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State Space

For the maze by itself there are $n * m - b$ possible states, where n is the width, m is the height, and b is the number of barriers. For the die by itself there are six sides and each can have four orientations depending upon which side is facing up. However, since the sixth side is not allowed to face up, the number of states for a die is $(6 - 1) * 4$. Now, when you combine the two you get $20(n * m - b)$ states total.

Problem Representation

Initial State: The initial state is the die lying on S space in the maze with the one on the die facing up, the two on the die facing north, and the three on the die facing east.

State Transition: A state transition consists of moving from one square to another while also rolling the die in that direction. Transitions that would require the six on the die to face up are not allowed. Additionally, a transition onto a square that acts as a barrier is not allowed.

Transition Costs: Transitions are measured in moves (e.g. it took four moves to reach the solution). Therefore each transition costs the same.

Goal State: The goal state is the die lying on G space in the maze with the one on the die facing up with any configuration of the other sides.

Output Interpretation

Output is formatted as follows for each of the three heuristics. The heuristic name is displayed on the top. Then the visited states are displayed describing the position on the grid, the north side of the die and the facing side of the die. From this information you can determine position and orientation. For each state after the first the direction used to reach that state is also presented. Next the results section is displayed which shows the number of states generated and visited. The last section is the solution, which displays the set of states and the transition necessary to move between them to get to the goal. The difference between this section and the visited states section is that in the visited states section, the state listed before each state may not be in the path to that state.

Heuristic Functions

Euclidean Distance

The first heuristic function I used was the Euclidean distance from the current state to the goal state. This heuristic is admissible because it calculates the shortest distance from a state to the goal state. It is not possible for this to overestimate because there is no distance that is shorter. This heuristic assumes:

- Die can move in a straight line to goal
- Die can slide (Doesn't need to roll)
- There are no obstacles between the die and the goal (Including no bounds on the size of the maze)

Manhattan Distance

The second heuristic function I used was the Manhattan distance from the current state to the goal state. This heuristic is admissible because it calculates the shortest amount of spaces from a state to the goal state. It is not possible for this to overestimate because there is no path through spaces that is shorter. This heuristic assumes:

- Die must move horizontally and vertically
- Die can slide (Doesn't need to roll)
- There are no obstacles between the die and the goal (Including no bounds on the size of the maze)

Die Roll Distance

The third heuristic function I used is a modification of the Manhattan distance to account for how the die is required to roll. It adds simple rules for different cases of whether the one side is facing up and the x and y distances. An important property of these rules is whether the die is on its side or not. The easiest way to think about a die being on its side is if you think of it like a bottle cap, where the open end is six, the top is one, and the other sides represent the round edge. When the die is on its side, that means that either two, three, four, or five are facing up. A die on its side can roll as many spaces as it needs in a straight line and then make one additional move to have the one side facing up. This behavior can be seen on sample puzzle one.

The first rule states, if a die is in the goal state then the distance is zero.

The second rule states, if a die is in the goal state except that a side other than one is facing up, then it will take exactly four moves to achieve the goal state.

The third and fourth rules state, if the die is in line with the goal (i.e. the change in x or y is zero) and has the one facing up, then it will take the Manhattan distance plus two moves to reach the solution optimally. One for it to get on its side, the Manhattan distance to move next to the goal, and one to get off its side onto the goal state.

The fifth and sixth rules are based on the same case as the third and fourth rules except the die has already moved to its side.

The seventh rule states that if the distance between the die and the goal are both greater than one and the die is not on its side, then it will require the Manhattan distance plus four moves to reach the goal. This can be thought of combining the cases of the third and fourth rule together, where the first task is to get in line with the goal, and then to move to the goal.

The eighth rule is like the seventh rule except the die is already on its side. In this case we assume the die has the optimal orientation to reach the goal in the shortest amount of moves. For example say the die is on its side with six facing west and the goal is east of the die. In order for the die to reorientate itself it would take two moves: one to move off its side moving in the y direction and another to get back on its side so that it can move towards the goal in the x direction. However, if the six is facing east then the die can set down in the direction of the goal, decreasing the Manhattan distance by one. Therefor instead of three moves plus the Manhattan distance which would be the case directly after the seventh rule when the die had moved one space, it is only the Manhattan distance plus two moves because we assume the die has the best orientation.

In each of these rules, either the exact distance or just short of the exact distance is generated. Therefore, none of these cases can overestimate the actual distance. Each of these rules is executed sequentially, so if any one matches than it is the only match considered. The assumptions for these rules are:

- Die must move horizontally and vertically
- Die must roll
- Die must never show the six side face up
- There are no obstacles between the die and the goal (Including no bounds on the size of the maze)
- If die is on its side, it is in the optimal orientation to achieve the goal in the least amount of steps

Performance Metrics

	Euclidean Distance		Manhattan Distance		Die Roll Distance	
States	Generated	Visited	Generated	Visited	Generated	Visited
Puzzle 1	21	14	21	14	13	9
Puzzle 2	82	59	79	57	37	31
Puzzle 3	3	3	3	3	3	3
Puzzle 4	128	103	128	101	63	51
Puzzle 5	1637	1072	1411	875	87	49

Discussion

The Euclidean distance was always the absolute shortest path and was always less than the actual cost of the path. The Manhattan distance can only occasionally calculate something near the actual cost. The only case which I am aware is when the die is on its side and the x or y distance is one. Because of this the Manhattan distance is generally always an underestimate as well. The Die Roll distance is the only method which takes how the die must roll into account. Due to this it generally makes an exact distance measurement or is very close. For the A* search to perform it must have a heuristic which never overestimates. If a heuristic could calculate the exact distance from any point to the goal then A* would only ever follow one path because it would be the optimal path. Therefore it is desirable to create a heuristic which is as close to always getting the exact distance at any point, because it would reduce the size of the path drastically.

This is the behavior that can be seen in the results above. As the size of the puzzle increases, the searches that do a poor job of estimating near the actual cost generate and explore more states than those that are more accurate in estimating.

Puzzle three seems peculiar in that all three methods generated and visited the same number of states. This is because there is no solution for puzzle three and therefore each search was required to try all possible states that were reachable.