Computational Physics Project / Stellar Structure

This project involves a simplified calculation of stellar structure, under spherical symmetry.

The general equations governing stellar structure in spherical symmetry are mass conservation, the hydrostatic equation, energy conservation, and energy transport. It turns out to be useful to consider these equations not as a function of radius, but as a function of enclosed mass within the radius.

A good introduction to the background behind the equations can be found in these lecture notes. To summarize, for a star with energy transport dominated by radiative diffusion:

$$
\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}m} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\mathrm{d}P}{\mathrm{d}m} &= -G\frac{m}{4\pi r^4} \\
\frac{\mathrm{d}L}{\mathrm{d}m} &= \mathcal{E} \\
\frac{\mathrm{d}T}{\mathrm{d}m} &= -\frac{3}{16\sigma}\frac{\kappa}{T^3}\frac{L}{(4\pi r^2)^2}
\end{aligned}
\tag{1}
$$

where r is the radius, m is the mass enclosed within r, L is the luminosity generated within r, P is the pressure, T is the temperature, and ρ is the density. $\mathcal{E}$ is the energy generated per unit mass and κ is the opacity of the gas. σ and c are the Stefan-Boltzman constant and the speed of light.

If we want to solve these equations, we need four boundary conditions; these are that $r = 0$ and $L = 0$ at $m = 0$, and that $P = 0$ and $T = 0$ at $m = M_*$.

We also need to know how $\mathcal{E}$ and κ behave. Generally:

$$
\mathcal{E} = \mathcal{E}_0 \rho^\alpha T^\beta
\tag{2}
$$

where here we will take $\alpha = 1$ and $\beta = 4$.

The opacity κ can be approximated as :

$$
\kappa = \kappa_0 \rho T^{-3.5}
\tag{3}
$$

Finally, we need the equation of state, which in the cases we consider here can be approximated as :

$$
P = \frac{\rho k T}{\mu m_p}
\tag{4}
$$

where μ is the mean molecular weight in units of proton masses, and is 0.6.

The parameters of the system that are not physical constants are M_*, κ_0, and $\mathcal{E}_0$.

1. Prep work

- Recast the above equations into a form with the minimum number of independent, unitless parameters. You should perform your numerical analysis in these variables, and then scale your solution after the fact to specific physical solutions.

- **polytrope solution**

2. Creating the integrator

- Build an integrator for the ODE system working from the center outwards, given some initial conditions at the center.

- Wrap the integration in a multidimensional minimization routine that adjusts the central conditions of T and P to fit the outer boundary conditions.

3. Testing the integrator

- Plot ρ, P, T, L and $\mathcal{E}$ as a function of enclosed radius for some choice of M_*.

- Given the Solar values of mass and luminosity, can you determine some combination of κ_0 and $\mathcal{E}_0$?