6.4 [Computational] **Freeze-in of particle species.** An interesting alternative to the usual freezeout production of relic particles, notably dark matter, is the freeze-*in* mechanism (reviewed in Bernal *et al.*, 2017). Here, unlike in the standard freezeout scenario, the dark-matter particle was never in thermal equilibrium. Rather, the dark-matter particle χ is produced from the decay of some heavier, Standard-Model particle σ, so that $\sigma \to \chi\chi$ and $\delta n_\chi = -2\delta n_\sigma$. The interaction is characterized by a very small rate $\Gamma \equiv \Gamma_{\sigma\to\chi\chi}$ which, unlike in the freezeout scenario, does not depend on the number density (i.e., it is some constant, possibly related to the mass of particles σ). For small enough Γ, the number density of dark-matter particles χ increases, then flattens off when the number density of σ particles becomes Boltzmann-suppressed ($n_\sigma \propto e^{-m_\sigma/T} \ll 1$). You will now demonstrate and briefly investigate the feasibility of this scenario.

(a) For the freeze-in scenario, the Boltzmann equation reads

$$\frac{1}{a^3}\frac{d(na^3)}{dt} = 2\Gamma h(t) n_{\sigma,\mathrm{eq}}(t),$$

where you can take $h(x) \simeq x/(x+2)$ in terms of our usual time variable $x \equiv m_\sigma/T$ below. Note that the density entering on the left-hand side is that of decay-product particles χ ($n \equiv n_\chi$), while the equilibrium density on the right-hand side is that of the decaying particles σ. And, as mentioned above, Γ is time independent.

Starting from this equation and using a derivation similar to that in the freezeout scenario that goes from Eq. (6.12) to Eq. (6.14), demonstrate that the equivalent of the latter equation in the freeze-in picture is

$$\frac{dY}{dx} = \lambda_1 x h(x) Y_{\mathrm{eq}}(x),$$

where λ_1 is some constant. *Hint:* Because I am not asking that you derive λ_1 in terms of Γ and the Hubble parameter at $m_\sigma = T$, all you need to do is track the x-dependence.

(b) Plot $Y(x)$ for $\lambda_1 = 10^{-6}, 10^{-8}, 10^{-10}$. Take a smallish initial value, say $Y(x_0) = 10^{-20}$. For the curves to look nice, start integrating at $x_0 = 0.01$, but plot only from $x = 0.1$ to $x = 100$. As in Problem 6.3, it may be easier to integrate $d \ln Y / d \ln x$. Also plot $Y_{\text{eq}}(x)$, thus again mimicking Fig. 6.1.

(c) Comparing the figure you generated in part (b) and Fig. 6.1: what is the key difference between the freezeout and freeze-in scenarios as the annihilation/decay rate Γ increases? Explain this qualitatively.