Computational Physics Project / Three-body problem

This project involves integrating gravitational orbits of the sort that you might find in planetary or stellar systems.

The general equations governing these systems are simple:

$$\begin{aligned} \dot{\vec{v}}_i &= -\sum_{j\neq i} \frac{Gm_j}{r_{ij}^2} \\ \dot{\vec{x}}_i &= \vec{v}_i \end{aligned} \tag{1}$$

For $N = 2$, this is a classic Keplerian problem with known solutions. A good introduction to this is given in the textbook *Principles of Astrophysics*, by Charles Keeton. Specifically, if r is the distance between the two particles, and ϕ is an angle in the plane of motion measured with respect to the apocenter of the orbit, then:

$$r = \frac{a(1-e^2)}{1+e\cos\phi} \tag{2}$$

where a is the semimajor axis of the orbit, and e is the eccentricity. The period of the orbit is:

$$P = \sqrt{\frac{4\pi^2 a^3}{G(m_1+m_2)}} \tag{3}$$

and the angle ϕ and time t are related by:

$$\frac{t}{P} = \frac{1}{2\pi}\left\{2\tan^{-1}\left[\left(\frac{1-e}{1+e}\right)^{1/2}\tan\frac{\phi}{2}\right] - \frac{e(1-e^2)^{1/2}\sin\phi}{1+e\cos\phi}\right\} \tag{4}$$

1. Testing the two-body problem

In the first section of this project, you will write a general program to integrate orbits, and test it on the two body problem.

- Write a module with an integrator using the techniques in Landau Chapter 8 that will solve the general system of equations for N bodies.
- Test the integrator on the two body problem. Use initial conditions for which the system is bound. Assess the period of the orbit, how well it approximates an ellipse, and showing that it (approximately) closes.

2. Exploring the three-body problem

Now you should be able explore how three-body systems behave.

- Start with two large, equal mass particles, and one much smaller mass particle. Start the equal mass particles on circular orbits, and verify sure that they stay on those orbits. Then explore a couple of different initial conditions for the third particle.
- Try the Lagrange Points (look the description up on Wikipedia) and put the small particle at those locations, with an angular velocity the same as the orbital angular velocity of the large mass particles. What happens?
- Make one large mass particle, one somewhat smaller mass particle (like a factor of 100) in orbit around the large mass particle, and then a third particle. Try different initial conditions for the third particle, including some which are close to the orbit of the second particle. Show what happens as the orbits get close together.
- Make three similar mass particles and follow their orbits. Can you find initial conditions which are stable?