Computational Physics / PHYS-GA 2000 / Problem Set #8
Due November 5, 2024

You *must* label all axes of all plots, including giving the *units*!!

1. We will do a simple likelihood maximization problem. Let's say you do a survey of the population, asking people a simple yes or no question, namely "Do you recognize the phrase 'Be Kind, Rewind', and know what it means?" You have a hypothesis that whether people answer yes should depend on age. The standard way people analyze the results to look for a correlation in situations like this is something called *logistic regression.* You model the probability as the logistic function:

$$p(x) = \frac{1}{1 + \exp\left[-(\beta_0 + \beta_1 x)\right]} \tag{1}$$

   where in this case x represents the age. You'll see that $p(x) = 0.5$ at $-\beta_0/\beta_1$ and the slope there is $1/\beta_1$. Then the likelihood for each resulting value is $p(\text{age})$ if the answer is "yes" and $1 - p(\text{age})$ if the answer is "no." Use this data, which gives the age of the respondent in years and their answer to the question, where "1" means yes and "0" means no. Note this is *not* from a real survey!! Find the maximum likelihood values and formal errors and covariance matrix of β_0 and β_1. Plot the logistic model and the answers on the same plot. Some points to keep in mind:

   - Using `jax`'s `autodiff` really helps for both the gradient and the Hessian calculation.
   - Remember you want to minimize the negative log likelihood.
   - Be careful about machine precision—you need to calculate the log likelihood in such a way that you are never taking the log of zero, which if you are not careful you can easily do.

   Do the answers make any sense?

   Note that although this logistic function regression is *standard* for problems where you are looking for a relationship between the probabilities from a bunch of 0s and 1s, just like fitting a line to data points is standard, that doesn't mean the logistic function is always the most sensible or correct model. (Though by construction it is in *this* case).

2. Exercise 7.3 of Newman (n.b. you may use `numpy` or `scipy`'s FFT routines).

3. Exercise 7.4 of Newman (n.b. you may use `numpy` or `scipy`'s FFT routines).