Frege on Existence

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Introduction

In several of his writings,[1] Frege provides an analysis of quantification under which existence is characterized as a second-order concept applying to first-order concepts. For example, the sentence “there exists a square root of 4” is understood to be true if and only if the second-order concept *being instantiated* is true of the first-order concept *being a square root of* 4.[2] In this paper, I present Frege’s analysis of quantification and evaluate three claims related to it:

(1) Existence is a second-order concept expressed by the quantifier “$\exists x$”.

(2) The quantifier “$\exists x$” and its natural-language counterparts (“there is”, “there are”, etc.) can range only over existing things.

(3) Existence is not a predicate.

I argue that these claims, taken individually or jointly, present difficulties.[3]

Exposition

In Frege’s system of logic (more accurately, a modern reconstruction of it),[4] a formal *language* consists of the following:

[1] [1] and [2].

[2] [2], 48–9. I am using the word “instantiated” instead of “nonempty”.

[3] For what it is worth, I outlined the arguments in this paper prior to reading the supplementary material in [3], although I did read it prior to drafting the paper for a sanity check.

[4] [4].

(a) A *vocabulary* consisting of *logical symbols* ("=", "¬", "∧", "∨", "→", "↔", "∀", "∃", etc.) and *nonlogical symbols* (constant symbols, predicate symbols, function symbols, etc.)

(b) Rules for recursively constructing *terms* like "$f(a)$" used to refer to objects.

(c) Rules for recursively constructing *formulas* like "$\exists x(\neg(x = y) \wedge Rxy)$" used to express statements about objects, and in particular for constructing *sentences* like "$\forall x(x = x)$" used to express general statements. (In sentences, there are no *free* variables; that is, all variables are bound to quantifiers.)

An *interpretation* of a language assigns meanings to its syntactic parts, allowing the sentences of the language to express statements. Under an interpretation, terms refer to objects, function symbols refer to functions on objects, and predicate symbols refer to *concepts*, which are just functions assigning *truth values* (truth or falsity) to objects. Sentences are assigned truth values as the statements they express relative to the interpretation are true or false.[5]

Of interest is the rule used to assign truth values to existentially quantified sentences. Note if $\varphi(x)$ is a formula with one free variable x, there is a concept C_φ corresponding to φ given by

$$C_\varphi(o) = \begin{cases} \text{true} & \text{if } \varphi(t) \text{ is true with some term } t \text{ referring to } o \\ \text{false} & \text{otherwise} \end{cases}$$

where o is an object and $\varphi(t)$ is the result of syntactically substituting t for all free occurrences of x in $\varphi(x)$. The existential quantifier is analyzed as follows:

(4) The sentence $\ulcorner \exists x \varphi(x) \urcorner$ is true if and only if $\varphi(t)$ is true with some term t referring to some object o, that is, if and only if the concept C_φ is true for some object o. The quantifier "$\exists x$" refers to a *second-order* concept $\mathscr{C}_\exists$ which takes (first-order) concepts C as arguments and

[5] For Frege, the truth values are special objects *the true* and *the false*, but I am ignoring this weirdness in my exposition.

assigns truth values as follows:

$$\mathscr{C}_\exists(C) = \begin{cases} \text{true} & \text{if } C(o) = \text{true for some object } o \\ \text{false} & \text{otherwise} \end{cases}$$

If $\mathscr{C}_\exists(C) = \text{true}$, C is said to be *instantiated*, so $\mathscr{C}_\exists$ is called *being instantiated*.

For Frege, existence *just is* the second-order concept *being instantiated* as defined in (4), so existence is a property of concepts, *not* a property of objects.[6] Truth conditions for existential sentences are defined in terms of this property of concepts in (4), and assertions about the existence of objects are understood to assert this property of concepts. Taking the example from earlier: asserting "there exists a square root of 4" just asserts a property of the concept *being a square root of* 4, namely that it falls under the concept *being instantiated*, and does not assert a property of an object (for example, of one of the numbers ±2). In other words, Frege holds (1) and (a certain reading of) (3).

Analysis

There are some difficulties with (1)–(3). I will start with (3).

Presumably, (3) is to be understood as stating that existence cannot be ascribed to objects, or that existence is not a property of objects, as Frege contends. But this seems false, or at the very least awkward, because it seems perfectly meaningful to assert, of an individual object, that it exists. For example, if the proper name "John" refers an individual (like me), it seems perfectly meaningful to assert the sentence "John exists", where the proposition expressed is understood to ascribe existence as a property to John. Note that we need not specify *what* the meaning of "exists" is in this context (that is, *what* it means for an object to exhibit existence) for us to recognize *that* the ascription of this property to an object seems meaningful. But if such an assertion is meaningful, there is nothing preventing us from treating existence as a property

[6] [1], p. 64–5; [2], p. 48–9.

of objects as opposed to, or in addition to, a property of concepts. This same objection poses a problem for (1), which limits the scope of existence to first-order concepts. If existence can hold of objects in the manner described, then (1) is false. Note that rejecting (1) and (3) does not commit us to rejecting (4) as an analysis of the existential quantifier or of existentially quantified sentences; it merely allows us to use "exists" as a predicate and assert existence holding as a property of individual objects.

It seems that, in addition to asserting the existence of objects, we can meaningfully assert the *nonexistence* of objects. For example, if John from the previous example were to die getting run over by a bus, it would seem perfectly meaningful to assert "John died getting run over by a bus, so no longer exists". This poses a problem for (2). Typically, for any singular term t and predicate P, if the proposition *t is P* is true, then we may conclude that the proposition *there is x such that x is P* is true (this is just an instance of *existential generalization*). But if nonexistence can be ascribed to objects (or, if nonexistence can be a property of objects), then there can be true statements of the form *there is x such that x does not exist*, which is impossible if quantifiers and their natural-language counterparts can only range over existing objects. Because it seems meaningful (and useful) to assert nonexistence of objects, it seems that (2) is also false. Again, this does not pose a problem for (4), although it clarifies our understanding of it by showing that we cannot restrict its scope to existing objects.

Finally, it is worth noting that even if these difficulties with each of (1)–(3) taken individually can be overcome, there are other difficulties with taking them jointly. For example, if (2) and (3) are taken jointly, it is not clear how "existing things" in (2) is to be understood. If existence is not a property of objects by (3), then "existing things" cannot be used to demarcate a domain of objects exhibiting a certain property. But if this is so, then our understanding of "existing things" must be based on our understanding of quantification, which is partly specified in (2). In other words, (2) is circular in this case. A similar problem prevents us from taking (1) and (2) jointly, insofar as (1) limits the scope of existence to first-order concepts.

Conclusion

Frege provides an insightful analysis of existential quantification in (4), but his commitment to (1) and (3) causes difficulties. It seems possible to meaningfully ascribe existence to objects (as opposed to, or in addition to, concepts), as illustrated by simple singular propositions. In addition, it seems possible to meaningfully ascribe nonexistence to objects, so our domain of quantification cannot be restricted to existing objects as in (2). Happily, rejecting (1)–(3) does not commit us to rejecting (4), and actually clarifies our understanding of it.

References

[1] Frege, G. *The Foundations of Arithmetic.* Trans. J. L. Austin. Northwestern, 1980.

[2] Frege, G. “On Concept and Object.” *Translations from the Philosophical Writings of Gottlob Frege.* Trans. Peter Geach and Max Black. Blackwell, 1960.

[3] Soames, S. *The Analytic Tradition, Volume I.* Forthcoming.

[4] Soames, S. “The symbolic language L_F” (course handout). USC, Fall 2011.