectively draw zero and one as new counter values (probability $(\frac{1}{2})^2$) which means that tag C transmits alone in slot 6, tag B transmits alone in slot 7 and tag A transmits alone in slot 8.

The probability of the reference realization to happen is the product of the probabilities of all the aforementioned events, that is, $P = (\frac{1}{2})^{13}$.

Exercise–6 (*Exam of July 28, 2014*)

A Dynamic Frame ALOHA system estimates the current backlog of unresolved tags to be 2. What is the single-frame throughput assuming that the system optimally set the frame length ACCORDING TO THE BACKLOG ESTIMATE and knowing that the real number of unresolved tags is 4.

Solution of Exercise–6

The frame size of the dynamic frame ALOHA is optimally set to the estimated value of the backlog, that is $r = \hat{n} = 2$. The average throughput of such system is then measured as the average number of tags (out of 4) that are resolved using a frame of 2 slots:

$$E[S] = 4 \times 2(\frac{1}{2})^4 = 0.5$$

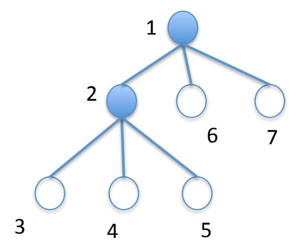


Figure 4.2: Ternary tree realization Ex. 7.

Exercise–7 (*Exam of September 13, 201*)

A RFID system is based on a ternary-tree arbitration system (after a collision, collided tags are split into three groups). Given the realization in the figure, find out the probability for this realization to happen and the corresponding efficiency. Empty circles represent successful slots, full circles represent collided slots, numbers are the slot numbers.

Solution of Exercise–7

Looking at the realization of the ternary tree algorithm, we realize that we have 5 tags in total. We then have two "random events": after the first slot (where everybody transmits and collide), three tags choose the following slot to re-attempt access, whereas the remaining two tags are partitioned into singleton groups (resolved later on); the probability for this event to happen is $2 \times \binom{5}{3}(\frac{1}{3})^5$. Note that we do not know which subgroup of three tags decide to re-attempt access straight away, that is, we have to count up all the possible ways to form subgroups of three tags out of five ($\binom{5}{3}$); then once the subgroup of three tags is fixed, the remaining two tags have two possibilities to "share" the remaining two slots (that's why we have the "2" in the probability formula).

After the collision at slot 2, the three tags choose to re-attempt access in three different slots, which happens with probability $3! \times (\frac{1}{3})^3$ (again we have to count up all the possible ways the three tags can be ordered in the three non-colliding subgroups, which is 3!).

The probability for this realization to happen is then given by: $P = 2 \times \binom{5}{3}(\frac{1}{3})^5 \times 3! \times (\frac{1}{3})^3$.

Exercise–8 (*Mock Exam 2016*)

A Dynamic Frame ALOHA system is used to arbitrate N=3 tags. Find out the average throughput after the first two frames of the tag arbitration process assuming that the first two frames have the following sizes r_1=3, r_2=2.

Solution of Exercise–8

Let's call S_1 andS_2 the throughput after frame 1 and 2, respectively. We want to find:

$$E[S_2] = \sum_{i=0}^{N} iP(S_2 = i) \tag{4.2}$$

In turn, we can write:

$$P(S_2 = i) = \sum_{j=0}^{N} P(S_2 = i/S_1 = j)P(S_1 = j) \tag{4.3}$$

We have:

$$P(S_1 = 0) = \frac{1}{9}$$
$$P(S_1 = 1) = \frac{2}{3}$$
$$P(S_1 = 2) = 0$$
$$P(S_1 = 3) = \frac{2}{9},$$

If S_1=0, we have:

$$P(S_2 = 0/S_1 = 0) = \frac{1}{4}$$
$$P(S_2 = 1/S_1 = 0) = \frac{3}{4}$$
$$P(S_2 = 2/S_1 = 0) = 0$$
$$P(S_2 = 3/S_1 = 0) = 0$$

If S_1=1, we have:

$$P(S_2 = 0/S_1 = 1) = 0$$
$$P(S_2 = 1/S_1 = 1) = \frac{1}{2}$$
$$P(S_2 = 2/S_1 = 1) = 0$$
$$P(S_2 = 3/S_1 = 1) = \frac{1}{2}$$

If S_1=3, then all the 3 tags have been resolved after frame 1:

$$P(S_2 = 3/S_1 = 3) = 1$$

By substituting the conditional probabilities in Eq. 4.3, we get:

$$P(S_2 = 0) = P(S_2 = 0/S_1 = 0)P(S_1 = 0) = \frac{1}{9}\frac{1}{4}$$

$$P(S_2 = 1) = P(S_2 = 1/S_1 = 0)P(S_1 = 0)+P(S_2 = 1/S_1 = 1)P(S_1 = 1) = \frac{1}{9}\frac{3}{4}+\frac{1}{2}\frac{2}{3}$$

$$P(S_2 = 2) = 0$$

$$P(S_2 = 3) = P(S_2 = 3/S_1 = 3)P(S_1 = 3)+P(S_2 = 3/S_1 = 1)P(S_1 = 1) = 1\frac{2}{9}+\frac{2}{3}\frac{1}{2},$$

and substituting everything in 4.2, we get:

$$E[S_2] = P(S_2 = 1) + 3P(S_2 = 3) \approx 2.08$$

Exercise–9 (*July 1, 2016*)

A Dynamic Frame ALOHA system is used to arbitrate 3 tags. What is the average duration of the arbitration time if the initial frame length is r_1=3 (assume that the frame size of the following frames may be optimally set to exact value of the current backlog)?

Solution of Exercise–9

The average duration of the arbitration process can be found by using the recursive formula:

$$L_3 = r_1 + \sum_{i=1}^{2} P(S=i)L_{3-i},$$

which leads to:

$$L_3 = r_1 + L_3P(S=0) + L_2P(S=1) + L_1P(S=2).$$

In our case, it is $P(S=0) = 3(\frac{1}{3})^3$, $P(S=2) = 0$, $P(S=3) = 3!(\frac{1}{3})^3$ and $P(S=1) = 1 - \frac{1}{9} - \frac{2}{9} = \frac{2}{3}$.

Iterating the process, we have:

$$L_2 = 2 + L_2P(S=0) + L_1P(S=1) = 2 + L_2\frac{1}{2},$$

thus $L_2 = 4$

Substituting the value of L_2 in the expression of L_3, we obtain:

$$L_3 = 3 + L_3\frac{1}{9} + 4\frac{2}{3},$$

which leads to $L_3 = \frac{9}{8}\frac{17}{3} = \frac{51}{8}$

Exercise–10 (*July 27, 2016*)

An RFID system is composed of 4 tags and uses a Dynamic Frame ALOHA access protocol. Assume that the initial frame size is r=4 and the tags choose the following slots for transmitting: tag 1 slot 1, tag 2, tag 3 and tag 4 slot 3. What is the backlog predicted by Schoute's estimate? Find out the probability that the resolved tags during the second frame is equal to i, with i=0, 1, 2, 3. 4 if the second frame length is set to Schoute's estimate.

Solution of Exercise–10

After the first frame one tag has been resolved (tag 1) and the other four tags have collided in one slot. Schoute's estimate observes c=1 collided slots, and thus the current backlog is estimated to be equal to round(2.39c) = 2. If the second frame length is set according to Schoute's estimate, we have the following probabilities for the number of tags resolved during the second frame, S_2:

$$P(S_2 = 0) = 2(\frac{1}{2})^3 = \frac{1}{4}$$

$$P(S_2 = 1) = 2\binom{3}{2}(\frac{1}{2})^3 = \frac{3}{4}$$

$$P(S_2 = 2) = 0$$

Exercise–11 (*September 2, 2016*)

A RFID system based on Dynamic Frame ALOHA is composed of 4 tags. Assuming that the initial frame size is r=2, find the overall collision resolution efficiency, .η

Solution of Exercise–11

The efficiency is defined as $\eta = \frac{n}{L_n}$, being n the initial population size and L_n the total number of slots to arbitrate all the tags. In our case, $n = 4$ and L_n can be found be applying the recursive formula:

$$L_n = r + \sum_{i=0}^{n-1} L_{n-i} P(S = i).$$

In our case it is:

$$L_4 = 2 + \sum_{i=0}^{3} L_{4-i} P(S = i).$$

It can be noted that, P(S=2)=P(S=3)=P(S=4)=0, thus the formula reduces to:

$$L_4 = 2 + L_4 P(S = 0) + L_3 P(S = 1),$$

where P(S=0)=P(S=1)=0.5.

Recursively, we can write:

$$L_3 = 3 + \sum_{i=0}^{2} L_{3-i} P(S = i).$$

We have: P(S=3)= 2/9, P(S=2)=0, P(S=0)=1/9, and P(S=1)=2/3, which leads to:

$$L_3 = 3 + 1/9L_3 + 2/3L_2$$

Again, L_2 can be found as:

$$L_2 = 2 + \sum_{i=0}^{1} L_{2-i} P(S = i),$$

where P(S=0)=1/2 and P(S=1)=0, which leads to:

$$L_2 = 2 + 1/2L_2.$$

Solving in L_2 we get L_2=4; substituting the value of L_2 in the expression of L_3 we get:

$$L_3 = 3 + 1/9L_3 + 8/3,$$

which leads to $L_3 = 6.375$. Finally, substituting the value of L_3 in the expression of L_4 we get:

$$L_4 = 2 + 1/2L_4 + 0.5 \times 6.375,$$

which leads to $L_4 = 16.75$. The efficiency is $\eta = \frac{4}{16.75} = 0.23$

Exercise–12 (*September 22, 2016*)

A RFID system based on Dynamic Frame ALOHA is composed of 3 tags. Assuming that the initial frame size is r=1, find the overall collision resolution efficiency η (assume that after the first frame, the frame size is correctly set to the current backlog size).

Solution of Exercise–12

The first frame is composed of 1 slot with three transmitting tags, thus throughput after the first frame is null.

The second frame, by assumption, is set to the correct backlog size, that is, $r_2 = 3$. We can then write:

$$L_3 = r_2 + \sum_{i=0}^{2} L_{3-i} P(S = i),$$

being $P(S = 0) = 3\frac{1}{3}^3$, $P(S = 1)$=2/3, $P(S = 2)$=0, $P(S = 3) = 6\frac{1}{3}^3$. We can the write:

$$L_3 = 3 + \frac{1}{9} L_3 + \frac{2}{3} L_2.$$

Iterating, we get:

$$L_2 = 2 + \sum_{i=0}^{1} L_{2-i} P(S = i),$$

being $P(S = 0) = 2\frac{1}{2}^2$, $P(S = 1)$=0, $P(S = 2)$=1/2, thus we get:

$$L_2 = 2 + 1/2 L_2,$$

which leads to $L_2 = 4$. Substituting the value of L_2 in the expression of L_3 we get:

$$L_3 = 3 + 1/9 L_3 + 8/3 = 6.375.$$

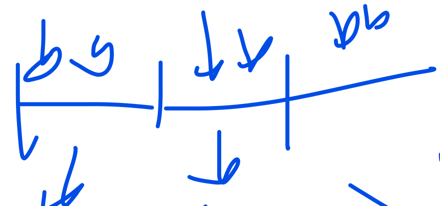
We finally have to add the single slot of the initial frame, thus the efficiency is:

$$\eta = \frac{3}{7.375} \approx 0.4.$$

Exercise–13 (*February 20, 2017 Como*)

A RFID system is composed of 2 tags. Find the collision resolution efficiency in the following two cases: (i) a Dynamic Frame ALOHA protocol is used with initial frame length r=3 (assume that after the first frame, the frame size is correctly set to the current backlog size), (ii) a binary tree protocol is used. Which approach has higher efficiency?

Solution of Exercise–13



(i) Dynamic Frame ALOHA

By applying the recursive formula we get:

$$L_2 = 3 + \sum_{i=0}^{1} L_{2-i} P(S = i),$$

$$\left(\frac{1}{3}\right)^2 \cdot \left(3 \cdot 2\right)$$

being $P(S = 0) = 0\cancel{.5}$ $1/3$ and $P(S = 1) = 0$; we can the write:

$$L_2 = 4.5.$$

$$\frac{2}{3}$$

The efficiency is: $\eta = 2/4.5$

(ii) Binary Tree.

In case of binary tree, the two motes keep colliding if they choose the same backoff value at every attempt (either 0 or 1). One first slot is wasted for the first (certain) collision, two slots are used for the final correct transmission; the number of attempts to the first success is $1/(1/2)$; at each one of this failed attempts either 1 or two slots are wasted, so the total number of required slots is: 1 + 1 x 1.5 + 2 = 4.5. The efficiency is 2/4.5

Exercise–14 (*February 6, 2017 Como*)

A RFID system based on Dynamic Frame ALOHA is composed of 2 tags. Assuming that the initial frame size is r=4, find the overall collision resolution efficiency, (assume that after the first frame, the frame size is correctly set to the current backlog size).

Solution of Exercise–14

Let's apply the recursive formula:

$$L_2 = 4 + \sum_{i=0}^{1} L_{2-i} P(S = i),$$

being $P(S = 0) = 4(\frac{1}{4})^2$ and $P(S = 1) = 0$; we can the write:

$$L_2 = 16/3.$$

The efficiency is: $\eta = 0.375$

Appendices

Appendix A

Unit measures

This is a brief leaflet with the most commonly used unit of measures throughput the exercises (and the whole course).

Information. Reference unit of measure *bit*, symbol [b] or [bit].

$$\begin{array}{rcl} 1 \quad byte \quad [byte] & = & 8 \quad bits \quad [b] \\ 1 \quad kbyte \quad [kbyte] & = & 8 \quad kbits \quad [kb] = 8 \times 10^3 \quad bits \quad [b] \\ 1 \quad Mbyte \quad [Mbyte] & = & 8 \quad Mbits \quad [Mb] = 8 \times 10^6 \quad bits \quad [b] \end{array}$$

Rate/Throughput. Reference unit of measure *bit per second*, symbol [b/s].

$$\begin{array}{rcl} 1 \quad [kb/s] & = & 10^3 \quad [b/s] \\ 1 \quad [Mb/s] & = & 10^6 \quad [b/s] \\ 1 \quad [kbyte/s] & = & 8 \quad [kb/s] = 8 \quad 10^3 \quad [b/s] \\ 1 \quad [Mbyte/s] & = & 8 \quad [Mb/s] = 8 \quad 10^6 \quad [b/s] \end{array}$$

Power/Energy. 1 [Joule] = 1 [A] 1[V] 1[s] 1 [Watt] = 1 [Joule] / 1 [s]

Appendix B

Frequently used probability distributions

This is a brief leaflet with the most commonly used probability distributions used throughput the exercises (and the whole course).

B.1 Poisson Distribution

The **Poisson distribution** is a discrete probability distribution that gives the probability of a given number of events occurring in a fixed interval of time (or space).

The parameter of the Poisson distribution, λ, is the event rate, that is the average number of events per unit of time.

If $X(\Delta T)$ representing the number of observed events in the interval ΔT is distributed according to Poisson, then it holds:

$$P(X(\Delta T) = k) = \frac{(\lambda \Delta T)^k}{k!} e^{-\lambda \Delta T} \qquad with \quad k = 1, \quad 2, \quad 3, ...$$

Property: $E[X(\Delta T)] = Var[X(\Delta T)] = \lambda \Delta T$.

B.2 Geometric Distribution

If a discrete random variable X follows a **geometric distribution** , then:

$$P(X = r) = (1-p)^{r-1} p \qquad with \quad r = 1, 2, ..$$

with p defined as the parameter of the distribution. The variable X counts the number of independent trials with binary possible results (ON-OFF, zero/one, success/fail) which are needed to obtain the first "positive" outcome (ON, one, success). In this interpretation, the parameter p of the distribution measures the probability to obtain a positive outcome (ON, one, success) out of the single trial.

Some useful properties: $E[X] = \frac{1}{p}$ and $Var[X] = \frac{1-p}{p^2}$.

The aforementioned geometric distribution can be leveraged to represent the number of **failed** trials up to the successful one. In this case, the definition of the probability distribution must be changed as follows:

$$P(Y = X - 1 = r) = (1-p)^r p \qquad with \quad r = 0, 1, 2, ..$$

It can be shown that: $E[Y] = E[X] - 1 = \frac{1-p}{p}$ and $E[Y] = Var[X] = \frac{1-p}{p^2}$