Errata from *Introduction to Real Analysis*

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Introduction

This document contains errata from [1]. Locations in the text are indicated by coordinates (p, n), where p is a page number and n is a line number on page p. Positive line numbers count from the top of the page, whereas negative line numbers count from the bottom of the page. Displayed equations, diagrams, and figures are counted as single lines.

Font styling errors that do not cause mathematical confusion (for example, f instead of $\boldsymbol{f}$, or $C[a,b]$ instead of $\mathscr{C}[a,b]$) and non-mathematical errors are not listed. Some of the items below refer to unclear or incomplete passages and may not be errors.

Chapter 1

- (10, 12): it should be noted that countably infinite sets are infinite, under the definitions given.
- (11, 5): this example uses the principle of "strong" induction, whereas only the principle of "weak" induction was presented; it should be noted that these are equivalent.
- (11, 16): n_{n+1} should be n_{i+1}.

Chapter 4

- (50, -1): in Exercise 33, it is false that the series diverges if $r > 1$ (for a counterexample, see Example 16); however, the series diverges if the limit *inferior* is greater than 1.

Chapter 5

- (54, 17): in Proposition 5, $i, \ldots, n$ should be $i = 1, \ldots, n$.

Chapter 6

- (60, 6): $f(\boldsymbol{x})$ should be $f(\boldsymbol{x}_k)$.

Chapter 8

- (80, 11): in Theorem 11, "K is a compact subset" should be "K is a nonempty compact subset".
- (80, -12): "compact set" should be "nonempty compact set".
- (82, 15): in Exercise 2, "bounded above and closed" should be "nonempty, bounded above, and closed".

Chapter 10

- (103, 2): it should be added "except $f(0,0) = 0$".
- (104, 5): in the displayed equation, x_h should be x_i.

Chapter 11

- (129, -14): $\boldsymbol{A}_k = \boldsymbol{f}(\boldsymbol{x}_k)$ should be $\boldsymbol{A}_k = \boldsymbol{f}_k(\boldsymbol{x}_0)$.

Chapter 12

- (141, -3): x_{-1} should be x_{i-1}.

Chapter 13

- (152, -9): "$f(t) = t\sin(1/t)$" should be "$f(t) = t^2\sin(1/t^2)$ with $f(0) = 0$", since the former function is neither defined nor differentiable at $t = 0$.
- (156, 2): $\int_i |f|$ should be $\int_I |f|$.
- (159, -9): in the displayed equation, $S(g_i, \mathscr{D})$ should be $S(g_1, \mathscr{D})$.
- (162, -7): "any partial tagged division" should be "any γ-fine partial tagged division".
- (164, 8): in the displayed equation, it is not clear why $f - f(x)$ is absolutely integrable. This follows from Corollary 33 later in the chapter.
- (165, -5): in the displayed equation, $\sum_{i=1}^n |f|$ should be $\sum_{i=1}^n \int_{x_{i-1}}^{x_i} |f|$.
- (166, -8): in the displayed equation, the subscript I'_j should be I'_i.
- (166, -6): in the displayed equation, $\int_{I'_i} |f|$ should be $\left|\int_{I'_i} f\right|$.
- (169, -6): "continuous functions of bounded variation" is true, but should be "continuous increasing functions".

Chapter 14

- (182, 16): "unbounded" has not been defined in $\overline{\mathbb{R}}$, so it should be noted that intervals containing $\pm\infty$ are considered unbounded in $\overline{\mathbb{R}}$ in this book (despite the fact that under standard constructions of $\overline{\mathbb{R}}$ as a metric space, they are bounded).
- (189, 11): in Exercise 10, $f(n+1) - f(n) - \int_1^n f$ should be $\sum_{k=1}^n f(k) - \int_1^n f$.

Chapter 16

- (213, 11): $I \cup E^c$ should be $I \cap E^c$.

Chapter 17

- (219, -3): in Definition 1, $x_i \in I$ should be $x_i \in I_i$.
- (223, 4): in Lemma 9, $\overline{\mathbb{R}}^p$ should be $\mathbb{R}^p$ because compactness has not been defined in $\overline{\mathbb{R}}^p$, and if we adopt a standard definition then $\overline{\mathbb{R}}^p$ is compact, which is clearly not intended here based on the proof and the remarks following.
- (223, 10): "diameter" has not yet been defined.
- (224, 1): in Theorem 10, $\overline{\mathbb{R}}^p$ should be $\mathbb{R}^p$.
- (227, -2): in the proof of Corollary 13, it is not clear why

  $$\int_H \int_G f(x,y)\,dx\,dy = \lim \int_H \int_G f_n(x,y)\,dx\,dy$$

  Specifically, in the context of this proof it seems like we want to show that $\{\int_G f_n(x,y)\,dx\}$ is bounded above for each $y \in H$ and apply the monotone convergence theorem once to conclude that $f(\cdot,y)$ is integrable over G and $\int_G f(x,y)\,dx = \lim \int_G f_n(x,y)\,dx$ for each $y \in H$. But it is not clear why the sequence of integrals is bounded above.

Chapter 18

- (239, -2): in Exercise 3, $f * g$ is not necessarily integrable. For example, if $f(x) = 1$ and $g(y) = e^{-y^2}$, then $f * g(x) = \sqrt{\pi}$, which is not integrable. However, $f * g$ is continuous.

Chapter 21

- (264, 5): in Definition 11, "Let $E \subseteq S$" should be "Let $E \subseteq S$ be nonempty".
- (266, -3): in Example 21, it is not clear why $T(X) \subseteq X$ (specifically, why the convolution product of two continuous, absolutely integrable functions is continuous).

Chapter 23

- (286, -3): in Example 5, it is not clear why $K(f) \in \mathscr{C}[a,b]$. (Theorem 15.5 does not allow variable limits of integration.)
- (288, 13): in Exercise 3, in the displayed equation, "$\forall x, y \in S$" should be "$\forall x, y \in S$ with $x \neq y$".

Chapter 24

- (294, -13): it is not clear why $f \in E_n$, since we may have $t \notin (0, 1-1/n)$.

Chapter 25

- (304, 1): in Exercise 14, "Let K_i be compact" should be "Let K_i be nonempty and compact".
- (304, -4): in Exercise 29, "$\forall x, y \in S$" should be "$\forall x, y \in S$ with $x \neq y$".

Chapter 27

- (314, 17): $G \cap E_{\alpha_0} = \emptyset$ should be $G \cap E_{\alpha_0} \neq \emptyset$.
- (314, -9): in the displayed equation, $S_1 \cup \{y_2\}$ should be $S_1 \times \{y_2\}$.

Chapter 29

- (325, 7): in the displayed equation, the second $\boldsymbol{e}_j$ should be $\boldsymbol{e}_i$.
- (332, -14): "when $m = k$" should be "when $n = k$".
- (333, -5): the proof of the continuity of df in Proposition 17 is flawed because (assuming $df = d_1 f = d_2 f = 0$ outside of D) we have

$$\begin{aligned} df &: X \times Y \to L(X \times Y, Z) \\ d_1 f \circ I_1 &: X \to L(X, Z) \\ d_2 f \circ I_2 &: Y \to L(Y, Z) \end{aligned}$$

so the functional equation $df = d_1 f \circ I_1 + d_2 f \circ I_2$ makes no sense.

By Proposition 16,

$$df(x, y)(h, k) = d_1 f(x, y)(h) + d_2 f(x, y)(k)$$

for all $(x, y), (h, k) \in X \times Y$, so

$$df(x, y) = d_1 f(x, y) \circ P_1 + d_2 f(x, y) \circ P_2$$

for all $(x, y) \in X \times Y$, where $P_1 \in L(X \times Y, X)$ and $P_2 \in L(X \times Y, Y)$ are just the projections $P_1(h, k) = h$ and $P_2(h, k) = k$. Define $\mathscr{P}_1 : L(X, Z) \to L(X \times Y, Z)$ by $\mathscr{P}_1(\varphi) = \varphi \circ P_1$ and $\mathscr{P}_2 : L(Y, Z) \to L(X \times Y, Z)$ by $\mathscr{P}_2(\psi) = \psi \circ P_2$. Then

$$df = \mathscr{P}_1 \circ d_1 f + \mathscr{P}_2 \circ d_2 f \tag{1}$$

Now $\mathscr{P}_1$ is linear and

$$\|\mathscr{P}_1(\varphi)\| = \|\varphi \circ P_1\| \leq \|\varphi\| \|P_1\| = \|\varphi\|$$

so $\mathscr{P}_1$ is also continuous, and similarly $\mathscr{P}_2$ is linear and continuous. It follows from (1) that df is continuous.

Chapter 30

- (338, -9): it is not clear why $\varphi(0) = 0$ is required in the definition of S.
- (342, 5): in the displayed equation, $d_1 F$ should be $d_1 f$.

References

[1] DePree, J. D. and C. W. Swartz. *Introduction to Real Analysis,* 1st printing (hardcover). Wiley, 1988.