DATA 607 Statistical and Machine Learning
Session 3: Kernel Smoothers; Nonparametric Classifiers

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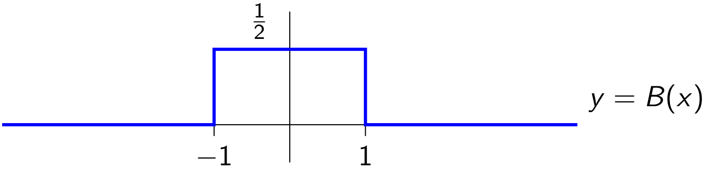
This Evening's Agenda

Indicator Functions

Define the **Boxcar Kernel** by

$$B(x) = \begin{cases} \frac{1}{2} & \text{if } -1 < x < 1, \\ 0 & \text{otherwise.} \end{cases} \qquad (\mathbf{x} \in \mathbb{R}).$$



Sliding Window Smoother

Data set:

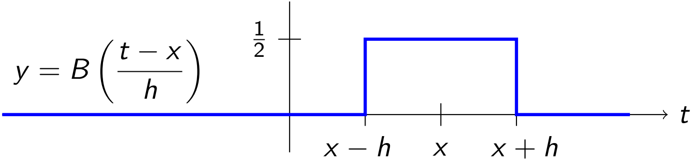
$$\mathcal{D} = \{(\mathbf{x}_1, y_1),\ (\mathbf{x}_1, y_1),\ \ldots,\ (\mathbf{x}_n, y_n)\}$$

Boxcar Kernel Smoother:

$$\widehat{r}_{h,\mathcal{D}}(\mathbf{x}) = \frac{\displaystyle\sum_{i=1}^{n} y_i\, B\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)}{\displaystyle\sum_{i=1}^{n} B\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)}$$

Generalization: Replace B by different "kernel" function.

$$
\begin{aligned}
B\left(\frac{x_i - x}{h}\right) &= \begin{cases} 1 & \text{if } -1 < \dfrac{x_i - x}{h} < 1 \\ 0 & \text{otherwise} \end{cases} \\
&= \begin{cases} 1 & \text{if } -h < x_i - x < h \\ 0 & \text{otherwise} \end{cases} \\
&= \begin{cases} 1 & \text{if } x - h < x_i < x + h \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$



$$\sum_{i=1}^{n} B\left(\frac{x_i - x}{h}\right) = \#\text{ of } x_i \text{ such that } x - h < x_i < x + h$$

Kernel Functions

$K(\mathbf{x})$ is a **kernel function** if

1. $K(\mathbf{x}) \geq 0$
2. $K(-\mathbf{x}) = K(\mathbf{x})$
3. $\int_{\mathbb{R}^n} K(\mathbf{x}) d\mathbf{x} = 1$

Popular Kernels

1. Boxcar:
$$B(x) = \frac{1}{2}\mathbf{1}_{(-1,1)}(x)$$
2. Triangular:
$$T(x) = (1 - |x|)\mathbf{1}_{(-1,1)}(x)$$
3. Epanechnikov:
$$E(x) = \frac{3}{4}(1 - x^2)\mathbf{1}_{(-1,1)}(x)$$
4. Gaussian:
$$G(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

Popular Kernels