Semantic Tableaux

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July, 2020

1 First Order Logic (FOL)

$$\begin{array}{c} \neg\neg\, A \\ \downarrow \\ A \end{array}$$

$$\begin{array}{c} A \wedge B \\ \downarrow \\ A \\ B \end{array} \qquad \begin{array}{ccc} & \neg(A \wedge B) & \\ \swarrow & & \searrow \\ \neg A & & \neg B \end{array}$$

$$\begin{array}{ccc} & A \vee B & \\ \swarrow & & \searrow \\ A & & B \end{array} \qquad \begin{array}{c} \neg(A \vee B) \\ \downarrow \\ \neg A \\ \neg B \end{array}$$

$$\begin{array}{ccc} & A \supset B & \\ \swarrow & & \searrow \\ \neg A & & B \end{array} \qquad \begin{array}{c} \neg(A \supset B) \\ \downarrow \\ A \\ \neg B \end{array}$$

$$\begin{array}{ccc} & A \equiv B & \\ \swarrow & & \searrow \\ A & & \neg A \\ B & & \neg B \end{array} \qquad \begin{array}{ccc} & \neg(A \equiv B) & \\ \swarrow & & \searrow \\ \neg A & & A \\ B & & \neg B \end{array}$$

Closure condition: A branch of closes if it contains both A and ¬A.

Counter-model procedure: Take any open branch, if P occurs at any node in the branch, assign it 1, and if ¬P occurs on any node, assign it 0.

Designated values: 1

2 First Degree Entailment (FDE)

$$\begin{array}{c} \neg\neg\, \mathrm{A}, + \\ \downarrow \\ \mathrm{A}, + \end{array} \qquad\qquad \begin{array}{c} \neg\neg\, \mathrm{A}, - \\ \downarrow \\ \mathrm{A}, - \end{array}$$

$$\begin{array}{c} \mathrm{A} \wedge \mathrm{B}, + \\ \downarrow \\ \mathrm{A}, + \\ \mathrm{B}, + \end{array} \qquad \begin{array}{ccc} & \neg(\mathrm{A} \wedge \mathrm{B}), + & \\ \swarrow & & \searrow \\ \neg\mathrm{A}, + & & \neg\mathrm{B}, + \end{array} \qquad \begin{array}{ccc} & \mathrm{A} \wedge \mathrm{B}, - & \\ \swarrow & & \searrow \\ \mathrm{A}, - & & \mathrm{B}, - \end{array} \qquad \begin{array}{ccc} & \neg(\mathrm{A} \wedge \mathrm{B}), - & \\ \swarrow & & \searrow \\ \neg\mathrm{A}, - & & \neg\mathrm{B}, - \end{array}$$

$$\begin{array}{ccc} & \mathrm{A} \vee \mathrm{B}, + & \\ \swarrow & & \searrow \\ \mathrm{A}, + & & \mathrm{B}, + \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), + \\ \downarrow \\ \neg\mathrm{A}, + \\ \neg\mathrm{B}, + \end{array} \qquad \begin{array}{c} \mathrm{A} \vee \mathrm{B}, - \\ \downarrow \\ \mathrm{A}, - \\ \mathrm{B}, - \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), - \\ \downarrow \\ \neg\mathrm{A}, - \\ \neg\mathrm{B}, - \end{array}$$

Closure condition: A branch closes if it contains both A, + and A, –.

Counter-model procedure: Take any open branch, if P, + occurs at any node in the branch assign it Pρ1, and if ¬P, + occurs on any node, assign it Pρ0.

Truth table:

$\neg$	
0	1
n	n
b	b
1	0

$\wedge$	0	n	b	1
0	0	0	0	0
n	0	n	0	n
b	0	0	b	b
1	0	n	b	1

$\vee$	0	n	b	1
0	0	n	b	1
n	n	n	1	1
b	b	1	b	1
1	1	1	1	1

Designated values: $1, b$

3 Kleene 3-valued logic (K_3)

$$\begin{array}{c} \neg\neg A, + \\ \downarrow \\ A, + \end{array} \qquad\qquad \begin{array}{c} \neg\neg A, - \\ \downarrow \\ A, - \end{array}$$

$$\begin{array}{c} A \wedge B, + \\ \downarrow \\ A, + \\ B, + \end{array} \quad \begin{array}{c} \neg(A \wedge B), + \\ \swarrow \quad \searrow \\ \neg A, + \quad \neg B, + \end{array} \quad \begin{array}{c} A \wedge B, - \\ \swarrow \quad \searrow \\ A, - \quad B, - \end{array} \quad \begin{array}{c} \neg(A \wedge B), - \\ \swarrow \quad \searrow \\ \neg A, - \quad \neg B, - \end{array}$$

$$\begin{array}{c} A \vee B, + \\ \swarrow \quad \searrow \\ A, + \quad B, + \end{array} \quad \begin{array}{c} \neg(A \vee B), + \\ \downarrow \\ \neg A, + \\ \neg B, + \end{array} \quad \begin{array}{c} A \vee B, - \\ \downarrow \\ A, - \\ B, - \end{array} \quad \begin{array}{c} \neg(A \vee B), - \\ \downarrow \\ \neg A, - \\ \neg B, - \end{array}$$

$$\begin{array}{c} A \supset B, + \\ \swarrow \quad \searrow \\ \neg A, + \quad B, + \end{array} \quad \begin{array}{c} \neg(A \supset B), + \\ \downarrow \\ A, + \\ \neg B, + \end{array} \quad \begin{array}{c} A \supset B, - \\ \downarrow \\ \neg A, - \\ B, - \end{array} \quad \begin{array}{c} \neg(A \supset B), - \\ \downarrow \\ A, - \\ \neg B, - \end{array}$$

Closure condition: A branch closes if it contains both A, + and A, − or both A, + and ¬A, +.

Counter-model procedure: Take any open branch, if P, + occurs at any node in the branch assign it Pρ1, and if ¬P, + occurs on any node, assign it Pρ0.

Truth table:

¬	
0	1
i	i
1	0

∧	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

∨	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

⊃	0	i	1
0	1	1	1
i	i	i	1
1	0	i	1

Designated values: 1

4 Łukasiewicz 3-valued logic (Ł$_3$)

$$\begin{array}{c} \neg\neg A, + \\ \downarrow \\ A, + \end{array} \qquad\qquad \begin{array}{c} \neg\neg A, - \\ \downarrow \\ A, - \end{array}$$

$$\begin{array}{c} A \wedge B, + \\ \downarrow \\ A, + \\ B, + \end{array} \qquad \begin{array}{ccc} & \neg(A \wedge B), + & \\ \swarrow & & \searrow \\ \neg A, + & & \neg B, + \end{array} \qquad \begin{array}{ccc} & A \wedge B, - & \\ \swarrow & & \searrow \\ A, - & & B, - \end{array} \qquad \begin{array}{ccc} & \neg(A \wedge B), - & \\ \swarrow & & \searrow \\ \neg A, - & & \neg B, - \end{array}$$

$$\begin{array}{ccc} & A \vee B, + & \\ \swarrow & & \searrow \\ A, + & & B, + \end{array} \qquad \begin{array}{c} \neg(A \vee B), + \\ \downarrow \\ \neg A, + \\ \neg B, + \end{array} \qquad \begin{array}{c} A \vee B, - \\ \downarrow \\ A, - \\ B, - \end{array} \qquad \begin{array}{c} \neg(A \vee B), - \\ \downarrow \\ \neg A, - \\ \neg B, - \end{array}$$

$$\begin{array}{ccccc} & & A \supset B, + & & \\ & \swarrow & \downarrow & \searrow & \\ \neg A, + & & B, + & & A \vee \neg A, - \\ & & & & B \vee \neg B, - \end{array} \qquad \begin{array}{c} \neg(A \supset B), + \\ \downarrow \\ A, + \\ \neg B, + \end{array} \qquad \begin{array}{ccc} & A \supset B, - & \\ \swarrow & & \searrow \\ A, + & & \neg A, - \\ B, - & & \neg B, + \end{array} \qquad \begin{array}{ccc} & \neg(A \supset B), - & \\ \swarrow & & \searrow \\ A, - & & \neg B, - \end{array}$$

Closure condition: A branch closes if it contains both A, + and A, – or both A, + and ¬A, +.

Counter-model procedure: Take any open branch, if P, + occurs at any node in the branch assign it Pρ1, and if ¬P, + occurs on any node, assign it Pρ0.

Designated values: 1

Truth table:

$\neg$	
0	1
i	i
1	0

$\wedge$	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

$\vee$	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

$\supset$	0	i	1
0	1	1	1
i	i	1	1
1	0	i	1

5 Logic of Paradox (LP)

$$\begin{array}{c} \neg\neg \mathrm{A}, + \\ \downarrow \\ \mathrm{A}, + \end{array} \qquad\qquad \begin{array}{c} \neg\neg \mathrm{A}, - \\ \downarrow \\ \mathrm{A}, - \end{array}$$

$$\begin{array}{c} \mathrm{A} \wedge \mathrm{B}, + \\ \downarrow \\ \mathrm{A}, + \\ \mathrm{B}, + \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \wedge \mathrm{B}), +} \\ \swarrow & \searrow \\ \neg\mathrm{A}, + & \neg\mathrm{B}, + \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \wedge \mathrm{B}, -} \\ \swarrow & \searrow \\ \mathrm{A}, - & \mathrm{B}, - \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \wedge \mathrm{B}), -} \\ \swarrow & \searrow \\ \neg\mathrm{A}, - & \neg\mathrm{B}, - \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \vee \mathrm{B}, +} \\ \swarrow & \searrow \\ \mathrm{A}, + & \mathrm{B}, + \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), + \\ \downarrow \\ \neg\mathrm{A}, + \\ \neg\mathrm{B}, + \end{array} \qquad \begin{array}{c} \mathrm{A} \vee \mathrm{B}, - \\ \downarrow \\ \mathrm{A}, - \\ \mathrm{B}, - \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), - \\ \downarrow \\ \neg\mathrm{A}, - \\ \neg\mathrm{B}, - \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \supset \mathrm{B}, +} \\ \swarrow & \searrow \\ \neg\mathrm{A}, + & \mathrm{B}, + \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \supset \mathrm{B}), + \\ \downarrow \\ \mathrm{A}, + \\ \neg\mathrm{B}, + \end{array} \qquad \begin{array}{c} \mathrm{A} \supset \mathrm{B}, - \\ \downarrow \\ \neg\mathrm{A}, - \\ \mathrm{B}, - \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \supset \mathrm{B}), - \\ \downarrow \\ \mathrm{A}, - \\ \neg\mathrm{B}, - \end{array}$$

Closure condition: A branch closes if it contains both A, + and A, − or both A, − and ¬A, −.

Counter-model procedure: Take any open branch, if P, − *does not* occur on any node of the branch assign it Pρ1, and if ¬P, − *does not* occur on any node, assign it Pρ0.

Designated values: $1, i$

Truth table:

$\neg$	
0	1
i	i
1	0

$\wedge$	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

$\vee$	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

$\supset$	0	i	1
0	1	1	1
i	i	i	1
1	0	i	1

6 Mix 3-valued logic (RM_3)

$$\begin{array}{c} \neg\neg \mathrm{A}, + \\ \downarrow \\ \mathrm{A}, + \end{array} \qquad \begin{array}{c} \neg\neg \mathrm{A}, - \\ \downarrow \\ \mathrm{A}, - \end{array}$$

$$\begin{array}{c} \mathrm{A} \wedge \mathrm{B}, + \\ \downarrow \\ \mathrm{A}, + \\ \mathrm{B}, + \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \wedge \mathrm{B}), +} \\ \swarrow & \searrow \\ \neg\mathrm{A}, + & \neg\mathrm{B}, + \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \wedge \mathrm{B}, -} \\ \swarrow & \searrow \\ \mathrm{A}, - & \mathrm{B}, - \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \wedge \mathrm{B}), -} \\ \swarrow & \searrow \\ \neg\mathrm{A}, - & \neg\mathrm{B}, - \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \vee \mathrm{B}, +} \\ \swarrow & \searrow \\ \mathrm{A}, + & \mathrm{B}, + \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), + \\ \downarrow \\ \neg\mathrm{A}, + \\ \neg\mathrm{B}, + \end{array} \qquad \begin{array}{c} \mathrm{A} \vee \mathrm{B}, - \\ \downarrow \\ \mathrm{A}, - \\ \mathrm{B}, - \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), - \\ \downarrow \\ \neg\mathrm{A}, - \\ \neg\mathrm{B}, - \end{array}$$

$$\begin{array}{ccc} \multicolumn{3}{c}{\mathrm{A} \supset \mathrm{B}, +} \\ \swarrow & \downarrow & \searrow \\ \mathrm{A}, - & \neg\mathrm{B}, - & \mathrm{A} \wedge \neg\mathrm{A}, + \\ & & \mathrm{B} \wedge \neg\mathrm{B}, + \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \supset \mathrm{B}), + \\ \downarrow \\ \mathrm{A}, + \\ \neg\mathrm{B}, + \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \supset \mathrm{B}, -} \\ \swarrow & \searrow \\ \mathrm{A}, + & \neg\mathrm{A}, - \\ \mathrm{B}, - & \neg\mathrm{B}, + \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \supset \mathrm{B}), -} \\ \swarrow & \searrow \\ \mathrm{A}, - & \neg\mathrm{B}, - \end{array}$$

Closure condition: A branch closes if it contains both A, + and A, − or both A, − and ¬A, −.

Counter-model procedure: Take any open branch, if P, − *does not* occur on any node of the branch assign it $\mathrm{P}\rho 1$, and if ¬P, − *does not* occur on any node, assign it $\mathrm{P}\rho 0$.

Designated values: $1, i$

Truth table:

$\neg$	
0	1
i	i
1	0

$\wedge$	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

$\vee$	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

$\supset$	0	i	1
0	1	1	1
i	0	i	1
1	0	0	1

7 Basic Modal Logic

$$\begin{array}{c} \neg\neg\ \mathrm{A}, i \\ \downarrow \\ \mathrm{A}, i \end{array}$$

$$\begin{array}{c} \mathrm{A} \wedge \mathrm{B}, i \\ \downarrow \\ \mathrm{A}, i \\ \mathrm{B}, i \end{array} \qquad \begin{array}{ccc} & \neg(\mathrm{A} \wedge \mathrm{B}), i & \\ \swarrow & & \searrow \\ \neg\mathrm{A}, i & & \neg\mathrm{B}, i \end{array} \qquad \begin{array}{ccc} & \mathrm{A} \vee \mathrm{B}, i & \\ \swarrow & & \searrow \\ \mathrm{A}, i & & \mathrm{B}, i \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), i \\ \downarrow \\ \neg\mathrm{A}, i \\ \neg\mathrm{B}, i \end{array}$$

$$\begin{array}{ccc} & \mathrm{A} \supset \mathrm{B}, i & \\ \swarrow & & \searrow \\ \neg\mathrm{A}, i & & \mathrm{B}, i \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \supset \mathrm{B}), i \\ \downarrow \\ \mathrm{A}, i \\ \neg\mathrm{B}, i \end{array} \qquad \begin{array}{ccc} & \mathrm{A} \equiv \mathrm{B}, i & \\ \swarrow & & \searrow \\ \mathrm{A}, i & & \neg\mathrm{A}, i \\ \mathrm{B}, i & & \neg\mathrm{B}, i \end{array} \qquad \begin{array}{ccc} & \neg(\mathrm{A} \equiv \mathrm{B}), i & \\ \swarrow & & \searrow \\ \neg\mathrm{A}, i & & \mathrm{A}, i \\ \mathrm{B}, i & & \neg\mathrm{B}, i \end{array}$$

$$\begin{array}{c} \Box\ \mathrm{A}, i \\ i \text{ r } j \\ \downarrow \\ \mathrm{A}, j \\ (\text{every } j) \end{array} \qquad \begin{array}{c} \neg\Box\ \mathrm{A}, i \\ \downarrow \\ \Diamond\neg\ \mathrm{A}, i \end{array} \qquad \begin{array}{c} \Diamond\ \mathrm{A}, i \\ \downarrow \\ i \text{ r } j \\ \mathrm{A}, j \\ (\text{new } j) \end{array} \qquad \begin{array}{c} \neg\Diamond\ \mathrm{A}, i \\ \downarrow \\ \Box\neg\ \mathrm{A}, i \end{array}$$

Closure condition: A branch of closes if it for some i, A, i and ¬ A, i occur on the same branch.

Counter-model procedure: For each i that occurs the word w_i exists. If i r j occurs on the branch then w_i R w_j. If P, i occurs on the branch then v_{w_i}(P) = 1, if ¬ P, i occurs on the branch then v_{w_i}(P) = 0.

Extensions

ρ reflexivity: for all w, wRw

σ symmetry: for all w_1, w_2, if w_1Rw_2, then w_2Rw_1

τ transitivity: for all w_1, w_2, w_3, if w_1Rw_2 and w_2Rw_3, then w_1Rw_3

η extendability: for all w_1, there is a w_2 such that w_1Rw_2

8 Tense Logic

$$\begin{array}{c} \neg\neg\, \mathrm{A}, i \\ \downarrow \\ \mathrm{A}, i \end{array}$$

$$\begin{array}{c} \mathrm{A} \wedge \mathrm{B}, i \\ \downarrow \\ \mathrm{A}, i \\ \mathrm{B}, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \wedge \mathrm{B}), i} \\ \swarrow & \searrow \\ \neg\mathrm{A}, i & \neg\mathrm{B}, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \vee \mathrm{B}, i} \\ \swarrow & \searrow \\ \mathrm{A}, i & \mathrm{B}, i \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \vee \mathrm{B}), i \\ \downarrow \\ \neg\mathrm{A}, i \\ \neg\mathrm{B}, i \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \supset \mathrm{B}, i} \\ \swarrow & \searrow \\ \neg\mathrm{A}, i & \mathrm{B}, i \end{array} \qquad \begin{array}{c} \neg(\mathrm{A} \supset \mathrm{B}), i \\ \downarrow \\ \mathrm{A}, i \\ \neg\mathrm{B}, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\mathrm{A} \equiv \mathrm{B}, i} \\ \swarrow & \searrow \\ \mathrm{A}, i & \neg\mathrm{A}, i \\ \mathrm{B}, i & \neg\mathrm{B}, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\mathrm{A} \equiv \mathrm{B}), i} \\ \swarrow & \searrow \\ \neg\mathrm{A}, i & \mathrm{A}, i \\ \mathrm{B}, i & \neg\mathrm{B}, i \end{array}$$

$$\begin{array}{c} [\mathrm{F}]\mathrm{A}, i \\ i \text{ r } j \\ \downarrow \\ \mathrm{A}, j \\ (\text{every } j) \end{array} \qquad \begin{array}{c} \neg[\mathrm{F}]\mathrm{A}, i \\ \downarrow \\ \langle \mathrm{F} \rangle \neg\, \mathrm{A}, i \end{array} \qquad \begin{array}{c} \langle \mathrm{F} \rangle \mathrm{A}, i \\ \downarrow \\ i \text{ r } j \\ \mathrm{A}, j \\ (\text{new } j) \end{array} \qquad \begin{array}{c} \neg\langle \mathrm{F} \rangle \mathrm{A}, i \\ \downarrow \\ [\mathrm{F}]\neg\, \mathrm{A}, i \end{array}$$

$$\begin{array}{c} [\mathrm{P}]\mathrm{A}, i \\ j \text{ r } i \\ \downarrow \\ \mathrm{A}, j \\ (\text{every } j) \end{array} \qquad \begin{array}{c} \neg[\mathrm{P}]\mathrm{A}, i \\ \downarrow \\ \langle \mathrm{P} \rangle \neg\, \mathrm{A}, i \end{array} \qquad \begin{array}{c} \langle \mathrm{P} \rangle \mathrm{A}, i \\ \downarrow \\ j \text{ r } i \\ \mathrm{A}, j \\ (\text{new } j) \end{array} \qquad \begin{array}{c} \neg\langle \mathrm{P} \rangle \mathrm{A}, i \\ \downarrow \\ [\mathrm{P}]\neg\, \mathrm{A}, i \end{array}$$

Closure condition: A branch of closes if it for some i, A, i and ¬ A, i occur on the same branch.

Counter-model procedure: For each i that occurs the word w_i exists. If i r j occurs on the branch then w_i R w_j. If P, i occurs on the branch then v_{w_i}(P) = 1, if ¬ P, i occurs on the branch then $v_{w_i}(\mathrm{P}) = 0$. If there are lines of the form $i = j$, $j = k$, ..., we only chose one of them, like i, and ignore the others.

Extensions

δ denseness: if iRj then for some k, iRk and kRj

ϕ forward convergence: if iRj and iRk then jRk or $j = k$ or kRj

β backward convergence: if jRi and kRi then jRk or $j = k$ or kRj

9 First Order Modal Logic

$$\begin{array}{c} \neg\neg\, A, i \\ \downarrow \\ A, i \end{array}$$

$$\begin{array}{c} A \wedge B, i \\ \downarrow \\ A, i \\ B, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(A \wedge B), i} \\ \swarrow & \searrow \\ \neg A, i & \neg B, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{A \vee B, i} \\ \swarrow & \searrow \\ A, i & B, i \end{array} \qquad \begin{array}{c} \neg(A \vee B), i \\ \downarrow \\ \neg A, i \\ \neg B, i \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{A \supset B, i} \\ \swarrow & \searrow \\ \neg A, i & B, i \end{array} \qquad \begin{array}{c} \neg(A \supset B), i \\ \downarrow \\ A, i \\ \neg B, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{A \equiv B, i} \\ \swarrow & \searrow \\ A, i & \neg A, i \\ B, i & \neg B, i \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(A \equiv B), i} \\ \swarrow & \searrow \\ \neg A, i & A, i \\ B, i & \neg B, i \end{array}$$

$$\begin{array}{c} \Box\, A, i \\ i \text{ r } j \\ \downarrow \\ A, j \\ \text{(every } j\text{)} \end{array} \qquad \begin{array}{c} \neg\Box\, A, i \\ \downarrow \\ \Diamond\neg\, A, i \end{array} \qquad \begin{array}{c} \Diamond\, A, i \\ \downarrow \\ i \text{ r } j \\ A, j \\ \text{(new } j\text{)} \end{array} \qquad \begin{array}{c} \neg\Diamond\, A, i \\ \downarrow \\ \Box\neg\, A, i \end{array}$$

Variable Domain

$$\begin{array}{cc} \multicolumn{2}{c}{\forall x\ A, i} \\ \swarrow & \searrow \\ A_x(a), i & \neg\ \mathfrak{E}a, i \\ \multicolumn{2}{c}{\text{(old } a \text{ if possible)}} \end{array} \qquad \begin{array}{c} \neg\forall x\ A, i \\ \downarrow \\ \exists x\ \neg A, i \end{array} \qquad \begin{array}{c} \exists x\ A, i \\ \downarrow \\ \mathfrak{E}a, i \\ A_x(a), i \\ \text{(new } a\text{)} \end{array} \qquad \begin{array}{c} \neg\exists x\ A, i \\ \downarrow \\ \forall x\ \neg A, i \end{array}$$

Constant Domain

$$\begin{array}{c} \forall x\ A, i \\ \downarrow \\ A_x(a), i \\ \text{(old } a \text{ if possible)} \end{array} \qquad \begin{array}{c} \neg\forall x\ A, i \\ \downarrow \\ \exists x\ \neg A, i \end{array} \qquad \begin{array}{c} \exists x\ A, i \\ \downarrow \\ A_x(a), i \\ \text{(new } a\text{)} \end{array} \qquad \begin{array}{c} \neg\exists x\ A, i \\ \downarrow \\ \forall x\ \neg A, i \end{array}$$

Counter-models: Need to define:

- which objects exist in which worlds: e.g. $D(w_1) = \{\partial_a, \partial_b\}$
- which predicates hold for which objects: e.g. $v_{w_1}(P) = \{\partial_c\}$

10 Fuzzy Logic

$$\neg x = 1 - x$$
$$x \wedge y = \min(x, y)$$
$$x \vee y = \max(x, y)$$
$$x \rightarrow y = \min(1, 1 - x + y)$$

Designated values: $A \models_{Ł_{0.5}}$ if $v(A) \geq 0.5$ then $v(B) \geq 0.5$
Special case: $Ł_{\aleph}$ - the only designated value is 1.

11 Default Logic

$$\beta = \frac{\phi : \psi_1, ..., \psi_n}{\chi} \quad \text{for example} \quad \frac{bird(X) : flies(X)}{flies(X)}$$

where:

$pre(\delta) = \phi$ (the prerequisites of δ)

$just(\delta) = \{\psi_1, ..., \psi_n\}$ (the justification of δ)

$cons(\delta) = \chi$ (the consequence of δ)

11.1 Processes

$\prod = (\delta_0, \delta_1, ...)$ can also be empty: ()

$Th(X)$ = all formulas that can be deduced from X.

$In(\prod) = Th(M)$, where $M = W \cup \{\, cons(\delta) \mid \delta \in \prod \,\}$

$Out(\prod) = Th(N)$, where $N = \{\, \neg just(\delta) \mid \delta \in \prod \,\}$

$\prod$ is *closed* iff every $\delta \in D$ that is applicable to $In(\prod)$ is also in $\prod$.

$\prod$ is *successful* iff $In(\prod) \cap Out(\prod) = \emptyset$

Extensions A set of formulas E is an extension of the default theory T iff there is some closed and successful process $\prod$ of T such that $E = In(\prod)$

Skeptical consequence $(W, D) \vdash_s \phi$ iff ϕ is in all extensions of (W, D).

Credulous consequence $(W, D) \vdash_c \phi$ iff ϕ is in at least one extension of (W, D).

12 Soundness and Completeness

12.1 Propositional Logic

$\Sigma \models A$ iff for all valuations v, for all $B \in \Sigma$, $v(B) = 1$, then $v(A) = 1$.

$\Sigma \vdash A$ iff there is a closed tree whose initial list comprises the members of Σ and the negation of A.

Faithful A valuation v is *faithful* to branch b iff for every formula D that occurs on b, $v(D) = 1$.

Induced A valuation v is *induced* by branch b, iff for every propositional parameter p that occurs on b, $v(p) = 1$ iff p is a node on b, and $v(p) = 0$ iff $\neg p$ is a node on b.

Soundness lemma If v is *faithful* to a branch b, and a tableau rule is applied to b, then v is faithful to at least *one* of of the branches generated by the application of the rule.

Completeness lemma If branch b is *complete* and *open*, and if v is the valuation *induced* by b, then for all formulas D: if D is on b, then $v(D) = 1$, and if $\neg D$ is on b, then $v(D) = 0$.

12.2 Modal Logic

Basic modal logic, extensions K_ρ, K_σ, K_τ, K_η, and any combination of extensions are all sound *and* complete.

$\Sigma \models A$ iff for all valuations v, for all $B \in \Sigma$, $v(B) = 1$, then $v(A) = 1$.

$\Sigma \vdash A$ iff there is a closed tree whose initial list comprises the members of Σ and the negation of A.

Faithful An interpretation $I = \langle W, R, v \rangle$ is *faithful* to branch b iff there is a map f from $\mathbb{N}$ to W such that:

- For every node D, i on b, D is true at world $f(i)$ in I.
- If i r j is on b, then $f(i)Rf(j)$ is in R.

Induced If branch b is *complete* and *open*, $I = \langle W, R, v \rangle$ is *induced* by b iff:

- W = the set of all worlds w_i such that i appears on b.
- $w_i R w_j$ iff i r j occurs on b.
- If p, i occurs on b, then $v_{w_i}(p) = 1$, and if $\neg p, i$ occurs on b, then $v_{w_i}(p) = 0$.

Soundness lemma If $I = \langle W, R, v \rangle$ is *faithful* to branch b, and a tableau rule is applied to b, then I is faithful to at least *one* of of the branches generated by the application of the rule.

Completeness lemma If branch b is *complete* and *open*, and $I = \langle W, R, v\rangle$ is *induced* by b, then for all formulas D and for all i: if D, i is on b, then $v_{w_i}(D) = 1$, and if $\neg D, i$ is on b, then $v_{w_i}(D) = 0$.