Sketch the graph $f(x)$, where

$$f(x) = \frac{x^2(x-1)}{x+2}$$

(1) Find the domain of f

$$Dom\_f = \left\{(-\infty,-2)\bigcup(-2,\infty)\right\} \tag{1}$$

(2) Find x and y intercepts

$$\begin{gathered} f(x_{int}) = 0 \\ \left(\frac{x_{int}^2(x_{int}-1)}{\cancel{x_{int}+2}}\right)(\cancel{x_{int}+2}) = 0(\cancel{x_{int}+2}) \\ x_{int}^2(x_{int}-1) = 0 \\ \boxed{x\_int = \{0,1\}} \end{gathered} \tag{2}$$

(3) Check for any symmetry with respect to the y-axis and its origin Let x be some arbitrary number say 4, then its reverse -4 so:

$$\begin{gathered} f(4) = f(-4) \\ \frac{4^2(4-1)}{4+2} = \frac{(-4)^2((-4)-1)}{(-4)+2} \\ 8 \neq 40 \end{gathered} \tag{3}$$

We conclude that f is not symmetrical.

(5) Check for asymptotes

Veritcal Asymptote: Let $x=-2$ and since $f(x)$ D.N.E, then -2 is the veritcal asymptote.

Horizontal Asymptote: Because the highest degree $\delta_{p(x)}(v)=3$ and $\delta_{q(x)}(v)=1$, where $p(x)=x^2(x-1)$ and $q(x)=x+2$

We can deduce that there is no horizontal asymptote in f

Oblique Asymptote Because $\delta_{p(x)} - \delta_{p(x)} = 2$, then there is no oblique asymptote

(6) Determine critical points c to determine if f has a relative extremum $f'(c)=0$ and inflection point $f''(c)=0$

Finding the critical point/s first of f'

$$
\begin{aligned}
f'(x) &= \frac{x^2(x-1)}{x+2} \\
&= \frac{x^3-x^2}{x+2} \\
&= \frac{(3x^2-2x)(x+2)-(x^3-x^2)}{(x+2)^2} \\
&= \frac{3x^3+6x^2-2x^2-4x-x^3+x^2}{(x+2)^2} \\
f'(x) &= \frac{2x^3+5x^2-4x}{(x+2)^2} \\
\implies f'(c) &= 0 \\
\frac{2c^3+5c^2-4c}{\cancel{(c+2)^2}}\cancel{(c+2)^2} &= 0\cancel{(c+2)^2} \\
2c^3+5c^2-4c &= 0 \\
c(2c^2+5c-4) &= 0 \\
c^2+\frac{5}{2}c-2 &= 0
\end{aligned} \tag{4}
$$

$$
\begin{array}{r|l}
(-\tfrac{5}{4}+u)(-\tfrac{5}{4}-u) = -2 & \tfrac{25}{16}-u^2 = -2 \\
 & u^2 = \tfrac{25}{16}+2 \\
 & u = \pm\tfrac{\sqrt{57}}{4} \\
\left(-\tfrac{5}{4}+\tfrac{\sqrt{57}}{4}\right)\left(-\tfrac{5}{4}+\tfrac{\sqrt{57}}{4}\right) = -2 &
\end{array}
$$

$$
c = \left\{-\frac{5-\sqrt{57}}{4}, 0, -\frac{5+\sqrt{57}}{4}\right\}
$$

Critical points of the first derivatives are $-\frac{5-\sqrt{57}}{4}, 0, -\frac{5+\sqrt{57}}{4}$

Finding critical point/s next of f''

$$
\begin{aligned}
f''(x) &= \frac{2x^3+5x^2-4x}{(x+2)^2} \\
&= \frac{(6x^2+10x-4)(x+2)^2 - \left[2(2x^3+5x^2-4x)(x+2)\right]}{(x+2)^4} \\
&= \frac{\cancel{(x+2)}\left[(6x^2+10x-4)(x+2)-(4x^3+10x^2-8x)\right]}{(x+2)^4} \\
&= \frac{6x^3+22x^2+16x-8-4x^3-10x^2+8x}{(x+2)^3} \\
f''(x) &= \frac{2x^3+12x^2+24x-8}{(x+2)^3} \\
\implies f''(c) &= 0 \\
\frac{\cancel{2}\left(c^3+6c^2+12c-4\right)}{\cancel{(c+2)^3}}\cancel{\frac{(c+2)^3}{2}} &= 0\left(\cancel{\frac{(c+2)^3}{2}}\right) \\
c^3+6c^2+12c-4 &= 0 \\
c(c^2+6c+12) &= 4
\end{aligned}
\tag{5}
$$

We have discovered something impossible. That is by solving $f''(c) = 0$ algebraically. Luckily, there is a method that we can use, and that is by using Newton's Method:

$$
x_n = x_{n-1} - \frac{f(x)}{f'(x)},\ f'(x) \neq 0 \tag{6}
$$

With this information, we now can at least approximate $f''(c) = 0$.

Create a program in **C** to find the critical value c such that $f''(c) \approx 0$