

Network Theory for the Social Sciences in Python

Methods Workshop: Social Sciences PhD Program (2025/2026)

4. Dynamics on Networks



https://github.com/blas-ko/uc3m_networks_workshop_2025

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Summary

1. Introduction
2. Models of dynamics on networks
 - Random walker
 - Independent Cascades
 - Epidemiological models (SIR)
3. Maximizing/minimizing spreading on networks
 - Immunization strategies
 - Seeding strategies
4. Complex contagion
5. Summary

Summary

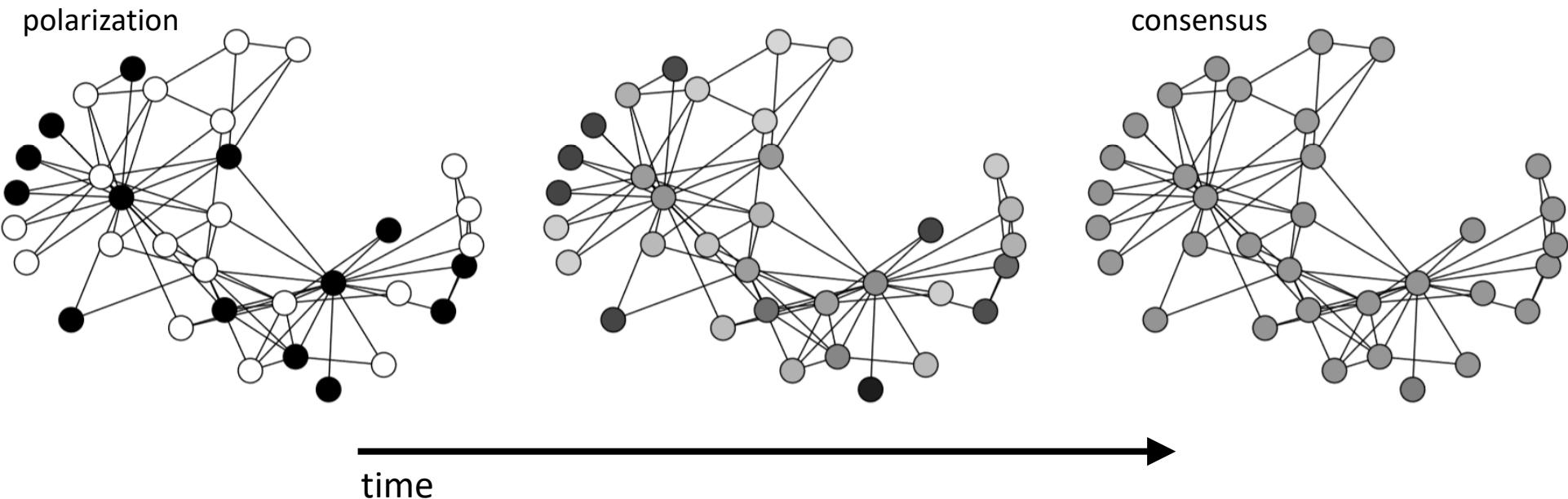
1. Introduction
2. Models of dynamics of networks
 - SI model
 - SIR model
3. Spreading on networks
4. Maximizing/Minimizing spreading
 - Immunization strategies in epidemic spreading
 - Seeding strategies in information spreading
5. Complex contagion

1 Introduction

Introduction

We have seen how networks may change over time.

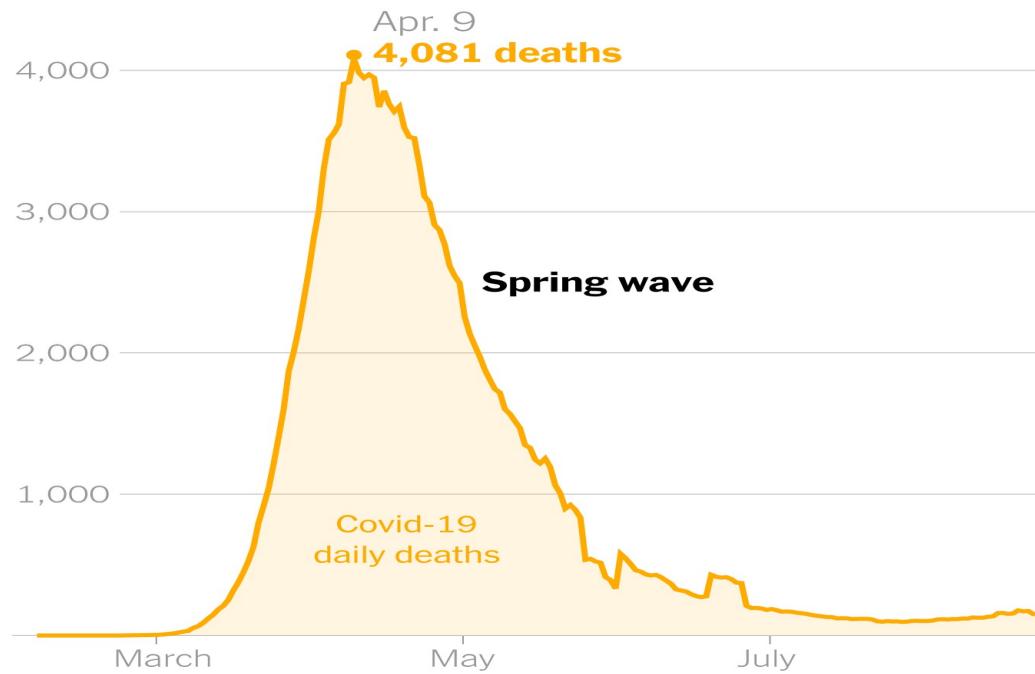
However, even for a fixed network structure
dynamical processes might unfold in them



Introduction

A large range of important problems in our society have to do with **spreading**: ideas, viruses, behavior, information, etc.

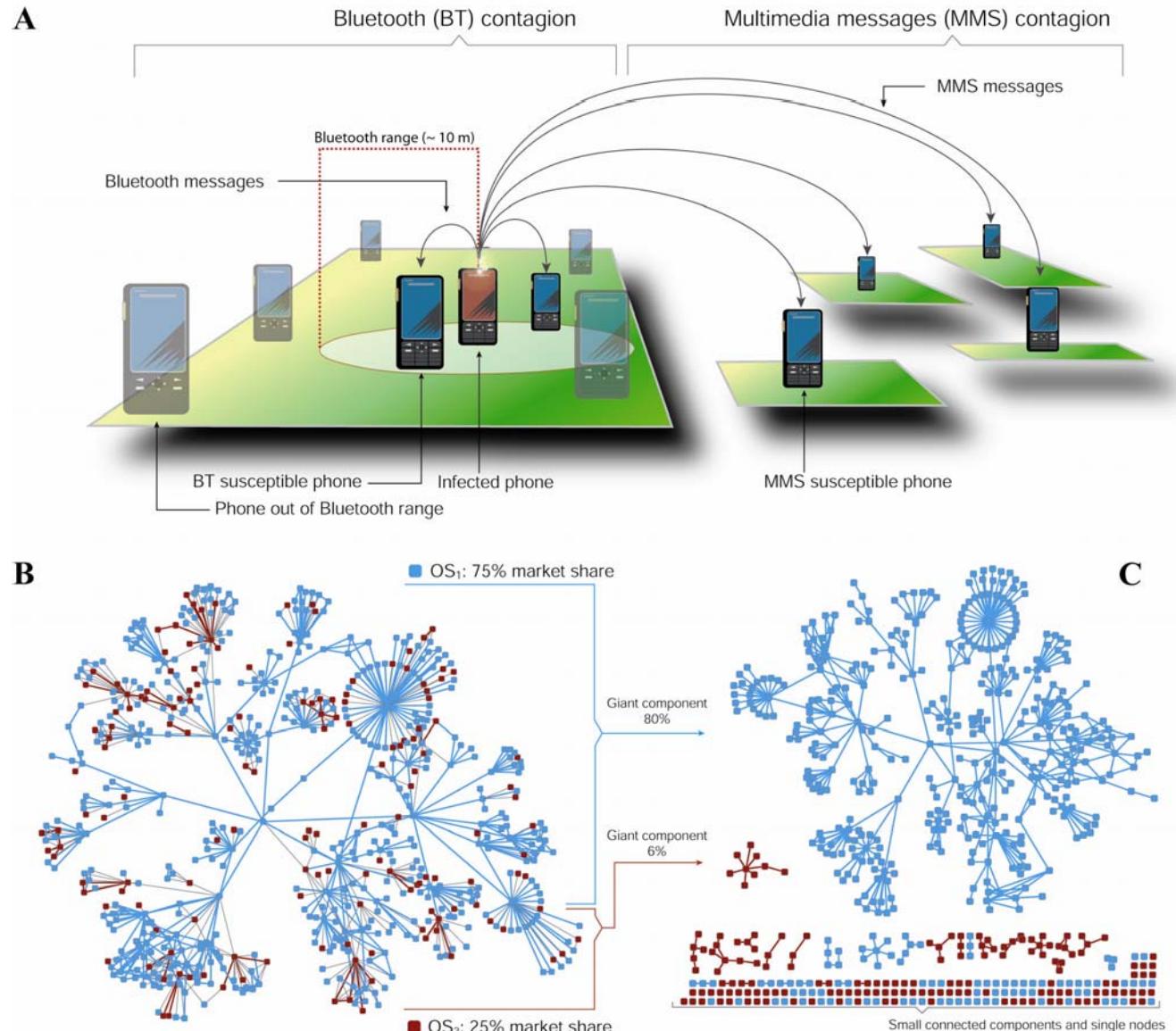
- What spreads and how it spreads is different in each case.
- **Example: Pathogens in diseases** like Influenza, SARS (COVID), tuberculosis, Ebola, AIDS, etc. They have different transmission mechanisms.



Jang, S., Han, S. H. & Rhee, J.-Y. Cluster of Coronavirus Disease Associated with Fitness Dance Classes, South Korea. *Emerg Infect Dis* **26**, 1917–1920 (2020).

Introduction

Computer:
viruses/malware



Wang, P., González, M. C., Hidalgo, C. A. & Barabási, A.-L. Understanding the Spreading Patterns of Mobile Phone Viruses. *Science* 324, 1071-1076 (2009).

Introduction

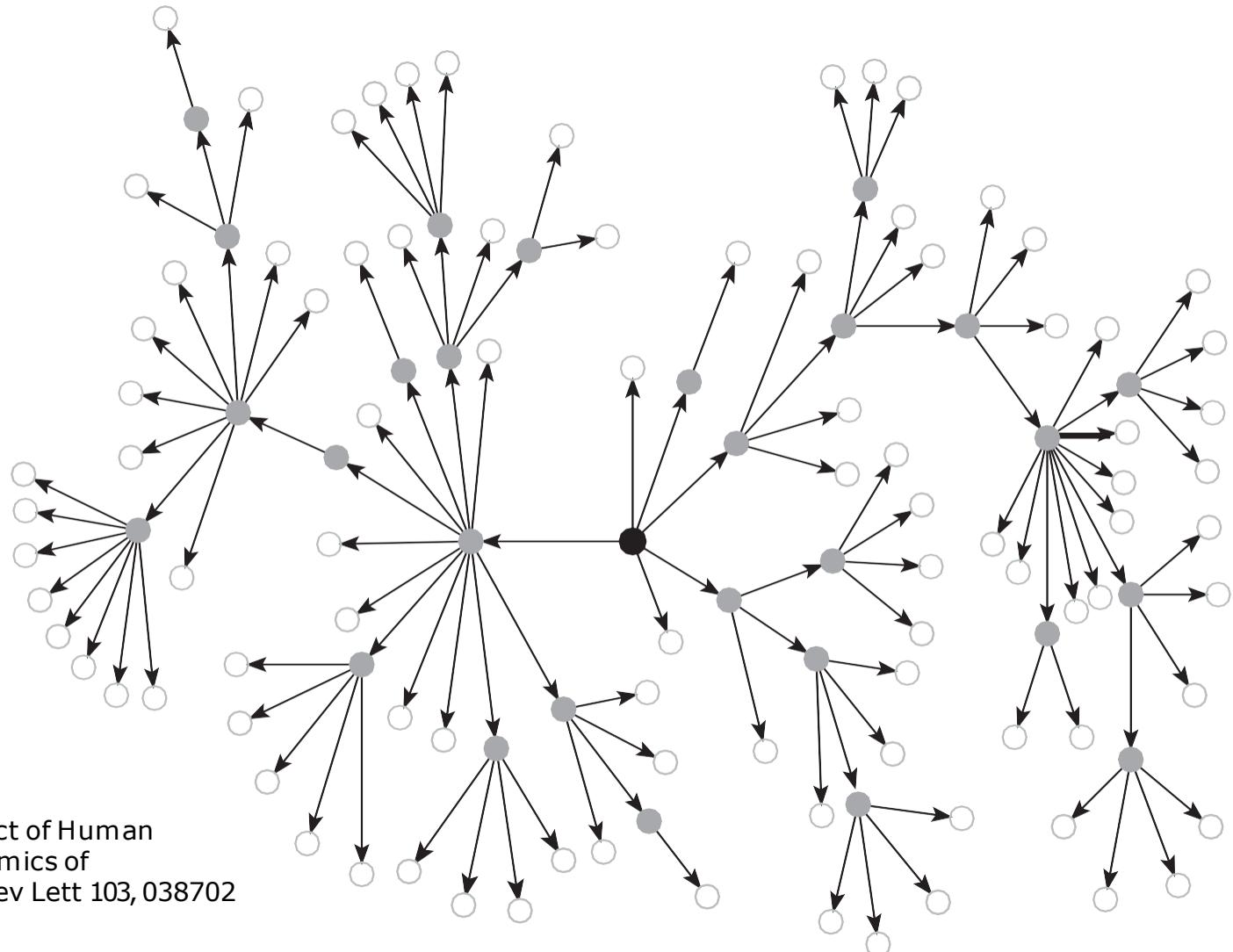
Information:
fake news

Vosoughi, S., Roy, D. & Aral, S. The spread of true and false news online. *Science* 359, 1146–1151 (2018).



Introduction

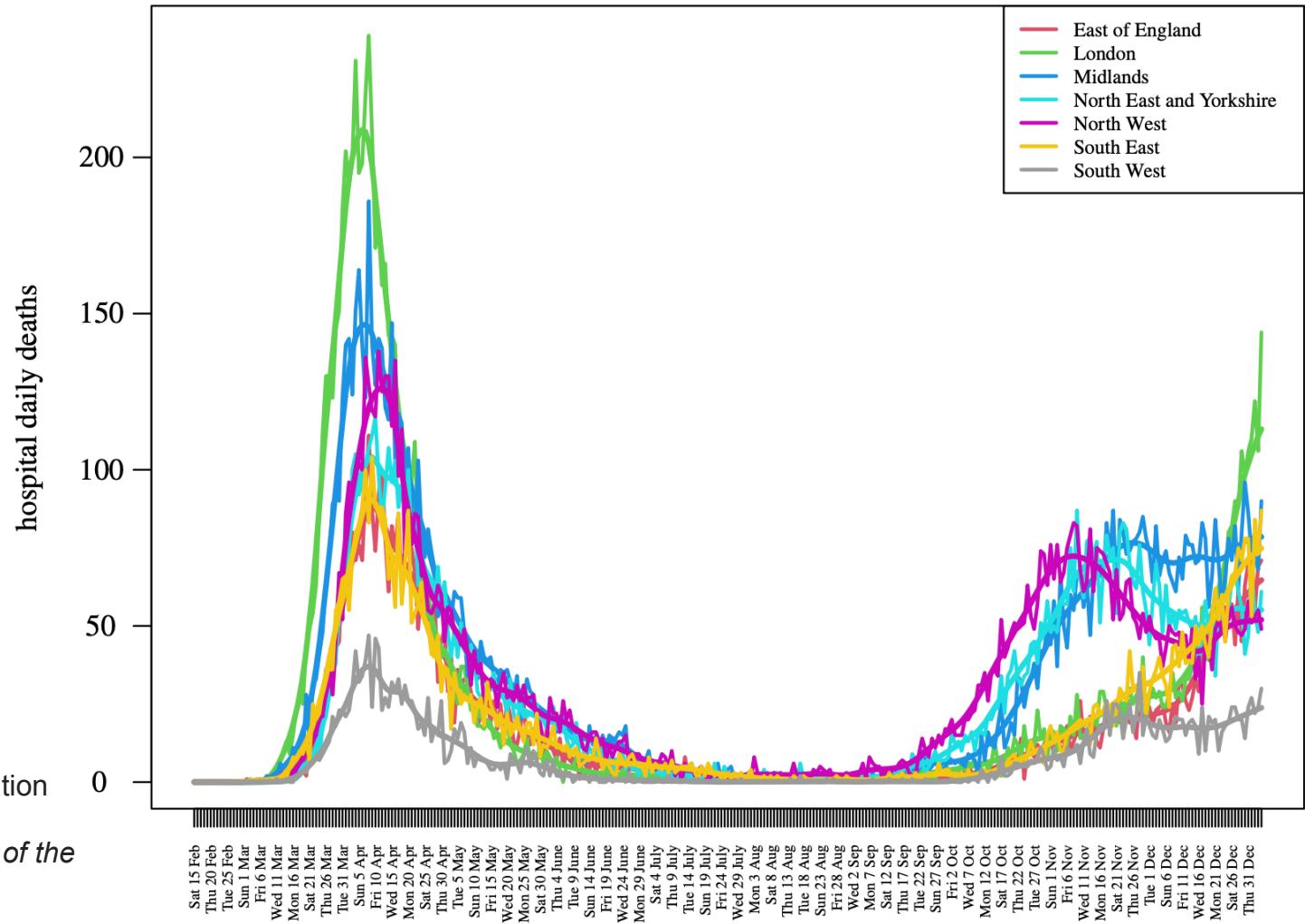
Information: Viral marketing



Iribarren, J. L. & Moro, E. Impact of Human Activity Patterns on the Dynamics of Information Diffusion. Phys Rev Lett 103, 038702 (2009).

Introduction

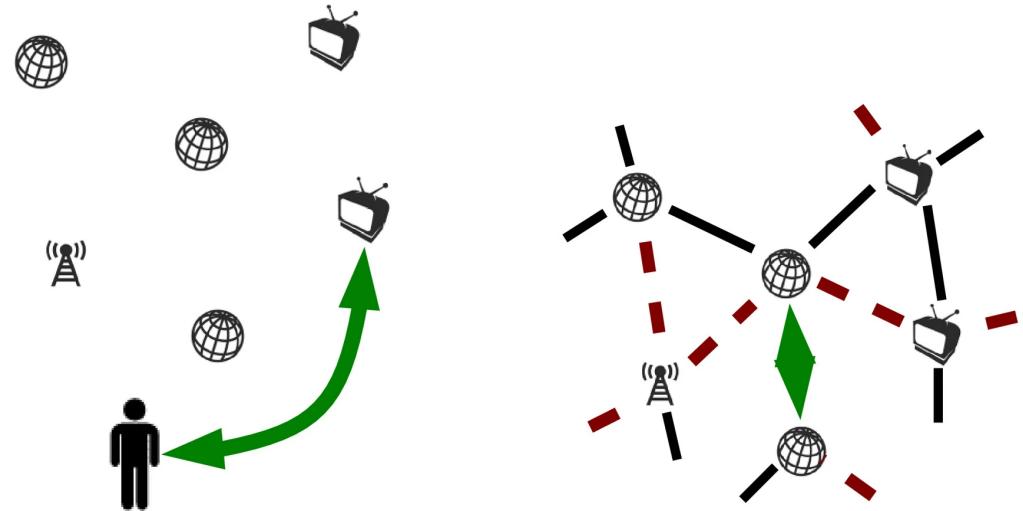
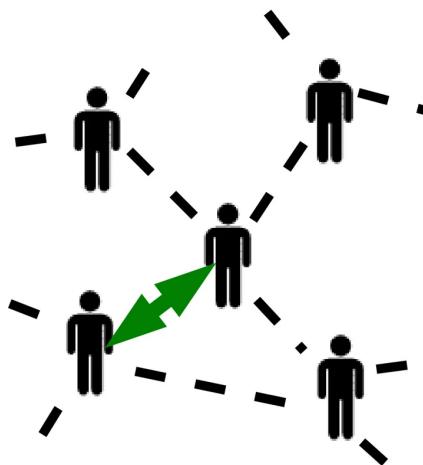
Diseases: Viruses and epidemics



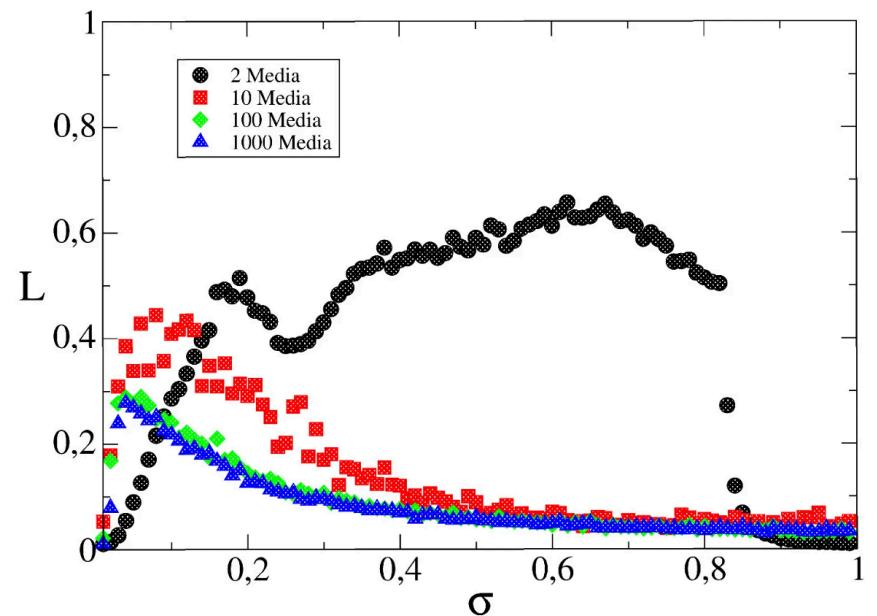
Vernon, Ian, et al. "Bayesian emulation and history matching of JUNE." *Philosophical Transactions of the Royal Society A* 380.2233 (2022): 20220039.

Introduction

Behavior: Opinion dynamics



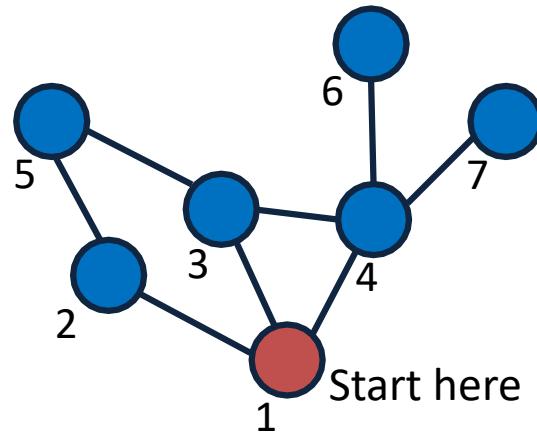
Quattrociocchi, Walter, Guido Caldarelli, and Antonio Scala. "Opinion dynamics on interacting networks: media competition and social influence." *Scientific reports* 4.1 (2014): 4938.



2 | Models of dynamics on networks

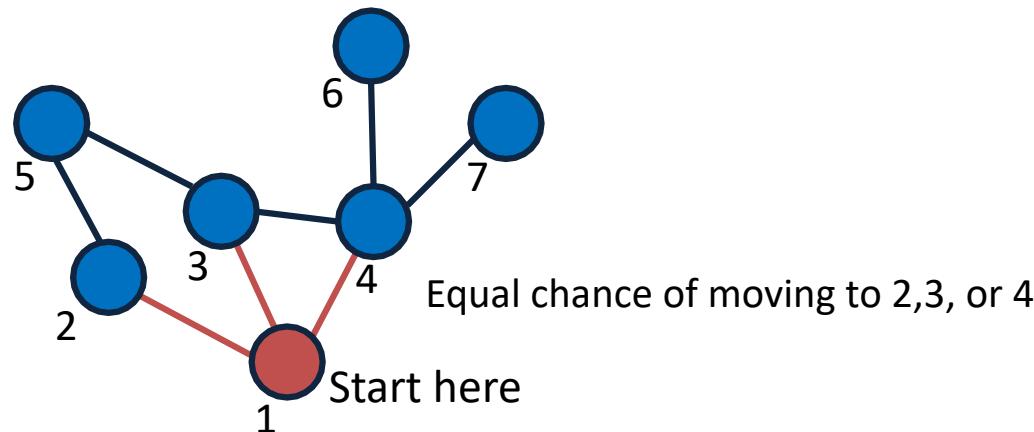
Dynamics on network

What's the simplest way to move in a network?



Dynamics on network

What's the simplest way to move in a network?
at random.

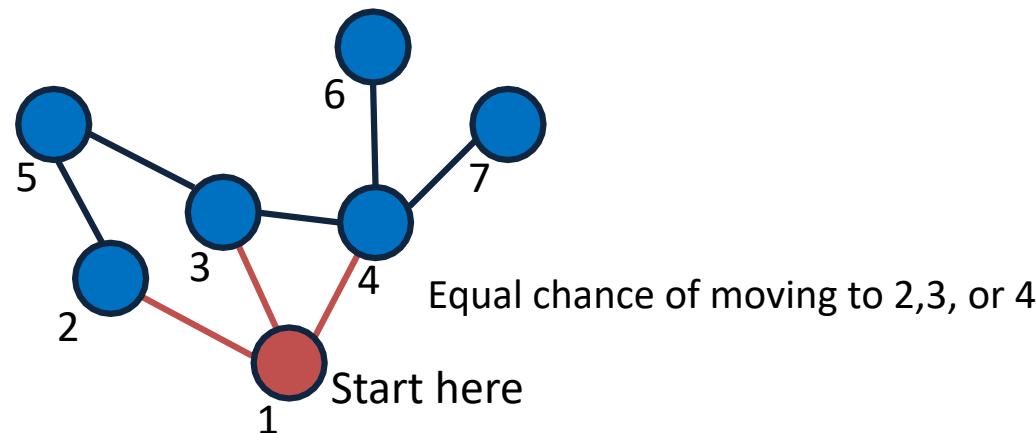


Dynamics on network

What's the simplest way to move in a network?
at random.

Let's keep a small counter

$$N(1) = \begin{cases} 1 : 1 \\ 2 : 0 \\ 3 : 0 \\ 4 : 0 \\ 5 : 0 \\ 6 : 0 \\ 7 : 0 \end{cases}$$

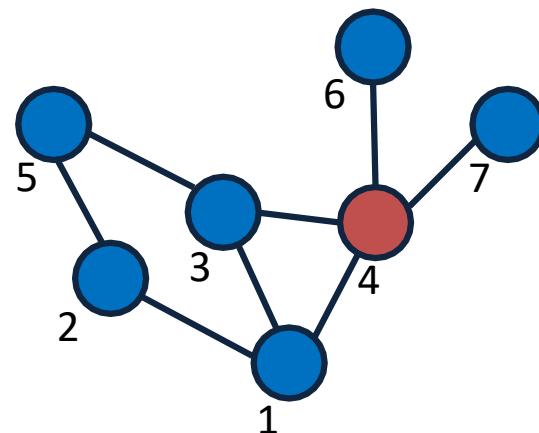


The random walker

What's the simplest way to move in a network?
at random.

Let's keep a small counter

$$N(2) = \begin{cases} 1 : 1 \\ 2 : 0 \\ 3 : 0 \\ 4 : 1 \\ 5 : 0 \\ 6 : 0 \\ 7 : 0 \end{cases}$$

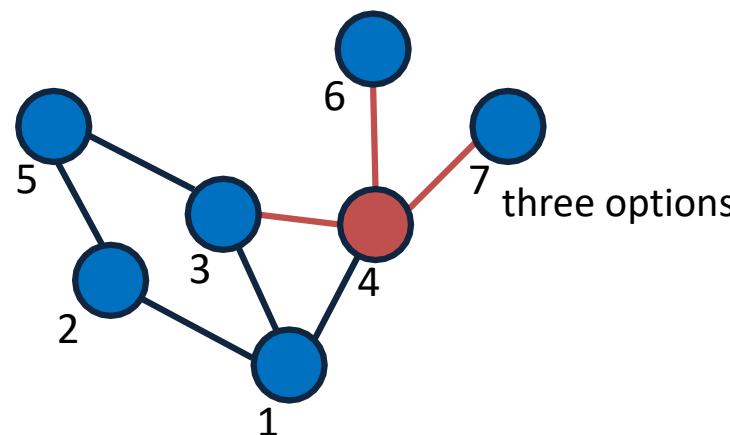


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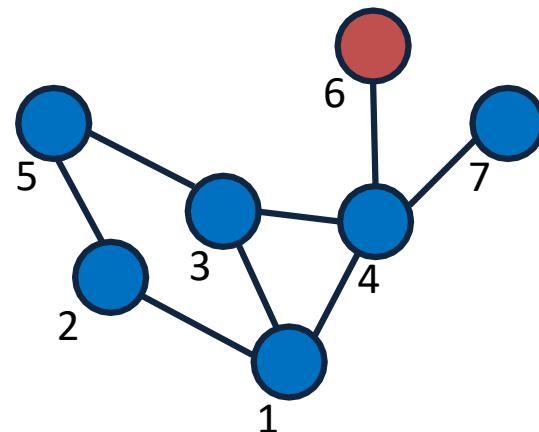


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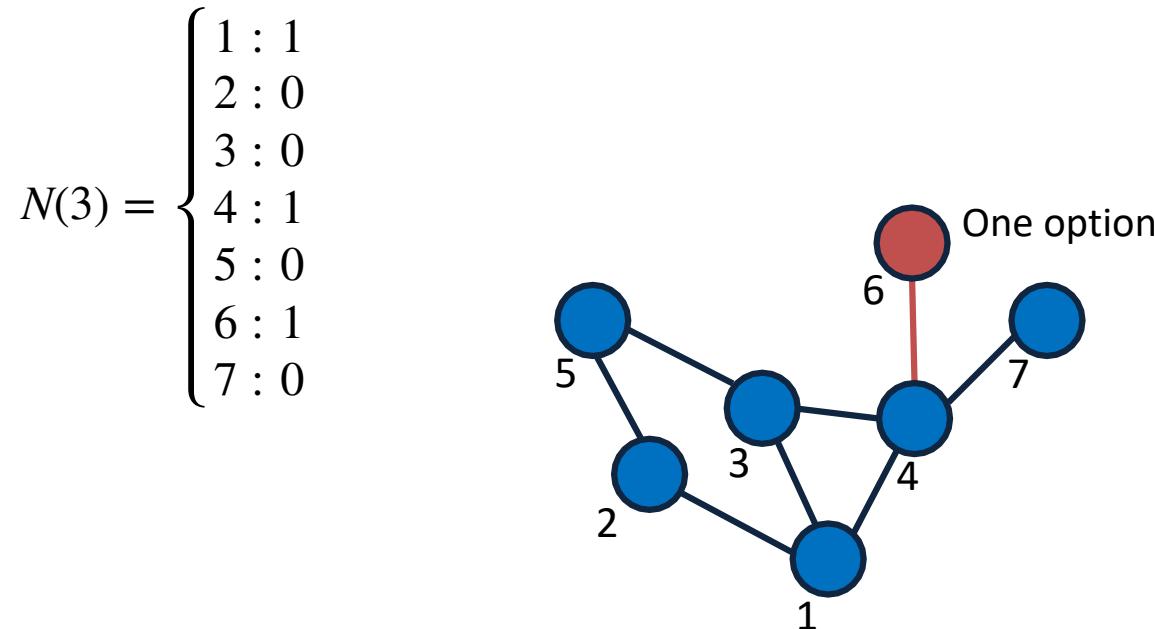
$$N(3) = \begin{cases} 1 : 1 \\ 2 : 0 \\ 3 : 0 \\ 4 : 1 \\ 5 : 0 \\ 6 : 1 \\ 7 : 0 \end{cases}$$



The random walker

What's the simplest way to move in a network?
at random.

Let's keep a small counter

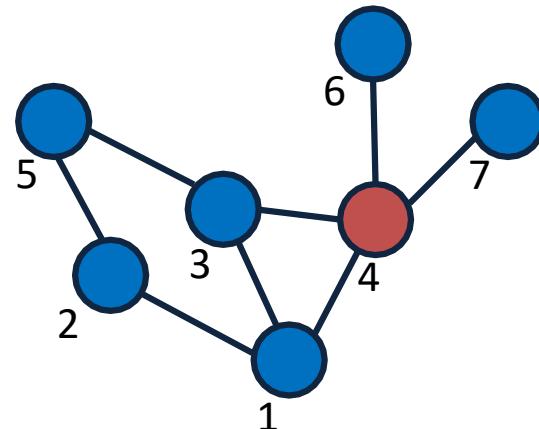


The random walker

What's the simplest way to move in a network?
at random.

Let's keep a small counter

$$N(4) = \begin{cases} 1 : 1 \\ 2 : 0 \\ 3 : 0 \\ 4 : 2 \\ 5 : 0 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

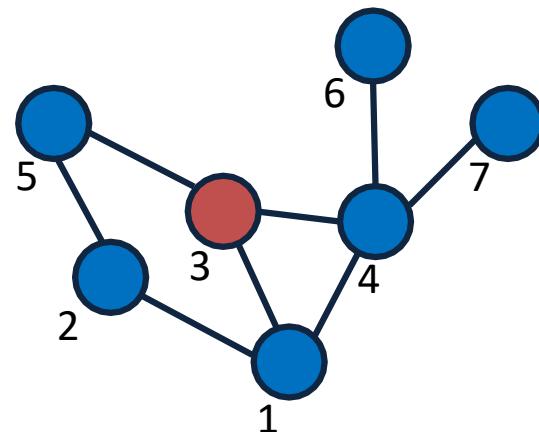


The random walker

What's the simplest way to move in a network?
at random.

Let's keep a small counter

$$N(5) = \begin{cases} 1 : 1 \\ 2 : 0 \\ 3 : 1 \\ 4 : 2 \\ 5 : 0 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

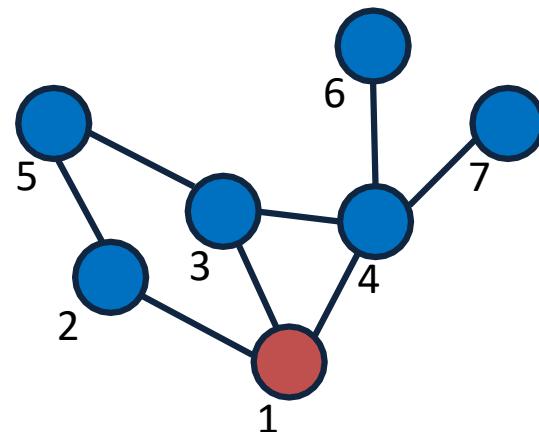


The random walker

What's the simplest way to move in a network?
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Let's keep a small counter

$$N(6) = \begin{cases} 1 : 2 \\ 2 : 0 \\ 3 : 1 \\ 4 : 2 \\ 5 : 0 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

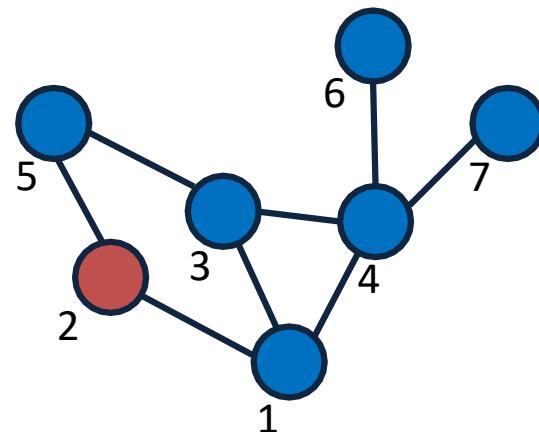


The random walker

What's the simplest way to move in a network?
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Let's keep a small counter

$$N(7) = \begin{cases} 1 : 2 \\ 2 : 1 \\ 3 : 1 \\ 4 : 2 \\ 5 : 0 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

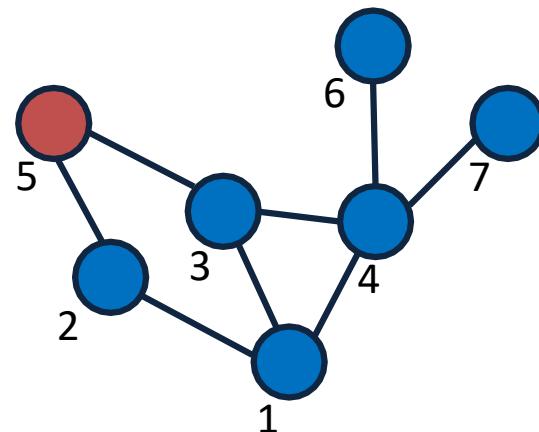


The random walker

What's the simplest way to move in a network?
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Let's keep a small counter

$$N(8) = \begin{cases} 1 : 2 \\ 2 : 1 \\ 3 : 1 \\ 4 : 2 \\ 5 : 1 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

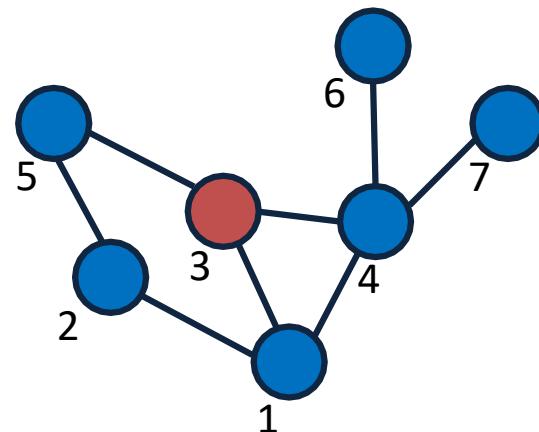


The random walker

What's the simplest way to move in a network?
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Let's keep a small counter

$$N(9) = \begin{cases} 1 : 2 \\ 2 : 1 \\ 3 : 2 \\ 4 : 2 \\ 5 : 1 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

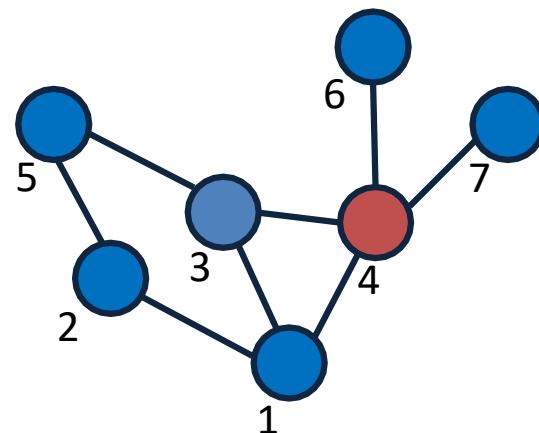


The random walker

What's the simplest way to move in a network?
at random.

Let's keep a small counter

$$N(10) = \begin{cases} 1 : 2 \\ 2 : 1 \\ 3 : 2 \\ 4 : 3 \\ 5 : 1 \\ 6 : 1 \\ 7 : 0 \end{cases}$$

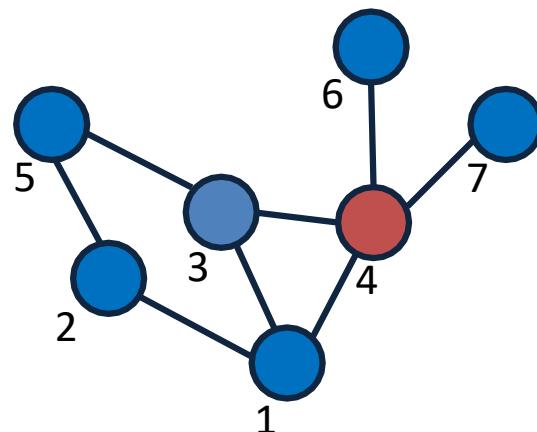


The random walker

What's the simplest way to move in a network?
at random.

Let's keep a small counter

$$N(10) = \begin{cases} 1 : 2 \\ 2 : 1 \\ 3 : 2 \\ 4 : 3 \\ 5 : 1 \\ 6 : 1 \\ 7 : 0 \end{cases}$$



Less connected nodes receive fewer visits

The random walker

Properties:

- **equilibrium**: if undirected, the walker reaches a stationary distribution. As T grows, $N(T)_i \propto k_i$

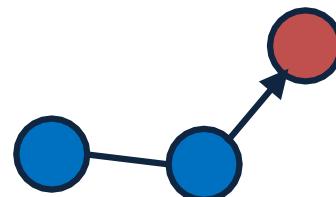
$$p_i = \frac{k_i}{2L}$$

Probability of finding the walker at node i
Number of edges

- **mean return time**: average time to go from node i back to itself.

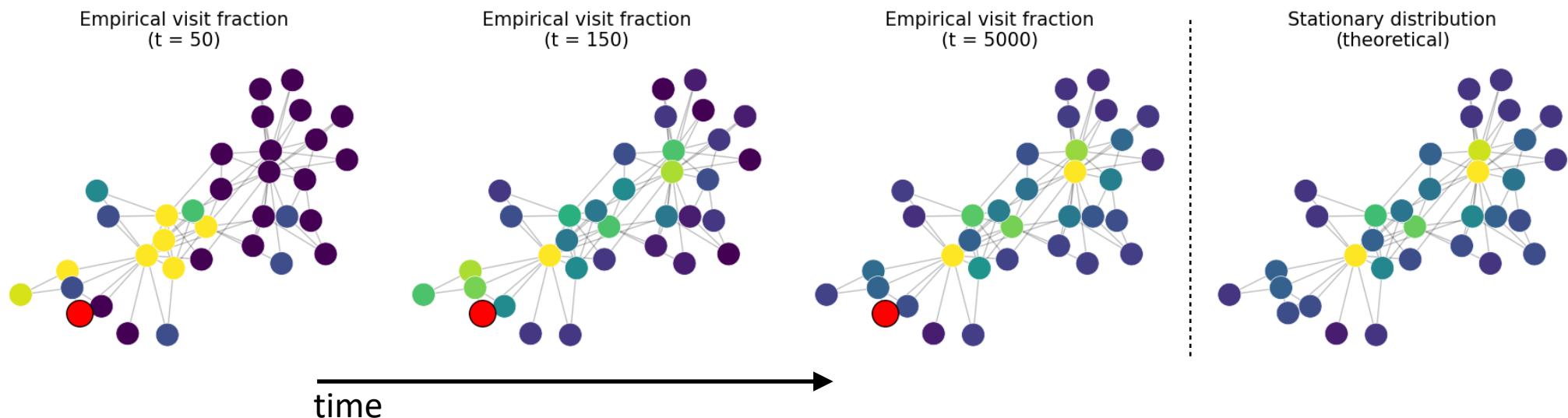
$$\langle T_{i \rightarrow i} \rangle = \frac{1}{p_i}$$

- **sinks**: if directed, the walker may get trapped because of ingoing edges with no outgoing ones



The random walker

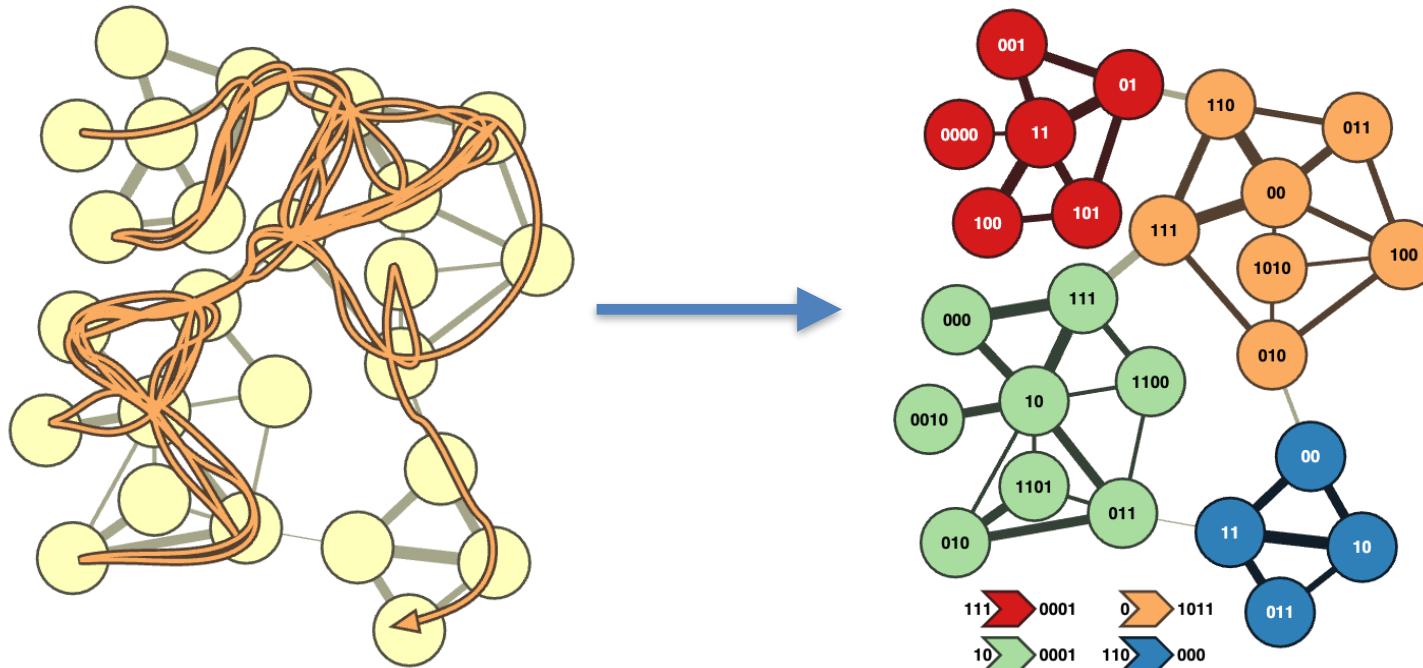
- **Timescales:**
 - Short walks reveal *local properties* of the network (initial node memory).
 - Longer walks show *global features* under equilibrium.



The random walker

Some applications

- **Community detection:** Communities are regions where random walkers get trapped (*Infomap*).

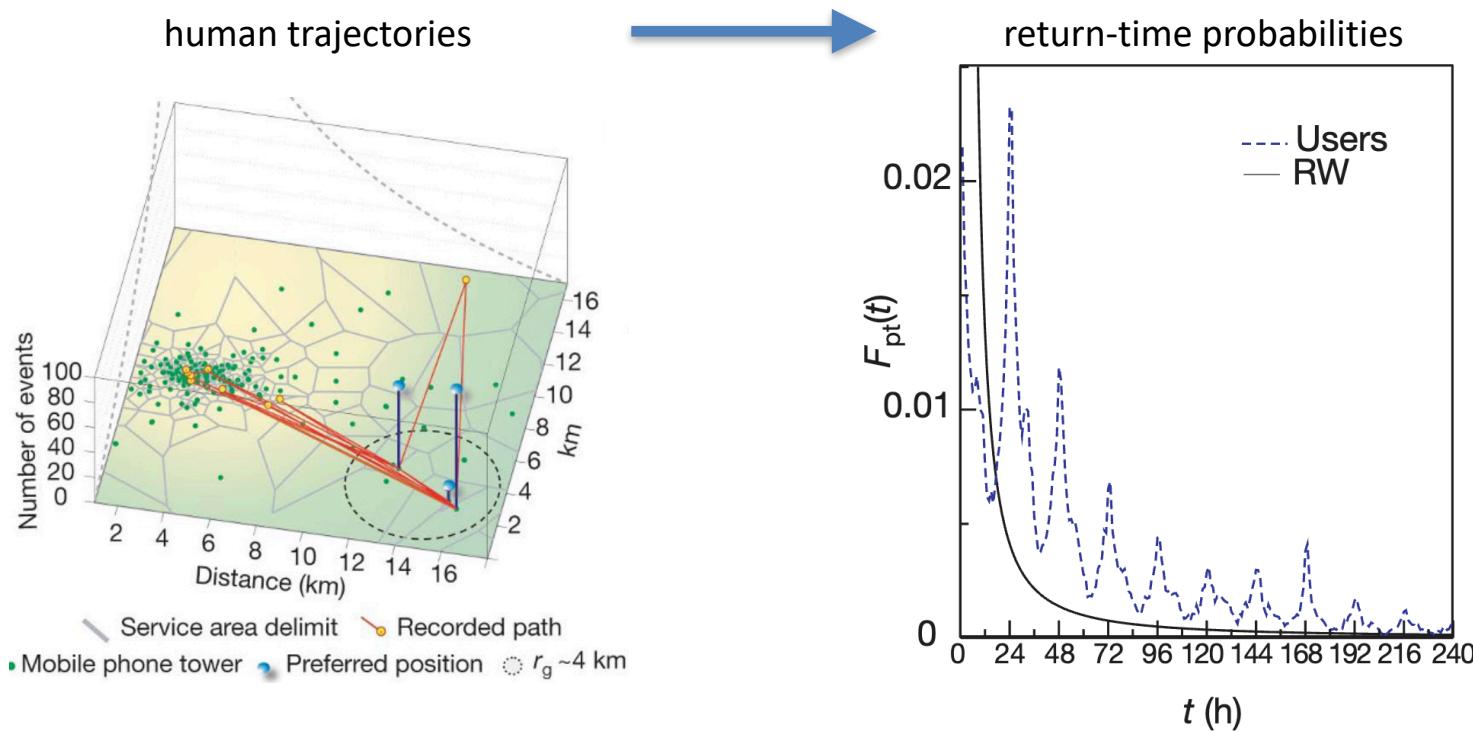


Rosvall, Martin, and Carl T. Bergstrom. "Maps of random walks on complex networks reveal community structure." *Proceedings of the national academy of sciences* 105.4 (2008): 1118-1123.

The random walker

Some applications

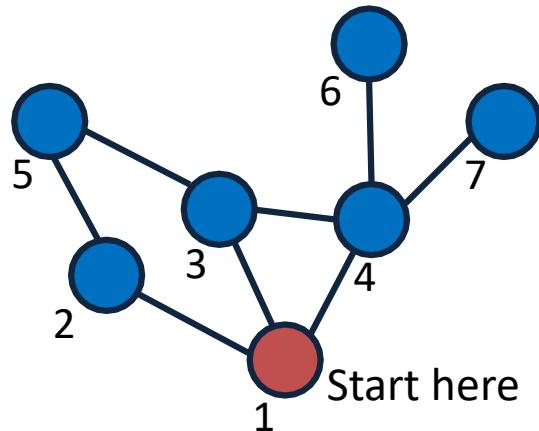
- **Human mobility:** Null model for human mobility patterns



Gonzalez, M. C., Hidalgo, C. A., & Barabasi, A. L. (2008). Understanding individual human mobility patterns. *nature*, 453(7196), 779-782.

Independent Cascade Model

Instead of *moving*, what if the point now is *spreading*?

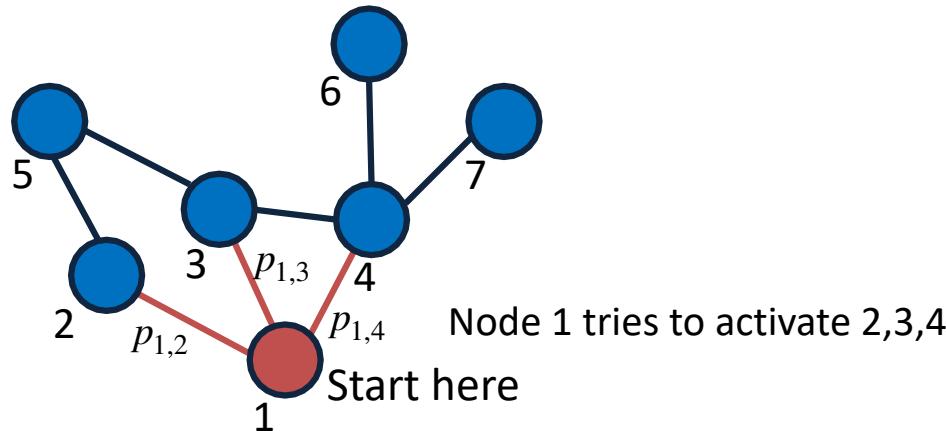


Independent Cascade Model

Instead of *moving*, what if the point now is *spreading*?

Model:

- p_{ij} : probability that node i reaches/activates neighbor j
- When activated, a node has a single chance to activate its neighbors
- The process continues until no one else is activated



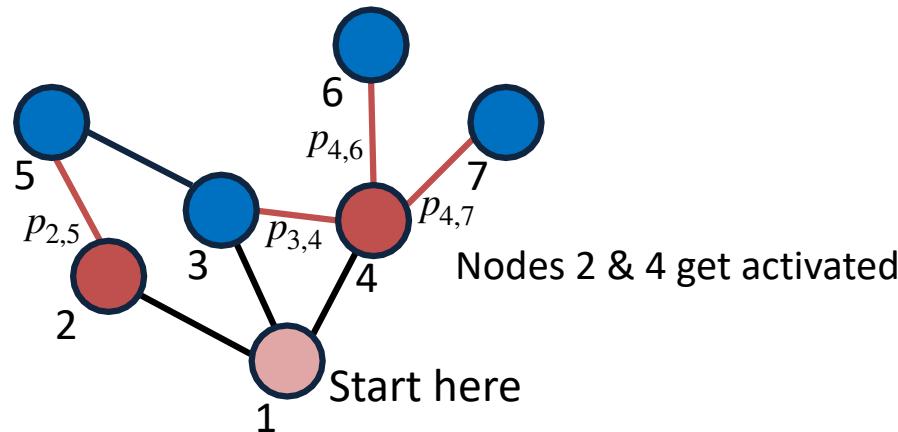
Kempe, David, Jon Kleinberg, and Éva Tardos. "Maximizing the spread of influence through a social network." *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*. 2003.

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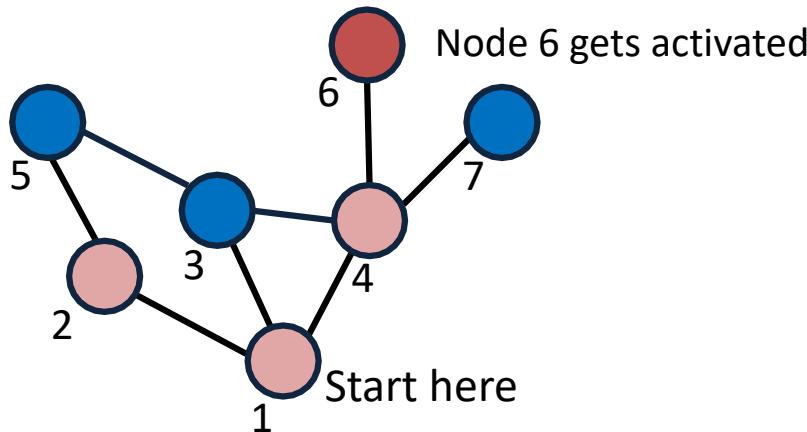


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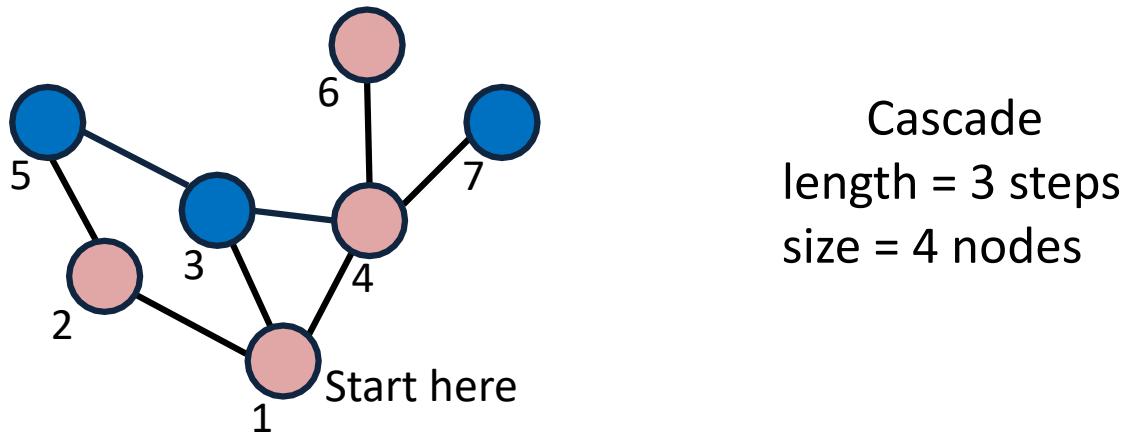


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- The process continues until no one else is activated



No new activations → cascade ends

Kempe, David, Jon Kleinberg, and Éva Tardos. "Maximizing the spread of influence through a social network." *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*. 2003.

Independent Cascade Model

Properties:

- **indicators**: cascade size (# activated nodes),
cascade length (# steps involved)
- **reproduction number**: at initial stages,
approximation of number of activations of node i

$$r_i \sim Bin(k_i, p)$$

of activations from i

$$R_0 = \langle r_i \rangle_i = p\langle k \rangle$$

Expected number of activations
(Reproduction number)

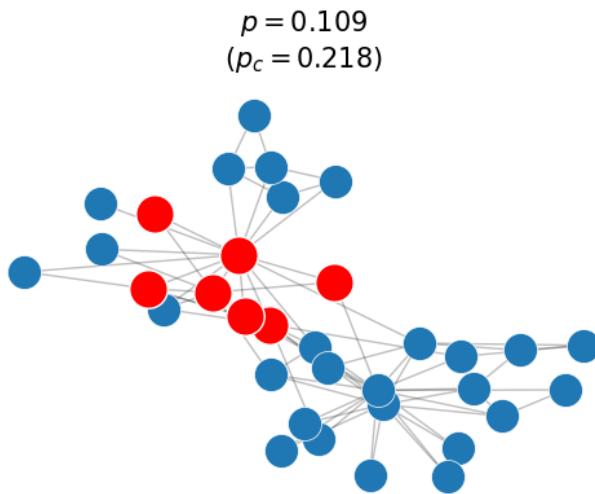
- $R < 1$ (subcritical): cascades die quickly
- $R = 1$ (critical): large fluctuations
- $R > 1$ (supercritical): large cascades

Independent Cascade Model

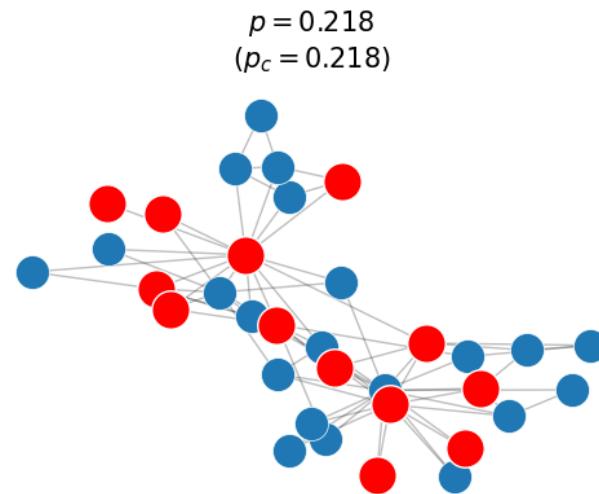
Reproduction number → critical activation probability

$$p_c = \frac{1}{\langle k \rangle}$$

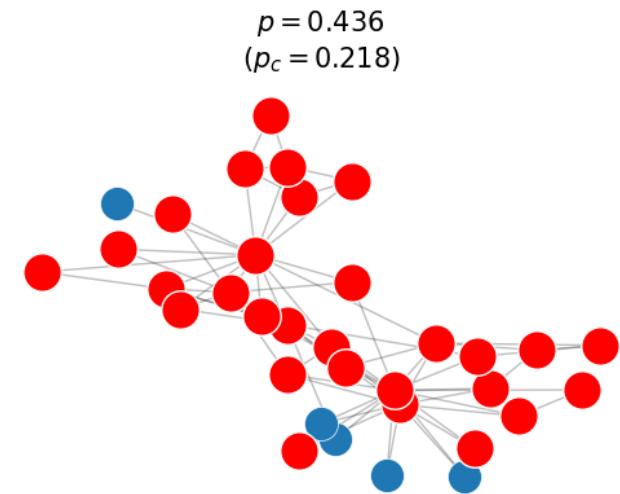
subcritical regime



critical regime



supercritical regime



Epidemiological models

In a similar fashion, epidemiologists have a long history studying how pathogens spread in a population, and how such population reacts to it over time.

Epidemiological models (SIR)

The idea is classifying the population based on their health state.
Typically, nodes can be:

- **Susceptible (S):** Healthy/pассивные узлы, которые могут быть заражены
- **Infected (I):** Contagious/активные узлы
- **Recovered (R):** Узлы, которые восстановились и приобрели иммунитет

Nodes can **transition** between states:



Epidemiological models (SIR)

At each time step, infected nodes infect each of their susceptible neighbors with probability β and recover with μ .

Transition probabilities:

- β : infection probability from infected to susceptible
- μ : recovery probability from an infected node



Epidemiological models (SIR)

Properties:

- **Reproduction number:** depends on infection AND recovery rates
 - Mixed populations (typical connectivity)

$$R_0 = \frac{\beta}{\mu} \langle k \rangle$$

- Heterogeneous populations (spread-out degrees)

$$R_0 = \frac{\beta}{\mu} \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

degree variance



Epidemiological models (SIR)

Reproduction number:

- $R_0 > 1$: exponential growth
- $R_0 < 1$: no epidemic

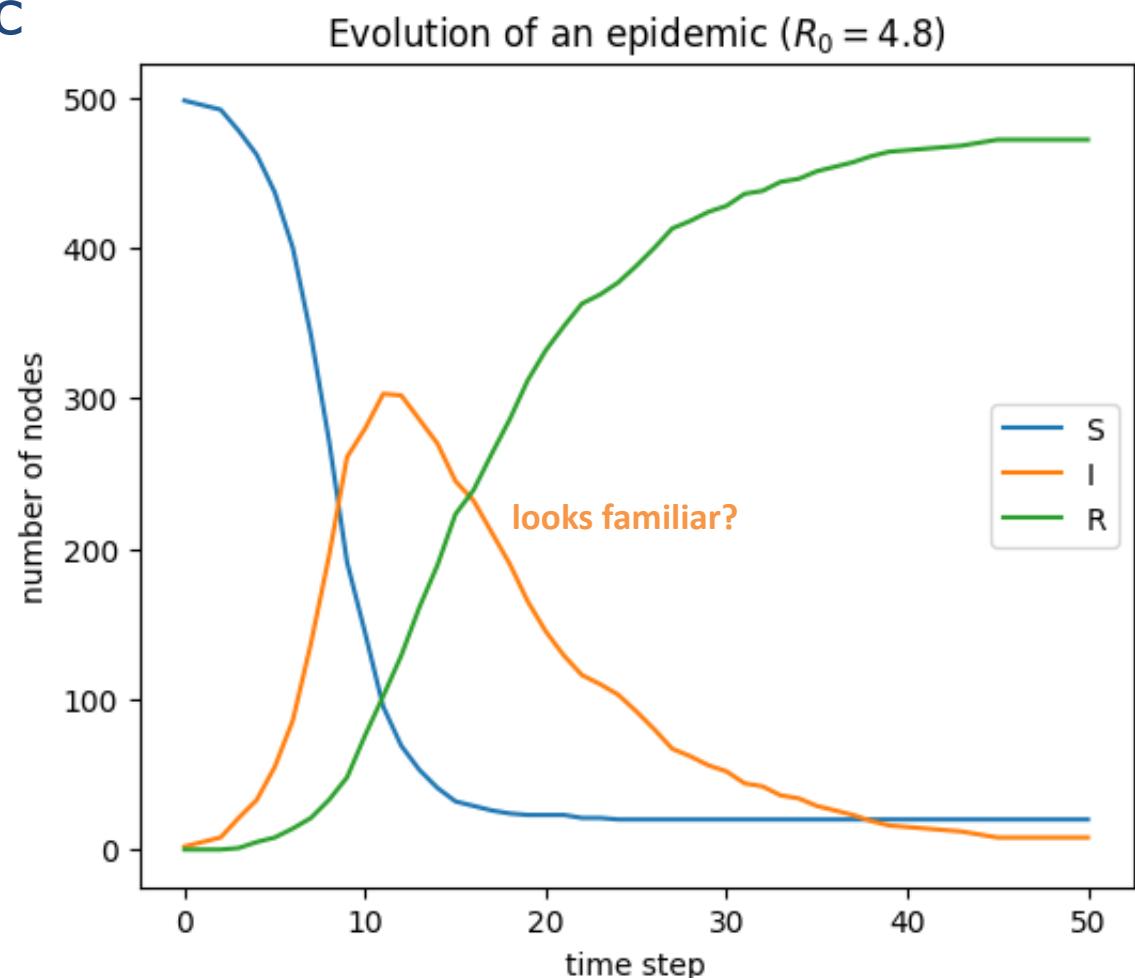
Critical infectivity:

$$\beta_c = \mu \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

Spread-out connectivities

$$\beta_c = \frac{\mu}{\langle k \rangle}$$

well-mixed populations



Spreading on mixed populations

SIR model:

- R_0 represents the average number of susceptibles infected by an infected during its infectious period in a fully susceptible population
- First thing epidemiologist measure to see how dangerous is the pathogen

Disease	Transmission	R_0
Measles	Airborne	12-18
Pertussis	Airborne droplet	12-17
Diphtheria	Saliva	6-7
Smallpox	Social contact	5-7
Polio	Fecal-oral route	5-7
Rubella	Airborne droplet	5-7
Mumps	Airborne droplet	4-7
HIV/AIDS	Sexual contact	2-5
SARS	Airborne droplet	2-5
Influenza (1918 strain)	Airborne droplet	2-3

3 | Minimizing/Maximizing spreading

How to design strategies?

The idea is identifying what effect nodes with certain characteristics have on network dynamics when removing/promoting them.

We will analyse two types of strategies:

- **Immunization**: to minimize spread
- **Seeding**: for censoring or maximizing spread

We'll take the SIR model as reference.

Immunization

Assume we *vaccinate* a node so it stops spreading.

- **Randomly vaccinating** a proportion g of the population is equivalent to reducing the degree

$$k \rightarrow k(1 - g)$$

Immunization

Assume we *vaccinate* a node so it stops spreading.

- **Randomly vaccinating** a proportion g of the population is equivalent to reducing the degree

$$k \rightarrow k(1 - g)$$

- This reduces the basic reproduction number:

$$R_0 \approx \frac{\beta}{\mu} \frac{\langle k^2 \rangle (1 - g)}{\langle k \rangle} \quad \xrightarrow[R_0=1]{} \quad g_c \approx 1 - \frac{\mu}{\beta} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

Critical proportion to stop epidemic

- In empirical networks $\langle k^2 \rangle \gg \langle k \rangle$, so $g_c \approx 1$
Most population would have to be vaccinated!

Immunization

Assume we *vaccinate* a node so it stops spreading.

- **Targeted vaccination:** instead, aim for the highest degree nodes to largely reduce variance $\langle k^2 \rangle$.

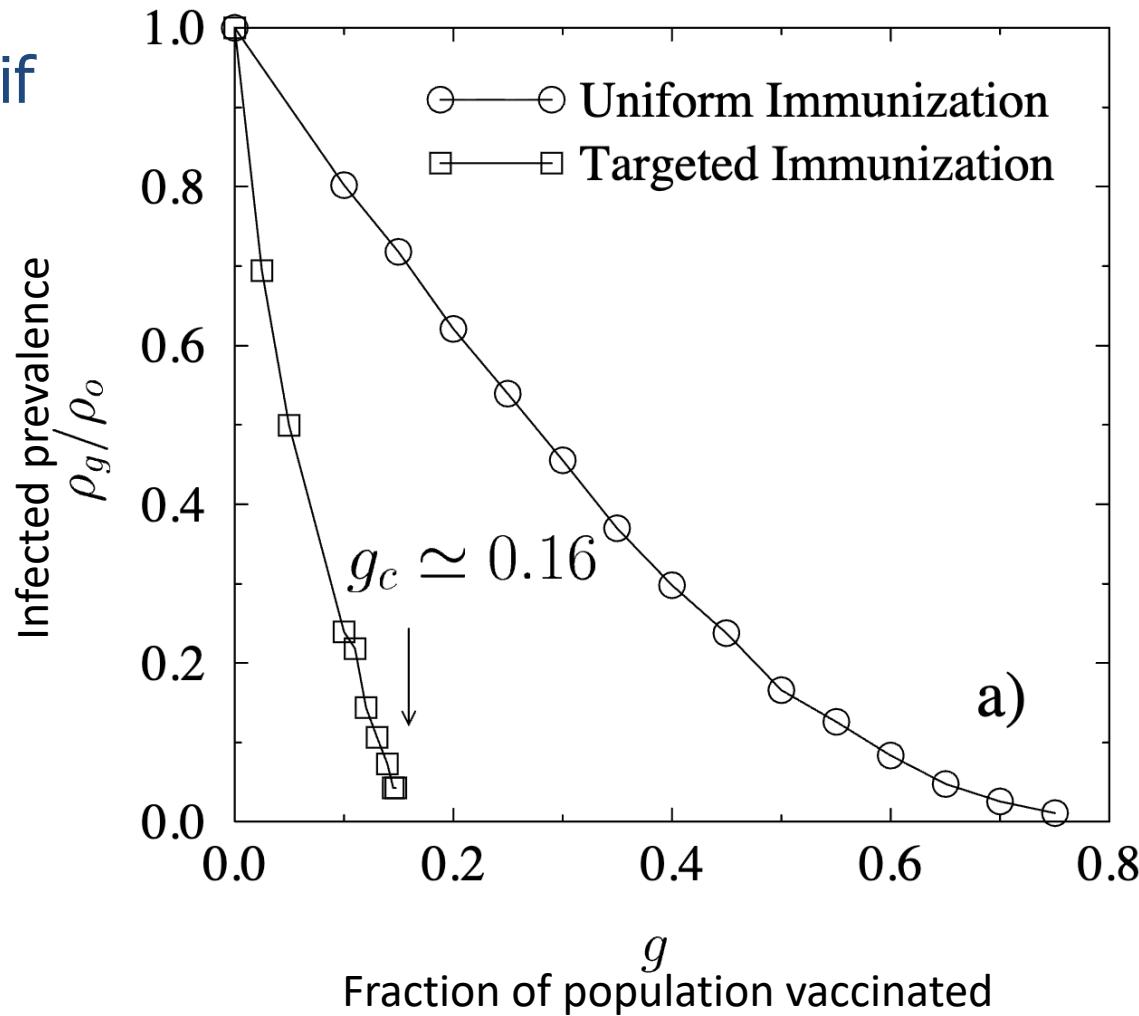
$$g_c \approx 1 - \frac{\mu}{\beta} \frac{\langle k \rangle_{k \leq k_M}}{\langle k^2 \rangle_{k \leq k_M}} \approx 1$$

Critical proportion to stop epidemic

$\langle k \rangle_{k \leq k_M}$: average degree when ignoring degrees above k_M .

Immunization

Infection decays way faster if vaccination is targeted.

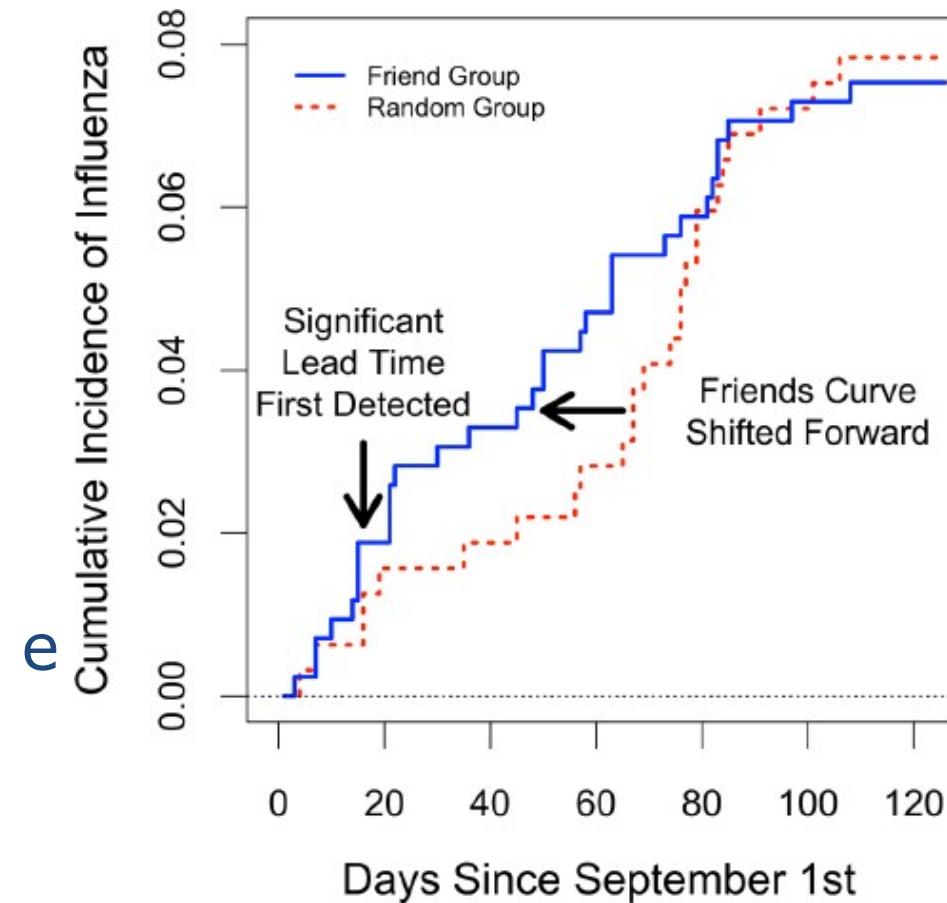


Sensors

On the other hand, nodes with a higher degree get infected faster.

We can use high-degree nodes as early sensors of the epidemic.

friendship paradox → select high-degree nodes



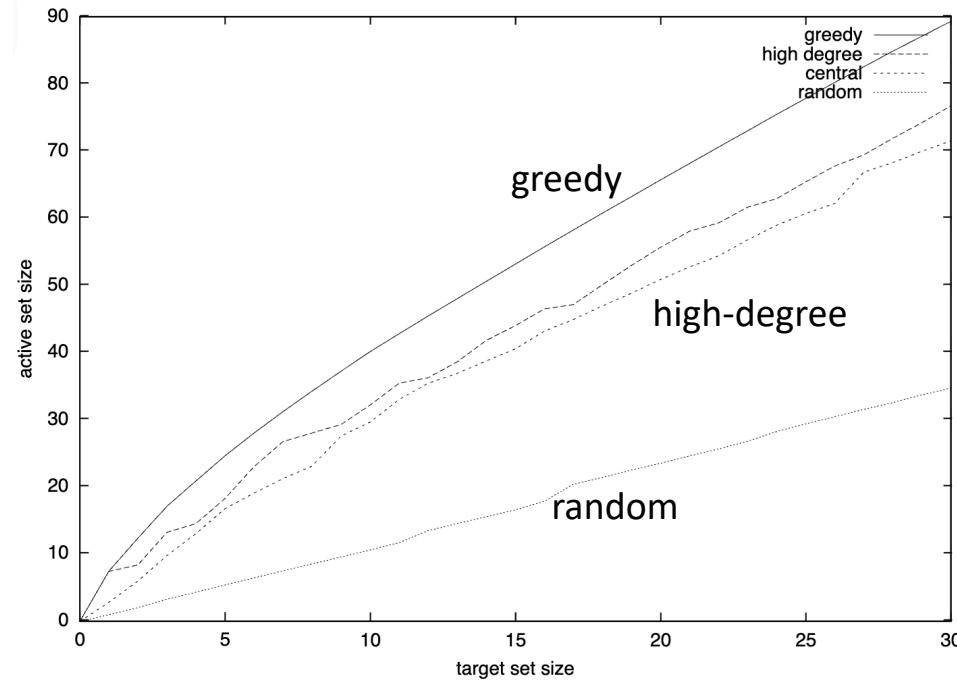
N. A. Christakis & J. H. Fowler (2010) PLoS ONE 5(9) e12948

Seeding

Similarly, important nodes will likely influence the network faster and wider.

High-degree nodes tend to be good, but

- No high-centrality of other types
- Connected only to other high-degree nodes (assortativity)



Kempe, David, Jon Kleinberg, and Éva Tardos. "Maximizing the spread of influence through a social network." *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*. 2003.

4 | Complex contagion

Complex contagion

So far, we have explored models of *simple contagion*: nodes move/spread just by getting in contact with their neighbors.

Complex contagion models consider more intricate kind of interactions, such as

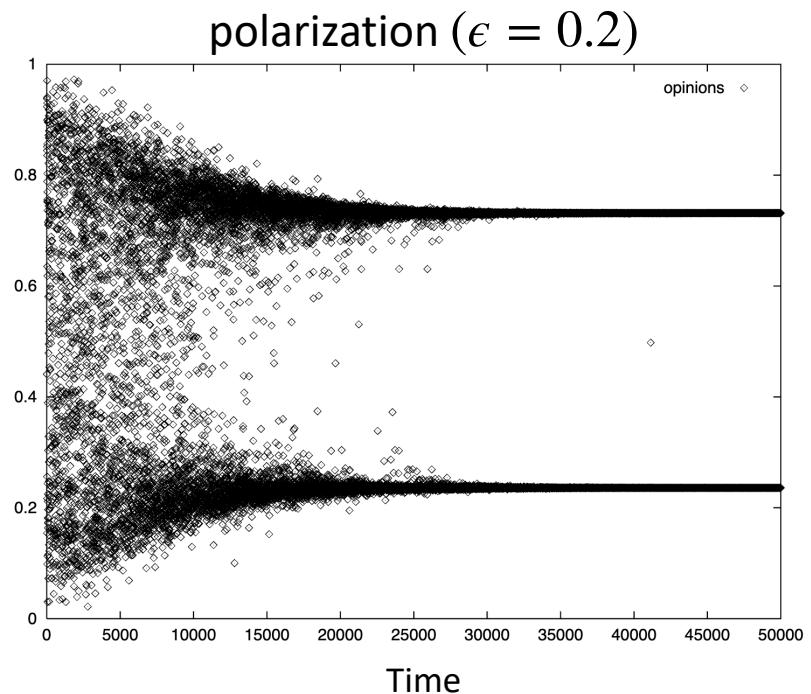
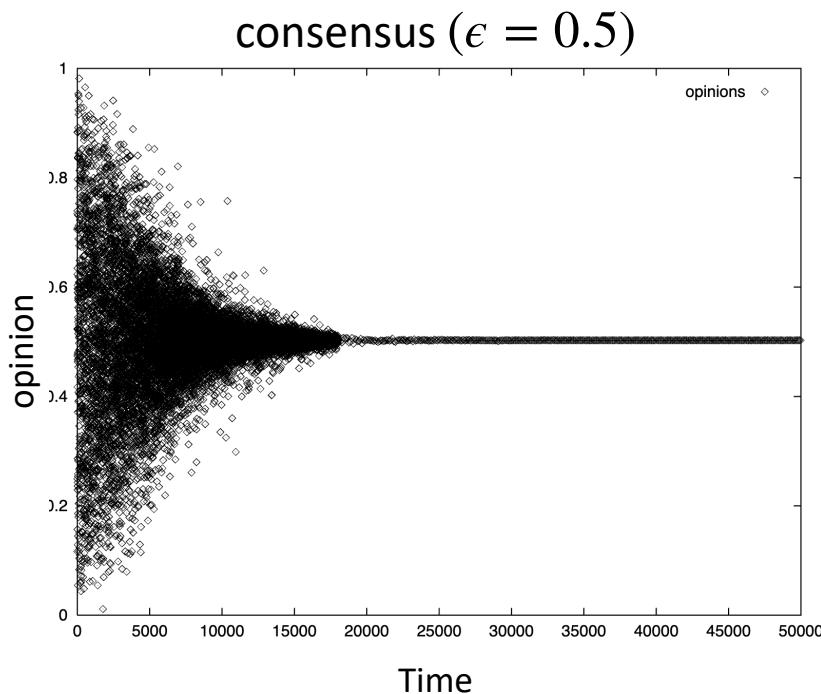
1. Bounded rationality
2. Heterogeneous node/agent actions
3. Thresholds for behavior

Complex contagion

Example 1: Bounded confidence opinion dynamics

Nodes interact only if their *opinions* are sufficiently close.

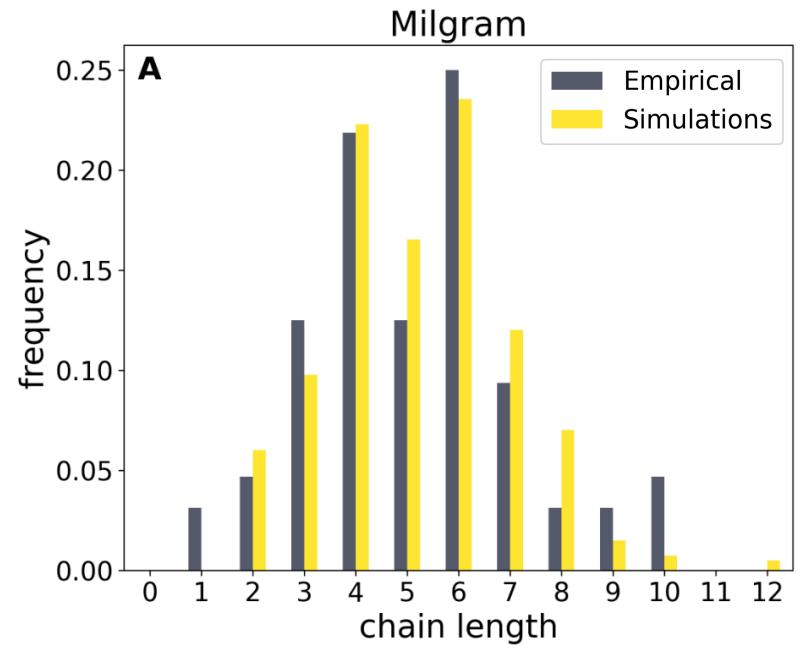
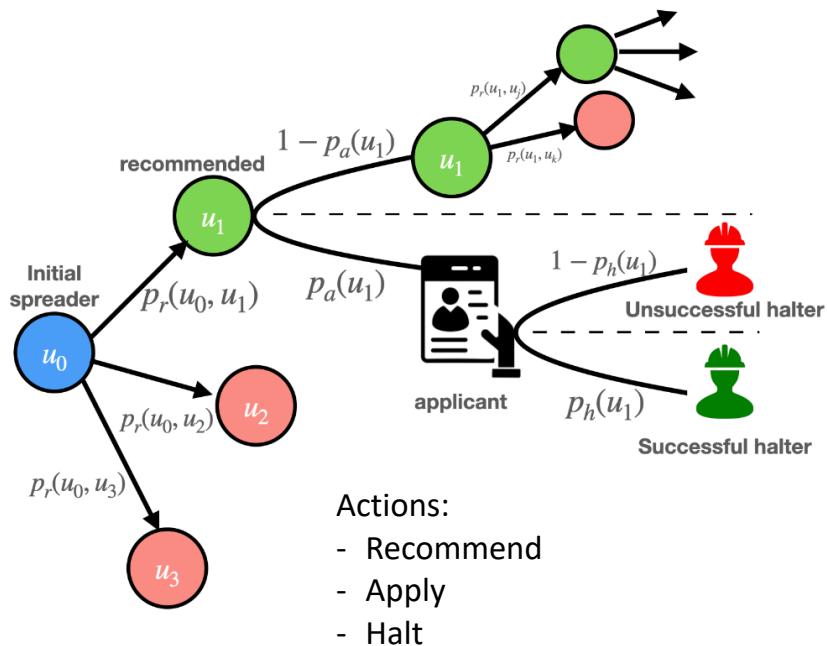
- ϵ_i : confidence level of node i



Complex contagion

Example 2: Independent Halting Cascades

Nodes may perform several *actions* to complete a task.

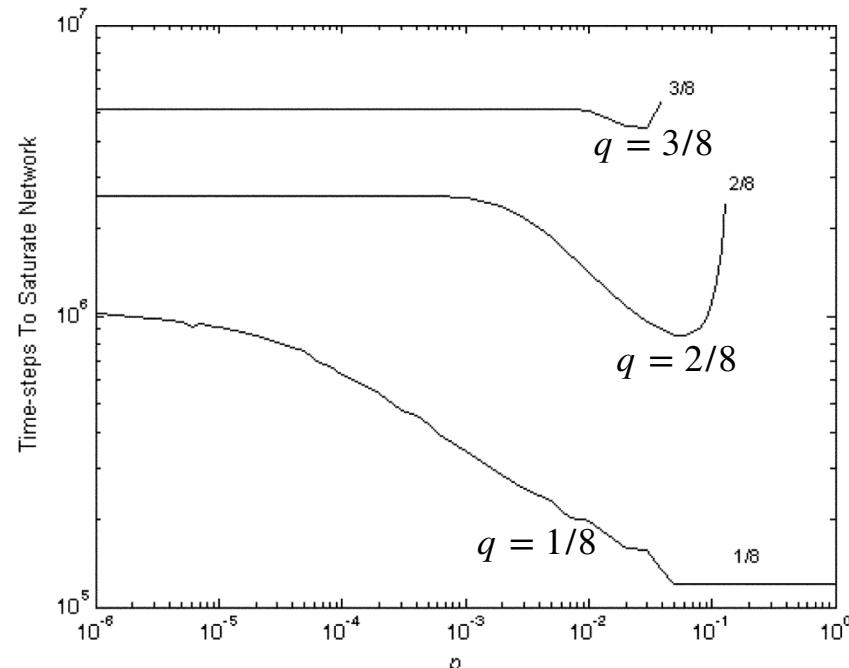


Complex contagion

Example 3: Thresholded diffusion

Contagion/infections happens only when enough neighbors are infected.

- q : fraction of infected neighbors to get infected
- The larger the q , the slower the propagation



Centola, D., & Macy, M. (2007). Complex contagions and the weakness of long ties. *American journal of Sociology*, 113(3), 702-734.

5 | Summary

Summary

1. Dynamics occur *on top* of network structures.
2. Simple models capture key spreading mechanisms
 - i. Random walks → exploration and visitation patterns
 - ii. Independent cascade → information spreading
 - iii. SIR model → epidemic spreading
3. Dynamics can be summarised through key indicators
 - i. Stationary distributions and return times
 - ii. Cascade length and size
 - iii. Reproduction numbers and critical thresholds
4. Network structure can be exploited to control spreading
5. Complex contagion models exist and are useful.