Omniscience and Cosmological Arguments

September 2008

Abstract

In this paper, I examine some set-theoretic implications of omniscience. I show that an omniscient being (if one can exist at all) must have an infinite collection of beliefs, and hence if an omniscient being exists, an infinite collection exists. I then examine the implications of this result for certain cosmological arguments for the existence of God—in particular Craig's *kalam*. I argue that the result presents a difficulty for any cosmological argument denying the possible existence of (actually) infinite collections. I also consider and respond to several potential objections to my argument.

Introduction

In several papers, Grim has presented Cantorian diagonal arguments suggesting (among other things) the impossibility of an omniscient being.[1] Recently, Spencer has continued along similar lines, arguing that the existence of an omniscient being is inconsistent with the axioms of Classical Extensional Mereology.[2]

In this paper, I do not pursue the possibility of omniscience, but instead examine some of its set-theoretic implications. I show that an omniscient being, if one can exist at all, must have an infinite collection of beliefs, and hence if an omniscient being exists, an infinite collection exists. I then examine the implications of this result for certain cosmological arguments for the existence of God—using as an example Craig's *kalam*. I argue that the result presents a difficulty for any cosmological argument denying the possible existence of (actually) infinite collections.

Language and Notation

My arguments below use some mathematical language and notation. I assume that the reader is familiar with basic set-theoretical concepts (*set*, *membership*, *subset*, *powerset*, *union*, *intersection*, *function*, etc.) and corresponding notation (X, $x \in X$, $X \subseteq Y$, $\mathscr{P}(X)$, $\bigcup$, $\bigcap$, $f : X \to Y$, etc.). I also assume familiarity with the notion of a (first-order) *property* and the corresponding notion of a *class*.

A subset $X \subseteq Y$ is *proper* iff $X \neq Y$.

A function $f : X \to Y$ is *injective* iff for all $w, x \in X$, $w \neq x$ implies $f(w) \neq f(x)$. It is *surjective* iff for all $y \in Y$ there exists $x \in X$ such that $f(x) = y$. It is *bijective* iff it is injective and surjective. The *range* of f refers to the set

$$f[X] = \{ y \in Y \mid (\exists x \in X)(y = f(x)) \}$$

The set $\omega = \{0, 1, 2, \ldots\}$ is the set of nonnegative integers (natural numbers).

A class is *proper* iff it does not form a set. If a class forms a set, we adopt the convention of identifying it with that set. In what follows, a *collection* is simply another term for a class.

Counting Beliefs

Grim presents[3] the following Cantorian argument that there is no set of all truths: Suppose T is the set of all truths and let $\mathscr{P}(T)$ be the powerset of T. Then by Cantor's Theorem, we know that $\mathscr{P}(T)$ must be strictly larger than T (that is, there does not exist an injection from $\mathscr{P}(T)$ into T). But now fix a truth $t_0 \in T$. For each $X \in \mathscr{P}(T)$, either $t_0 \in X$ or $t_0 \not\in X$—that is, either '$t_0 \in X$' is a truth, or '$t_0 \not\in X$' is a truth. Thus corresponding to each $X \in \mathscr{P}(T)$ there exists a distinct truth, which is just an element of T. But that means that T is at least as large as $\mathscr{P}(T)$ (that is, there is an injection from $\mathscr{P}(T)$ into T). This contradicts that $\mathscr{P}(T)$ must be strictly larger than T. Hence our assumption that T is a set must be false—that is, there is no set of all truths.

Grim claims that this gives us a 'short and sweet' Cantorian argument against omniscience; he notes, 'were there an omniscient being, what that being would know would constitute a set of all truths. But there can be no set of all truths and so can be no omniscient being.'[4]

This is an important and powerful argument. I repeat it here because, if Grim is correct, then clearly cosmological (as well as other) arguments for the existence of God are troubled from the start—if omniscience is impossible, then God (understood as an omniscient being) simply cannot exist. But Grim's argument raises many other issues, and significant objections have been raised against it.[5] Because many of these issues are beyond the scope of this paper, I do not address Grim's argument directly. Instead, in what follows, I will operate under the assumption that—contrary to Grim's conclusion—omniscience is possible. I will show that nevertheless omniscience poses a problem for certain cosmological arguments, because if an omniscient being does exist, then it must possess an infinite collection of beliefs.

Let us suppose then that an omniscient being does exist. Call it G. Let B_G denote the collection of G's beliefs.[6] Either B_G forms a set or it does not. If B_G does not form a set, then it is a proper class, and thus trivially it is an infinite collection (proper classes are too large to form sets, so they certainly cannot be finite). On the other hand, it remains to be considered whether B_G can be a finite set.

If B_G is a set, it can be shown to be infinite in several ways. A simple iterative argument is perhaps the most intuitive. Note that since G is omniscient, for each $b \in B_G$, G knows, and hence believes, that G believes b. Therefore we may define a function F on B_G by the following mapping:

$$b \mapsto G\text{'s belief that } G \text{ believes } b \qquad (b \in B_G)$$

Note F is injective, since if G's belief that G believes b is the same as G's belief that

G believes b', then $b = b'$. Now let b_0 be G's belief that G exists. By weak recursion on ω, there exists a mapping $H : \omega \to B_G$ satisfying[7]

$$H(0) = b_0$$
$$H(n+1) = F(H(n)) \qquad (n \in \omega)$$

Moreover, it is verified by induction on ω that H is injective since F is injective, and since b_0 is not in the range of F (G's belief that G exists is not a belief that G believes something). Since there cannot be an injection from ω into a finite set, it follows that B_G must be infinite.

While this argument is perfectly legitimate, it relies on set-theoretic machinery that I do not wish to introduce into the discussion. More specifically, it uses weak recursion on ω, which requires (among other things) the existence of the infinite set ω. Since some cosmological arguments call into question the possible existence of (actually) infinite collections, I would rather not have my argument depend so critically on an infinite set (even though the former sense of 'existence' is weaker than the latter sense). Instead, I will approach the problem from the other direction.

Note that the range of F is given by

$$F[B_G] = \{\, y \in B_G \mid (\exists x \in B_G)(y = \text{`}G \text{ believes } x\text{'}) \,\}$$

That is, the range consists of all of G's beliefs of the form 'G believes x' for some belief x of G. Now $F[B_G]$ is a proper subset of B_G since, as we noted above, G has beliefs which fall outside of the range of F. But then since F is injective (and trivially surjective on its range), *F is a bijective mapping from B_G onto a proper subset of B_G*. The result now follows immediately from a theorem of set theory:

Theorem (Pigeonhole Principle). *If X is a finite set and Y is a proper subset of X, then there does not exist a bijection $f : X \to Y$.*

Since we have exhibited a bijection from B_G onto a proper subset of B_G, it follows that B_G cannot be finite.

It is important to note that this argument does not require use of an infinite set. It requires only a definition of 'finite' under which the Pigeonhole Principle holds. Now it is true that many common developments of set theory define 'finite' indirectly by way of an infinite set. For example, it is often said that a set X is finite iff there exists some $n \in \omega$ and some bijection $f : n \to X$. And then the Pigeonhole Principle is proven by induction on ω. But this is not the only definition of 'finite' available.

The most straightforward definition for my argument is Dedekind's definition:

Definition (Dedekind). A set X is *Dedekind-finite* iff for all proper subsets Y of X, there does not exist a bijection $f : X \to Y$.

Thus the Pigeonhole Principle holds for Dedekind-finite sets by definition. But this may seem too slick of a definition for the purposes of my argument.

A slightly more complex definition due to Kuratowski defines the finite subsets of a given set in terms of a closure property:

Definition (Kuratowski). Fix a set A. For any $\mathscr{C} \subseteq \mathscr{P}(A)$, consider the operation

$$\mathscr{O}_A(\mathscr{C}) = \{X \cup \{a\} \mid X \in \mathscr{C} \text{ and } a \in A\}$$

So $\mathscr{O}_A(\mathscr{C})$ consists of all subsets of A obtainable by adjoining a single element $a \in A$ to some set $X \in \mathscr{C}$. Now define

$$\mathrm{FIN}_A = \bigcap \{\mathscr{C} \subseteq \mathscr{P}(A) \mid \emptyset \in \mathscr{C} \text{ and } \mathscr{O}_A(\mathscr{C}) \subseteq \mathscr{C}\}$$

So FIN_A is the smallest set of subsets of A containing the empty set and closed under the adjoining (to its elements) of single elements of A.

Then a set X is said to be *Kuratowski-finite* iff $X \in \mathrm{FIN}_X$.

While this definition is formally complex, it is intuitively very simple. The idea is that, given a set X, to obtain any finite subset of X we should be able to start with the empty set and repeatedly adjoin elements of X, one by one, until we arrive at the desired set. The set X is finite if we can carry out this procedure and eventually arrive at X itself. It is straightforward to prove that the Pigeonhole Principle holds for for Kuratowski-finite sets, so this definition will also work for the argument above.

The point here is not to get bogged down in set-theoretic definitions. It is simply to show that my second argument above can be presented without implicit reliance on a background infinite set. Either of the above two definitions of 'finite' may be used for the argument, neither of which make reference to an infinite set.[8]

Thus the above arguments show that if an omniscient being does exist, it must have an infinite collection of beliefs. I record this for future reference:

(1) If an omniscient being exists, then an infinite collection exists (namely the collection of beliefs of the omniscient being).

Implications for Cosmological Arguments

One consequence of the above result, assuming it is correct, is that any argument for the existence of God must accommodate, at least to some degree, the existence of infinite collections. This poses a particular challenge for some cosmological arguments which object on general grounds to the possibility of an infinite causal history for the universe. To illustrate, I examine as an example part of Craig's *kalam* cosmological argument.

Craig presents the following argument in support of the claim that the universe began to exist:[9]

(2) An actual infinite cannot exist.

(3) An infinite temporal regress of events is an actual infinite.

(4) Therefore, an infinite temporal regress of events cannot exist.

Here I am only interested in (2), the claim that an actual infinite cannot exist. For Craig, an *actual infinite* is understood as 'a determinate whole actually possessing an infinite number of members.'[10] This concept is meant to stand in contrast to that of a *potential infinite*, which is characterized as something admitting of indefinite extension.[11] By 'cannot exist', Craig means cannot exist *in reality*.[12] Note that Craig is not challenging the logical consistency or logical possibility of actual infinites, only the notion that they can *really* exist.[13]

At first glance, it seems like (1) poses a difficulty for (2) in the context of Craig's argument. Clearly the collection of beliefs of an omniscient being forms a determinate whole, and as I have shown it must actually possess an infinite number of members. Hence such a collection is actually infinite. But then if an actual infinite cannot (really) exist, the collection of beliefs of an omniscient being cannot (really) exist. And of course if this is the case, then an omniscient being cannot (really) exist either. In particular, God (understood as an omniscient being) cannot (really) exist, which is clearly not the result that Craig desires.[14]

But things might not be so stark for Craig. It becomes increasingly important to determine just which sorts of actually infinite collections Craig has in mind in (2)—or, more accurately, just which sorts of objects constitute the collections that Craig rejects in (2). Does (2) apply only to collections of physical objects, events, etc.? Or does it also apply to collections of abstract objects like numbers, properties, beliefs (if they are abstract objects), and so on?

Interestingly, Craig does not seem to restrict (2) to collections of physical objects (etc.), but includes collections of (at least some) abstract objects within its scope. This is evidenced, for example, by his implicit assumption that the truth of Platonism in set theory would pose a threat to (2):

> If it could be proved that the Platonist-realist view of the ontological status of

> mathematical entities is correct and that such a view could escape the logical antinomies implicit in naive set theory, then our argument that an actual infinite cannot exist in reality would stand opposed to Cantor's analysis.[15]

This assumption motivates his subsequent rejection of Platonism on the basis of antinomies from naive set theory.[16]

On the other hand, Craig does qualify (2) in a potentially critical way relative to (1) by identifying (or at least associating) 'real' existence with 'extra-mental' or 'mind-independent' existence.[17] It seems, in the context of Craig's argument, that this just means something like:

(5) For all x, x 'really' exists iff x does not exist only as part of the content of some thought in a mind or minds.

If something like (5) is the correct reading, then (1) is still problematic to (2) since an existing omniscient being's beliefs do exist as objects independently of thought.[18] But if this qualification is read in such a way that (2) does not apply to collections of mental objects (e.g. beliefs), then Craig could argue that the existence of an actually infinite collection of beliefs is permissible, while the existence of an actually infinite collection of non-mental things (e.g. causal events) is still impermissible.

Of course, this solution would be extremely bizarre and *ad hoc*. What sort of metaphysics would permit us to have actually infinite collections of mental objects in reality, but *not* actually infinite collections of non-mental objects (even abstract ones)? If we have good general reasons for believing that actually infinite collections of non-mental objects cannot exist, then it seems equally reasonable to think that actually infinite collections of mental objects cannot exist either. So this reading cannot be right.

It seems then that (1), if correct, does present a difficulty for Craig's argument after all. If an actually infinite collection of even abstract objects cannot exist in reality, then neither can God (understood as an omniscient being).

It is important to note that this difficulty is not specific to Craig's argument alone. Indeed, it arises for any cosmological argument which casts general doubts on the possible real existence of actually infinite collections (or equivalently infinite sequences, series, regresses, and so on). If such general considerations are accurate, then by (1) they show us that an omniscient being cannot exist in reality. So even if they inform us about the causal history of the universe, they rule out the existence of God.

Of course, this certainly does not mean that the difficulty arises for all forms of cosmological argument. For example, arguments using direct, empirical premises to support claims like (4) will not suffer from the above problem if they do not rule out the existence of infinite collections. This is even true for Craig's *kalam*, taken in its entirety, where additional empirical arguments are presented for the claim that the universe began to exist (for example, those based on Big Bang cosmology).[19] But the above results show that any successful cosmological argument must use premises narrower and more direct than (2). The focus must shift from general philosophical considerations about infinite collections to specific facts about the causal history of the universe.

Responses to Potential Objections

I now consider and respond to possible objections to my argument above.

One seemingly popular objection to the type of argument given above is that it is somehow fundamentally mistaken to understand or analyze an omniscient being's knowledge (and in particular God's knowledge) in set-theoretic terms. Specifically, an omniscient being's knowledge should not be seen as constituted by some collection of discrete beliefs, but instead as one unified, complete, all-encompassing totality.[20] As finite beings, we (attempt to) break an omniscient being's knowledge into units of individual beliefs, but this does not mean that it is actually constituted by beliefs. And if it is not constituted by beliefs, then the set-theoretic argument for (1) cannot even get off the ground.

This is a very important objection, and it raises many questions that are beyond the scope of this paper, so here I will only suggest possible ways around it. First, it might be responded that even if an omniscient being's knowledge does form one unified totality and is not *constituted* by individual beliefs, it might still be meaningful to talk about 'parts' or 'regions' of the being's unified knowledge corresponding to individual true propositions, provided that such parts can be demarcated into definite and distinct units. That this is possible is suggested by the intuition that an omniscient being's knowledge of particular pieces of information, however that is understood in terms of the unified totality, is nevertheless still reflected structurally somehow. And if it is meaningful to talk about such parts, then it seems legitimate to count them. The arguments above would show that the number of parts must be infinite in precisely the same manner.

Another possible response, which is similar to one given by Spencer,[21] is that even if an omniscient being's knowledge is unified in nature, for each fact x there must be something external which grounds or justifies the being's knowledge that x (we could identify this with the truth-maker for x). If this is so, then my argument above could be rephrased in terms of a collection of grounds or justifications (truth-makers) for an omniscient being's knowledge. The argument would show that this collection cannot be finite in precisely the same way.

A slightly different objection allows for an omniscient being to have infinitely many beliefs, but argues that these need not form a collection. And if the beliefs do not form a collection, then the argument for (1) does not work.

First, let me say that there is some merit to the objection that, if an omniscient being can exist at all, its beliefs might not form a *set*. But even if they do not form a set, it is still legitimate to talk about the *class* because it is legitimate to talk about the property of *being one of the omniscient being's beliefs* (classes in general being merely a manner of speaking about properties). However, even if for some reason it were not acceptable to talk about any sort of collection of the beliefs, this would not help things for Craig's argument. If there could be infinitely many beliefs which do not form a collection, then by the same token there could be an infinite temporal regress of events which do not form a collection. And this would not be good news for Craig. Either such an infinite counts as an actual infinite under (2) or it does not. If it does not, then Craig's argument fails because (3) is false. If it does, then the conflict between omniscience and (2) remains. So in either case, the argument is troubled, and the objection does not help to resolve this.

Another objection concerns the belief iteration function F used in the arguments above. Note that for an omniscient being G and a proposition P, it is necessarily true that: G believes P if and only if G believes 'G believes P'. This suggests, it is argued, that G's belief that P is actually *identical to* (that is, the same belief as) G's belief that G believes P. And if this is the case, then my argument does not show that G has infinitely many beliefs. This objection is also supported by a position which identifies propositions with sets of possible worlds. The propositions 'G believes P' and 'G believes that G believes P' will be true in precisely the same possible worlds. Hence, on this account, the propositions are actually the same. This suggests that the two beliefs are the same, which again poses a problem for my argument.

This objection is important, and also raises issues that are beyond the scope of this paper. But let me say that it seems at least intuitively wrong that G's belief that P should in general be the same as G's belief that G believes P, even if they are in the above sense equivalent. To take a simple example: this would mean that if G exists, G's belief that I exist is the same as G's belief that G believes that I exist. This seems to be false because the two beliefs are about different things—the first is about me, while the second is about G. We can also approach this from another direction: it seems pretty clear that my own belief that I exist is distinct from my belief that I believe that I exist—for example, I can imagine a time when I believed the former without believing the latter. So why would this change in the case that I become omniscient? It seems odd to say that my presently distinct beliefs would 'collapse' into an identical residue; it seems more reasonable to think they would simply remain distinct, along with the rest of my presently distinct (true) beliefs. So it seems at least intuitively false that omniscience demands the proposed identity.

Examples like these also seem to tell against the position that propositions can simply be identified with the set of possible worlds in which they are true. Indeed, such examples echo in epistemological terms one of the difficulties faced by the position: namely, that the set of possible worlds in which a proposition is true does not adequately individuate the proposition.[22] However, I cannot address all of the important issues that arise here within the scope of this paper. I will therefore only record my feeling that there are difficulties with this position as a general account of propositions.

Another objection to my argument is that an omniscient being's knowledge, while infinite, is not actually but only potentially infinite.[23] Even though we may be able to indefinitely continue the process of picking out distinct things that an omniscient being believes, this does not imply that such a being believes *actually* infinitely many distinct things. It just means that the collection of such a being's beliefs can be indefinitely extended, or in other words that the collection is *potentially* infinite. But if this is the case, then (1) does not pose a difficulty for (2)—and more generally omniscience does not present a difficulty for any cosmological argument rejecting actually infinite collections.

First, let me say that this objection might not be entirely off the mark if it were raised against my recursive argument. In that argument, I utilized recursive iteration of the 'believes that' function in order to construct an infinite subset of an omniscient being's beliefs. It might be argued that this procedure is only potentially infinite: given an omniscient being G and a belief b of G, I can indefinitely continue the process of picking out beliefs: e.g., G's belief that G believes b, G's belief that G believes that G believes b, and so on *ad infinitum*. But, it might be said, this does not imply that G believes actually infinitely many things.

However, this is not the entire story. My argument also used weak recursion on ω in order to obtain from this potentially infinite procedure an actually infinite set of beliefs. In a specific, technical sense, all forms of transfinite recursion in set theory can be seen as producing (guaranteeing the existence of) actually infinite collections (sets or classes) from potentially infinite procedures (functions or operations satisfying certain properties). So remarking that the procedure above is potentially infinite is not enough to ensure that it cannot be used to produce an actually infinite set, as it was in my argument.

On the other hand, as I noted earlier, this recursion does depend on the existence of an infinite set (namely ω), hence the objector might claim that this procedure is in some sense illegitimate, at least in a context where the possible (real) existence of actually infinite collections is in question. This is an interesting claim, but I do not wish to address it here. It is for this reason that I presented the pigeonhole argument above.

The current objection is not successful against the pigeonhole argument. The problem is that, in light of that argument, it does not work to say that the collection of all beliefs of an omniscient being is potentially but not actually infinite. First, it seems the only way to interpret this claim meaningfully is to say that the collection does not actually contain an infinite number of members, but can be indefinitely extended. Unfortunately neither of these claims is true. If the collection does not actually contain an infinite number of members, then it must qualify as a finite set by any of the above definitions.[24] But the pigeonhole argument above shows that it *cannot* be a finite set, so this cannot be right. And the collection cannot be indefinitely extended, at least by other beliefs of the being, since by hypothesis *all* of the being's beliefs are already in the collection.

More generally, it seems the only way it makes sense to say that a collection is potentially but not actually infinite—let us call this *strictly potentially infinite*—in the present context, is by using something like the following:

(6) Given a property $\varphi(x)$, a class X may be said to be *strictly potentially infinite* relative to φ iff (i) X forms a finite set and (ii) for all $x \in X$, $\varphi(x)$ is true, but (iii) it is (logically) possible to construct arbitrarily large finite sets Y such that $X \subseteq Y$ and for all $y \in Y$, $\varphi(y)$ is true.

Here 'finite' is to be defined using one of the definitions given above. Note that I use logical possibility in (iii) to allow for a set to be strictly potentially infinite in the actual world even if, owing to contingencies in the actual world, it is not actually possible to construct arbitrarily large extensions. For example, we should be able to say that the set of books in a library is strictly potentially infinite (since it is finite and we can add books indefinitely) even if there is an upper bound on the number of possible books in the universe. Or a stick should be strictly potentially infinitely divisible even if it is impossible to perform (or even nearly perform) a supertask in reality. And so on.

For our purposes, given an omniscient being G, we define $\varphi(x)$ to state 'G believes x', and then the claim that a collection X of G's beliefs is strictly potentially infinite can be read according to (6). But while this claim is true for, say, $X = \emptyset$, since we can construct arbitrarily large finite sets of G's beliefs starting with the empty set, this is not true for $X = B_G$, the collection of all of G's beliefs, since my argument above shows that B_G cannot itself be a finite set.

Hence the claim that the collection of an omniscient being's beliefs is potentially but not actually infinite does not seem right, at least under any obvious definition that works in the present context.

A different type of objection to (1) appeals to intuitions about logical possibility. First, note that it seems logically possible that there should only be finitely many true propositions. Next, note that it seems logically possible that there should only be finitely many true propositions *and* that some being should know these propositions. Such a being would of course be omniscient.[25] But if this is possible, then (1) seems to be false—omniscience does not necessarily require infinitely many beliefs. And if this is the case, then my arguments do not establish an inconsistency between omniscience and cosmological arguments.

While interesting, this objection suffers from a number of difficulties, one of them being that it begs the question against my set theoretic arguments. Indeed, it supposes that it is possible for an omniscient being to exist when there are only finitely many things for it to believe. But the set theory seems to suggest that this is not possible, and the objection does not address that at all.

More directly, however, the objection encounters the serious difficulty that *there cannot be only finitely many true propositions*. This can be seen by running through the ontological versions of my set theoretic arguments where 'truths' are substituted for 'beliefs'. Or if the objector is skeptical of my own arguments, we can make use of Grim's: if there are only finitely many true propositions, then they surely form a set. But Grim's Cantorian argument shows that there cannot be a set of all truths. Hence the suggestion that there are only finitely many true propositions leads us quickly into contradictions.[26] All in all, it seems like this objection is problematic.[27]

The final objection that I will consider, related to my analysis of cosmological arguments, is that even if Craig's own interpretation of (2) does not explicitly restrict it to collections of physical objects (etc.), (2) can (or should) be so restricted, in which case (1) poses no problem. More generally: cosmological arguments of the above sort need not (or should not) deny the possible existence of *all* actually infinite collections, but only those containing physical objects, events, and the like. If this done, then the existence of infinite sets of abstract objects, such as those of numbers, propositions, beliefs, etc. does not pose any problem.

In general, I am not opposed to this approach, although I do feel that it runs the risk of becoming *ad hoc*. If general reasons are provided to support the claim that actually infinite collections cannot exist—such as Craig's claims that their infinite cardinality would entail unacceptable 'absurdities',[28]—then it seems as if such reasons would tell equally against the existence of all types of actually infinite collections. For this approach to work, the reasons provided would have to be narrower in scope. They would need to outline features of the universe which make it impossible for specific types of actually infinite collections to exist. At that point, however, it might be easier to avoid this approach altogether and present more direct empirical arguments regarding the causal history of the universe.

Notes

[1]See, e.g., [3], [4], and [7].
[2]See [8]. Note that Spencer actually thinks that omniscience is possible, and hence that one or more of the axioms of Classical Extensional Mereology is false.
[3][4], p. 395.
[4][4], p. 395–6.
[5]See, e.g., [7].
[6]For the purposes of this argument, I am assuming that the omniscient being has beliefs. I consider an objection to this assumption later on.
[7]I do not cover the details of recursion formally here, but note that it just guarantees the existence of the function H satisfying $H(0) = b_0$, $H(1) = F(b_0)$, $H(2) = F(F(b_0))$, etc.
[8]It is important to note that all of these definitions are equivalent in ZFC. The advantage of the latter two definitions for my purposes, however, are that they do not rely on the Axiom of Infinity for their statement.
[9][1], p. 9.
[10][1], p. 9.
[11][1], p. 5.
[12][1], p. 9.
[13]See [1], p. 12 and [2], p. 95, and other passages.
[14]Recall given Craig's usage of 'cannot exist', these statements do not concern the logical possibility of an omniscient being.
[15][1], p. 21.
[16][1], p. 18–21.
[17][1], p. 9.
[18]Recall I am here assuming that the analysis implicit in (1) is correct, which includes the assumption that an omniscient being has beliefs which can be treated as distinct objects. In the next section I consider objections to my argument for (1).
[19]See [1] and [2].
[20]Craig endorses such a view in [2], p. 94, citing in particular William Alston, 'Does God Have Beliefs?' *Religious Studies* 22 (1986).
[21]See [8], p. 67. Note however that Spencer only suggests this in response to the objection that God's knowledge is dispositional in nature, or that God is infallible.
[22]As an example, on this account, all necessary propositions are identical.
[23]This objection actually runs in parallel to the first.
[24]This claim seems to be consistent with at least Craig's understanding of the potential infinite; he notes that 'technically speaking, then, the potential infinite at any particular point is always finite' in [1], p. 5. Of course this statement also introduces a notion of temporality which would be inappropriate in the present context.
[25]The being may not be necessarily or even essentially omniscient, but that is fine for the purposes of this objection.
[26]Of course nothing about Grim's argument is specific to a finite set as opposed to an infinite set. But if we insist that there are only finitely many true propositions, then it becomes very hard to escape Grim's argument, as it is much harder to pursue even the objection that perhaps the truths do not form a set.
[27]It might be noted here that the ontological versions of my arguments would seem to provide a more direct attack against Craig's argument. Indeed, I think this is so. But the versions provided which hinge directly on omniscience also illustrate something important—namely, while God is not himself a mathematical infinite, at least some of his attributes seem to involve mathematical infinites, so it is troublesome to sweepingly reject such entities while arguing for the existence of God.
[28]See [1] and [2].

References

[1] Craig, William Lane. 'The Finitude of the Past and the Existence of God.' *Theism, Atheism, and Big Bang Cosmology.* Eds. William Lane Craig and Quentin Smith. New York: Oxford, 1993. First pub-

lished in *The Kalam Cosmological Argument*, Library of Philosophy and Religion. London: Macmillan, 1979.

[2] Craig, William Lane. 'Time and Infinity.' *Theism, Atheism, and Big Bang Cosmology.* First published in *International Philosophical Quarterly* 31/4 (1991).

[3] Grim, Patrick. 'Logic and the Limits of Knowledge and Truth.' *The Impossibility of God.* Eds. Michael Martin and Ricki Monnier. Amherst, New York: Prometheus, 2003. First published in *Noûs* 22 (1988).

[4] Grim, Patrick. 'The Being That Knew Too Much.' *The Impossibility of God.* First published in *International Journal for Philosophy of Religion* 47 (2000).

[5] Hajnal, András and Peter Hamburger. *Set Theory.* Cambridge, 1999.

[6] Harrington, Leo. *Introduction to Set Theory* (course notes by John Peloquin). UC Berkeley, Spring 2007.

[7] Plantinga, Alvin and Patrick Grim. 'Truth, Omniscience, and Cantorian Arguments: An Exchange'. *Philosophical Studies* 71 (1993).

[8] Spencer, Joshua T. 'Two Mereological Arguments Against the Possibility of an Omniscient Being.' *Philo* 9.1 (2006).