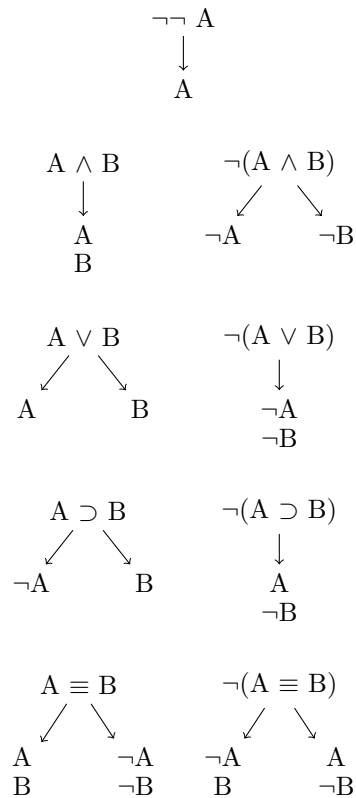


Semantic Tableaux

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July, 2020

1 First Order Logic (FOL)

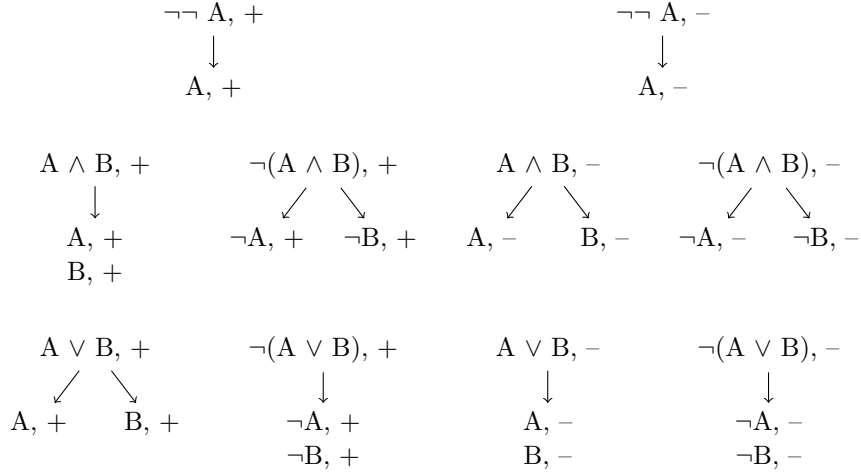


Closure condition: A branch closes if it contains both A and $\neg A$.

Counter-model procedure: Take any open branch, if P occurs at any node in the branch, assign it 1, and if $\neg P$ occurs on any node, assign it 0.

Designated values: 1

2 First Degree Entailment (FDE)



Closure condition: A branch closes if it contains both $A, +$ and $A, -$.

Counter-model procedure: Take any open branch, if $P, +$ occurs at any node in the branch assign it $P\rho 1$, and if $\neg P, +$ occurs on any node, assign it $P\rho 0$.

Truth table:

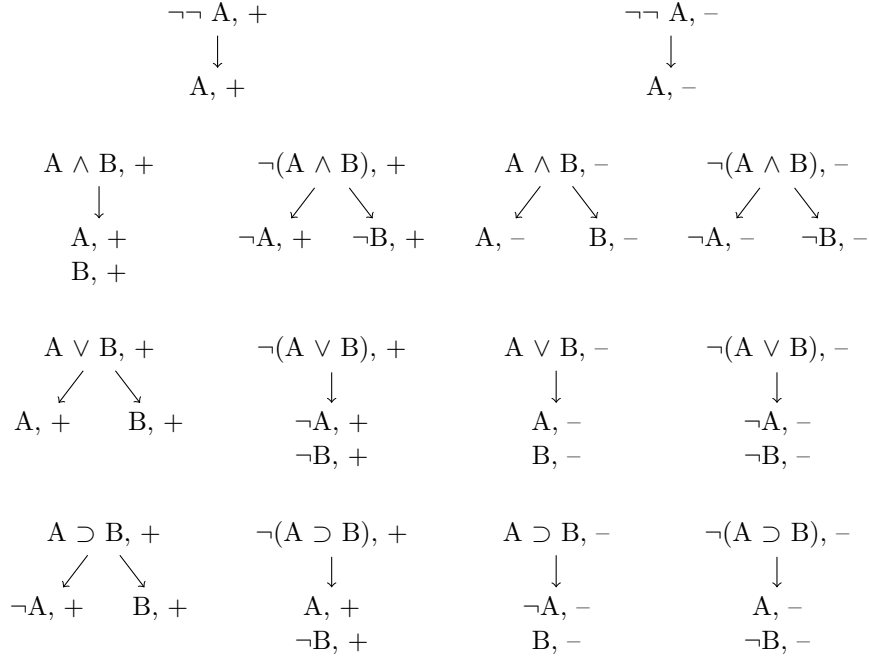
\neg	
0	1
n	n
b	b
1	0

\wedge	0	n	b	1
0	0	0	0	0
n	0	n	0	n
b	0	0	b	b
1	0	n	b	1

\vee	0	n	b	1
0	0	n	b	1
n	n	n	1	1
b	b	1	b	1
1	1	1	1	1

Designated values: 1, b

3 Kleene 3-valued logic (K_3)



Closure condition: A branch closes if it contains both $A, +$ and $A, -$ or both $A, +$ and $\neg A, +$.

Counter-model procedure: Take any open branch, if $P, +$ occurs at any node in the branch assign it $P\rho 1$, and if $\neg P, +$ occurs on any node, assign it $P\rho 0$.

Truth table:

\neg	
0	1
i	i
1	0

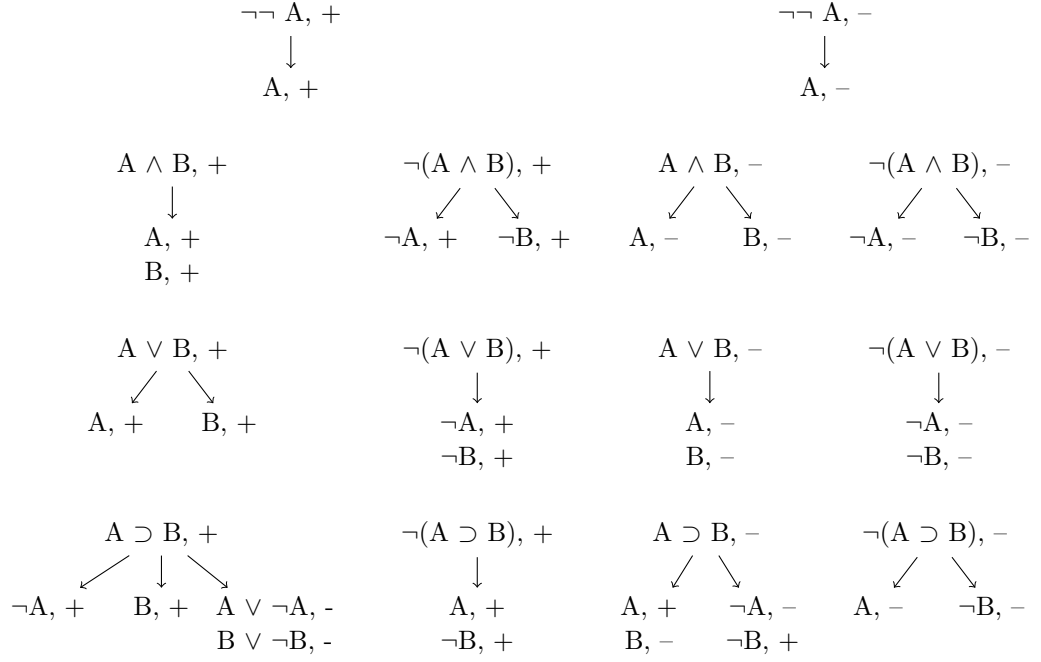
\wedge	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

\vee	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

\supset	0	i	1
0	1	1	1
i	i	i	1
1	0	i	1

Designated values: 1

4 Łukasiewicz 3-valued logic (\mathbf{L}_3)



Closure condition: A branch closes if it contains both $A, +$ and $A, -$ or both $A, +$ and $\neg A, +$.

Counter-model procedure: Take any open branch, if $P, +$ occurs at any node in the branch assign it $P\rho 1$, and if $\neg P, +$ occurs on any node, assign it $P\rho 0$.

Designated values: 1

Truth table:

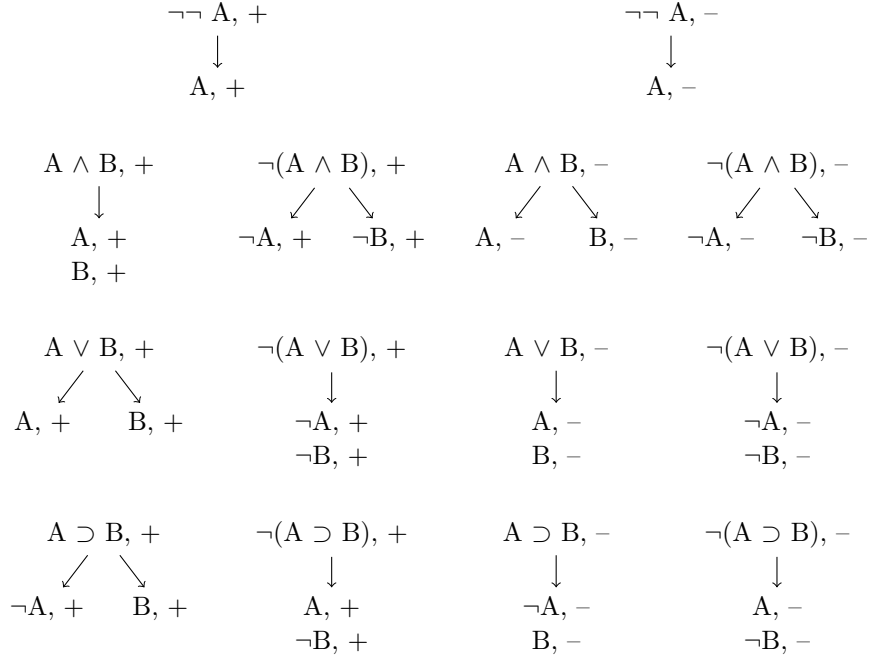
\neg	
0	1
<i>i</i>	<i>i</i>
1	0

\wedge	0	<i>i</i>	1
0	0	0	0
<i>i</i>	0	<i>i</i>	<i>i</i>
1	0	<i>i</i>	1

\vee	0	<i>i</i>	1
0	0	<i>i</i>	1
<i>i</i>	<i>i</i>	<i>i</i>	1
1	1	1	1

\supset	0	<i>i</i>	1
0	1	1	1
<i>i</i>	<i>i</i>	1	1
1	0	<i>i</i>	1

5 Logic of Paradox (LP)



Closure condition: A branch closes if it contains both $A, +$ and $A, -$ or both $A, -$ and $\neg A, -$.

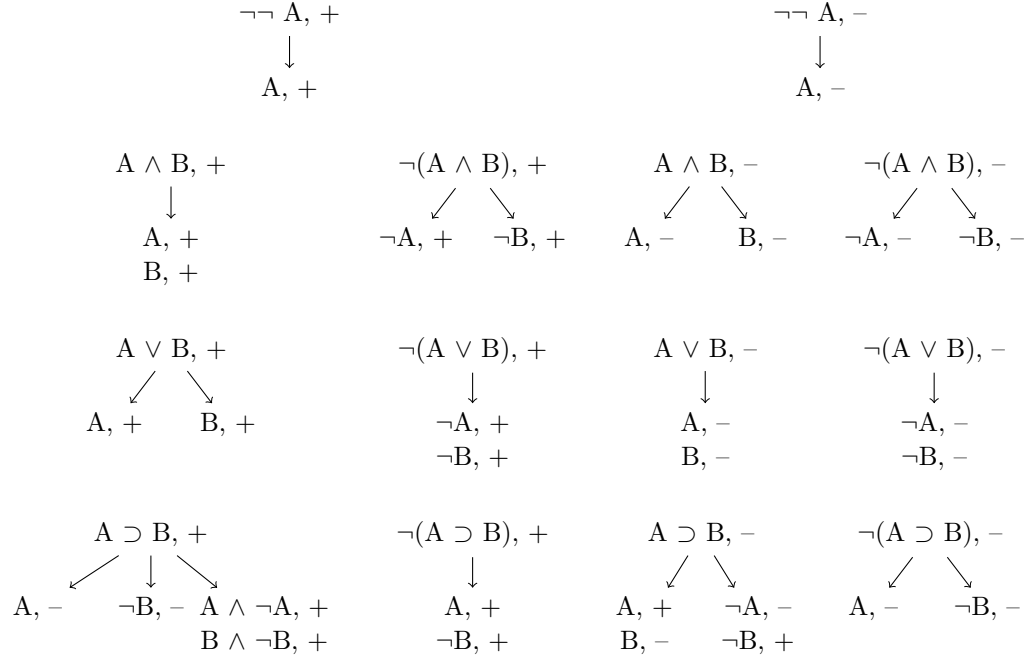
Counter-model procedure: Take any open branch, if $P, -$ *does not* occur on any node of the branch assign it $P\rho 1$, and if $\neg P, -$ *does not* occur on any node, assign it $P\rho 0$.

Designated values: $1, i$

Truth table:

\neg		\wedge	0	i	1	\vee	0	i	1	\supset	0	i	1
0	1	0	0	0	0	0	0	i	1	0	1	1	1
i	i	i	0	i	i	i	i	i	1	i	i	i	1
1	0	1	0	i	1	1	1	1	1	1	0	i	1

6 Mix 3-valued logic (RM₃)



Closure condition: A branch closes if it contains both $A, +$ and $A, -$ or both $A, -$ and $\neg A, -$.

Counter-model procedure: Take any open branch, if $P, -$ *does not* occur on any node of the branch assign it $P\rho 1$, and if $\neg P, -$ *does not* occur on any node, assign it $P\rho 0$.

Designated values: $1, i$

Truth table:

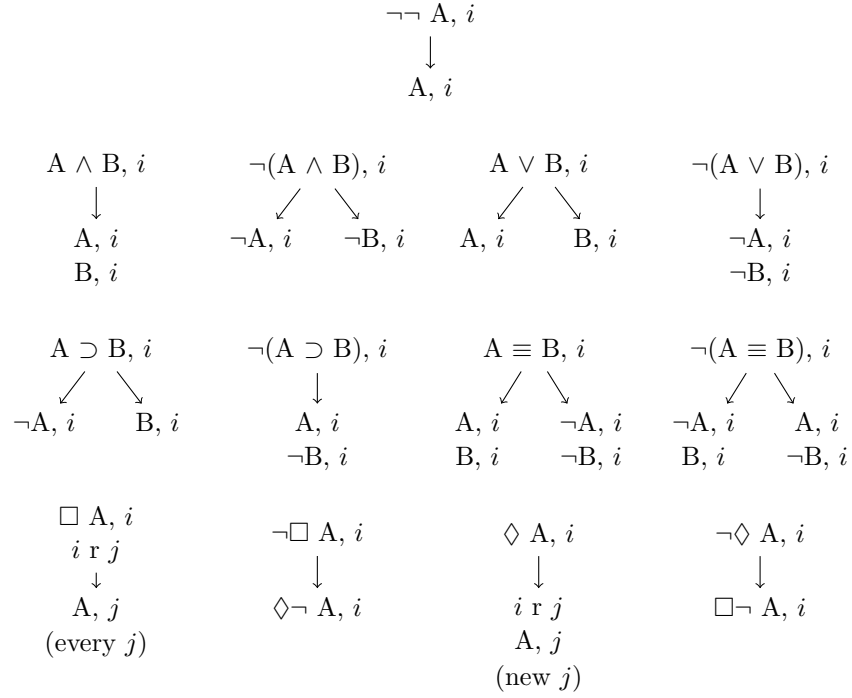
\neg	
0	1
i	i
1	0

\wedge	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

\vee	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

\supset	0	i	1
0	1	1	1
i	0	i	1
1	0	0	1

7 Basic Modal Logic



Closure condition: A branch of closes if it for some i , A, i and $\neg A, i$ occur on the same branch.

Counter-model procedure: For each i that occurs the word w_i exists. If $i \text{ r } j$ occurs on the branch then $w_i R w_j$. If P, i occurs on the branch then $v_{w_i}(P) = 1$, if $\neg P, i$ occurs on the branch then $v_{w_i}(P) = 0$.

Extensions

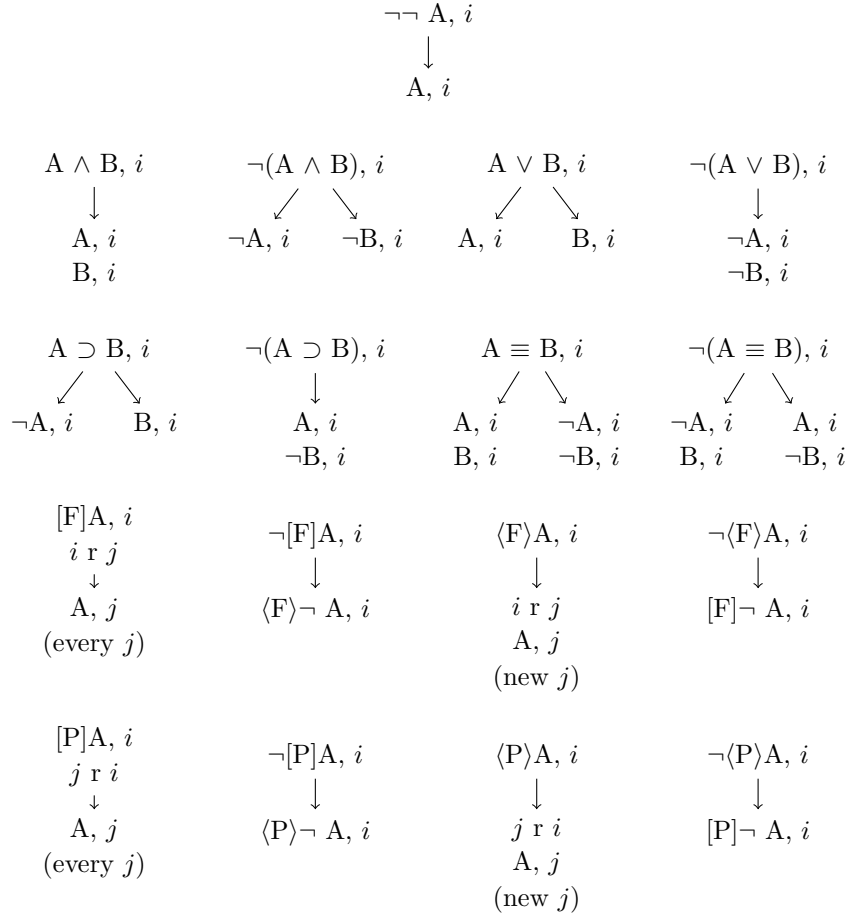
ρ reflexivity: for all w , wRw

σ symmetry: for all w_1, w_2 , if w_1Rw_2 , then w_2Rw_1

τ transitivity: for all w_1, w_2, w_3 , if w_1Rw_2 and w_2Rw_3 , then w_1Rw_3

η extendability: for all w_1 , there is a w_2 such that w_1Rw_2

8 Tense Logic



Closure condition: A branch closes if it for some i , A, i and $\neg A, i$ occur on the same branch.

Counter-model procedure: For each i that occurs the word w_i exists. If $i \text{ r } j$ occurs on the branch then $w_i R w_j$. If P, i occurs on the branch then $v_{w_i}(P) = 1$, if $\neg P, i$ occurs on the branch then $v_{w_i}(P) = 0$. If there are lines of the form $i = j, j = k, \dots$, we only chose one of them, like i , and ignore the others.

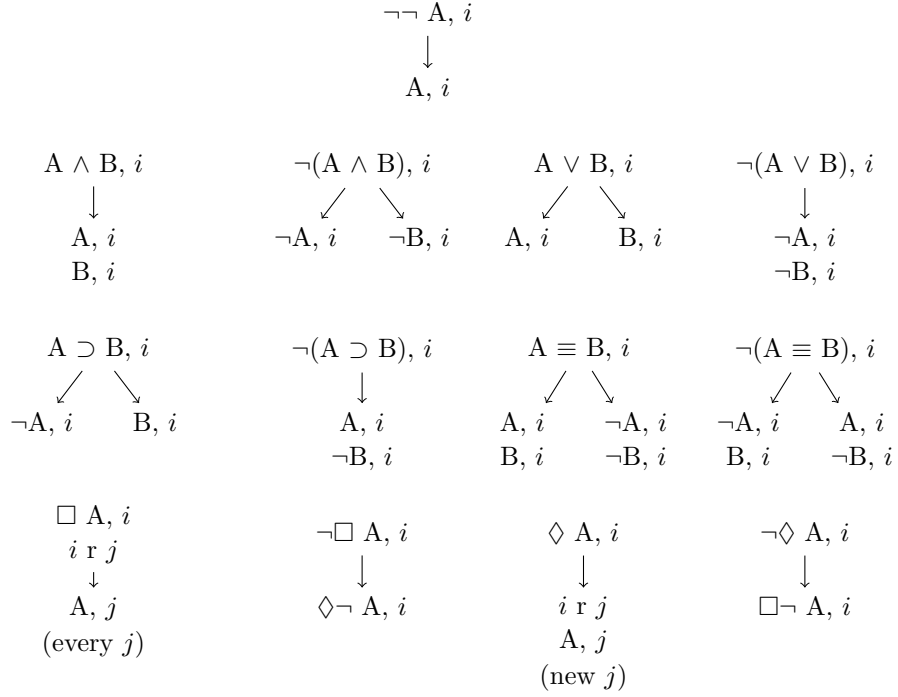
Extensions

δ denseness: if iRj then for some k , iRk and kRj

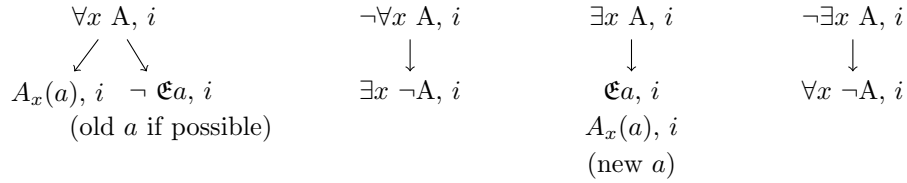
ϕ forward convergence: if iRj and iRk then jRk or $j = k$ or kRj

β backward convergence: if jRi and kRi then jRk or $j = k$ or kRj

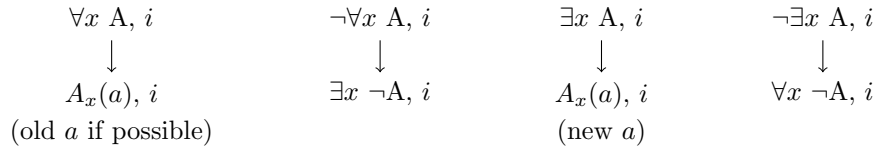
9 First Order Modal Logic



Variable Domain



Constant Domain



Counter-models: Need to define:

- which objects exist in which worlds: e.g. $D(w_1) = \{\partial_a, \partial_b\}$
- which predicates hold for which objects: e.g. $v_{w_1}(P) = \{\partial_c\}$

10 Fuzzy Logic

$$\begin{aligned}\neg x &= 1 - x \\ x \wedge y &= \min(x, y) \\ x \vee y &= \max(x, y) \\ x \rightarrow y &= \min(1, 1 - x + y)\end{aligned}$$

Designated values: $A \models_{L_{0.5}}$ if $v(A) \geq 0.5$ then $v(B) \geq 0.5$
Special case: L_N - the only designated value is 1.

11 Default Logic

$$\beta = \frac{\phi : \psi_1, \dots, \psi_n}{\chi} \quad \text{for example} \quad \frac{bird(X) : flies(X)}{flies(X)}$$

where:

$$\begin{aligned}pre(\delta) &= \phi \text{ (the prerequisites of } \delta) \\ just(\delta) &= \{\psi_1, \dots, \psi_n\} \text{ (the justification of } \delta) \\ cons(\delta) &= \chi \text{ (the consequence of } \delta)\end{aligned}$$

11.1 Processes

$\Pi = (\delta_0, \delta_1, \dots)$ can also be empty: $()$

$Th(X)$ = all formulas that can be deduced from X .

$In(\Pi) = Th(M)$, where $M = W \cup \{cons(\delta) \mid \delta \in \Pi\}$

$Out(\Pi) = Th(N)$, where $N = \{\neg just(\delta) \mid \delta \in \Pi\}$

Π is *closed* iff every $\delta \in D$ that is applicable to $In(\Pi)$ is also in Π .

Π is *successful* iff $In(\Pi) \cap Out(\Pi) = \emptyset$

Extensions A set of formulas E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Skeptical consequence $(W, D) \vdash_s \phi$ iff ϕ is in all extensions of (W, D) .

Credulous consequence $(W, D) \vdash_c \phi$ iff ϕ is in at least one extension of (W, D) .

12 Soundness and Completeness

12.1 Propositional Logic

$\Sigma \models A$ iff for all valuations v , for all $B \in \Sigma$, $v(B) = 1$, then $v(A) = 1$.

$\Sigma \vdash A$ iff there is a closed tree whose initial list comprises the members of Σ and the negation of A .

Faithful A valuation v is *faithful* to branch b iff for every formula D that occurs on b , $v(D) = 1$.

Induced A valuation v is *induced* by branch b , iff for every propositional parameter p that occurs on b , $v(p) = 1$ iff p is a node on b , and $v(p) = 0$ iff $\neg p$ is a node on b .

Soundness lemma If v is *faithful* to a branch b , and a tableau rule is applied to b , then v is faithful to at least *one* of the branches generated by the application of the rule.

Completeness lemma If branch b is *complete* and *open*, and if v is the valuation *induced* by b , then for all formulas D : if D is on b , then $v(D) = 1$, and if $\neg D$ is on b , then $v(D) = 0$.

12.2 Modal Logic

Basic modal logic, extensions K_ρ , K_σ , K_τ , K_η , and any combination of extensions are all sound *and* complete.

$\Sigma \models A$ iff for all valuations v , for all $B \in \Sigma$, $v(B) = 1$, then $v(A) = 1$.

$\Sigma \vdash A$ iff there is a closed tree whose initial list comprises the members of Σ and the negation of A .

Faithful An interpretation $I = \langle W, R, v \rangle$ is *faithful* to branch b iff there is a map f from \mathbb{N} to W such that:

- For every node D, i on b , D is true at world $f(i)$ in I .
- If $i \text{ r } j$ is on b , then $f(i)Rf(j)$ is in R .

Induced If branch b is *complete* and *open*, $I = \langle W, R, v \rangle$ is *induced* by b iff:

- W = the set of all worlds w_i such that i appears on b .
- $w_i R w_j$ iff $i \text{ r } j$ occurs on b .
- If p, i occurs on b , then $v_{w_i}(p) = 1$, and if $\neg p, i$ occurs on b , then $v_{w_i}(p) = 0$.

Soundness lemma If $I = \langle W, R, v \rangle$ is *faithful* to branch b , and a tableau rule is applied to b , then I is faithful to at least *one* of the branches generated by the application of the rule.

Completeness lemma If branch b is *complete* and *open*, and $I = \langle W, R, v \rangle$ is *induced* by b , then for all formulas D and for all i : if D, i is on b , then $v_{w_i}(D) = 1$, and if $\neg D, i$ is on b , then $v_{w_i}(D) = 0$.