

Errata from *Linear Algebra and Multilinear Algebra*

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Introduction

This document contains errata from [1] and [2]. Locations within each text are indicated by coordinates (p, n) , where p is a page number and n is a line number on page p . Positive line numbers count from the top of the page, whereas negative line numbers count from the bottom of the page. Displayed equations, diagrams, and figures are counted as single lines.

Linear Algebra

Errata are currently only listed for Chapters 0–V, VI (except § 3), VII, VIII (except § 7), XI (except § 4–5), XII (except § 4), and XIII.

Minor, purely typographical inconsistencies like those between “ \rightarrow ” and “ \mapsto ”, or between “ $1 \dots n$ ” and “ $1, \dots, n$ ”, or between “family x_α ” and “family (x_α) ” and “family $\{x_\alpha\}$ ”, or between “ $f \mid g$ ” and “ f/g ”, are not listed below but occur frequently.

Chapter 0

- (2, 8): in the definition of subgroup, “subset H ” should be “nonempty subset H ”.
- (3, -12): a homomorphism between fields must also preserve the multiplicative identity.
- (3, -13): a subfield must also contain the multiplicative identity.

Chapter I

- (9, 16): in the definition of linear dependence, “non-trivial linear combination of the x_α ” should be “non-trivial linear combination of the x_α equal to zero”.
- (12, 1): $\lambda^i = 0$ should be $\lambda^i \neq 0$.
- (12, 5): throughout the proof of (ii), n should be m .
- (12, -5): “The a partial order” should be “A partial order”.
- (12, -4): “chain” should be “nonempty chain”, and it should be noted that R is an upper bound for the empty chain.
- (12, -4): “maximal element” should be “upper bound”.
- (13, 8): $x \in E$ should be $x \in S - T$.
- (13, 9), (13, 10): $T \cup x$ should be $T \cup \{x\}$ and $x \cup T$ should be $\{x\} \cup T$.

- (14, 1): “element f_a ” should be “elements f_a ”.
- (14, 3), (14, 5): in the displayed equations, $j = 1$ should be $i = 1$.
- (15, -2): in problem 10, $\{x_\alpha\}_{\alpha \neq \beta}$ should be $\{a\} \cup \{x_\alpha\}_{\alpha \neq \beta}$.
- (20, 16): “ $\varphi(S)$ is a system of generators for $\varphi(S)$ ” should be “ $\varphi(S)$ is a system of generators for F ”.
- (22, -15), (22, -11): in part (v) of Problem 5, the concepts of generated subspace and kernel have not yet been defined.
- (23, 7): in the definition of subspace, “subset of E ” should be “nonempty subset of E ”.
- (27, -11): in the displayed equation, λ_i should be λ^i (two occurrences). Also, it should be noted that $y_i \in E_1$ and $z_i \in E_2$.
- (29, -16): “*canonical projection* of E onto E_1 ” should be “*canonical projection* of E onto E/E_1 ”.
- (31, 14): in problem 7, “composition” should be “decomposition”.
- (31, 19): in problem 8, the period at the end of the second sentence should be a question mark.
- (40, -8): in the second displayed equation in problem 6, $u \in E'_2$ should be $y \in E'_2$.

Chapter II

- (47, -6): “assume that there” should be “assume that”.
- (52, -8): $\psi : E_1 \leftarrow F$ should be $\psi : E \leftarrow F$.
- (52, -3): “left inverse” should be “left inverse ψ ”.
- (53, 13): “inverse” should be “left inverse”.
- (53, -1): in problem 1, the inclusion $L(E; F) \subset C(E; F)$ is wrong.

- (59, -5): in (2.25), it should be noted that $\delta_{\rho\sigma}$ is a Kronecker delta.¹
- (62, 10): in (2.34), y should be y_j .
- (63, -7): in problem 7, “second set” should be “disjoint set”.
- (67, -1): in the displayed equation, $y^* \in F$ should be $y^* \in F^*$.
- (68, -7): in the displayed equation, $\langle y^*(\varphi + \psi)x \rangle$ should be $\langle y^*, (\varphi + \psi)x \rangle$.
- (70, 14): “bilinear functions” should be “bilinear function”.
- (75, -2): in problem 10, the induced mapping should be

$$\overline{\varphi}^* : E^* / E_1^\perp \leftarrow F^* / F_1^\perp$$

- (76, 9): “imension” should be “dimension”.
- (76, -7): in the displayed equation (and really the rest of subsection 2.31), φ_v^μ should be φ_μ^v .
- (77, -7): φ should be Φ .

Chapter III

- (83, -10): in the displayed equation, $(b_1^\mu \dots b_n^\mu)$ should be $(\alpha_1^\mu \dots \alpha_n^\mu)$.
- (84, -7): “columns of the matrix α_v^μ ” should be “columns of the matrix $\alpha_v^{*\mu}$ ”.
- (84, -6): $y = y^{*\mu}$ should be $y^* = y^{*\mu}$.
- (85, 13): in the displayed equation, it should be noted that $A = (\alpha_v^\mu)$.
- (88, 1): in the main theorem, “system of n equations in m unknowns” should be “system of m equations in n unknowns”.
- (89, 12): it should be noted that $\dim E = n$ and $\dim F = m$.
- (89, -8): in the displayed equation, a_v^μ should be α_v^μ .
- (91, 5): “ E automorphism of E ” should be “automorphism of E ”.

¹This notation is defined later in a footnote on p. 76.

- (91, 9): in the displayed equation, $M(\varphi)^{-1}$ should be $M(\varphi^{-1})$.
- (91, -3): in problem 3, “be linear transformation” should be “be a linear transformation”.
- (93, -13): “inverse of the matrix of the transformation $x_v \rightarrow \bar{x}_v$ ” should be “transpose of the inverse of the matrix of the transformation $x_v \rightarrow \bar{x}_v$ ”.
- (95, 12): $\mu = 1, \dots, n$ should be $\mu = 1, \dots, m$.
- (96, 1): multiplication of basis vectors by nonzero scalars should be added to the list of elementary basis transformations.
- (96, -7): $2 \leq v \leq m$ should be $2 \leq v \leq n$.
- (98, 3): (3.36) should be

$$\xi^r = (\kappa_r^r)^{-1} \left(\omega^r - \sum_{v=r+1}^n \kappa_v^r \xi^v \right)$$

Chapter IV

- (103, 15): in (4.6), it should be noted that \hat{x}_j indicates that the vector x_j is deleted.²
- (105, -8): “Proposition II” should be “Proposition III”.
- (106, 5): “(4.14)” should be “(4.12)”.
- (107, 13): in (4.14), $p = \dim E$ should be $p = \dim E_1$.
- (109, 1): the problem numbers on this page should be incremented by 1.
- (109, 10): in problem 6, it should be assumed that E is real.
- (109, -11): in problem 8, the trace of a linear transformation has not yet been defined.
- (113, 1): “(4.21)” should be “(4.22)”.
- (113, -10): in the displayed equation, x^*_i should be x^{*i} .

²This notation is defined later in a footnote on p. 198.

- (115, 1): “(4.14)” should be “(4.12)”.
- (115, 4): in the displayed equation, $\widehat{\varphi a_j}$ should be $\widehat{\varphi a_i}$.
- (115, 9): it should be noted that $M(\varphi) = (\alpha_v^\mu)$.
- (115, -10): the displayed equation should be $\beta_j^i = \det C_i^j$.
- (116, 11): “(4.36)” should be “(4.38)”.
- (116, -1): in the displayed matrix B_i^j , the first column should have entries $1, \alpha_1^j, \dots, \alpha_{i-1}^j, \alpha_{i+1}^j, \dots, \alpha_n^j$.
- (117, 3): “(4.38)” should be “(4.16)”, or a reference to Problem 6.
- (117, 7): “(4.35) and (4.30)” should be “(4.38) and (4.40)”.
- (117, -14): “minor” should be “submatrix”.
- (118, 4): in problem 1, in the Laplace expansion, $v_1 < \dots < v_n$ should be $v_1 < \dots < v_p$, and it should be noted that $v_{p+1} < \dots < v_n$ is the complementary ordered $(n - p)$ -tuple.
- (122, 2): on the third line of the displayed equation, σ should be ρ .
- (130, -14): in problem 11, it should be assumed that E is real.
- (130, -10): in problem 12, $\varphi : E_1 \rightarrow E_1$ should be $\varphi_1 : E_1 \rightarrow E_1$.
- (132, 13): n should be E .
- (132, 14): “orientated” should be “oriented”.
- (135, -3): “4.30” should be “4.29”.
- (136, -6): “4.17” should be “4.16”, and it should be noted that ξ_v^i are the components of x_v with respect to some basis.
- (139, -13): $(a'_1, b_2 \dots b_n)$ should be $(a'_1, b_3 \dots b_n)$.
- (139, -10): in the displayed equation, $i = 1 \dots n$ should be $i = 1 \dots n - 1$.
- (142, -11): in problem 5, “isotopices” should be “isotopies”.

Chapter V

- (146, 1): $(\varphi, \psi) \rightarrow \psi\varphi$ should be $(\psi, \varphi) \rightarrow \psi\varphi$.
- (148, 10): the displayed list should also include elements of the form asb for $s \in S$ and $a, b \in A$.
- (148, -17): it must be assumed that $A \neq 0$ (equivalently $e \neq 0$) for it to follow that $\lambda = 0$.
- (151, 2): the extra “can be” should be deleted.
- (158, 14): “cheeked” should be “checked”.
- (160, -13): “let be” should be “be”.
- (160, -8): it must be assumed that $E \neq 0$ for $A(E; E)^2 \neq 0$.
- (161, -1): “non-zero, I , ideal” should be “non-zero ideal I ,”.
- (165, 12), (166, 7): it should be clarified that Γ is assumed to be a field under the restrictions of the operations in $A_\Delta(E; E)$.

Chapter VI

- (168, -16): it should be noted that the zero map is homogeneous of every degree (hence *the* degree is not well-defined in that case).
- (172, -4): “product” should be “products”.
- (173, -3): in problem 6, $\deg \varphi^* = -k$.
- (175, 4): the displayed statement should be $xe_k \in A_{l+k}$.
- (176, 1): E should be A .
- (177, 2): in the second part of problem 1, it must be assumed that $A \neq 0$ to conclude that $k = 0$.
- (177, 14): in problem 5, “ ≤ 0 (≥ 0)” should be “ ≥ 0 (≤ 0)”.

Chapter VII

- (189, -12): in the displayed equation, $\lambda > 0$ should be $\lambda \geq 0$.
- (192, 5): $x = x_\mu$ should be $y = x_\mu$.
- (192, 14): “basisvectors” should be “basis vectors”.
- (193, 14): “orthogonal bases” should be “orthonormal bases”.
- (193, -8): (α_φ^μ) should be (α_ν^μ) .
- (195, 4): in problem 3, it should be assumed that E is finite-dimensional or else established that orthogonal projection still works as long as E_1 is finite-dimensional.
- (204, 13): in the displayed equation, $\sum_\nu g_{\nu\lambda} \xi^\lambda$ should be $\sum_\nu g_{\nu\lambda} \xi^\nu$.
- (206, 14): “least upper bound” should be “least nonnegative upper bound” to account for the case $E = 0$ where there are no unit vectors.
- (207, 7): “natural topology” should be “natural topology”.
- (210, 15): throughout subsection 7.24, it should be assumed that $A \neq 0$.

Chapter VIII

- (217, 13): the concept of a metric tensor has not yet been defined.
- (217, 14): in the displayed equation, \sum_ν should be \sum_λ .
- (220, -1): in problem 1, equality holds only if $\psi = \lambda\varphi$ or $\varphi = \lambda\psi$.
- (223, -2): in (8.22), $\lambda - \lambda_i$ should be $\lambda_i - \lambda$.
- (226, 17): in problem 11, the concept of a rotation has not yet been defined.
- (227, 14): $E_1 \in E_2^\perp$ should be $E_1 \subset E_2^\perp$.

- (231, 7): the proof of the normal form (8.35) is incorrect because it is not true in general that the a_n defined form an orthonormal basis of the space.
- (231, -5): “Proposition II” should be “Proposition III”.
- (234, 4): in the displayed equation, a_μ^κ should be α_μ^κ .
- (234, -16): “*rotation*” should be “*rotation* φ ”.
- (234, -10): it must be assumed that $e \neq 0$.
- (234, -9): “sec. 4, 17” should be “sec. 4.17”.
- (238, -7): in (8.40), the second equation should be $\sin \theta = -\frac{1}{2} \operatorname{tr}(j \circ \varphi)$.
- (239, 17): in the displayed equation, “ \equiv ” should be “=”.
- (240, -13): “see 8.21” should be “sec. 8.21”.
- (241, 6): in (8.49), x should be u .
- (242, -13): in the displayed equation, “ \equiv ” should be “=”.
- (243, 7): E should be e .
- (243, -5): “see 8.21” should be “sec. 8.23”.
- (243, -4): in the displayed equation, (z, τ, z) should be $(z, \tau z)$.
- (243, -2): it should be noted that Δ is the positive normed determinant function in E_1 .
- (244, -11): F_1 should be F .
- (245, 8): in the displayed equation, b should be b_1 .
- (245, -10), (245, -5): “(8.53)” should be “(8.54)”.
- (246, 10), (246, -10): in problems 3 and 5, “plane” should be “plane E ”.
- (247, -1): f should be defined so that $f(0) = 0$.
- (248, -8): in problem 16, E should be \mathbb{H} .

Chapter XI

- (328, 14), (328, -7): the proof of the equality condition for the triangle inequality does not properly account for the case $x = 0$; in particular, “ $y = \lambda x$ ” should be “ $y = \lambda x$ or $x = \lambda y$ ”.
- (332, 11): in problem 3, the displayed equation should be

$$(z_1, z_2) = ((x_1, x_2) + (y_1, y_2)) + i((x_2, y_1) - (x_1, y_2))$$

and it should be noted that $z_k = x_k + i y_k$.

- (334, -7): “automorphism” should be “involution”.
- (335, -12): in the displayed equation, Φ should be φ .
- (335, -5): the concept of a conjugate determinant function is not defined.
- (335, -3): in the displayed equation, \bar{x}_p should be \bar{x}_n .
- (336, -8): in the displayed equation, $\sum_{\mu, \nu}$ should be $\sum_{\mu, \lambda}$.
- (337, -16): “normal mapping” should be “normal mappings”.
- (337, -10): in the displayed equation, λ should be λ_1 .
- (339, 8): it must be assumed that $e \neq 0$.
- (339, 14): in problem 1, “bilinear” should be “binary”.

Chapter XII

- (352, -11): the degree(s) of the zero polynomial should be clarified.
- (353, -8): $A \in \Gamma[t]$ should be $A = \Gamma[t]$.
- (356, 3): in problem 1(d), it is false that $f_1(g) = f_2(g)$ implies $f_1 = f_2$.
- (356, 9): in problem 2, it is false that if f_1 and f_2 are two linear mappings on $\Gamma[t]$ with $d \circ f_1 = \iota = d \circ f_2$ then there is a fixed $\alpha \in \Gamma$ with $(f_1 - f_2)f = \alpha$ for all $f \in \Gamma[t]$.

- (358, 6): “second polynomial” should be “second monic polynomial”.
- (358, 10): in subsection 12.6, greatest common divisor and relative primality should be defined for arbitrary nonzero polynomials, and the rest of the chapter updated to reflect this.
- (358, -13): irreducible polynomials should be required to be nonconstant, and the rest of the chapter updated to reflect this.
- (358, -12): “ar” should be “or”.
- (358, -4): in subsection 12.7, least common multiple should be defined for arbitrary nonzero polynomials and the rest of the chapter updated to reflect this.
- (360, -5): $g \leq 0$ should be $0 \leq g$.
- (361, 3): it should be clarified that $k_i \geq 1$.
- (363, 8): in the proof of Proposition IV, it must be assumed (without loss of generality) that f and g are monic.
- (363, -4): “common factor” should be “nonconstant common factor”.
- (364, 10): in the displayed equation, n should be r .
- (366, -3): “commutative” should be deleted.
- (367, 14), (367, 16): in Examples I and II, it must be assumed that $A \neq 0$.
- (367, -13): in the displayed equation, $\Phi(f)$ and $\pi(f)$ should be swapped.
- (367, -3): “Proposition V” should be “Proposition I”.
- (369, 8), (369, 13): the Q’s should all be \mathbb{Q} ’s.

Chapter XIII

- (387, 5): “Proposition II” should be “Proposition I”.
- (387, 7): the displayed equation should be

$$K(f) = K(1) = 0$$

- (387, 15): “non-zero” should be deleted.
- (388, -11): φ_0 should be φ .
- (389, 4): in problem 2, “Given” should be “Give”.
- (389, -15): in problem 6, “minimal polynomial” should be “minimum polynomial”.
- (389, -11): in problem 6, in the displayed equation, it should be clarified that the λ_v are not all zero.
- (390, 8): in problem 8, in the displayed equation, α_v should be α_ν .
- (390, -13): in problem 13, $L(E, E)$ should be $L(E; E)$.
- (393, -7): “vektor” should be “vector”.
- (395, 14): in problem 1, “stable subspace” should be “non-zero stable subspace”.
- (396, -17): in problem 7, “unique” should be deleted.
- (397, -1): “ f irreducible” should be “ f monic irreducible”.
- (398, 4): $a \in K(f)$ should be $a \in K(h)$.
- (398, 7): “Thus $K(g) = K_\mu$. Now it follows from (13.28) that $a \in K_\mu$.” should be “Thus $g = \mu$. Now it follows that $h \in I_\mu$.”
- (398, 10): it should be clarified that this is the prime decomposition of μ (so the f_i are distinct monic and $k_i \geq 1$).
- (400, -3): “Proposition III” should be “Proposition II”.
- (402, 12): it should be clarified that $m = \deg \mu$. Also, for this reason, μ should not be used as the index variable!
- (402, 13): $1 \dots m - 1$ should be $0 \dots m - 1$.
- (402, -13): π^* should be π .
- (403, 11): in the displayed equation, F_i should be F_j .

- (403, 16): in the displayed equation, F_r should be F_j , and it should be clarified that $\dim F_j > 0$.
- (403, -15): $j = 1$ should be $r = 1$.
- (403, -12): “ φ is cyclic” should be “ E is cyclic”.
- (403, -8): in part (i) of Theorem I, it should be clarified that this is the prime decomposition of μ (so f is monic and $k \geq 1$).
- (404, 10): $\mu \mid f^{k_i}$ should be $\mu \mid f^{k_1}$.
- (404, 13): “13.10” should be “13.11”.
- (404, -1): “ f is an irreducible” should be “ f is a monic irreducible”.
- (406, 3): “irreducible” should be deleted.
- (406, 5): “13.12” should be “13.13”.
- (407, 5): in the matrix, there should be a 0 in the upper right corner.
- (407, -16): “13.12” should be “13.13”.
- (407, -11): it should be clarified that this is the prime decomposition of μ (so f is monic and $k \geq 1$).
- (407, -10): “irreducible subspaces” should be “non-zero irreducible subspaces”.
- (407, -8): “13.12” should be “13.14”.
- (410, -10): ψ should be ψ_i^j .
- (412, -10): it should be clarified that this is the prime decomposition of f (so the f_i are distinct monic and $m_i \geq 1$).
- (413, 9): in problem 1, “ $\varphi \in A(E; E)$ ” should be moved from (i) to (ii).
- (414, 3): in problem 6(a), “ φ_j is cyclic” should be “ F_j is cyclic”.
- (414, 18): in problem 8(a), $K(f^{k-1})$ should be $K(f^k)$.
- (416, -9): it should be clarified that $j_1 \neq j_2$.

- (417, 5): the first displayed equation should be

$$f_1^{l_2}(\varphi)f_1^{l_1-l_2}(\varphi)y_1 = 0$$

- (418, 5): τ need not be a surjection.
- (419, 5): $f \mid \mu$ should be $\mu \mid f$.
- (420, 13): F_0 should be F .
- (422, 5): “subalgebra of (cf. example I, see 5.2)” should be “subalgebra of $A(E; E)$ (cf. example I, sec. 5.2)”.
- (422, -2): φ should be φ_i .
- (423, 5): E_j should be F_j .
- (425, -6): “13.14” should be “13.15”.
- (427, 15): “12.17” should be “12.16”.
- (427, 16): “decompositions” should be “decomposition”.
- (427, -7): “12.14” should be “12.13”.
- (428, 2): it should be clarified that this is the prime decomposition of μ (so the f_i are monic).
- (428, -14): the displayed equation should be

$$E = F_1 \oplus \cdots \oplus F_s$$

- (429, 3): g should be q .
- (429, 5): “over $\Delta[t]$ ” should be “in $\Delta[t]$ ”.
- (429, -9), (429, -8): in Theorem I, it should be clarified that the prime decomposition of the minimum polynomial of φ is $f_1^{k_1} \cdots f_r^{k_r}$.
- (429, -3): $\psi_N = \psi_N$ should be $\psi_N = \varphi_N$.
- (430, 12): “see. 13.15” should be “sec. 13.14”.

- (431, 4): “polynomial” should be “polynomials”.
- (432, 3): $\varphi_i : E \rightarrow E$ should be $\varphi_i : E_i \rightarrow E_i$.
- (432, 5): E should be E_i .
- (432, 7): in the displayed equation, E should be E_i (twice).
- (432, -1): it should be clarified that this is the prime decomposition of μ_1 (so the f_i are monic).
- (437, -11), (439, 8): in Theorems I and II, the definition of *homothetic* must be loosened slightly to allow for possibly negative scalars.

Multilinear Algebra

Errata are currently only listed for Chapters 1, 2 (except § 10–17), and 3–6. Minor purely typographical inconsistencies are not listed.

Chapter 1

- (4, 11): in problem 3, it should be assumed that $E \neq 0$.
- (4, -7): in problem 5(b), the claim is false.
- (7, -5): in the displayed equation, $\lambda_\alpha^\vee \in \Gamma$ should be $\lambda_\vee^\alpha \in \Gamma$.
- (11, -10): “satisfis” should be “satisfies”.
- (11, -3): The function name f should not be reused.
- (12, 9): in problem 1,

$$(\xi^1, \dots, \xi^n) \times (\eta^1, \dots, \eta^m)$$

should be

$$((\xi^1, \dots, \xi^n), (\eta^1, \dots, \eta^m))$$

- (12, -6): in problem 6(a), the tensor product of linear maps has not yet been defined.
- (13, 15): “ φ_1 factors over \otimes ” should be “ φ_1 factors over \otimes' ”.
- (14, 6): in the displayed equation, $y \in F_1$ should be $y_1 \in F_1$.
- (14, -5): $\tilde{E} = \otimes_\alpha E_\alpha$ should be $\tilde{E} = \oplus_\alpha E_\alpha$ and $\tilde{F} = \otimes_\beta F_\beta$ should be $\tilde{F} = \oplus_\beta F_\beta$.
- (14, -1): $\pi_\alpha : E \rightarrow E_\alpha$ should be $\pi_\alpha : \tilde{E} \rightarrow E_\alpha$.
- (17, 11): in the displayed equation, it should be noted that $x_\alpha \in E_\alpha$ and $y_\beta \in F_\beta$.
- (22, -9): in Corollary III, $L(E \otimes F; E' \otimes F)$ should be $L(E \otimes F; E' \otimes F')$.

- (22, -8): in Corollary III, the claim is not shown in subsection 1.27, where it is also assumed that E' and F' are finite-dimensional.
- (24, 5): in the displayed equation, $\tilde{\psi} \otimes \psi$ should be $\tilde{\psi} \circ \psi$.
- (25, 9): in problem 1(a), the “only if” part of the claim is false.
- (30, 13), (31, 11): it is false that $\Phi \otimes \Psi$ is nondegenerate only if Φ and Ψ are nondegenerate.
- (34, -2): it should be assumed that $x^* \neq 0$.
- (36, 12): $\varphi \times \psi$ should be (φ, ψ) .
- (36, 14): in the displayed equation, $\dim(F; F')$ should be $\dim L(F; F')$.
- (37, 3): in the commutative diagram, \otimes should be the linear map induced by \otimes .
- (37, 16): $(\varphi \times \psi)$ should be (φ, ψ) .
- (37, -3): (α, β) should be $\alpha \otimes \beta$.
- (38, 3): $F : A \times A \rightarrow L(A; A)$ should be $F : A \otimes A \rightarrow L(A; A)$.
- (38, 8): $L(A \otimes A)$ should be $A \otimes A$.
- (38, -5): in subsection 1.30, it should be assumed that $E \neq 0$.
- (39, 3): Ω should be the bilinear map induced by Ω .
- (40, 5): in problem 2, it should be assumed that $E, F \neq 0$.
- (40, -5): in problem 3, it should be assumed that E is oriented.

Chapter 2

- (42, 18): if $A \neq 0$, it must be assumed that $B \neq 0$ for φ to be injective.
- (43, -1): in (2.12), $\Omega_{A \otimes B}$ should be $\Omega_{A \otimes A}$.

- (48, 14): in Proposition 2.9.1, it should be clarified that Ω_A is an antiderivation with respect to ω_A , and similarly Ω_B with respect to the canonical involution ω_B , and $\Omega_{A \widehat{\otimes} B}$ with respect to $\omega_{A \otimes B}$ (which might as well be written $\omega_{A \widehat{\otimes} B}$).
- (48, -9): $\Omega_{A \otimes B}$ should be $\Omega_{A \widehat{\otimes} B}$.
- (49, -6): in the first equality of the displayed equation, $(-1)^{p+q'}$ should be $(-1)^{p+p'}$.
- (49, -2): in the displayed equation, Ω should be θ .

Chapter 3

- (60, -3): it should be noted that $v \in \bigotimes^q E$.
- (61, 3): $\dim E = 1$ should be $\dim E \leq 1$.
- (62, 4): in the displayed equation, it should be noted that $u_p \in \bigotimes^p E$ and $v_q \in \bigotimes^q E$.
- (63, -2): in **T**, “unit element” should be “unit element e ”.
- (64, -7): in the uniqueness theorem, proof, and remarks after, it should be noted that the homomorphisms preserve the unit.
- (65, 1): in the proof of the uniqueness theorem, the unit element is also needed to generate U .
- (68, 8): in problem 2, $\lambda_1, \dots, \lambda_p = 1$ should be $\lambda_1 \cdots \lambda_p = 1$.
- (68, 9): in problem 3, it should be asked to prove that $\text{tr}(\varphi\psi) = \text{tr}(\psi\varphi)$.
- (68, -8): in the displayed equation, it should be noted that $u^{*p} \in \bigotimes^p E^*$ and $v_p \in \bigotimes^p E$.
- (69, -10): the reference to (3.8) should be deleted.
- (70, -2): (3.14) is not used to establish the displayed equation.
- (72, 6): “noncommutative” should be “noncommutative in general”.

- (72, -5): $\Phi_j^i(\omega)$ should be $C_j^i(\omega)$.
- (74, 1): in many problems in this subsection, the order of dual spaces should be swapped to be consistent with the rest of the chapter; for example, $\otimes(E, E^*)$ should be $\otimes(E^*, E)$.
- (74, -13): in problem 6, the result does not hold for arbitrary fields Γ .
- (74, -9): in problem 7, it should be assumed that E is finite-dimensional.
- (74, -5): in problem 8, it should be assumed that E is finite-dimensional.
- (75, 10): in problem 11(a), the result does not hold for arbitrary fields Γ .
- (77, 9): in several problems in this subsection, it could be made clearer that E is assumed to be finite-dimensional.
- (77, 15): in problem 2(a), $g \otimes g^*$ should be $g^* \otimes g$ to be consistent with notational conventions in the rest of the book.
- (77, -3): in problem 5, the scalars $g_{\nu\mu} = (e_\nu, e_\mu)$ should be defined.
- (78, -6): “noncommutative” should be “noncommutative in general”.
- (79, 3): it should be noted that $\Phi \in T^p(E)$.
- (79, 6): it should be noted that $\Phi \in T^p(E)$ and clarified how $i_\nu(h)$ acts if $\nu > p$.
- (79, -8), (79, -6): the definitions of the operators $i_A(h)$ and $i_S(h)$ given in this subsection are inappropriate for use with the algebras of skew-symmetric and symmetric multilinear functions, respectively. See below.
- (80, 11): in the displayed equation, $E^{*\nu}$ should be E^* .
- (82, 6): in the displayed equation, $\langle f_1 \cdots f_p \Psi \rangle$ should be $\langle f_1 \cdots f_p, \Psi \rangle$.
- (82, -10): in the displayed equation,

$$\Psi(x_1, \dots, x_r, x_1^*, \dots, x_s^*)$$

should be

$$\Psi(x_{p+1}, \dots, x_{p+r}, x_{q+1}^*, \dots, x_{q+s}^*)$$

- (82, -5): “Section 1.20” should be “Section 1.27”.
- (83, -1): y^{*j} should be y_j^* .

Chapter 4

- (85, 1): it should be assumed that $i < j$.
- (87, 12): in the displayed equation, $u = x^{*1} \otimes \cdots \otimes x^{*p}$ should be $u^* = x^{*1} \otimes \cdots \otimes x^{*p}$.
- (90, 6): “restriction of π_X ” should be “restriction of π_A ”.
- (91, 2): in the displayed equation, $X(E)$ should be $X^p(E)$.
- (93, 11): in (4.31), $\pi_r v$ should be $\pi_S v$.
- (95, -5): in (4.36), $\otimes E^*$ should be $\otimes^p E^*$ and $\otimes E$ should be $\otimes^p E$.

Chapter 5

- (97, 15): it should be assumed that $i < j$.
- (99, -15): in problem 2(a), in the displayed equation, x_p should be x_n .
- (99, -10): in problem 2(b), in the definition of φ , $(-1)^{n-1}$ should be $(-1)^{n-i}$.
- (100, -5): “ p -linear mappings” should be “skew-symmetric p -linear mappings”.
- (101, -12): “it reduces to zero” should be “ h reduces to zero”.
- (101, -2): “ p th exterior algebra” should be “ p th exterior power”.
- (105, -13): it should be assumed that $(\epsilon x)^2 = 0$ for all $x \in E$.
- (106, 4): the codomain of g should be $\wedge E^*$.
- (108, 3): in problem 3, the matrix of a 2-vector has not been defined.

- (108, 4): in problem 4, the rightmost matrix in the Lagrange identity should be

$$\begin{pmatrix} \eta_{v_1}^1 & \cdots & \eta_{v_p}^1 \\ \vdots & & \vdots \\ \eta_{v_1}^p & \cdots & \eta_{v_p}^p \end{pmatrix}$$

Also, in the hint, the order of the two factors in the scalar product should be swapped, to be consistent with the rest of the chapter.

- (108, -2): $\varphi_\wedge, \wedge E \rightarrow \wedge F$ should be $\varphi_\wedge : \wedge E \rightarrow \wedge F$.
- (110, 6), (110, 14): $v^* \in \wedge E^*$ should be $v^* \in \wedge F^*$.
- (112, -9): (5.26) is not used to establish the displayed equation.
- (114, -8): “an antiderivation” should be “a homogeneous antiderivation”.
- (115, 4): “ α -Antiderivations” should be “ α_\wedge -Antiderivations”.
- (115, 6): it should be clarified that φ and ψ are homogeneous with respect to the trivial gradations in E and F , respectively, where all elements are homogeneous of degree 1.
- (115, 8): it should be clarified that $\omega_E = \omega$.
- (115, 12): in (5.36), it should be clarified that $\Omega_E = \Omega$ and $\Omega_F(\psi)$ is the induced antiderivation with respect to ω_F in $\wedge F$.
- (115, 14), (115, 15): “ α -antiderivations” should be “ α_\wedge -antiderivations”.
- (115, -4): in problem 1(b), in the displayed equation, π_F should be π_E .
- (116, 4): in problem 4, the claim is false.
- (116, 13), (116, 15): in problem 6, “degree 1” should be “degree -1 ”, and $\tilde{\Omega}u = p \cdot \Omega u$ should be $\tilde{\Omega}u = p^{-1} \cdot \Omega u$ for $p > 0$.
- (116, -8): in problem 8, the transformations $(\psi - \lambda\iota)_\wedge$ and ψ_i should all be restricted to $\wedge^n E$. Also, $\psi_0 = (-1)_n \iota$ should be $\psi_0 = (-1)^n \iota$.
- (117, 8): it must be assumed that the dual map exists.
- (117, 11): in the displayed equation, it should be noted that $u^* \in \wedge E^*$.

- (118, 13): $\Omega(h)$ should be Ω_h .
- (119, 8): in the proof of Proposition 5.14.2, it should be assumed that $\nu \in \bigwedge^p E$.
- (119, -4): in problem 3, it should be assumed that $e_\nu, e^{*\nu}$ is a pair of dual bases of E, E^* .
- (120, 7): in problem 6, in the displayed equation, the order of the two factors in the scalar product should be swapped to be consistent with the rest of the chapter.
- (120, -1): it should be noted that f is homogeneous of degree zero by (5.42).
- (121, -10): in the displayed equation, $\eta(i_1 x) = x$ should be $\eta(i_1 x) = x \otimes 1$.
- (121, -9): in the displayed equation, $\eta(i_2 y) = y$ should be $\eta(i_2 y) = 1 \otimes y$.
- (121, -6): $x \oplus y$ should be $x + y$.
- (123, -3): in the displayed equation, $k \in E$ should be $k \in F$.
- (123, -1): in (5.50), $h \oplus k$ should be $h + k$.
- (125, 7): in the displayed equation, $u_\rho \in E_\rho$ should be $u_\rho \in \bigwedge E_\rho$.
- (125, -8): it should be assumed that $k_i \geq 1$.
- (126, -2): in problem 2(b), the top arrow of the diagram should be labeled as φ_\wedge .
- (127, 6): it should be noted that $\nu_q \in \bigwedge^q E$.
- (127, -9): “and hence $p = q$ ” should be “and hence $p = q$ if $A, B \neq 0$ ”.
- (129, 10): $(\varphi'_\wedge)_\wedge$ should be φ'_\wedge .
- (130, -13): in the Corollary, “If E has finite dimension” should be “If E has finite dimension n ”.
- (130, -6): in the proof of the second part of the Corollary, it should be noted that $I^q \neq 0$.
- (131, 5): $U \in \bigwedge E$ should be $U \subset \bigwedge E$.

- (131, -4): in (5.62), $b_i \in H$ should be $b_i \in \wedge H$, as in (5.63).
- (132, 2): in the displayed equation, $v_j \in F$ should be $v_j \in \wedge F$.
- (132, 5): u' should be the sum of all terms in the equation with parts from v_j of positive degree.
- (132, 7): in the displayed equation, b_j should be b'_j , a scalar multiple of b_j .
- (132, 11): it should be assumed that $p \leq m$.
- (135, 11): in Proposition 5.27.2, it should be clarified that φ is homogeneous with respect to the trivial gradation in E , where all elements are homogeneous of degree 1.
- (135, 15): in the displayed equations, it should be noted that $u_1, u_2 \in \wedge^k E$.
- (136, 15): “by the Corollary to Proposition 5.27.1” should be “by Proposition 5.27.2”.
- (136, -7): “ideal” should be “ideal in $\wedge E$ ”.
- (137, 4): in problem 6, “be linear transformation” should be “be a linear transformation”.
- (138, -6): $\wedge(F^{*\perp}) = I_{F^*}$ should be $\wedge(F^{*\perp}) = (I_{F^*})^\perp$.
- (138, -3): in Proposition 5.28.2, it should be assumed that $A \neq 0$.
- (139, 6): $u^* \in \wedge E$ should be $u^* \in \wedge E^*$.
- (141, -8): in Proposition 5.30.1(B), $A(E)$ (which should be $A^*(E)$) has not been defined.
- (141, -1): in the proof of Proposition 5.30.1(B), the first and second equalities in the first displayed equation should be swapped.
- (142, 13): in the displayed equation, it should be noted that $\Phi \in A^p(E)$.
- (142, -1): in the displayed equation, it should be noted that $\Phi \in A^p(E)$ and $\Psi \in A^q(E)$, and in the third equality, $\theta^T(\varphi) \cdot \Psi$ should be $\theta^T(\varphi)\Phi \cdot \Psi$.

- (143, 1): in subsection 5.32, the substitution operator $i_A(h)$ should be re-defined on $T^p(E)$ as

$$i_A(h) = \frac{1}{p} \sum_{v=1}^p (-1)^{v-1} i_v(h)$$

The missing factor of $1/p$ introduces errors throughout the subsection.

- (143, -3): in the displayed equation, the tensor product symbols should be function product symbols.
- (145, 4): in the displayed equation, $i(h)$ should be $i_A(h)$.
- (145, -11): in the heading of subsection 5.33, $T^\bullet(E)$ should be $A^\bullet(E)$.
- (145, -1): in the displayed equation, $v \in X^q(E^*)$ should be $v \in X^q(E^*)$.
- (146, 12): $\varphi^A : A(E) \leftarrow A(F)$ should be $\varphi^A : A^\bullet(E) \leftarrow A^\bullet(F)$.
- (146, -16): “mappings” should be “functions”.
- (146, -8): in the displayed equation, $A^p(E)$ should be $T^p(E)$ and $A_p(E)$ should be $T_p(E)$.
- (147, 9): the proof should be

$$\begin{aligned} \langle \Phi, x_1 \wedge \cdots \wedge x_p \rangle_A &= \frac{1}{p!} \langle \Phi, x_1 \wedge \cdots \wedge x_p \rangle \\ &= \langle \Phi, A(x_1 \cdot \dots \cdot x_p) \rangle \\ &= \langle A\Phi, x_1 \cdot \dots \cdot x_p \rangle \\ &= \langle \Phi, x_1 \cdot \dots \cdot x_p \rangle \\ &= \Phi(x_1, \dots, x_p) \end{aligned}$$

- (147, -1): in the displayed equation, $A(x_1 \otimes \cdots \otimes x_p)$ should be $A(x_1 \cdot \dots \cdot x_p)$ and $f_1(x_{\sigma(p)})$ should be $f_1(x_{\sigma(1)})$.

Chapter 6

- (148, -8): in Proposition 6.1.1, it should be noted that ψ maps into H .

- (149, -7): in the first equality of the displayed equation, $\pi_2(x_1 \wedge \cdots \wedge x_q)$ should be $\pi_2(x_1 \otimes \cdots \otimes x_q)$.
- (150, 10): the displayed equation should be $w^0 = 1 \otimes 1$.
- (150, -2): it must be assumed that the dual operator exists.
- (151, 2): “if $r \geq p$ and $s \geq q$ ” should be “if $r \geq q$ and $s \geq p$ ”.
- (152, 13): in the displayed equation, $\mu[\psi^\wedge \otimes \varphi_\wedge]z$ should be $\mu[(\psi^\wedge \otimes \varphi_\wedge)z]$.
- (153, 3): $T_E(b \otimes a^*)$ should be $T_{E^*}(b \otimes a^*)$.
- (153, -9): “This show that the box product is symmetric,” should be “This shows that the box product is symmetric.”.
- (153, -7): in the displayed equation, $\sigma\tau$ should be $\tau\sigma$.
- (153, -6): $\sigma\tau = \rho$ should be $\tau\sigma = \rho$.
- (153, -3): in (6.6), the notation $\wedge^p \varphi$ has not yet been defined.³
- (154, 3): in the fifth equality of the displayed equation, the T ’s should be T_E ’s.
- (155, 11): in the displayed equation, $\wedge_p^{p-p}(E^*, E)$ should be $\wedge_p^p(E^*, E)$.
- (155, -11): “for which $w_1 \circ w_2 \neq 0$ ” should be deleted.
- (157, -10): “Section 6.5” should be “(6.5)”.
- (157, -7): in (6.12), $T_q^p(E^*, E)$ should be $\wedge_q^p(E^*, E)$ and $T_p^q(E^*, E)$ should be $\wedge_p^q(E^*, E)$.
- (157, -6): $w_2 \in b^* \otimes a$ should be $w_2 = b^* \otimes a$.
- (159, 13): the corollary to Proposition 6.7.1 should be labeled.
- (161, 2): in the proof of (2), D_e should be D_p .

³This notation is defined later on p. 169.

- (161, 5), (161, 6): in the proof of (3), $v \in \bigwedge^{n-p} E$ should be replaced by $v^* \in \bigwedge^{n-p} E^*$.
- (161, 8): “way” should be “way.”.
- (161, 12): it should be noted that $e^* \in \bigwedge^n E^*$ and $\langle e^*, e \rangle = 1$.
- (161, -2): e^*, e should be e, e^* .
- (164, -7): it should be noted that $e_1 \wedge \cdots \wedge e_n = e$.
- (164, -4): “ $(n-p)$ -tuple” should be “ordered $(n-p)$ -tuple”.
- (165, 6): in the displayed equation, e^{*v_n} should be e^{*v_p} .
- (166, 2): in the displayed equation, $0 \leq i, j \leq n-1$ should be $1 \leq i, j \leq n-1$.
- (166, 4): in the displayed equation, e_2 should be e_n .
- (167, 5): in problem 2, the first relation should be $i(1 \otimes u) \sharp^p = 1 \otimes u$.
- (167, 13): in problem 4, $\Delta(E^*, E)$ should be ΔE .
- (168, 1): in problem 8, it should be noted that $e_1 \wedge \cdots \wedge e_n = e$.
- (168, 2): in problem 8, the displayed equation should be

$$D_e(x_1 \wedge \cdots \wedge x_p) = \sum_{v_1 < \cdots < v_p} (-1)^{\sum_{i=1}^p (v_i - i)} \det \langle e^{*v_j}, x_i \rangle e^{v_{p+1} \cdots v_n}$$

- (168, 3): in problem 8, “complement (v_1, \dots, v_p) ” should be “complement of (v_1, \dots, v_p) ”.
- (169, -7): in the displayed equation, $\varphi \otimes \iota$ should be $\iota \otimes \varphi$.
- (170, 12): in (6.30), $\psi \otimes \iota$ should be $\iota \otimes \psi$.
- (170, 13): it is circular (and unnecessary) to prove that T and \tilde{T} are linear isomorphisms with the aid of (6.28) and (6.30).
- (171, 2): $v^* \in \bigwedge^r E$ should be $v^* \in \bigwedge^r E^*$.
- (171, -13): in the displayed equation, “ pq even” should be “ $p+q$ even”.

- (171, -5): in the title of subsection 6.18, $E^* \widehat{\otimes} E$ should be $\wedge E^* \widehat{\otimes} \wedge E$.
- (172, 5): in the displayed equation, $x^* \oplus x$ should be $x^* + x$ and $y^* \oplus y$ should be $y^* + y$.
- (172, -3): in the displayed equations, $\langle\langle x_\alpha^*, y_\delta \rangle\rangle$ should be $\langle x_\alpha^*, y_\delta \rangle$ and $\langle\langle y_\gamma^*, x_\beta \rangle\rangle$ should be $\langle y_\gamma^*, x_\beta \rangle$.

References

- [1] Greub, W. *Linear Algebra*, 4th ed. Springer, 1975.
- [2] Greub, W. *Multilinear Algebra*, 2nd ed. Springer, 1978.