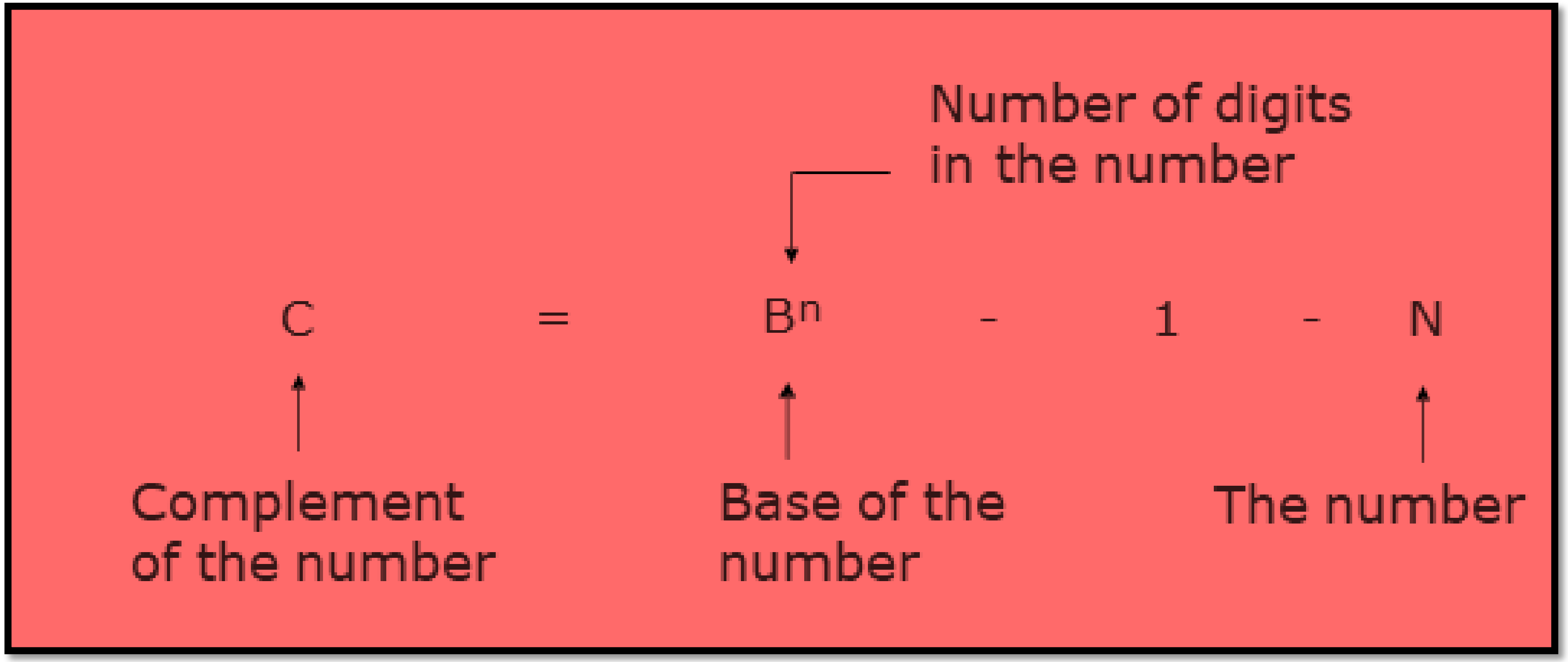
DIGITAL LOGIC DESIGN

Lecture 2

COMPLEMENTS OF NUMBERS

- Complements in digital computers **simplify the subtraction operation**
- Simplifying operations leads to simpler, less expensive circuits to implement the operations.
- There are two types of complements for each base-*r* system:
- The radix complement and the diminished radix complement.
- The first is referred to as the *r*'s complement and the second as the (*r* - 1)'s complement.
- When the value of the base *r* is substituted in the name, the two types are referred to as the 2's complement and 1's complement for binary numbers and the 10's complement and 9's complement for decimal numbers.

Diminished radix complement



Diminished radix complement

- **Example**
    - Find the complement of 37_{10}
- **Solution**
    - Since the number has 2 digits and the value of base is 10,
        - $(Base)^n - 1 = 10^2 - 1 = 99$
        - Now $99 - 37 = 62$
- Hence, complement of $37_{10} = 62_{10}$

Diminished radix complement

Example

Find the complement of 6_8

Solution

Since the number has 1 digit and the value of base is 8,

$(\text{Base})^n - 1 = 8^1 - 1 = 7_{10} = 7_8$

Now $7_8 - 6_8 = 1_8$

Hence, complement of $6_8 = 1_8$

COMPLEMENTS OF NUMBERS

Complement of a binary number can be obtained by transforming all its 0's to 1's and all its 1's to 0's

Example

Complement of 1 0 1 1 0 1 0 is

↓ ↓ ↓ ↓ ↓ ↓ ↓

0 1 0 0 1 0 1

Note: Verify by conventional complement

Complementary Method of Subtraction

Involves following 3 steps:

Step 1: Find the complement of the number you are subtracting (subtrahend)

Step 2: Add this to the number from which you are taking away (minuend)

Step 3: If there is a carry of 1, add it to obtain the result; if there is no carry, recomplement the sum and attach a negative sign

Complementary subtraction is an additive approach of subtraction

Complementary Subtraction (Example 1)

Example:

Subtract 56_{10} from 92_{10} using complementary method.

Solution

Step 1: Complement of 56_{10}

$= 10^2 - 1 - 56 = 99 - 56 = 43_{10}$

Step 2: 92 + 43 (complement of 56)

= 135 (note 1 as carry)

Step 3: 35 + 1 (add 1 carry to sum)

Result = 36

The result may be verified using the method of normal subtraction:

92 - 56 = 36

Complementary Subtraction (Example 2)

Example

Subtract 35_{10} from 18_{10} using complementary method.

Solution

Step 1: Complement of 35_{10}

$= 10^2 - 1 - 35$

$= 99 - 35$

$= 64_{10}$

Step 2:

$$\begin{array}{r} 18 \\ +\ 64 \\ \hline 82 \\ \hline \end{array}$$

(complement of 35)

Step 3: Since there is no carry, re-complement the sum and attach a negative sign to obtain the result.

Result $= -(99 - 82)$

$= -17$

The result may be verified using normal subtraction:

$18 - 35 = -17$

Binary Subtraction Using Complementary Method (Example 1)

Example

Subtract 0111000_2 (56_{10}) from 1011100_2 (92_{10}) using complementary method.

Solution

$$
\begin{array}{rl}
1011100 & \\
+1000111 & \text{(complement of 0111000)} \\
\hline
10100011 & \\
1 & \text{(add the carry of 1)} \\
\hline
0100100 & \\
\hline
\end{array}
$$

Result = $0100100_2 = 36_{10}$

Binary Subtraction Using Complementary Method (Example 2)

Example

Subtract 100011_2 (35_{10}) from 010010_2 (18_{10}) using complementary method.

Solution

$$\begin{array}{r} 010010 \\ +011100 \\ \hline 101110 \\ \hline \end{array} \quad \text{(complement of 100011)}$$

Since there is no carry, we have to complement the sum and attach a negative sign to it. Hence,

Result = -010001_2 (complement of 101110_2)

$= -17_{10}$

Diminished Radix Complement

- Given a number N in base r having n digits, the $(r - 1)$'s complement of N , i.e. its diminished radix complement, is defined as $(r^n - 1) - N$.
- For decimal numbers, $r = 10$ and $r - 1 = 9$, so the 9's complement of N is $(10^n - 1) - N$.
- $10^n - 1$ is a number represented by n 9's.
- For example, if $n = 4$, we have $10^4 = 10{,}000$ and $10^4 - 1 = 9999$.
- It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9.
- The 9's complement of 546700 is 999999 - 546700 = 453299

Diminished Radix Complement

- For binary numbers, r = 2 and r - 1 = 1, so the 1's complement of N is $(2^n - 1) - N$.
- $2^n – 1$ is a binary number represented by n 1's.
- For example, if $n = 4$, we have $2^4 =(16)_2=(10000)_2$ and $2^4 - 1 = (1111)2$.
- Thus, the 1's complement of a binary number is obtained by subtracting each digit from 1. However, when subtracting binary digits from 1, we can have either 1 - 0 = 1 or 1 - 1 = 0, which causes the bit to change from 0 to 1 or from 1 to 0, respectively.
- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.**
- The (r - 1)'s complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively

Diminished Radix Complement

- So we can say that diminished radix complement of a number is obtained by performing following:
- In binary subtract from 1
- In decimal subtract from 9
- In octal subtract from 7
- In hexadecimal subtract from F or 15

Radix Complement

- The *r*'s complement of an *n*-digit number *N* in base *r* is defined as $r^n - N$ for *N* *not equal to* 0 and as 0 for $N = 0$.
- Comparing with the $(r - 1)$'s complement, we note that the r's complement is obtained by adding 1 to the $(r - 1)$'s complement, since $r^n - N = [(r^n - 1) - N] + 1$.
- Thus, the 10's complement of decimal 2389 is 7610 + 1 = 7611 and is obtained by adding 1 to the 9's complement value.
- The 2's complement of binary 101100 is 010011 + 1 = 010100 and is obtained by adding 1 to the 1's-complement value.
- Since 10 is a number represented by a 1 followed by *n* 0's, $10^n - N$, which is the 10's complement of *N*, can be formed also by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9.

Radix Complement

- Thus, the 10's complement of 012398 is 987602 and
- the 10's complement of 246700 is 753300
- The 10's complement of the first number is obtained by subtracting 8 from 10 in the least significant position and subtracting all other digits from 9.
- The 10's complement of the second number is obtained by leaving the two least significant 0's unchanged, subtracting 7 from 10, and subtracting the other three digits from 9.

Radix Complement

- Similarly, the 2’s complement can be formed by leaving all least significant 0’s and the first 1 unchanged and replacing 1’s with 0’s and 0’s with 1’s in all other higher significant digits.
- For example, the 2’s complement of 1101100 is 0010100 and
- the 2’s complement of 0110111 is 1001001
- The 2’s complement of the first number is obtained by leaving the two least significant 0’s and the first 1 unchanged and then replacing 1’s with 0’s and 0’s with 1’s in the other four most significant digits.
- The 2’s complement of the second number is obtained by leaving the least significant 1 unchanged and complementing all other digits.

Subtraction with Complements

The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n - N) = M - N + r^n$.
2. If $M \geq N$, the sum will produce an end carry r^n, which can be discarded; what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Subtraction with Complements

EXAMPLE 1.5

Using 10's complement, subtract 72532 − 3250.

$$\begin{aligned} M &= 72532 \\ \text{10's complement of } N &= + \;\; 96750 \\ \text{Sum} &= 169282 \\ \text{Discard end carry } 10^5 &= - \;\underline{100000} \\ \textit{Answer} &= 69282 \end{aligned}$$

Note that M has five digits and N has only four digits. Both numbers must have the same number of digits, so we write N as 03250. Taking the 10's complement of N produces a 9 in the most significant position. The occurrence of the end carry signifies that $M \geq N$ and that the result is therefore positive.

Subtraction with Complements

EXAMPLE 1.6

Using 10's complement, subtract 3250 − 72532.

$$\begin{aligned} M &= 03250 \\ \text{10's complement of } N &= +\ \underline{27468} \\ \text{Sum} &= 30718 \end{aligned}$$

There is no end carry. Therefore, the answer is −(10's complement of 30718) = −69282.

Note that since 3250 < 72532, the result is negative. Because we are dealing with unsigned numbers, there is really no way to get an unsigned result for this case. When subtracting with complements, we recognize the negative answer from the absence of the end carry and the complemented result. When working with paper and pencil, we can change the answer to a signed negative number in order to put it in a familiar form.

Subtraction with complements is done with binary numbers in a similar manner, using the procedure outlined previously.

Subtraction with Complements

EXAMPLE 1.7

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

$$
\begin{array}{lrr}
\text{(a)} & X = & 1010100 \\
 & \text{2's complement of } Y = & +\ \ 0111101 \\
 & \text{Sum} = & 10010001 \\
 & \text{Discard end carry } 2^7 = & -\underline{10000000} \\
 & \textit{Answer}: X - Y = & 0010001
\end{array}
$$

$$
\begin{array}{lrr}
\text{(b)} & Y = & 1000011 \\
 & \text{2's complement of } X = & +\ \ 0101100 \\
 & \text{Sum} = & 1101111
\end{array}
$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{2's complement of } 1101111) = -0010001$.

■

Subtraction of unsigned numbers can also be done by means of the $(r - 1)$'s complement. Remember that the $(r - 1)$'s complement is one less than the r's complement. Because of this, the result of adding the minuend to the complement of the subtrahend produces a sum that is one less than the correct difference when an end carry occurs. Removing the end carry and adding 1 to the sum is referred to as an *end-around carry*.

Subtraction with Complements

EXAMPLE 1.8

Repeat Example 1.7, but this time using 1's complement.

(a) $X - Y = 1010100 - 1000011$

$$\begin{aligned} X &= 1010100 \\ \text{1's complement of } Y &= +\ 0111100 \\ \text{Sum} &= 10010000 \\ \text{End-around carry} &= +\ \underline{1} \\ \textit{Answer}: X - Y &= 0010001 \end{aligned}$$

(b) $Y - X = 1000011 - 1010100$

$$\begin{aligned} Y &= 1000011 \\ \text{1's complement of } X &= +\ \underline{0101011} \\ \text{Sum} &= 1101110 \end{aligned}$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{1's complement of } 1101110) = -0010001$.

■

Note that the negative result is obtained by taking the 1's complement of the sum, since this is the type of complement used. The procedure with end-around carry is also applicable to subtracting unsigned decimal numbers with 9's complement.

SIGNED BINARY NUMBERS

- Positive integers (including zero) can be represented as unsigned numbers. However, to represent negative integers, we need a notation for negative values.
- In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign.
- Because of hardware limitations, computers must represent everything with binary digits.
- It is customary to represent the sign with a bit placed in the leftmost position of the number.
- The convention is to make the sign bit 0 for positive and 1 for negative.
- It is important to realize that both signed and unsigned binary numbers consist of a string of bits when represented in a computer.

SIGNED BINARY NUMBERS

- The user determines whether the number is signed or unsigned.
- If the binary number is signed, then the leftmost bit represents the sign and the rest of the bits represent the number.
- If the binary number is assumed to be unsigned, then the leftmost bit is the most significant bit of the number.
- For example, the string of bits 01001 can be considered as 9 (unsigned binary) or as +9 (signed binary) because the leftmost bit is 0.
- The string of bits 11001 represents the binary equivalent of 25 when considered as an unsigned number and the binary equivalent of -9 when considered as a signed number.
- This is because the 1 that is in the leftmost position designates a negative and the other four bits represent binary 9.

SIGNED BINARY NUMBERS

- Usually, there is no confusion in interpreting the bits if the type of representation for the number is known in advance.
- The representation of the signed numbers in the last example is referred to as the *signed-magnitude* convention.
- In this notation, the number consists of a magnitude and a symbol (+ or -) or a bit (0 or 1) indicating the sign.
- This is the representation of signed numbers used in ordinary arithmetic.
- When arithmetic operations are implemented in a computer, it is more convenient to use a different system, referred to as the *signed-complement* system, for representing negative numbers.
- In this system, a negative number is indicated by its complement.

SIGNED BINARY NUMBERS

- Whereas the signed-magnitude system negates a number by changing its sign, the signed-complement system negates a number by taking its complement.
- Since positive numbers always start with 0 (plus) in the leftmost position, the complement will always start with a 1, indicating a negative number.
- The signed-complement system can use either the 1’s or the 2’s complement, but the 2’s complement is the most common.
- As an example, consider the number 9, represented in binary with eight bits.
- +9 is represented with a sign bit of 0 in the leftmost position, followed by the binary equivalent of 9, which gives 00001001.
- Note that all eight bits must have a value; therefore, 0’s are inserted following the sign bit up to the first 1.

SIGNED BINARY NUMBERS

- Although there is only one way to represent +9, there are three different ways to represent -9 with eight bits:
- signed-magnitude representation: 10001001
- signed-1’s-complement representation: 11110110
- signed-2’s-complement representation: 11110111
- Table 1.3 lists all possible four-bit signed binary numbers in the three representations.
- The equivalent decimal number is also shown for reference.

SIGNED BINARY NUMBERS

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

SIGNED BINARY NUMBERS (Arithmetic Addition)

- **The addition of two signed binary numbers with negative numbers represented in signed- 2's-complement form is obtained from the addition of the two numbers, including their sign bits. A carry out of the sign-bit position is discarded.**

Numerical examples for addition follow:

+ 6	00000110	− 6	11111010
+13	00001101	+13	00001101
+19	00010011	+ 7	00000111
+ 6	00000110	− 6	11111010
−13	11110011	−13	11110011
− 7	11111001	−19	11101101

SIGNED BINARY NUMBERS (Arithmetic Subtraction)

- Subtraction of two signed binary numbers when negative numbers are in 2's-complement form is simple and can be stated as follows:
- Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit).
- A carry out of the sign-bit position is discarded.
- This procedure is adopted because a subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed, as is demonstrated by the following relationship:

$$(\pm A) - (+B) = (\pm A) + (-B);$$
$$(\pm A) - (-B) = (\pm A) + (+B).$$

SIGNED BINARY NUMBERS (Arithmetic Subtraction)

- It is worth noting that binary numbers in the signed-complement system are added and subtracted by the same basic addition and subtraction rules as unsigned numbers.
- Therefore, **computers need only one common hardware circuit to handle both types of arithmetic.**
- This consideration has resulted in the signed-complement system being used in virtually all arithmetic units of computer systems.
- The user or programmer must interpret the results of such addition or subtraction differently, depending on whether it is assumed that the numbers are signed or unsigned.