Partial differential equations (PDEs) are distinguished from ordinary differential equations by the presence of more than one independent variable, which may be time or space. Generally speaking, they are applicable to continuous systems: fluids, electromagnetic fields, gravity, general relativity, etc.

There are several types of PDEs, which can be classified as follows:

- Hyperbolic: second-order derivatives, with opposite signs, like the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} = 0 \tag{1}$$

- Elliptic: second-order derivatives, with like signs, like the Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{2}$$

- Parabolic: first-order and second-order mixed, like the diffusion equation:

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(D \frac{\partial u}{\partial x^2}\right) = 0 \tag{3}$$

These sorts of equations are more complex than ODEs, mainly because you cannot reduce the equations to a single derivative of the state variable with the single independent variable. Furthermore, you cannot as easily set the boundary conditions; at least, there are many more ways to.

The simplest way to see this is the case of the Laplace equation. To solve Laplace for an electric potential within some region requires setting the potential at the surface of that region (or its derivative, or some combination). Clearly this is a bit complicated. Also, it is not at all obvious whether a solution with the desired boundary conditions is necessarily allowed by the equations (with Laplace, this is not really a problem of course, but it can be with more general equations).

The setting of the boundary conditions is an extremely important feature of any PDE problem, and usually defines the basic watersheds between numerical approaches. For example, you can imagine setting $u(x = 0, t)$ and using the wave or diffusion equations to integrate in time. These are *Cauchy* or *initial value* problems. But for Laplace, you need to specify *boundary values*, either of u (*Dirichlet*) or its derivative (*Neumann*).

1. Boundary value problems

We will concern ourselves first with boundary value problems.