

# Errata from *Linear Algebra and Multilinear Algebra*

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## **Introduction**

This document contains errata from [1] and [2]. Locations within each text are indicated by coordinates  $(p, n)$ , where  $p$  is a page number and  $n$  is a line number on page  $p$ . Positive line numbers count from the top of the page, whereas negative line numbers count from the bottom of the page. Displayed equations, diagrams, and figures are counted as single lines.

# Linear Algebra

Errata are currently only listed for Chapters 0–V, VI (except § 3), VII, VIII (except § 7), XI (except § 4–5), XII (except § 4), and XIII.

Minor, purely typographical inconsistencies like those between “ $\rightarrow$ ” and “ $\mapsto$ ”, or between “ $1 \dots n$ ” and “ $1, \dots, n$ ”, or between “family  $x_\alpha$ ” and “family  $(x_\alpha)$ ” and “family  $\{x_\alpha\}$ ”, or between “ $f \mid g$ ” and “ $f/g$ ”, are not listed below but occur frequently.

## Chapter 0

- (2, 8): in the definition of subgroup, “subset  $H$ ” should be “nonempty subset  $H$ ”.
- (3, -12): a homomorphism between fields must also preserve the multiplicative identity.
- (3, -13): a subfield must also contain the multiplicative identity.

## Chapter I

- (9, 16): in the definition of linear dependence, “non-trivial linear combination of the  $x_\alpha$ ” should be “non-trivial linear combination of the  $x_\alpha$  equal to zero”.
- (12, 1):  $\lambda^i = 0$  should be  $\lambda^i \neq 0$ .
- (12, 5): throughout the proof of (ii),  $n$  should be  $m$ .
- (12, -5): “The a partial order” should be “A partial order”.
- (12, -4): “chain” should be “nonempty chain”, and it should be noted that  $R$  is an upper bound for the empty chain.
- (12, -4): “maximal element” should be “upper bound”.
- (13, 8):  $x \in E$  should be  $x \in S - T$ .
- (13, 9), (13, 10):  $T \cup x$  should be  $T \cup \{x\}$  and  $x \cup T$  should be  $\{x\} \cup T$ .

- (14, 1): “element  $f_a$ ” should be “elements  $f_a$ ”.
- (14, 3), (14, 5): in the displayed equations,  $j = 1$  should be  $i = 1$ .
- (15, -2): in problem 10,  $\{x_\alpha\}_{\alpha \neq \beta}$  should be  $\{a\} \cup \{x_\alpha\}_{\alpha \neq \beta}$ .
- (20, 16): “ $\varphi(S)$  is a system of generators for  $\varphi(S)$ ” should be “ $\varphi(S)$  is a system of generators for  $F$ ”.
- (22, -15), (22, -11): in part (v) of Problem 5, the concepts of generated subspace and kernel have not yet been defined.
- (23, 7): in the definition of subspace, “subset of  $E$ ” should be “nonempty subset of  $E$ ”.
- (27, -11): in the displayed equation,  $\lambda_i$  should be  $\lambda^i$  (two occurrences). Also, it should be noted that  $y_i \in E_1$  and  $z_i \in E_2$ .
- (29, -16): “*canonical projection* of  $E$  onto  $E_1$ ” should be “*canonical projection* of  $E$  onto  $E/E_1$ ”.
- (31, 14): in problem 7, “composition” should be “decomposition”.
- (31, 19): in problem 8, the period at the end of the second sentence should be a question mark.
- (40, -8): in the second displayed equation in problem 6,  $u \in E'_2$  should be  $y \in E'_2$ .

## Chapter II

- (47, -6): “assume that there” should be “assume that”.
- (52, -8):  $\psi : E_1 \leftarrow F$  should be  $\psi : E \leftarrow F$ .
- (52, -3): “left inverse” should be “left inverse  $\psi$ ”.
- (53, 13): “inverse” should be “left inverse”.
- (53, -1): in problem 1, the inclusion  $L(E; F) \subset C(E; F)$  is wrong.

- (59, -5): in (2.25), it should be noted that  $\delta_{\rho\sigma}$  is a Kronecker delta.<sup>1</sup>
- (62, 10): in (2.34),  $y$  should be  $y_j$ .
- (63, -7): in problem 7, “second set” should be “disjoint set”.
- (67, -1): in the displayed equation,  $y^* \in F$  should be  $y^* \in F^*$ .
- (68, -7): in the displayed equation,  $\langle y^*(\varphi + \psi)x \rangle$  should be  $\langle y^*, (\varphi + \psi)x \rangle$ .
- (70, 14): “bilinear functions” should be “bilinear function”.
- (75, -2): in problem 10, the induced mapping should be

$$\overline{\varphi}^* : E^* / E_1^\perp \leftarrow F^* / F_1^\perp$$

- (76, 9): “imension” should be “dimension”.
- (76, -7): in the displayed equation (and really the rest of subsection 2.31),  $\varphi_v^\mu$  should be  $\varphi_\mu^v$ .
- (77, -7):  $\varphi$  should be  $\Phi$ .

## Chapter III

- (83, -10): in the displayed equation,  $(b_1^\mu \dots b_n^\mu)$  should be  $(\alpha_1^\mu \dots \alpha_n^\mu)$ .
- (84, -7): “columns of the matrix  $\alpha_v^\mu$ ” should be “columns of the matrix  $\alpha_v^{*\mu}$ ”.
- (84, -6):  $y = y^{*\mu}$  should be  $y^* = y^{*\mu}$ .
- (85, 13): in the displayed equation, it should be noted that  $A = (\alpha_v^\mu)$ .
- (88, 1): in the main theorem, “system of  $n$  equations in  $m$  unknowns” should be “system of  $m$  equations in  $n$  unknowns”.
- (89, 12): it should be noted that  $\dim E = n$  and  $\dim F = m$ .
- (89, -8): in the displayed equation,  $a_v^\mu$  should be  $\alpha_v^\mu$ .
- (91, 5): “ $E$  automorphism of  $E$ ” should be “automorphism of  $E$ ”.

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<sup>1</sup>This notation is defined later in a footnote on p. 76.

- (91, 9): in the displayed equation,  $M(\varphi)^{-1}$  should be  $M(\varphi^{-1})$ .
- (91, -3): in problem 3, “be linear transformation” should be “be a linear transformation”.
- (93, -13): “inverse of the matrix of the transformation  $x_v \rightarrow \bar{x}_v$ ” should be “transpose of the inverse of the matrix of the transformation  $x_v \rightarrow \bar{x}_v$ ”.
- (95, 12):  $\mu = 1, \dots, n$  should be  $\mu = 1, \dots, m$ .
- (96, 1): multiplication of basis vectors by nonzero scalars should be added to the list of elementary basis transformations.
- (96, -7):  $2 \leq v \leq m$  should be  $2 \leq v \leq n$ .
- (98, 3): (3.36) should be

$$\xi^r = (\kappa_r^r)^{-1} \left( \omega^r - \sum_{v=r+1}^n \kappa_v^r \xi^v \right)$$

## Chapter IV

- (103, 15): in (4.6), it should be noted that  $\hat{x}_j$  indicates that the vector  $x_j$  is deleted.<sup>2</sup>
- (105, -8): “Proposition II” should be “Proposition III”.
- (106, 5): “(4.14)” should be “(4.12)”.
- (107, 13): in (4.14),  $p = \dim E$  should be  $p = \dim E_1$ .
- (109, 1): the problem numbers on this page should be incremented by 1.
- (109, 10): in problem 6, it should be assumed that  $E$  is real.
- (109, -11): in problem 8, the trace of a linear transformation has not yet been defined.
- (113, 1): “(4.21)” should be “(4.22)”.
- (113, -10): in the displayed equation,  $x^*_i$  should be  $x^{*i}$ .

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<sup>2</sup>This notation is defined later in a footnote on p. 198.

- (115, 1): “(4.14)” should be “(4.12)”.
- (115, 4): in the displayed equation,  $\widehat{\varphi a_j}$  should be  $\widehat{\varphi a_i}$ .
- (115, 9): it should be noted that  $M(\varphi) = (\alpha_v^\mu)$ .
- (115, -10): the displayed equation should be  $\beta_j^i = \det C_i^j$ .
- (116, 11): “(4.36)” should be “(4.38)”.
- (116, -1): in the displayed matrix  $B_i^j$ , the first column should have entries  $1, \alpha_1^j, \dots, \alpha_{i-1}^j, \alpha_{i+1}^j, \dots, \alpha_n^j$ .
- (117, 3): “(4.38)” should be “(4.16)”, or a reference to Problem 6.
- (117, 7): “(4.35) and (4.30)” should be “(4.38) and (4.40)”.
- (117, -14): “minor” should be “submatrix”.
- (122, 2): on the third line of the displayed equation,  $\sigma$  should be  $\rho$ .
- (130, -14): in problem 11, it should be assumed that  $E$  is real.
- (130, -10): in problem 12,  $\varphi : E_1 \rightarrow E_1$  should be  $\varphi_1 : E_1 \rightarrow E_1$ .
- (132, 13):  $n$  should be  $E$ .
- (132, 14): “orientated” should be “oriented”.
- (135, -3): “4.30” should be “4.29”.
- (136, -6): “4.17” should be “4.16”, and it should be noted that  $\xi_v^i$  are the components of  $x_v$  with respect to some basis.
- (139, -13):  $(a'_1, b_2 \dots b_n)$  should be  $(a'_1, b_3 \dots b_n)$ .
- (139, -10): in the displayed equation,  $i = 1 \dots n$  should be  $i = 1 \dots n - 1$ .
- (142, -11): in problem 5, “isotopices” should be “isotopies”.

## Chapter V

- (146, 1):  $(\varphi, \psi) \rightarrow \psi\varphi$  should be  $(\psi, \varphi) \rightarrow \psi\varphi$ .
- (148, 10): the displayed list should also include elements of the form  $asb$  for  $s \in S$  and  $a, b \in A$ .
- (148, -17): it must be assumed that  $A \neq 0$  (equivalently  $e \neq 0$ ) for it to follow that  $\lambda = 0$ .
- (151, 2): the extra “can be” should be deleted.
- (158, 14): “cheeked” should be “checked”.
- (160, -13): “let be” should be “be”.
- (160, -8): it must be assumed that  $E \neq 0$  for  $A(E; E)^2 \neq 0$ .
- (161, -1): “non-zero,  $I$ , ideal” should be “non-zero ideal  $I$ ,”.
- (165, 12), (166, 7): it should be clarified that  $\Gamma$  is assumed to be a field under the restrictions of the operations in  $A_\Delta(E; E)$ .

## Chapter VI

- (168, -16): it should be noted that the zero map is homogeneous of every degree (hence *the* degree is not well-defined in that case).
- (172, -4): “product” should be “products”.
- (173, -3): in problem 6,  $\deg \varphi^* = -k$ .
- (175, 4): the displayed statement should be  $xe_k \in A_{l+k}$ .
- (176, 1):  $E$  should be  $A$ .
- (177, 2): in the second part of problem 1, it must be assumed that  $A \neq 0$  to conclude that  $k = 0$ .
- (177, 14): in problem 5, “ $\leq 0$  ( $\geq 0$ )” should be “ $\geq 0$  ( $\leq 0$ )”.

## Chapter VII

- (189, -12): in the displayed equation,  $\lambda > 0$  should be  $\lambda \geq 0$ .
- (192, 5):  $x = x_\mu$  should be  $y = x_\mu$ .
- (192, 14): “basisvectors” should be “basis vectors”.
- (193, 14): “orthogonal bases” should be “orthonormal bases”.
- (193, -8):  $(\alpha_\varphi^\mu)$  should be  $(\alpha_\nu^\mu)$ .
- (195, 4): in problem 3, it should be assumed that  $E$  is finite-dimensional or else established that orthogonal projection still works as long as  $E_1$  is finite-dimensional.
- (204, 13): in the displayed equation,  $\sum_\nu g_{\nu\lambda} \xi^\lambda$  should be  $\sum_\nu g_{\nu\lambda} \xi^\nu$ .
- (206, 14): “least upper bound” should be “least nonnegative upper bound” to account for the case  $E = 0$  where there are no unit vectors.
- (207, 7): “natural topology” should be “natural topology”.
- (210, 15): throughout subsection 7.24, it should be assumed that  $A \neq 0$ .

## Chapter VIII

- (217, 13): the concept of a metric tensor has not yet been defined.
- (217, 14): in the displayed equation,  $\sum_\nu$  should be  $\sum_\lambda$ .
- (220, -1): in problem 1, equality holds only if  $\psi = \lambda\varphi$  or  $\varphi = \lambda\psi$ .
- (223, -2): in (8.22),  $\lambda - \lambda_i$  should be  $\lambda_i - \lambda$ .
- (226, 17): in problem 11, the concept of a rotation has not yet been defined.
- (227, 14):  $E_1 \in E_2^\perp$  should be  $E_1 \subset E_2^\perp$ .



- (231, 7): the proof of the normal form (8.35) is incorrect because it is not true in general that the  $a_n$  defined form an orthonormal basis of the space.
- (231, -5): “Proposition II” should be “Proposition III”.
- (234, 4): in the displayed equation,  $a_\mu^\kappa$  should be  $\alpha_\mu^\kappa$ .
- (234, -16): “*rotation*” should be “*rotation*  $\varphi$ ”.
- (234, -10): it must be assumed that  $e \neq 0$ .
- (234, -9): “sec. 4, 17” should be “sec. 4.17”.
- (238, -7): in (8.40), the second equation should be  $\sin \theta = -\frac{1}{2} \operatorname{tr}(j \circ \varphi)$ .
- (239, 17): in the displayed equation, “ $\equiv$ ” should be “=”.
- (240, -13): “see 8.21” should be “sec. 8.21”.
- (241, 6): in (8.49),  $x$  should be  $u$ .
- (242, -13): in the displayed equation, “ $\equiv$ ” should be “=”.
- (243, 7):  $E$  should be  $e$ .
- (243, -5): “see 8.21” should be “sec. 8.23”.
- (243, -4): in the displayed equation,  $(z, \tau, z)$  should be  $(z, \tau z)$ .
- (243, -2): it should be noted that  $\Delta$  is the positive normed determinant function in  $E_1$ .
- (244, -11):  $F_1$  should be  $F$ .
- (245, 8): in the displayed equation,  $b$  should be  $b_1$ .
- (245, -10), (245, -5): “(8.53)” should be “(8.54)”.
- (246, 10), (246, -10): in problems 3 and 5, “plane” should be “plane  $E$ ”.
- (247, -1):  $f$  should be defined so that  $f(0) = 0$ .
- (248, -8): in problem 16,  $E$  should be  $\mathbb{H}$ .

## Chapter XI

- (328, 14), (328, -7): the proof of the equality condition for the triangle inequality does not properly account for the case  $x = 0$ ; in particular, “ $y = \lambda x$ ” should be “ $y = \lambda x$  or  $x = \lambda y$ ”.
- (332, 11): in problem 3, the displayed equation should be

$$(z_1, z_2) = ((x_1, x_2) + (y_1, y_2)) + i((x_2, y_1) - (x_1, y_2))$$

and it should be noted that  $z_k = x_k + i y_k$ .

- (334, -7): “automorphism” should be “involution”.
- (335, -12): in the displayed equation,  $\Phi$  should be  $\varphi$ .
- (335, -5): the concept of a conjugate determinant function is not defined.
- (335, -3): in the displayed equation,  $\bar{x}_p$  should be  $\bar{x}_n$ .
- (336, -8): in the displayed equation,  $\sum_{\mu, \nu}$  should be  $\sum_{\mu, \lambda}$ .
- (337, -16): “normal mapping” should be “normal mappings”.
- (337, -10): in the displayed equation,  $\lambda$  should be  $\lambda_1$ .
- (339, 8): it must be assumed that  $e \neq 0$ .
- (339, 14): in problem 1, “bilinear” should be “binary”.

## Chapter XII

- (352, -11): the degree(s) of the zero polynomial should be clarified.
- (353, -8):  $A \in \Gamma[t]$  should be  $A = \Gamma[t]$ .
- (356, 3): in problem 1(d), it is false that  $f_1(g) = f_2(g)$  implies  $f_1 = f_2$ .
- (356, 9): in problem 2, it is false that if  $f_1$  and  $f_2$  are two linear mappings on  $\Gamma[t]$  with  $d \circ f_1 = \iota = d \circ f_2$  then there is a fixed  $\alpha \in \Gamma$  with  $(f_1 - f_2)f = \alpha$  for all  $f \in \Gamma[t]$ .

- (358, 6): “second polynomial” should be “second monic polynomial”.
- (358, 10): in subsection 12.6, greatest common divisor and relative primality should be defined for arbitrary nonzero polynomials, and the rest of the chapter updated to reflect this.
- (358, -13): irreducible polynomials should be required to be nonconstant, and the rest of the chapter updated to reflect this.
- (358, -12): “ar” should be “or”.
- (358, -4): in subsection 12.7, least common multiple should be defined for arbitrary nonzero polynomials and the rest of the chapter updated to reflect this.
- (360, -5):  $g \leq 0$  should be  $0 \leq g$ .
- (361, 3): it should be clarified that  $k_i \geq 1$ .
- (363, 8): in the proof of Proposition IV, it must be assumed (without loss of generality) that  $f$  and  $g$  are monic.
- (363, -4): “common factor” should be “nonconstant common factor”.
- (364, 10): in the displayed equation,  $n$  should be  $r$ .
- (366, -3): “commutative” should be deleted.
- (367, 14), (367, 16): in Examples I and II, it must be assumed that  $A \neq 0$ .
- (367, -13): in the displayed equation,  $\Phi(f)$  and  $\pi(f)$  should be swapped.
- (367, -3): “Proposition V” should be “Proposition I”.
- (369, 8), (369, 13): the Q’s should all be  $\mathbb{Q}$ ’s.

## Chapter XIII

- (387, 5): “Proposition II” should be “Proposition I”.
- (387, 7): the displayed equation should be

$$K(f) = K(1) = 0$$

- (387, 15): “non-zero” should be deleted.
- (388, -11):  $\varphi_0$  should be  $\varphi$ .
- (389, 4): in problem 2, “Given” should be “Give”.
- (389, -15): in problem 6, “minimal polynomial” should be “minimum polynomial”.
- (389, -11): in problem 6, in the displayed equation, it should be clarified that the  $\lambda_v$  are not all zero.
- (390, 8): in problem 8, in the displayed equation,  $\alpha_v$  should be  $\alpha_\nu$ .
- (390, -13): in problem 13,  $L(E, E)$  should be  $L(E; E)$ .
- (393, -7): “vektor” should be “vector”.
- (395, 14): in problem 1, “stable subspace” should be “non-zero stable subspace”.
- (396, -17): in problem 7, “unique” should be deleted.
- (397, -1): “ $f$  irreducible” should be “ $f$  monic irreducible”.
- (398, 4):  $a \in K(f)$  should be  $a \in K(h)$ .
- (398, 7): “Thus  $K(g) = K_\mu$ . Now it follows from (13.28) that  $a \in K_\mu$ .” should be “Thus  $g = \mu$ . Now it follows that  $h \in I_\mu$ .”
- (398, 10): it should be clarified that this is the prime decomposition of  $\mu$  (so the  $f_i$  are distinct monic and  $k_i \geq 1$ ).
- (400, -3): “Proposition III” should be “Proposition II”.
- (402, 12): it should be clarified that  $m = \deg \mu$ . Also, for this reason,  $\mu$  should not be used as the index variable!
- (402, 13):  $1 \dots m - 1$  should be  $0 \dots m - 1$ .
- (402, -13):  $\pi^*$  should be  $\pi$ .
- (403, 11): in the displayed equation,  $F_i$  should be  $F_j$ .

- (403, 16): in the displayed equation,  $F_r$  should be  $F_j$ , and it should be clarified that  $\dim F_j > 0$ .
- (403, -15):  $j = 1$  should be  $r = 1$ .
- (403, -12): “ $\varphi$  is cyclic” should be “ $E$  is cyclic”.
- (403, -8): in part (i) of Theorem I, it should be clarified that this is the prime decomposition of  $\mu$  (so  $f$  is monic and  $k \geq 1$ ).
- (404, 10):  $\mu \mid f^{k_i}$  should be  $\mu \mid f^{k_1}$ .
- (404, 13): “13.10” should be “13.11”.
- (404, -1): “ $f$  is an irreducible” should be “ $f$  is a monic irreducible”.
- (406, 3): “irreducible” should be deleted.
- (406, 5): “13.12” should be “13.13”.
- (407, 5): in the matrix, there should be a 0 in the upper right corner.
- (407, -16): “13.12” should be “13.13”.
- (407, -11): it should be clarified that this is the prime decomposition of  $\mu$  (so  $f$  is monic and  $k \geq 1$ ).
- (407, -10): “irreducible subspaces” should be “non-zero irreducible subspaces”.
- (407, -8): “13.12” should be “13.14”.
- (410, -10):  $\psi$  should be  $\psi_i^j$ .
- (412, -10): it should be clarified that this is the prime decomposition of  $f$  (so the  $f_i$  are distinct monic and  $m_i \geq 1$ ).
- (413, 9): in problem 1, “ $\varphi \in A(E; E)$ ” should be moved from (i) to (ii).
- (414, 3): in problem 6(a), “ $\varphi_j$  is cyclic” should be “ $F_j$  is cyclic”.
- (414, 18): in problem 8(a),  $K(f^{k-1})$  should be  $K(f^k)$ .
- (416, -9): it should be clarified that  $j_1 \neq j_2$ .

- (417, 5): the first displayed equation should be

$$f_1^{l_2}(\varphi)f_1^{l_1-l_2}(\varphi)y_1 = 0$$

- (418, 5):  $\tau$  need not be a surjection.
- (419, 5):  $f \mid \mu$  should be  $\mu \mid f$ .
- (420, 13):  $F_0$  should be  $F$ .
- (422, 5): “subalgebra of (cf. example I, see 5.2)” should be “subalgebra of  $A(E; E)$  (cf. example I, sec. 5.2)”.
- (422, -2):  $\varphi$  should be  $\varphi_i$ .
- (423, 5):  $E_j$  should be  $F_j$ .
- (425, -6): “13.14” should be “13.15”.
- (427, 15): “12.17” should be “12.16”.
- (427, 16): “decompositions” should be “decomposition”.
- (427, -7): “12.14” should be “12.13”.
- (428, 2): it should be clarified that this is the prime decomposition of  $\mu$  (so the  $f_i$  are monic).
- (428, -14): the displayed equation should be

$$E = F_1 \oplus \cdots \oplus F_s$$

- (429, 3):  $g$  should be  $q$ .
- (429, 5): “over  $\Delta[t]$ ” should be “in  $\Delta[t]$ ”.
- (429, -9), (429, -8): in Theorem I, it should be clarified that the prime decomposition of the minimum polynomial of  $\varphi$  is  $f_1^{k_1} \cdots f_r^{k_r}$ .
- (429, -3):  $\psi_N = \psi_N$  should be  $\psi_N = \varphi_N$ .
- (430, 12): “see. 13.15” should be “sec. 13.14”.

- (431, 4): “polynomial” should be “polynomials”.
- (432, 3):  $\varphi_i : E \rightarrow E$  should be  $\varphi_i : E_i \rightarrow E_i$ .
- (432, 5):  $E$  should be  $E_i$ .
- (432, 7): in the displayed equation,  $E$  should be  $E_i$  (twice).
- (432, -1): it should be clarified that this is the prime decomposition of  $\mu_1$  (so the  $f_i$  are monic).
- (437, -11), (439, 8): in Theorems I and II, the definition of *homothetic* must be loosened slightly to allow for possibly negative scalars.

# Multilinear Algebra

Errata are currently only listed for Chapters 1, 2 (except § 10–17), 3–4, and 5. Minor purely typographical inconsistencies are not listed.

## Chapter 1

- (4, 11): in problem 3, it should be assumed that  $E \neq 0$ .
- (4, -7): in problem 5(b), the claim is false.
- (7, -5): in the displayed equation,  $\lambda_\alpha^\vee \in \Gamma$  should be  $\lambda_\vee^\alpha \in \Gamma$ .
- (11, -10): “satisfis” should be “satisfies”.
- (11, -3): The function name  $f$  should not be reused.
- (12, 9): in problem 1,

$$(\xi^1, \dots, \xi^n) \times (\eta^1, \dots, \eta^m)$$

should be

$$((\xi^1, \dots, \xi^n), (\eta^1, \dots, \eta^m))$$

- (12, -6): in problem 6(a), the tensor product of linear maps has not yet been defined.
- (13, 15): “ $\varphi_1$  factors over  $\otimes$ ” should be “ $\varphi_1$  factors over  $\otimes'$ ”.
- (14, 6): in the displayed equation,  $y \in F_1$  should be  $y_1 \in F_1$ .
- (14, -5):  $\tilde{E} = \otimes_\alpha E_\alpha$  should be  $\tilde{E} = \oplus_\alpha E_\alpha$  and  $\tilde{F} = \otimes_\beta F_\beta$  should be  $\tilde{F} = \oplus_\beta F_\beta$ .
- (14, -1):  $\pi_\alpha : E \rightarrow E_\alpha$  should be  $\pi_\alpha : \tilde{E} \rightarrow E_\alpha$ .
- (17, 11): in the displayed equation, it should be noted that  $x_\alpha \in E_\alpha$  and  $y_\beta \in F_\beta$ .
- (22, -9): in Corollary III,  $L(E \otimes F; E' \otimes F)$  should be  $L(E \otimes F; E' \otimes F')$ .



- (22, -8): in Corollary III, the claim is not shown in subsection 1.27, where it is also assumed that  $E'$  and  $F'$  are finite-dimensional.
- (24, 5): in the displayed equation,  $\tilde{\psi} \otimes \psi$  should be  $\tilde{\psi} \circ \psi$ .
- (25, 9): in problem 1(a), the “only if” part of the claim is false.
- (30, 13), (31, 11): it is false that  $\Phi \otimes \Psi$  is nondegenerate only if  $\Phi$  and  $\Psi$  are nondegenerate.
- (34, -2): it should be assumed that  $x^* \neq 0$ .
- (36, 12):  $\varphi \times \psi$  should be  $(\varphi, \psi)$ .
- (36, 14): in the displayed equation,  $\dim(F; F')$  should be  $\dim L(F; F')$ .
- (37, 3): in the commutative diagram,  $\otimes$  should be the linear map induced by  $\otimes$ .
- (37, 16):  $(\varphi \times \psi)$  should be  $(\varphi, \psi)$ .
- (37, -3):  $(\alpha, \beta)$  should be  $\alpha \otimes \beta$ .
- (38, 3):  $F : A \times A \rightarrow L(A; A)$  should be  $F : A \otimes A \rightarrow L(A; A)$ .
- (38, 8):  $L(A \otimes A)$  should be  $A \otimes A$ .
- (38, -5): in subsection 1.30, it should be assumed that  $E \neq 0$ .
- (39, 3):  $\Omega$  should be the bilinear map induced by  $\Omega$ .
- (40, 5): in problem 2, it should be assumed that  $E, F \neq 0$ .
- (40, -5): in problem 3, it should be assumed that  $E$  is oriented.

## Chapter 2

- (42, 18): if  $A \neq 0$ , it must be assumed that  $B \neq 0$  for  $\varphi$  to be injective.
- (43, -1): in (2.12),  $\Omega_{A \otimes B}$  should be  $\Omega_{A \otimes A}$ .

- (48, 14): in Proposition 2.9.1, it should be clarified that  $\Omega_A$  is an antiderivation with respect to  $\omega_A$ , and similarly  $\Omega_B$  with respect to the canonical involution  $\omega_B$ , and  $\Omega_{A \widehat{\otimes} B}$  with respect to  $\omega_{A \otimes B}$  (which might as well be written  $\omega_{A \widehat{\otimes} B}$ ).
- (48, -9):  $\Omega_{A \otimes B}$  should be  $\Omega_{A \widehat{\otimes} B}$ .
- (49, -6): in the first equality of the displayed equation,  $(-1)^{p+q'}$  should be  $(-1)^{p+p'}$ .
- (49, -2): in the displayed equation,  $\Omega$  should be  $\theta$ .

## Chapter 3

- (60, -3): it should be noted that  $v \in \bigotimes^q E$ .
- (61, 3):  $\dim E = 1$  should be  $\dim E \leq 1$ .
- (62, 4): in the displayed equation, it should be noted that  $u_p \in \bigotimes^p E$  and  $v_q \in \bigotimes^q E$ .
- (63, -2): in **T**, “unit element” should be “unit element  $e$ ”.
- (64, -7): in the uniqueness theorem, proof, and remarks after, it should be noted that the homomorphisms preserve the unit.
- (65, 1): in the proof of the uniqueness theorem, the unit element is also needed to generate  $U$ .
- (68, 8): in problem 2,  $\lambda_1, \dots, \lambda_p = 1$  should be  $\lambda_1 \cdots \lambda_p = 1$ .
- (68, 9): in problem 3, it should be asked to prove that  $\text{tr}(\varphi\psi) = \text{tr}(\psi\varphi)$ .
- (68, -8): in the displayed equation, it should be noted that  $u^{*p} \in \bigotimes^p E^*$  and  $v_p \in \bigotimes^p E$ .
- (69, -10): the reference to (3.8) should be deleted.
- (70, -2): (3.14) is not used to establish the displayed equation.
- (72, 6): “noncommutative” should be “noncommutative in general”.

- (72, -5):  $\Phi_j^i(\omega)$  should be  $C_j^i(\omega)$ .
- (74, 1): in many problems in this subsection, the order of dual spaces should be swapped to be consistent with the rest of the chapter; for example,  $\otimes(E, E^*)$  should be  $\otimes(E^*, E)$ .
- (74, -13): in problem 6, the result does not hold for arbitrary fields  $\Gamma$ .
- (74, -9): in problem 7, it should be assumed that  $E$  is finite-dimensional.
- (74, -5): in problem 8, it should be assumed that  $E$  is finite-dimensional.
- (75, 10): in problem 11(a), the result does not hold for arbitrary fields  $\Gamma$ .
- (77, 9): in several problems in this subsection, it could be made clearer that  $E$  is assumed to be finite-dimensional.
- (77, 15): in problem 2(a),  $g \otimes g^*$  should be  $g^* \otimes g$  to be consistent with notational conventions in the rest of the book.
- (77, -3): in problem 5, the scalars  $g_{\nu\mu} = (e_\nu, e_\mu)$  should be defined.
- (78, -6): “noncommutative” should be “noncommutative in general”.
- (79, 3): it should be noted that  $\Phi \in T^p(E)$ .
- (79, 6): it should be noted that  $\Phi \in T^p(E)$  and clarified how  $i_\nu(h)$  acts if  $\nu > p$ .
- (79, -8), (79, -6): the definitions of the operators  $i_A(h)$  and  $i_S(h)$  given in this subsection are inappropriate for use with the algebras of skew-symmetric and symmetric multilinear functions, respectively. See below.
- (80, 11): in the displayed equation,  $E^{*\nu}$  should be  $E^*$ .
- (82, 6): in the displayed equation,  $\langle f_1 \cdots f_p \Psi \rangle$  should be  $\langle f_1 \cdots f_p, \Psi \rangle$ .
- (82, -10): in the displayed equation,

$$\Psi(x_1, \dots, x_r, x_1^*, \dots, x_s^*)$$

should be

$$\Psi(x_{p+1}, \dots, x_{p+r}, x_{q+1}^*, \dots, x_{q+s}^*)$$

- (82, -5): “Section 1.20” should be “Section 1.27”.
- (83, -1):  $y^{*j}$  should be  $y_j^*$ .

## Chapter 4

- (85, 1): it should be assumed that  $i < j$ .
- (87, 12): in the displayed equation,  $u = x^{*1} \otimes \cdots \otimes x^{*p}$  should be  $u^* = x^{*1} \otimes \cdots \otimes x^{*p}$ .
- (90, 6): “restriction of  $\pi_X$ ” should be “restriction of  $\pi_A$ ”.
- (91, 2): in the displayed equation,  $X(E)$  should be  $X^p(E)$ .
- (93, 11): in (4.31),  $\pi_r v$  should be  $\pi_S v$ .
- (95, -5): in (4.36),  $\otimes E^*$  should be  $\otimes^p E^*$  and  $\otimes E$  should be  $\otimes^p E$ .

## Chapter 5

- (97, 15): it should be assumed that  $i < j$ .
- (99, -15): in problem 2(a), in the displayed equation,  $x_p$  should be  $x_n$ .
- (99, -10): in problem 2(b), in the definition of  $\varphi$ ,  $(-1)^{n-1}$  should be  $(-1)^{n-i}$ .
- (100, -5): “ $p$ -linear mappings” should be “skew-symmetric  $p$ -linear mappings”.
- (101, -12): “it reduces to zero” should be “ $h$  reduces to zero”.
- (101, -2): “ $p$ th exterior algebra” should be “ $p$ th exterior power”.
- (105, -13): it should be assumed that  $(\epsilon x)^2 = 0$  for all  $x \in E$ .
- (106, 4): the codomain of  $g$  should be  $\wedge E^*$ .
- (108, 3): in problem 3, the matrix of a 2-vector has not been defined.

- (108, 4): in problem 4, the rightmost matrix in the Lagrange identity should be

$$\begin{pmatrix} \eta_{v_1}^1 & \cdots & \eta_{v_p}^1 \\ \vdots & & \vdots \\ \eta_{v_1}^p & \cdots & \eta_{v_p}^p \end{pmatrix}$$

Also, in the hint, the order of the two factors in the scalar product should be swapped, to be consistent with the rest of the chapter.

- (108, -2):  $\varphi_\wedge, \wedge E \rightarrow \wedge F$  should be  $\varphi_\wedge : \wedge E \rightarrow \wedge F$ .
- (110, 6), (110, 14):  $v^* \in \wedge E^*$  should be  $v^* \in \wedge F^*$ .
- (112, -9): (5.26) is not used to establish the displayed equation.
- (114, -8): “an antiderivation” should be “a homogeneous antiderivation”.
- (115, 4): “ $\alpha$ -Antiderivations” should be “ $\alpha_\wedge$ -Antiderivations”.
- (115, 6): it should be clarified that  $\varphi$  and  $\psi$  are homogeneous with respect to the trivial gradations in  $E$  and  $F$ , respectively, where all elements are homogeneous of degree 1.
- (115, 8): it should be clarified that  $\omega_E = \omega$ .
- (115, 12): in (5.36), it should be clarified that  $\Omega_E = \Omega$  and  $\Omega_F(\psi)$  is the induced antiderivation with respect to  $\omega_F$  in  $\wedge F$ .
- (115, 14), (115, 15): “ $\alpha$ -antiderivations” should be “ $\alpha_\wedge$ -antiderivations”.
- (115, -4): in problem 1(b), in the displayed equation,  $\pi_F$  should be  $\pi_E$ .
- (116, 4): in problem 4, the claim is false.
- (116, 13), (116, 15): in problem 6, “degree 1” should be “degree  $-1$ ”, and  $\tilde{\Omega}u = p \cdot \Omega u$  should be  $\tilde{\Omega}u = p^{-1} \cdot \Omega u$  for  $p > 0$ .
- (116, -8): in problem 8, the transformations  $(\psi - \lambda\iota)_\wedge$  and  $\psi_i$  should all be restricted to  $\wedge^n E$ . Also,  $\psi_0 = (-1)_n \iota$  should be  $\psi_0 = (-1)^n \iota$ .
- (117, 8): it must be assumed that the dual map exists.
- (117, 11): in the displayed equation, it should be noted that  $u^* \in \wedge E^*$ .

- (118, 13):  $\Omega(h)$  should be  $\Omega_h$ .
- (119, 8): in the proof of Proposition 5.14.2, it should be assumed that  $\nu \in \bigwedge^p E$ .
- (119, -4): in problem 3, it should be assumed that  $e_\nu, e^{*\nu}$  is a pair of dual bases of  $E, E^*$ .
- (120, 7): in problem 6, in the displayed equation, the order of the two factors in the scalar product should be swapped to be consistent with the rest of the chapter.
- (120, -1): it should be noted that  $f$  is homogeneous of degree zero by (5.42).
- (121, -10): in the displayed equation,  $\eta(i_1 x) = x$  should be  $\eta(i_1 x) = x \otimes 1$ .
- (121, -9): in the displayed equation,  $\eta(i_2 y) = y$  should be  $\eta(i_2 y) = 1 \otimes y$ .
- (121, -6):  $x \oplus y$  should be  $x + y$ .
- (123, -3): in the displayed equation,  $k \in E$  should be  $k \in F$ .
- (123, -1): in (5.50),  $h \oplus k$  should be  $h + k$ .
- (125, 7): in the displayed equation,  $u_\rho \in E_\rho$  should be  $u_\rho \in \bigwedge E_\rho$ .
- (125, -8): it should be assumed that  $k_i \geq 1$ .
- (126, -2): in problem 2(b), the top arrow of the diagram should be labeled as  $\varphi_\wedge$ .
- (127, 6): it should be noted that  $\nu_q \in \bigwedge^q E$ .
- (127, -9): “and hence  $p = q$ ” should be “and hence  $p = q$  if  $A, B \neq 0$ ”.
- (129, 10):  $(\varphi'_\wedge)_\wedge$  should be  $\varphi'_\wedge$ .
- (130, -13): in the Corollary, “If  $E$  has finite dimension” should be “If  $E$  has finite dimension  $n$ ”.
- (130, -6): in the proof of the second part of the Corollary, it should be noted that  $I^q \neq 0$ .
- (131, 5):  $U \in \bigwedge E$  should be  $U \subset \bigwedge E$ .

- (131, -4): in (5.62),  $b_i \in H$  should be  $b_i \in \wedge H$ , as in (5.63).
- (132, 2): in the displayed equation,  $v_j \in F$  should be  $v_j \in \wedge F$ .
- (132, 5):  $u'$  should be the sum of all terms in the equation with parts from  $v_j$  of positive degree.
- (132, 7): in the displayed equation,  $b_j$  should be  $b'_j$ , a scalar multiple of  $b_j$ .
- (132, 11): it should be assumed that  $p \leq m$ .
- (135, 11): in Proposition 5.27.2, it should be clarified that  $\varphi$  is homogeneous with respect to the trivial gradation in  $E$ , where all elements are homogeneous of degree 1.
- (135, 15): in the displayed equations, it should be noted that  $u_1, u_2 \in \wedge^k E$ .
- (136, 15): “by the Corollary to Proposition 5.27.1” should be “by Proposition 5.27.2”.
- (136, -7): “ideal” should be “ideal in  $\wedge E$ ”.
- (137, 4): in problem 6, “be linear transformation” should be “be a linear transformation”.
- (138, -6):  $\wedge(F^{*\perp}) = I_{F^*}$  should be  $\wedge(F^{*\perp}) = (I_{F^*})^\perp$ .
- (138, -3): in Proposition 5.28.2, it should be assumed that  $A \neq 0$ .
- (139, 6):  $u^* \in \wedge E$  should be  $u^* \in \wedge E^*$ .
- (141, -8): in Proposition 5.30.1(B),  $A(E)$  (which should be  $A^*(E)$ ) has not been defined.
- (141, -1): in the proof of Proposition 5.30.1(B), the first and second equalities in the first displayed equation should be swapped.
- (142, 13): in the displayed equation, it should be noted that  $\Phi \in A^p(E)$ .
- (142, -1): in the displayed equation, it should be noted that  $\Phi \in A^p(E)$  and  $\Psi \in A^q(E)$ , and in the third equality,  $\theta^T(\varphi) \cdot \Psi$  should be  $\theta^T(\varphi)\Phi \cdot \Psi$ .

- (143, 1): in subsection 5.32, the substitution operator  $i_A(h)$  should be re-defined on  $T^p(E)$  as

$$i_A(h) = \frac{1}{p} \sum_{v=1}^p (-1)^{v-1} i_v(h)$$

The missing factor of  $1/p$  introduces errors throughout the subsection.

- (143, -3): in the displayed equation, the tensor product symbols should be function product symbols.
- (145, 4): in the displayed equation,  $i(h)$  should be  $i_A(h)$ .
- (145, -11): in the heading of subsection 5.33,  $T^\bullet(E)$  should be  $A^\bullet(E)$ .
- (145, -1): in the displayed equation,  $v \in X^q(E^*)$  should be  $v \in X^q(E^*)$ .
- (146, 12):  $\varphi^A : A(E) \leftarrow A(F)$  should be  $\varphi^A : A^\bullet(E) \leftarrow A^\bullet(F)$ .
- (146, -16): “mappings” should be “functions”.
- (146, -8): in the displayed equation,  $A^p(E)$  should be  $T^p(E)$  and  $A_p(E)$  should be  $T_p(E)$ .
- (147, 9): the proof should be

$$\begin{aligned} \langle \Phi, x_1 \wedge \cdots \wedge x_p \rangle_A &= \frac{1}{p!} \langle \Phi, x_1 \wedge \cdots \wedge x_p \rangle \\ &= \langle \Phi, A(x_1 \cdot \dots \cdot x_p) \rangle \\ &= \langle A\Phi, x_1 \cdot \dots \cdot x_p \rangle \\ &= \langle \Phi, x_1 \cdot \dots \cdot x_p \rangle \\ &= \Phi(x_1, \dots, x_p) \end{aligned}$$

- (147, -1): in the displayed equation,  $A(x_1 \otimes \cdots \otimes x_p)$  should be  $A(x_1 \cdot \dots \cdot x_p)$  and  $f_1(x_{\sigma(p)})$  should be  $f_1(x_{\sigma(1)})$ .

## References

- [1] Greub, W. *Linear Algebra*, 4th ed. Springer, 1975.
- [2] Greub, W. *Multilinear Algebra*, 2nd ed. Springer, 1978.