Errata from
Linear Algebra and *Multilinear Algebra*

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Introduction

This document contains errata from [1] and [2]. Locations within each text are indicated by coordinates (p, n), where p is a page number and n is a line number on page p. Positive line numbers count from the top of the page, whereas negative line numbers count from the bottom of the page. Displayed equations, diagrams, and figures are counted as single lines.

Linear Algebra

Errata are currently only listed for Chapters 0–VIII (except § 3 of Chapter VI and § 7 of Chapter VIII) and Chapter XI (except § 4–5).

Minor, purely typographical inconsistencies like those between "→" and "↦", or between "$1 \ldots n$" and "$1, \ldots, n$", or between "family x_α" and "family (x_α)" and "family $\{x_\alpha\}$", are not listed but occur frequently.

Chapter 0

- (2, 8): in the definition of subgroup, "subset H" should be "nonempty subset H".

- (3, -12): a homomorphism between fields must also preserve the multiplicative identity.

- (3, -13): a subfield must also contain the multiplicative identity.

Chapter I

- (9, 16): in the definition of linear dependence, "non-trivial linear combination of the x_α" should be "non-trivial linear combination of the x_α equal to zero".

- (12, 1): $\lambda^i = 0$ should be $\lambda^i \neq 0$.

- (12, 5): throughout the proof of (ii), n should be m.

- (12, -5): "The a partial order" should be "A partial order".

- (12, -4): "maximal element" should be "upper bound".

- (13, 8): $x \in E$ should be $x \in S - T$.

- (13, 9), (13, 10): $T \cup x$ should be $T \cup \{x\}$ and $x \cup T$ should be $\{x\} \cup T$.

- (14, 1): "element f_a" should be "elements f_a".

- (14, 3), (14, 5): in the displayed equations, $j = 1$ should be $i = 1$.

- (15, -2): in problem 10, $\{x_\alpha\}_{\alpha \neq \beta}$ should be $\{a\} \cup \{x_\alpha\}_{\alpha \neq \beta}$.

- (20, 16): "$\varphi(S)$ is a system of generators for $\varphi(S)$" should be "$\varphi(S)$ is a system of generators for F".

- (22, -15): in part (v) of Problem 5, the concept of a generated subspace has not yet been defined.

- (23, 7): in the definition of subspace, "subset of E" should be "nonempty subset of E".

- (27, -11): in the displayed equation, λ_i should be λ^i (two occurrences). Also, it should be noted that $y_i \in E_1$ and $z_i \in E_2$.

- (29, -16): "*canonical projection* of E onto E_1" should be "*canonical projection* of E onto E/E_1".

- (40, -8): in the second displayed equation in problem 6, $u \in E_2'$ should be $y \in E_2'$.

Chapter II

- (47, -6): "assume that there" should be "assume that".

- (52, -8): $\psi : E_1 \leftarrow F$ should be $\psi : E \leftarrow F$.

- (52, -3): "left inverse" should be "left inverse ψ".

- (53, 13): "inverse" should be "left inverse".

- (53, -1): in problem 1, the inclusion $L(E; F) \subset C(E; F)$ is wrong.

- (59, -5): in (2.25), it should be noted that $\delta_{\varrho\sigma}$ is a Kronecker delta.[1]

- (62, 10): in (2.34), y should be y_j.

- (63, -7): in problem 7, "second set" should be "disjoint set".

- (67, -1): in the displayed equation, $y^* \in F$ should be $y^* \in F^*$.

- (68, -7): in the displayed equation, $\langle y^* (\varphi + \psi) x \rangle$ should be $\langle y^*, (\varphi + \psi) x \rangle$.

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[1]This notation is defined later in a footnote on p. 76.

- (76, 9): "imension" should be "dimension".

- (76, -7): in the displayed equation (and really the rest of subsection 2.31), φ_v^μ should be φ_μ^v.

- (77, -7): φ should be Φ.

Chapter III

- (83, -10): in the displayed equation, $(b_1^\mu \ldots b_n^\mu)$ should be $(\alpha_1^\mu \ldots \alpha_n^\mu)$.

- (84, -7): "columns of the matrix α_v^μ" should be "columns of the matrix $\alpha_v^{*\mu}$".

- (84, -6): $y = y^{*\mu}$ should be $y^* = y^{*\mu}$.

- (85, 13): in the displayed equation, it should be noted that $A = (\alpha_v^\mu)$.

- (88, 1): in the main theorem, "system of n equations in m unknowns" should be "system of m equations in n unknowns".

- (89, 12): it should be noted that $\dim E = n$ and $\dim F = m$.

- (89, -8): in the displayed equation, a_v^μ should be α_v^μ.

- (91, 5): "E automorphism of E" should be "automorphism of E".

- (91, 9): in the displayed equation, $M(\varphi)^{-1}$ should be $M(\varphi^{-1})$.

- (91, -3): in problem 3, "linear transformation" should be "a linear transformation".

- (93, -13): "inverse of the matrix of the transformation $x_v \to \bar{x}_v$" should be "transpose of the inverse of the matrix of the transformation $x_v \to \bar{x}_v$".

- (95, 12): $\mu = 1,\ldots,n$ should be $\mu = 1,\ldots,m$.

- (96, 1): multiplication of basis vectors by nonzero scalars should be added to the list of elementary basis transformations.

- (96, -7): $2 \leqq v \leqq m$ should be $2 \leqq v \leqq n$.

- (98, 3): (3.36) should be

$$\xi^r = (\kappa_r^r)^{-1}\left(\omega^r - \sum_{v=r+1}^{n} \kappa_v^r \xi^v\right)$$

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Chapter IV

- (103, 15): in (4.6), it should be noted that $\hat{x}_j$ indicates that the vector x_j is deleted.[2]

- (105, -8): "Proposition II" should be "Proposition III".

- (106, 5): "(4.14)" should be "(4.12)".

- (107, 13): in (4.14), $p = \dim E$ should be $p = \dim E_1$.

- (109, 1): the problem numbers on this page should be incremented by 1.

- (109, 10): in problem 6, it must be assumed that E is real.

- (109, -11): in problem 8, the trace of a linear transformation has not yet been defined.

- (113, 1): "(4.21)" should be "(4.22)".

- (113, -10): in the displayed equation, $x^*{}_i$ should be x^{*i}.

- (115, 1): "(4.14)" should be "(4.12)".

- (115, 4): in the displayed equation, $\widehat{\varphi a_j}$ should be $\widehat{\varphi a_i}$.

- (115, 9): it should be noted that $M(\varphi) = (\alpha_\nu^\mu)$.

- (115, -10): the displayed equation should be $\beta_j^i = \det C_i^j$.

- (116, 11): "(4.36)" should be "(4.38)".

- (116, -1): in the displayed matrix B_i^j, the first column should have entries $1, \alpha_1^j, \ldots, \alpha_{i-1}^j, \alpha_{i+1}^j, \ldots, \alpha_n^j$.

- (117, 3): "(4.38)" should be "(4.16)", or a reference to Problem 6.

- (117, 7): "(4.35) and (4.30)" should be "(4.38) and (4.40)".

- (117, -14): "minor" should be "submatrix".

- (122, 2): on the third line of the displayed equation, σ should be ϱ.

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[2]This notation is defined later in a footnote on p. 198.

- (130, -14): in problem 11, it must be assumed that E is real.

- (135, -3): "4.30" should be "4.29".

- (136, -6): "4.17" should be "4.16", and it should be noted that ξ_ν^i are the components of x_ν with respect to some basis.

- (139, -13): $(a_1', b_2 \ldots b_n)$ should be $(a_1', b_3 \ldots b_n)$.

- (139, -10): in the displayed equation, $i = 1 \ldots n$ should be $i = 1 \ldots n - 1$.

Chapter V

- (146, 1): $(\varphi, \psi) \to \psi\varphi$ should be $(\psi, \varphi) \to \psi\varphi$.

- (148, 10): the displayed list should also include elements of the form asb for $s \in S$ and $a, b \in A$.

- (148, -17): it must be assumed that $A \neq 0$ (equivalently $e \neq 0$) for it to follow that $\lambda = 0$.

- (151, 2): the extra "can be" should be deleted.

- (158, 14): "cheeked" should be "checked".

- (160, -13): "let be" should be "be".

- (160, -8): it must be assumed that $E \neq 0$ for $A(E; E)^2 \neq 0$.

- (161, -1): "non-zero, I, ideal" should be "non-zero ideal I,".

- (165, 12), (166, 7): it should be clarified that Γ is assumed to be a field under the restrictions of the operations in $A_\Delta(E; E)$.

Chapter VI

- (168, -16): it should be noted that the zero map is homogeneous of every degree (hence *the* degree is not well-defined in that case).

- (172, -4): "product" should be "products".

- (173, -3): in problem 6, $\deg \varphi^* = -k$.

- (175, 4): the displayed statement should be $xe_k \in A_{l+k}$.

- (176, 1): E should be A.

- (177, 2): in the second part of problem 1, it must be assumed that $A \neq 0$ to conclude that $k = 0$.

- (177, 14): in problem 5, "$\leqq 0$ ($\geqq 0$)" should be "$\geqq 0$ ($\leqq 0$)".

Chapter VII

- (189, -12): in the displayed equation, $\lambda > 0$ should be $\lambda \geqq 0$.

- (192, 5): $x = x_\mu$ should be $y = x_\mu$.

- (192, 14): "basisvectors" should be "basis vectors".

- (193, 14): "orthogonal bases" should be "orthonormal bases".

- (193, -8): (α_φ^μ) should be (α_ν^μ).

- (195, 4): in problem 3, it should be assumed that E is finite-dimensional or else established that orthogonal projection still works as long as E_1 is finite-dimensional.

- (204, 13): in the displayed equation, $\sum_\nu g_{\nu\lambda} \xi^\lambda$ should be $\sum_\nu g_{\nu\lambda} \xi^\nu$.

- (206, 14): "least upper bound" should be "least nonnegative upper bound" to account for the case $E = 0$ where there are no unit vectors.

- (207, 7): "naturaltopology" should be "natural topology".

- (210, 15): throughout subsection 7.24, it should be assumed that $A \neq 0$.

Chapter VIII

- (217, 13): the concept of a metric tensor has not yet been defined.

- (217, 14): in the displayed equation, $\sum\limits_{\nu}$ should be $\sum\limits_{\lambda}$.

- (220, -1): in problem 1, equality holds only if $\psi = \lambda\varphi$ or $\varphi = \lambda\psi$.

- (223, -2): in (8.22), $\lambda - \lambda_i$ should be $\lambda_i - \lambda$.

- (226, 17): in problem 11, the concept of a rotation has not yet been defined.

- (227, 14): $E_1 \in E_2^\perp$ should be $E_1 \subset E_2^\perp$.

- (231, 7): the proof of the normal form (8.35) is incorrect because it is not true in general that the a_n defined form an orthonormal basis of the space.

- (231, -5): "Proposition II" should be "Proposition III".

- (234, 4): in the displayed equation, a_μ^κ should be α_μ^κ.

- (234, -16): "*rotation*" should be "*rotation φ*".

- (234, -10): it must be assumed that $e \neq 0$.

- (234, -9): "sec. 4, 17" should be "sec. 4.17".

- (238, -7): in (8.40), the second equation should be $\sin\theta = -\frac{1}{2}\operatorname{tr}(j \circ \varphi)$.

- (239, 17): in the displayed equation, "$\equiv$" should be "$=$".

- (240, -13): "see 8.21" should be "sec. 8.21".

- (241, 6): in (8.49), x should be u.

- (242, -13): in the displayed equation, "$\equiv$" should be "$=$".

- (243, 7): E should be e.

- (243, -5): "see 8.21" should be "sec. 8.23".

- (243, -4): in the displayed equation, (z, τ, z) should be $(z, \tau z)$.

- (243, -2): it should be noted that Δ is the positive normed determinant function in E_1.

- (244, -11): F_1 should be F.

- (245, 8): in the displayed equation, b should be b_1.

- (245, -10), (245, -5): "(8.53)" should be "(8.54)".

- (246, 10), (246, -10): in problems 3 and 5, "plane" should be "plane E".

- (247, -1): f should be defined so that $f(0) = 0$.

- (248, -8): in problem 16, E should be $\mathbb{H}$.

Chapter XI

- (328, 14), (328, -7): the proof of the equality condition for the triangle inequality does not properly account for the case $x = 0$; in particular, "$y = \lambda x$" should be "$y = \lambda x$ or $x = \lambda y$".

- (332, 11): in problem 3, the displayed equation should be

$$(z_1, z_2) = \big((x_1, x_2) + (y_1, y_2)\big) + i\big((x_2, y_1) - (x_1, y_2)\big)$$

  and it should be noted that $z_k = x_k + i y_k$.

- (334, -7): "automorphism" should be "involution".

- (335, -12): in the displayed equation, Φ should be φ.

- (335, -5): the concept of a conjugate determinant function is not defined.

- (335, -3): in the displayed equation, $\bar{x}_p$ should be $\bar{x}_n$.

- (336, -8): in the displayed equation, $\sum\limits_{\mu,\nu}$ should be $\sum\limits_{\mu,\lambda}$.

- (337, -16): "normal mapping" should be "normal mappings".

- (337, -10): in the displayed equation, λ should be λ_1.

- (339, 8): it must be assumed that $e \neq 0$.

- (339, 14): in problem 1, "bilinear" should be "binary".

Multilinear Algebra

Errata are currently only listed for Chapters 1–5 (except § 4,8–17 of Chapter 2 and § 15–28 of Chapter 5). Again, minor purely typographical inconsistencies are not listed.

Chapter 1

- (4, 11): in problem 3, it should be assumed that $E \neq 0$.

- (4, -7): in problem 5(b), the claim is false.

- (7, -5): in the displayed equation, $\lambda_\alpha^\nu \in \Gamma$ should be $\lambda_\nu^\alpha \in \Gamma$.

- (11, -10): "satisfis" should be "satisfies".

- (11, -3): The function name f should not be reused.

- (12, 9): in problem 1,

$$(\xi^1, \ldots, \xi^n) \times (\eta^1, \ldots, \eta^m)$$

  should be

$$\left((\xi^1, \ldots, \xi^n), (\eta^1, \ldots, \eta^m) \right)$$

- (12, -6): in problem 6(a), the tensor product of linear maps has not yet been defined.

- (13, 15): "φ_1 factors over $\otimes$" should be "φ_1 factors over $\otimes'$".

- (14, 6): in the displayed equation, $y \in F_1$ should be $y_1 \in F_1$.

- (14, -5): $\widetilde{E} = \bigotimes_\alpha E_\alpha$ should be $\widetilde{E} = \bigoplus_\alpha E_\alpha$ and $\widetilde{F} = \bigotimes_\beta F_\beta$ should be $\widetilde{F} = \bigoplus_\beta F_\beta$.

- (14, -1): $\pi_\alpha : E \to E_\alpha$ should be $\pi_\alpha : \widetilde{E} \to E_\alpha$.

- (17, 11): in the displayed equation, it should be noted that $x_\alpha \in E_\alpha$ and $y_\beta \in F_\beta$.

- (22, -9): in Corollary III, $L(E \otimes F; E' \otimes F)$ should be $L(E \otimes F; E' \otimes F')$.

- (22, -8): in Corollary III, the claim is not shown in subsection 1.27, where it is also assumed that E' and F' are finite-dimensional.

- (25, 9): in problem 1(a), the "only if" part of the claim is false.

- (30, 13), (31, 11): it is false that $\Phi \otimes \Psi$ is nondegenerate only if Φ and Ψ are nondegenerate.

- (34, -2): it should be assumed that $x^* \neq 0$.

- (36, 12): $\varphi \times \psi$ should be (φ, ψ).

- (36, 14): in the displayed equation, $\dim(F; F')$ should be $\dim L(F; F')$.

- (37, 3): in the commutative diagram, $\otimes$ should be the linear map induced by $\otimes$.

- (37, 16): $(\varphi \times \psi)$ should be (φ, ψ).

- (37, -3): (α, β) should be $\alpha \otimes \beta$.

- (38, 3): $F : A \times A \to L(A; A)$ should be $F : A \otimes A \to L(A; A)$.

- (38, 8): $L(A \otimes A)$ should be $A \otimes A$.

- (38, -5): in subsection 1.30, it should be assumed that $E \neq 0$.

- (39, 3): Ω should be the bilinear map induced by Ω.

- (40, 5): in problem 2, it should be assumed that $E, F \neq 0$.

- (40, -5): in problem 3, it should be assumed that E is oriented.

Chapter 2

- (42, 18): if $A \neq 0$, it must be assumed that $B \neq 0$ for φ to be injective.

Chapter 3

- (60, -3): it should be noted that $v \in \bigotimes^q E$.

- (61, 3): $\dim E = 1$ should be $\dim E \leq 1$.

- (62, 4): in the displayed equation, it should be noted that $u_p \in \bigotimes^p E$ and $v_q \in \bigotimes^q E$.

- (63, -2): in **T**, "unit element" should be "unit element e".

- (64, -7): in the uniqueness theorem, proof, and remarks after, it should be noted that the homomorphisms preserve the unit.

- (65, 1): in the proof of the uniqueness theorem, the unit element is also needed to generate U.

- (68, 8): in problem 2, $\lambda_1, \ldots, \lambda_p = 1$ should be $\lambda_1 \cdots \lambda_p = 1$.

- (68, 9): in problem 3, it should be asked to prove that $\operatorname{tr}(\varphi\psi) = \operatorname{tr}(\psi\varphi)$.

- (68, -8): in the displayed equation, it should be noted that $u^{*p} \in \bigotimes^p E^*$ and $v_p \in \bigotimes^p E$.

- (69, -10): the reference to (3.8) should be deleted.

- (72, 6): "noncommutative" should be "noncommutative in general".

- (72, -5): $\Phi^i_j(\omega)$ should be $C^i_j(\omega)$.

- (74, 1): in many problems in this subsection, the order of dual spaces should be swapped to be consistent with the rest of the chapter; for example, $\bigotimes(E, E^*)$ should be $\bigotimes(E^*, E)$.

- (74, -9): in problem 7, it should be assumed that E is finite-dimensional.

- (74, -5): in problem 8, it should be assumed that E is finite-dimensional.

- (77, -3): in problem 5, the scalars $g_{\nu\mu} = (e_\nu, e_\mu)$ should be defined.

- (78, -6): "noncommutative" should be "noncommutative in general".

- (79, 6): it should be clarified how $i_\nu(h)$ acts if $\nu > p$.

- (79, -8), (79, -6): the definitions of the operators $i_A(h)$ and $i_S(h)$ given in this subsection are inappropriate for use with the algebras of skew-symmetric and symmetric multilinear functions, respectively. See below.

- (80, 11): in the displayed equation, $E^{*\nu}$ should be E^*.

- (82, 6): in the displayed equation, $\langle f_1 \cdot \ldots \cdot f_p \Psi \rangle$ should be $\langle f_1 \cdot \ldots \cdot f_p, \Psi \rangle$.

- (82, -10): in the displayed equation,

$$\Psi(x_1, \ldots, x_r, x_1^*, \ldots, x_s^*)$$

  should be
$$\Psi(x_{p+1}, \ldots, x_{p+r}, x_{q+1}^*, \ldots, x_{q+s}^*)$$

- (83, -1): y^{*j} should be y_j^*.

Chapter 4

- (85, 1): it should be assumed that $i < j$.

- (87, 12): in the displayed equation, $u = x^{*1} \otimes \cdots \otimes x^{*p}$ should be $u^* = x^{*1} \otimes \cdots \otimes x^{*p}$.

- (90, 6): "restriction of π_X" should be "restriction of π_A".

- (91, 2): in the displayed equation, $X(E)$ should be $X^p(E)$.

- (93, 11): in (4.31), $\pi_r \nu$ should be $\pi_S \nu$.

- (95, -5): in (4.36), $\otimes E^*$ should be $\otimes^p E^*$ and $\otimes E$ should be $\otimes^p E$.

Chapter 5

- (97, 15): it should be assumed that $i < j$.

- (99, -15): in problem 2(a), in the displayed equation, x_p should be x_n.

- (99, -10): in problem 2(b), in the definition of φ, $(-1)^{n-1}$ should be $(-1)^{n-i}$.

- (100, -5): "p-linear mappings" should be "skew-symmetric p-linear mappings".

- (101, -2): "pth exterior algebra" should be "pth exterior power".

- (105, -13): it should be assumed that $(\epsilon x)^2 = 0$ for all $x \in E$.

- (106, 4): the codomain of g should be $\bigwedge E^*$.

- (108, 3): in problem 3, the matrix of a 2-vector has not been defined.

- (108, 4): in problem 4, the rightmost matrix in the Lagrange identity should be
$$\begin{pmatrix} \eta^1_{v_1} & \cdots & \eta^1_{v_p} \\ \vdots & & \vdots \\ \eta^p_{v_1} & \cdots & \eta^p_{v_p} \end{pmatrix}$$
Also, in the hint, the order of the two factors in the scalar product should be swapped, to be consistent with the rest of the chapter.

- (108, -2): $\varphi_\wedge, \bigwedge E \to \bigwedge F$ should be $\varphi_\wedge : \bigwedge E \to \bigwedge F$.

- (115, 5): it should be clarified that $\omega_E = \omega$ and $\Omega_E = \Omega$.

- (115, -4): in problem 1(b), in the displayed equation, π_F should be π_E.

- (116, -8): in problem 8, the transformations $(\psi - \lambda \iota)_\wedge$ and ψ_i should all be restricted to $\bigwedge^n E$. Also, $\psi_0 = (-1)_n \iota$ should be $\psi_0 = (-1)^n \iota$.

- (117, 8): it must be assumed that the dual map exists.

- (118, 13): $\Omega(h)$ should be Ω_h.

- (119, 8): in the proof of Proposition 5.14.2, it should be assumed that $v \in \bigwedge^p E$.

- (119, -4): in problem 3, it should be assumed that e_v, e^{*v} is a pair of dual bases of E, E^*.

- (120, 7): in problem 6, in the displayed equation, the order of the two factors in the scalar product should be swapped to be consistent with the rest of the chapter.

- (141, -8): in Proposition 5.30.1(B), $A(E)$ has not been defined.

- (141, -1): in the proof of Proposition 5.30.1(B), the first and second equalities in the first displayed equation should be swapped.

- (142, -1): in the third equality of the displayed equation, $\theta^T(\varphi) \cdot \Psi$ should be $\theta^T(\varphi)\Phi \cdot \Psi$.

- (143, 1): in subsection 5.32, the substitution operator $i_A(h)$ should be redefined on $T^p(E)$ as

$$i_A(h) = \frac{1}{p} \sum_{\nu=1}^{p} (-1)^{\nu-1} i_\nu(h)$$

  The missing factor of $1/p$ introduces errors throughout the subsection.

- (143, -3): in the displayed equation, the tensor product symbols should be function product symbols.

- (145, 4): in the displayed equation, $i(h)$ should be $i_A(h)$.

- (145, -11): in the heading of subsection 5.33, $T^\bullet(E)$ should be $A^\bullet(E)$.

- (146, -16): "mappings" should be "functions".

- (146, -8): in the displayed equation, $A^p(E)$ should be $T^p(E)$ and $A_p(E)$ should be $T_p(E)$.

- (147, 9): the proof should be

$$\begin{aligned}
\langle \Phi, x_1 \wedge \cdots \wedge x_p \rangle_A &= \frac{1}{p!} \langle \Phi, x_1 \wedge \cdots \wedge x_p \rangle \\
&= \langle \Phi, A(x_1 \cdot \ldots \cdot x_p) \rangle \\
&= \langle A\Phi, x_1 \cdot \ldots \cdot x_p \rangle \\
&= \langle \Phi, x_1 \cdot \ldots \cdot x_p \rangle \\
&= \Phi(x_1, \ldots, x_p)
\end{aligned}$$

- (147, -1): in the displayed equation, $A(x_1 \otimes \cdots \otimes x_p)$ should be $A(x_1 \cdot \ldots \cdot x_p)$ and $f_1(x_{\sigma(p)})$ should be $f_1(x_{\sigma(1)})$.

References

[1] Greub, W. *Linear Algebra*, 4th ed. Springer, 1975.

[2] Greub,W. *Multilinear Algebra*, 2nd ed. Springer, 1978.