Graph Eigenvalue Sound Generator: README

1 Introduction

This document serves as a README for the netsound script that generates sound and visuals based on graph eigenvalues.

2 Eigenvalues and Audio Signal Generation

Eigenvalues of the graph's adjacency or other spectral matrices are used as a foundational element in generating the audio signals. These eigenvalues are extracted by solving the characteristic equation associated with the given spectrum type (Adjacency Matrix, Laplacian, or Modularity). The real parts of the eigenvalues are then normalized to a frequency range between 200 Hz and 4000 Hz.

2.1 Audio Types and Eigenvalue Application

Different types of audio waveforms are generated based on user selection: Sine Wave, Square Wave, Sawtooth Wave, FM Synthesis, and Waveshaping Synthesis. In each case, the eigenvalue-derived frequencies serve as the basic frequency components of the audio signal. Modulation techniques and additional harmonics are applied depending on the selected audio type.

- **Sine Wave**: A pure sine wave is generated for each eigenvalue, which is then modulated by a frequency derived from the eigenvalue and modulation index.
- **Square Wave**: A square wave is generated using the frequencies derived from the eigenvalues.
- **Sawtooth Wave**: A sawtooth waveform is generated where the frequency components are based on the eigenvalues.
- **FM Synthesis**: Frequency modulation is applied, where the carrier frequency is an eigenvalue-derived frequency and the modulating frequency is determined by the modulation index.
- **Waveshaping Synthesis**: The eigenvalue-derived frequencies are used as the fundamental frequencies upon which waveshaping techniques are applied.

2.2 Harmonics

The first three harmonics (2nd, 3rd, and 4th) are added to enrich the audio signal. Each harmonic frequency is an integer multiple of the base frequency derived from the eigenvalue.

2.3 Modulation

The modulation index controls the extent of frequency modulation applied to the audio signal. A higher modulation index results in a more complex waveform, making the sound richer but potentially less recognizable in terms of its relation to the original graph.

2.4 Spectrogram

A spectrogram of the generated audio signal is also displayed, providing a time-frequency representation that visualizes how the eigenvalues (now frequencies) are distributed over time in the audio signal.

3 Graph Eigenvalues

For a graph G, eigenvalues (λ) of its adjacency matrix A are obtained by solving the characteristic equation:

$$\det(A - \lambda I) = 0$$

The eigenvalues can also be computed for the normalized Laplacian and modularity matrices.

3.1 Normalized Laplacian

The normalized Laplacian is defined as:

$$L = D^{-1/2} A D^{-1/2}$$

3.2 Modularity Matrix

The modularity matrix B is defined as:

$$B = A - \frac{k \times k}{2m}$$

where A is the adjacency matrix, k is the degree vector, and m is the total number of edges.

4 Audio Generation

4.1 Sine Wave

$$f(t) = \sin(2\pi f t)$$

4.2 Square Wave

$$f(t) = \mathrm{sgn}(\sin(2\pi f t))$$

4.3 Sawtooth Wave

$$f(t) = \frac{1}{\pi} \arctan(\tan(\pi f t))$$

4.4 FM Synthesis

$$f(t) = \sin(2\pi f t + I \sin(2\pi f_m t))$$

where I is the modulation index, f is the carrier frequency, and f_m is the modulating frequency.

4.5 Waveshaping Synthesis

$$f(t) = \mathrm{sgn}(\sin(2\pi f t)) \times (1 - e^{-|\sin(2\pi f t)|})$$

5 Types of Graphs

In this script, multiple types of graphs are provided as options for generating sound and visuals. Below is a brief description of each graph type:

5.1 Complete Graph

A graph where every pair of distinct vertices is connected by a unique edge.

5.2 Cycle Graph

A graph that forms a single cycle or a closed chain.

5.3 Random Graph

A graph generated using the Erdős–Rényi model, where each edge exists independently with probability p. In the script $p = 0.5$

5.4 Star Graph

A graph where all nodes are connected to a central node.

5.5 Wheel Graph

Formed by connecting a single vertex to all vertices of a cycle graph.

5.6 Lollipop Graph

A graph comprising a complete graph and a path graph connected by a single edge.

5.7 Barabási–Albert Graph

A scale-free graph generated using preferential attachment.

5.8 Ladder Graph

Consists of two parallel lines with rungs between them, resembling a ladder.

5.9 Circular Ladder Graph

A ladder graph where the ends are also connected, forming a cycle.

5.10 Path Graph

A linear graph where vertices are connected end to end, but no cycles exist.

5.11 Binominal Tree

A balanced tree graph where each node has exactly two children.

5.12 Karate Club Graph

A social network graph representing relationships between members of a karate club.

5.13 Florentine Family Graph

A graph representing the relationships between prominent families in Renaissance Florence.

5.14 Les Miserables Graph

A co-appearance graph of characters in the novel Les Miserables.