Contents

Exercise 1

1.1 point a

Since PRP in by definition computationally close to U breaking CPA security implies to distinguish between the PRP and a uniform distribution.

T0D0 1: Find a correct game

1.2 point b

Let's construct a contrived PRP family in the following way:

$\forall k, P_k^{-1}(0) = 0^{n/2}||k$ this is still CPA since A cannot do decryption queries however an adversary playing a CCA game can query the decryption of 0 and thus obtain the key.

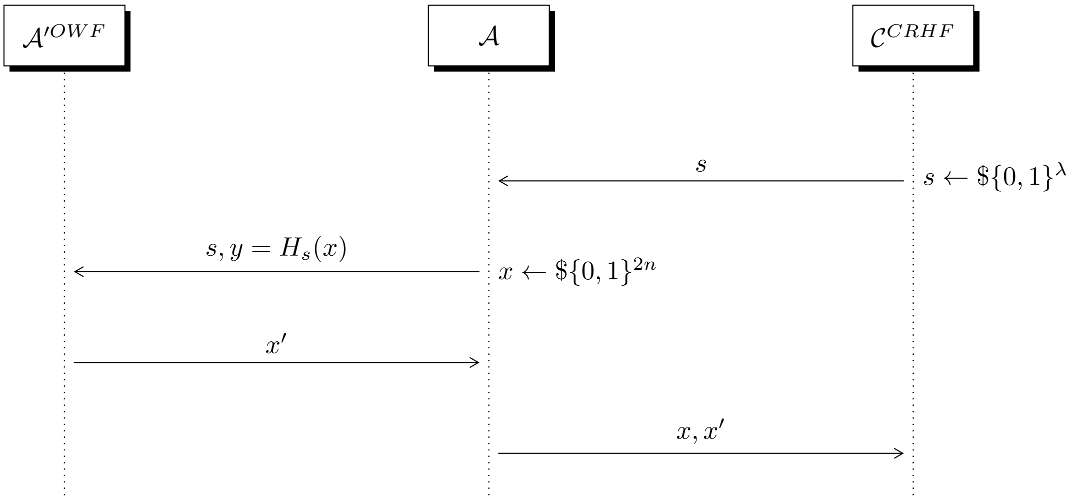
Exercise 2

2.1 point a

2.1.1 point i

$$\mathcal{H} \text{ is CRHF } \Rightarrow \mathcal{H} \text{ is OWF}$$

To show this property, let's make a reduction:



When does not $\mathcal{A}$ win?
Since CRHF game wants the final couple (x, x') with $x \neq x'$, if $\mathcal{A}'^{OWF}$ returns $x' = x$ the CRHF game doesn't work.

This **BAD** event happens with

$$\mathcal{P}[x = x'] = Col(X, X') = \sum_x \mathcal{P}[X = x \wedge X' = x] = \sum_x \mathcal{P}[X = x]\mathcal{P}[X' = x] = \frac{1}{2^{2n}}$$

.

2.1.2 ii

If functions from $\mathcal{H}$ family aren't compressing, the probability of **BAD** event changes:

$$\mathcal{P}[x = x'] = Col(X, X') = \sum_x \mathcal{P}[X = x \wedge X' = x] = \sum_x \mathcal{P}[X = x]\mathcal{P}[X' = x] = \frac{1}{2^n}$$

.

Now, if our functions from $\mathcal{H}$ were compressing (from 2n bits to n bits), the best CRHF function (the function with the minimum number of collisions) had $2^n + 1$ inputs generating a collision (in the same codomain's element).

In this case, the best possible CRHF function is bijective (since it could be a permutation over 2^n elements).

In general, for non-compressing functions we can show that

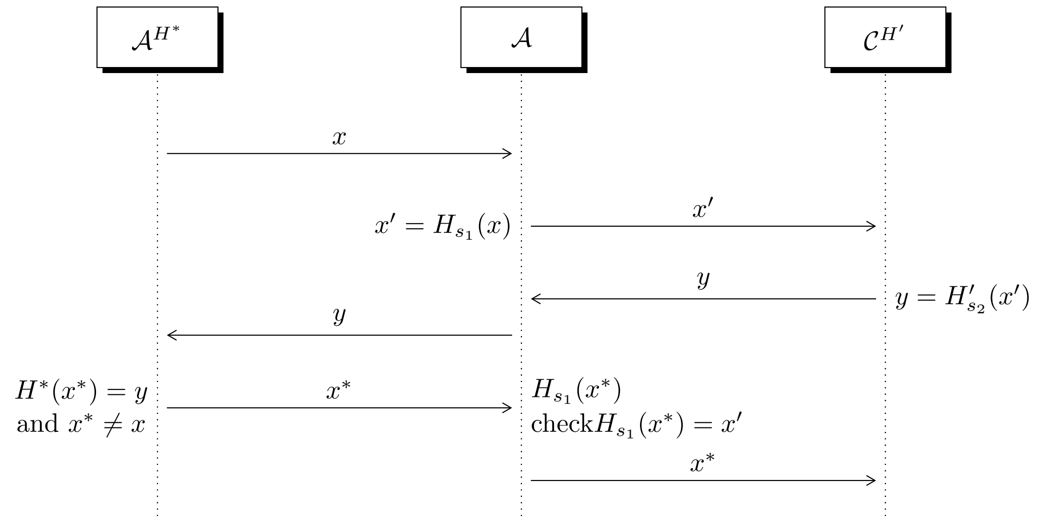
$$\mathcal{H} \text{ is CRHF mapping n bits to n bits} \Rightarrow \mathcal{H} \text{ is OWF}$$

with the same reduction of the above **point i** .

2.2 point b

Given $H^*_{s_1,s_2}(x) = H'_{s_2}(H_{s_1}(x))$ with $H^* : 4n \to n$. Suppose $\exists A^{H^*}$ which is able to find a collision in H^*.

Consider the following two Games:

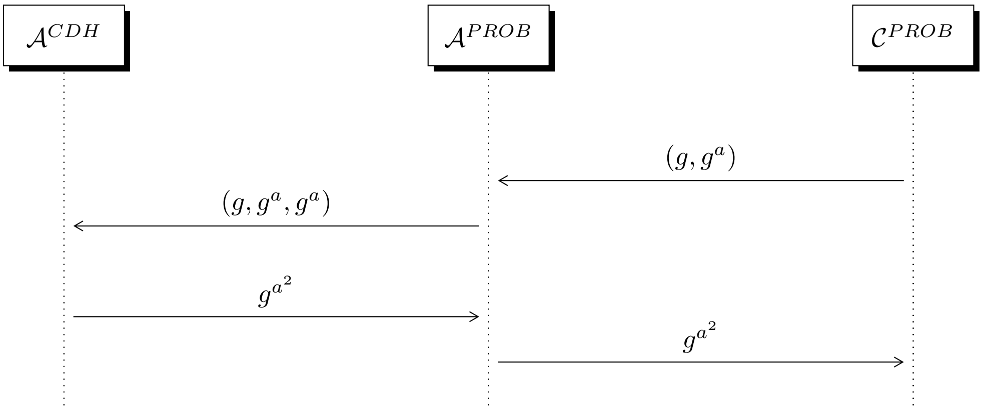


There is a "BAD event" in which A^{H^*} outputs a collision for H_{s_1}, meaning that $H_{s_1}(x) = H_{s_1}(x')$ in this case the second part of the reduction doesn't work. But $Pr[BAD]$ is negligible since H_{s_1} is collision resistant by definition. But now $H^*(x') = H^*(x)$ since x' was a collision for H^* but this must be a collision also for H'_{s_2} which was a CRHF for hypothesis.

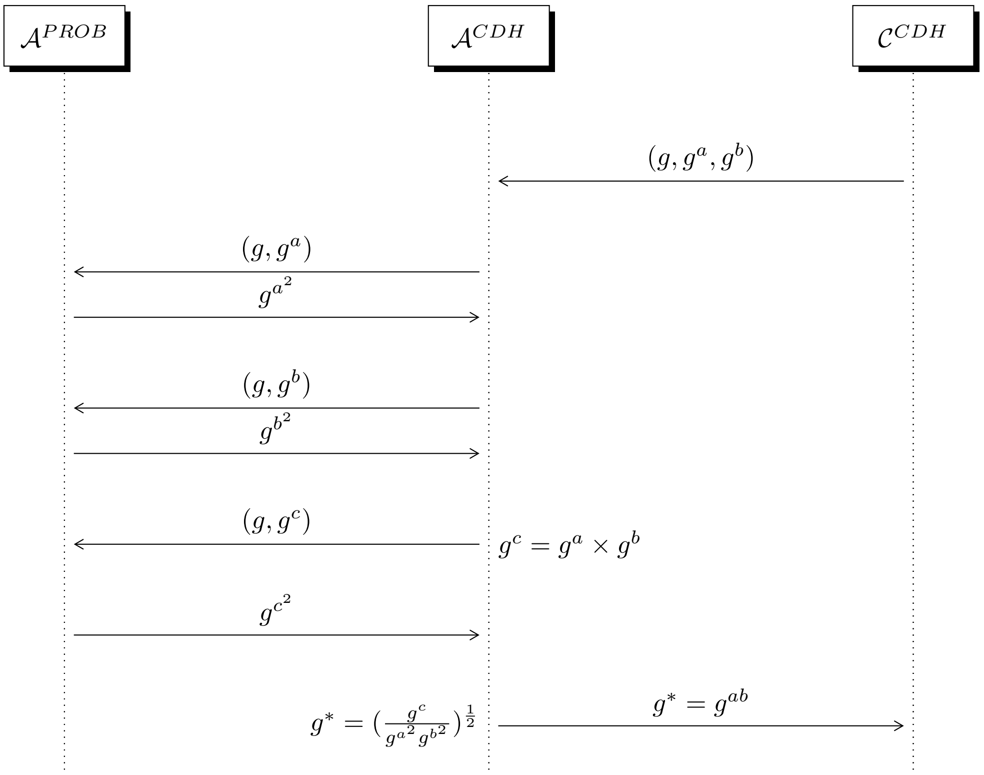
Exercise 3

3.1 point a

Call (g, g^a) compute g^{a^2} PROBLEM.
Now we want to prove that PROBLEM $\Leftrightarrow$ CDH
PROBLEM $\implies$ CDH:
Suppose $\exists A^{CDH}$ which is able to break CDH with non negligible probability.



CDH $\implies$ PROBLEM



3.2 point b

$$\begin{cases} N = pq \\ \varphi(N) = (p-1)(q-1) = pq - p - q + 1 \end{cases} \Rightarrow \begin{cases} N = pq \\ p + q = N - \varphi(N) + 1 = \beta \end{cases}$$

$$\Rightarrow \begin{cases} N = \beta q - q^2 \Rightarrow -q^2 + \beta q - N = 0 \\ \beta = N - \varphi(N) + 1 \end{cases}$$

$$\Rightarrow q^2 - \beta q + N = 0 \Rightarrow p, q = \frac{\beta \pm \sqrt{\beta^2 - 4N}}{2}$$

Now with $N = 18830129$ and $\varphi(N) = 18819060$

$$\beta = 18830129 - 18819060 + 1 = 11070$$
$$\Delta = 11070^2 - 4 * 18830129 = 122544900 - 75320516 = 47224384$$
$$q_1 = \frac{11070 + 6872}{2} = 8971$$
$$q_2 = \frac{11070 - 6872}{2} = 2099$$

3.3 point c

The Bézout's identity states that it is always possible to find x,y s.t. $ax+by=d$ if GCD(a,b)=d.

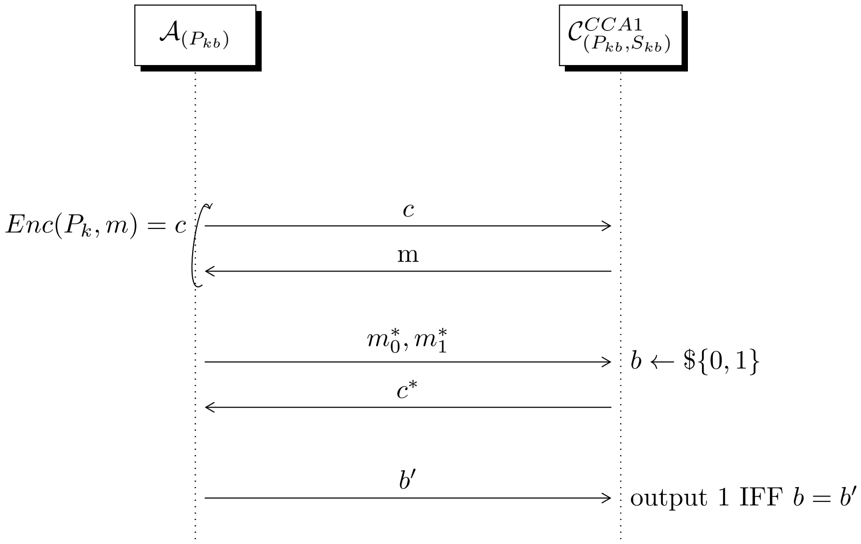
Now since e_A, e_b are coprime, GCD(e_A, e_b)=1 and then we can (using Euclidean algorithm) polinomially retrieve x,y s.t. $e_Ax+e_by=1$.

Since Eve has e_a, e_b, c_a, c_b and also x, y he can calculate $(m^{e_a})^x(m^{e_b})^y = m^{e_Ax+e_by} = m$

Exercise 4

4.1 point a

Formal definition of CCA1. Consider the following $GAME^{CCA}$



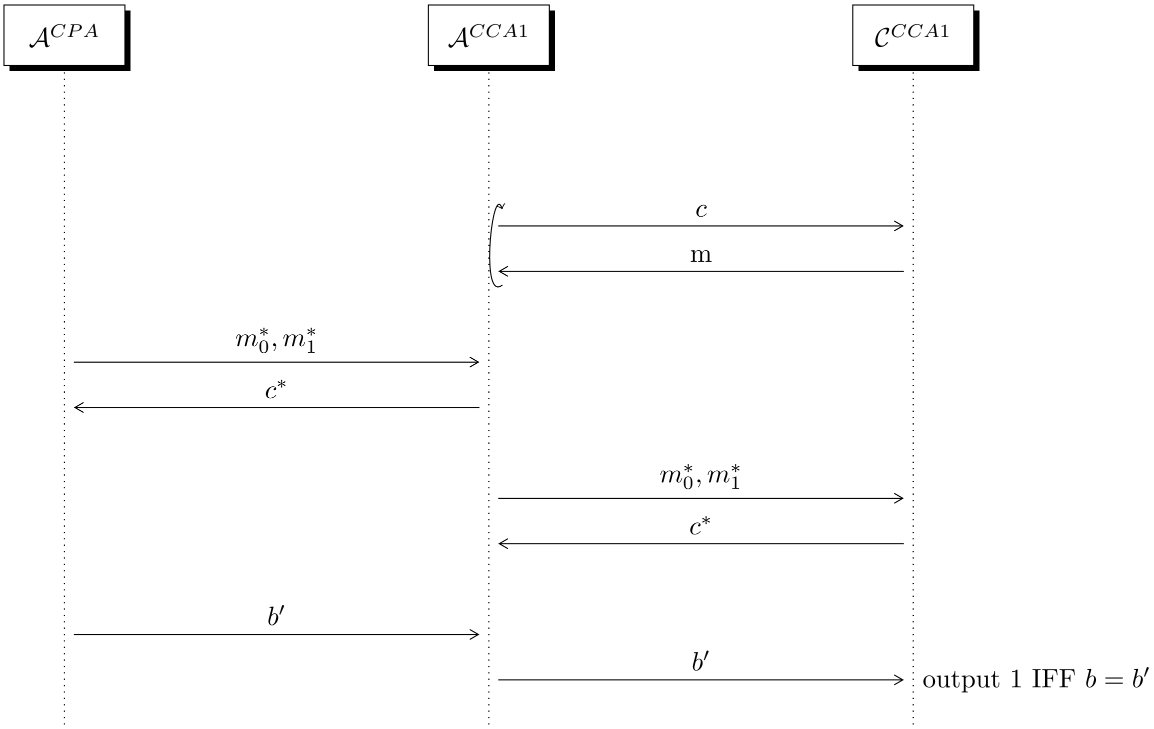
$$Pr[A(\lambda, 0) = 1] - Pr[A(\lambda, 1) = 1]$$

T0D0 2: Put an explanation and finalize probability

4.2 point b

CCA1 $\implies$ CPA

Assume $\exists \mathcal{A}^{CPA}$ which is able to break CPA. $\mathcal{A}^{CCA1}$ will use this $\mathcal{A}^{CPA}$ to break $CCA1$

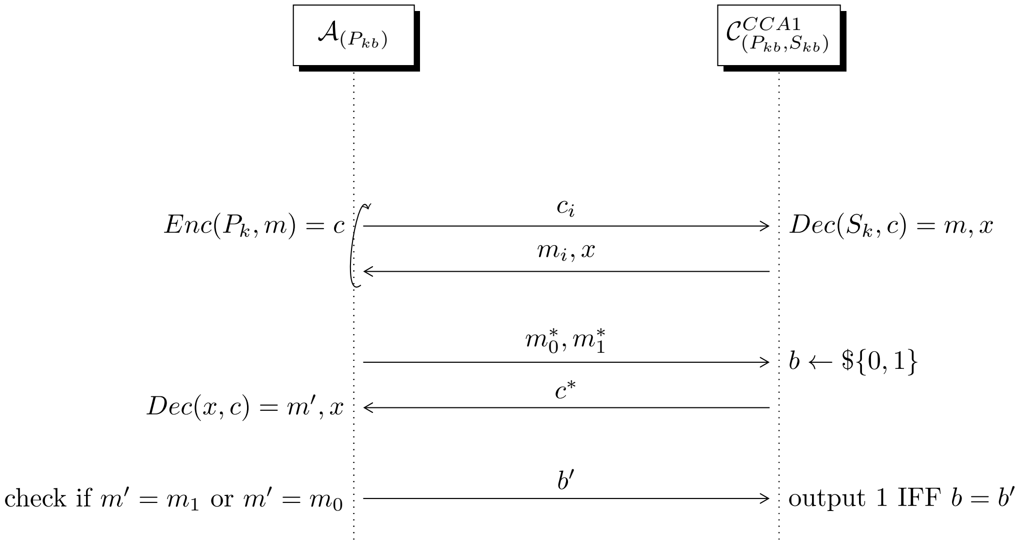


Intuitively this works because we used CPA to define CCA security. Therefore if an attacker is able to break CPA he is also "automatically" able to break CCA (the challenge part is the same for both games).

$PKE^{CPA} \implies CCA1$

Now consider the following Game which is still CPA secure but on the other hand it leaks the key whenever C receives a decryption query.

The scheme is defined as follows



4.3 point c

4.3.1 point i

My goal is to demostrate if PI is CCA1 -¿ PI' is also CCA1, in order to this observe the following reduction scheme:

A'(PI') A C

A'sends a message m composed by 't' elements. A takes every single elements and sends to C to get the cyphertexts of each. Then recombines the cyphertext and sends back the single cyphertext to A'.

At the start of the challenge, A' sends to messages: m0, m1 of 't' bytes to A. A sends to C the t-1 bytes of m0 and receives thr corrispondent t-1 cyphertext then A sends to C two bytes: m0t and 1, and receive the cyphertext c*' of one of these. At this point A recombines all of the t-1 cyphertext + the last received, c*' and sends back to A'. Now if A' distinguishes that c* is the cyphertext of m0 sends b' in the other case C has enrypted the fixed byte and send BOT.

At the end if A receives b', sends b', if receives BOT sends 0.

point ii

Π CCA2 $\implies \Pi'\neg$CCA2

Consider the following PKE Scheme:

- $Enc(P_k, m[t]) = Enc(P_k, m_1)||...||Enc(P_k, m_t)$
- $Dec(S_k, c[t]) = Dec(S_k, c_1)||...||Dec(S_k, c_t)$

Since in CCA2 I can make decryption queries after the challenge, I can create a $c' \neq c^*$ just by inverting the first two bits of c^* ($c* = c^*_1||c^*_2||...||c^*_t$ now $c' = c^*_2||c^*_1||...||c^*_t$). Now when I receive the decrypted message I can simply switch the first two bits again and discover which of the two challenge messages was encrypted.

4.4 point d

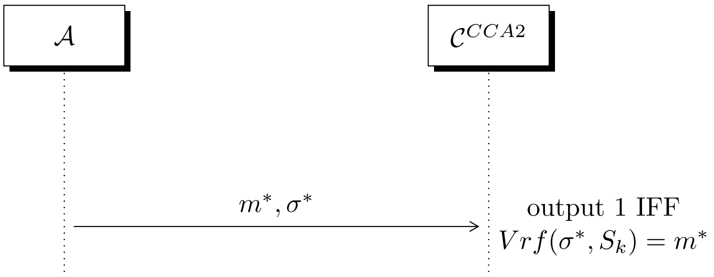
From the definition of padded-RSA I can construct the following attack:

Suppose we do the challenge query, when I receive C^* I will have something in this form: $c^* = (r||m_b)^e modN$. Now, since RSA is malleable, I can change C^* in order to be able to ask a valid decryption query, therefore $C' = C^* \times (r')^e modN = ((r||m_b) \times r')^e \neq C^*$. Now when I ask for the decryption of c' I will get $m' = ((r||m)r')^{ed} = (r||m)r'$ since I know r' I can simply divide $\frac{m'}{r'}$ and take the last l bits. This was the encrypted message in the challenge.

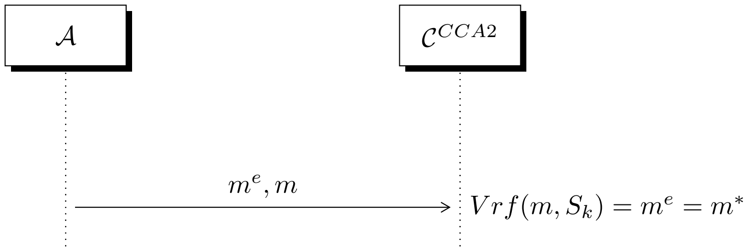
Exercise 5

5.0.1 point a

We want to break the following game:



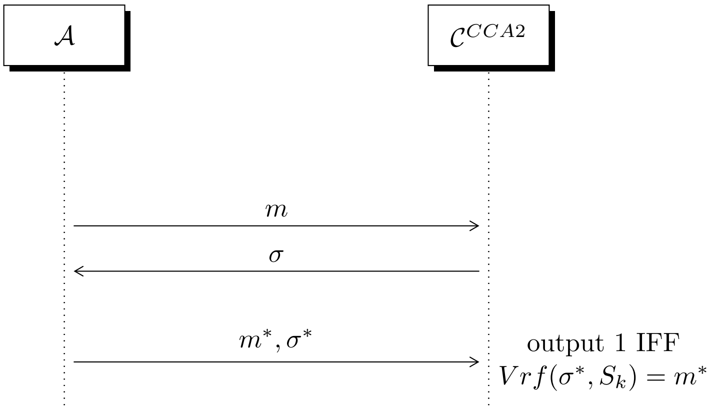
We win in this way:

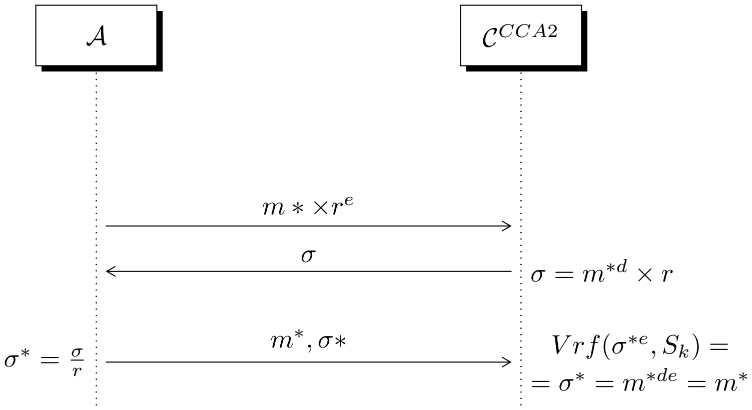


5.0.2 point b

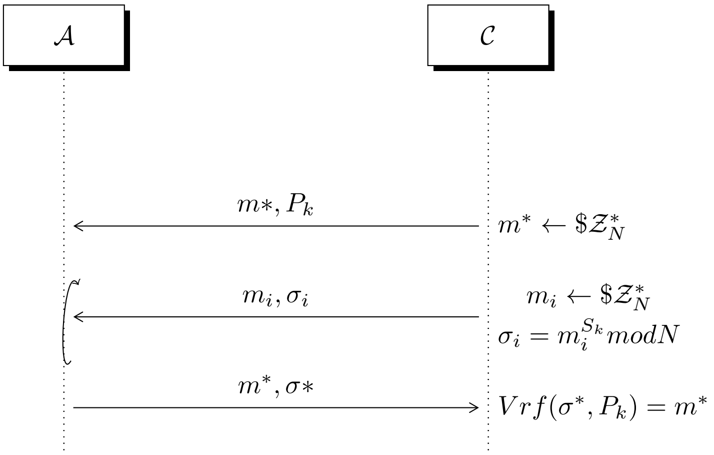
We want to break the following game when m^* is fixed:
We can win in the following way:

- Select $r \in \mathcal{Z}_n^*$
- Compute r^{-1}




$\mathcal{A}$
$\mathcal{C}^{CCA2}$
$m * \times r^e$
σ
$\sigma = m^{*d} \times r$
$\sigma^* = \frac{\sigma}{r}$
$m^*, \sigma*$
$Vrf(\sigma^{*e}, S_k) =$
$= \sigma^* = m^{*de} = m^*$

5.0.3 point c

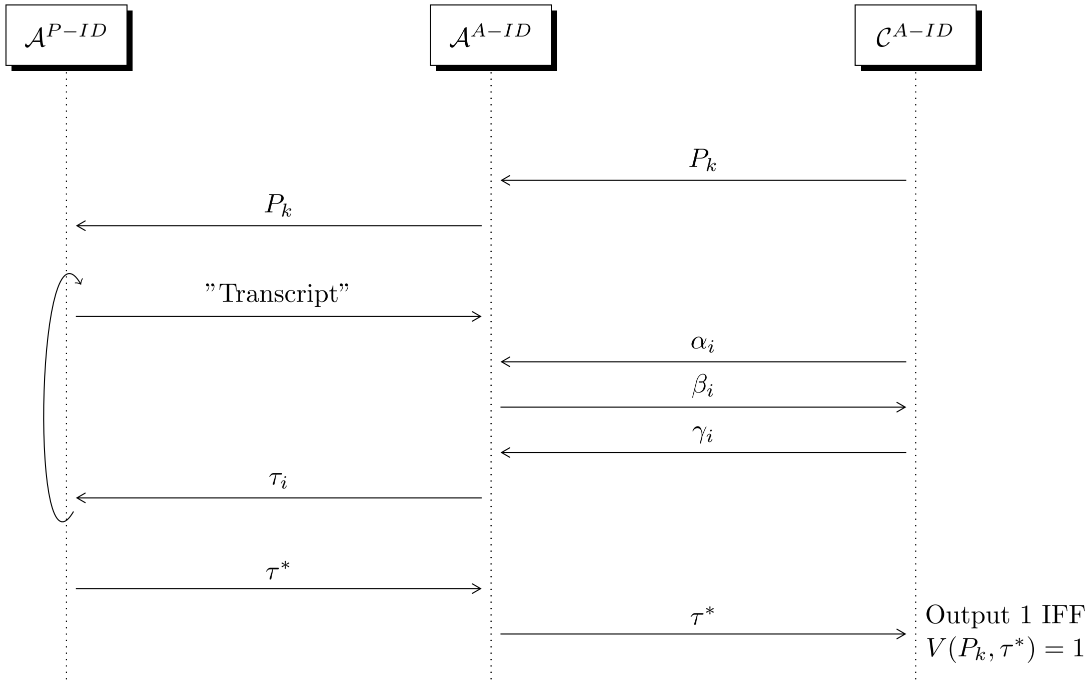

$\mathcal{A}$
$\mathcal{C}$
$m*, P_k$
$m^* \leftarrow \$\mathcal{Z}_N^*$
m_i, σ_i
$m_i \leftarrow \$\mathcal{Z}_N^*$
$\sigma_i = m_i^{S_k} modN$
$m^*, \sigma*$
$Vrf(\sigma^*, P_k) = m^*$

T0D0 3: Finish the proofs from the pictures

Exercise 6

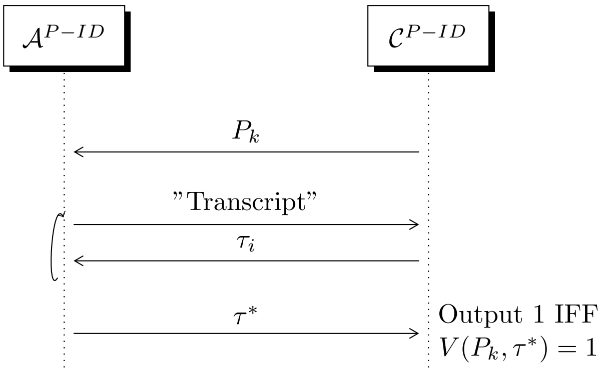
6.1 point a

Active $\implies$ Passive



We can construct a $\Pi_B AD$ s.t. Passive $\not\Leftarrow$ Active, where if we play as a dishonest Verifier we can leak the entire S_k.

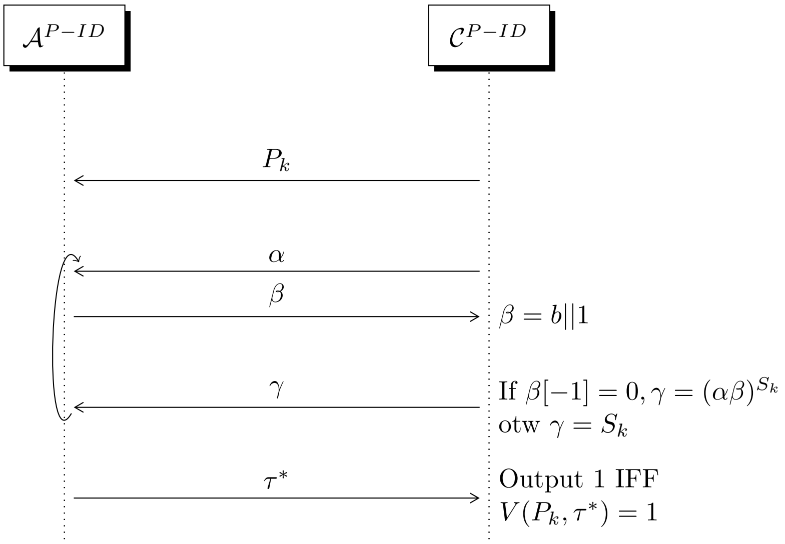
For the passive Game:



Now τ_i will be as follows, from the point of view of A:

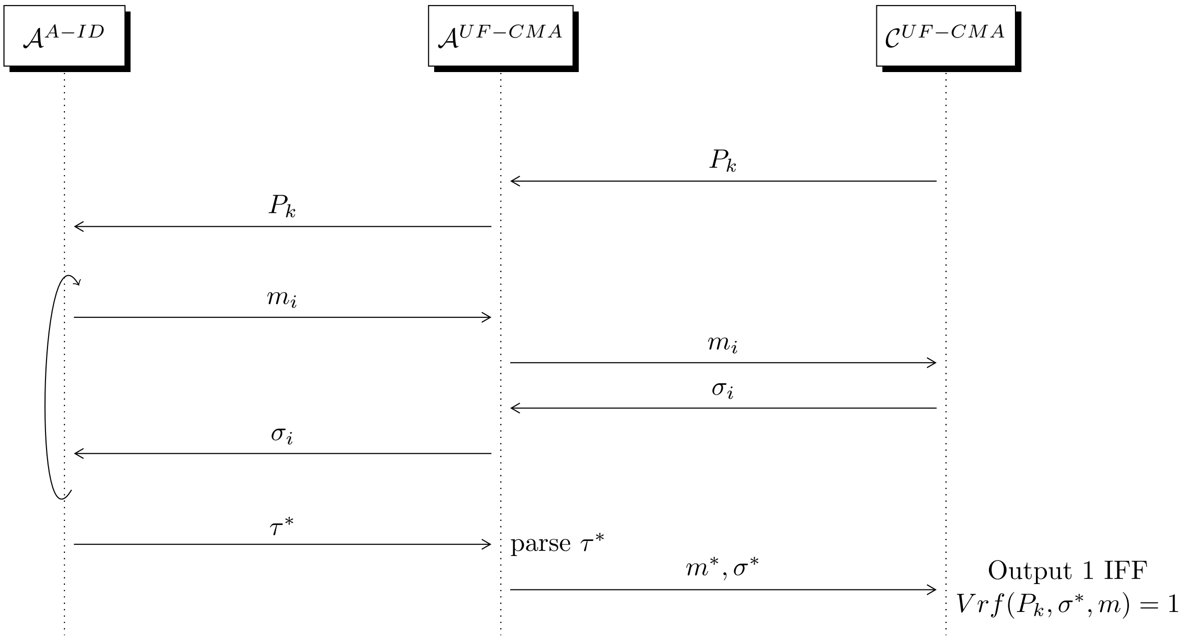
- $\alpha \leftarrow U$
- $\beta = b||0$ where $b \leftarrow \$U$
- $\gamma = \beta[-1](S_k) + (1 - \beta[-1])(\alpha\beta)^{S_k}$

So the above ID-scheme has still Passive Security. Instead for the Active Security we can play the following interaction:



Now that we have S_k we can always create a valid τ^*.

6.2 point b



6.3 point c

The HVZK property comes from the fact that the signature scheme used in the following way:

- The Verifier sends the message
- The Prover signs the message

doesn't leak any information about the secret used for signing the message (assuming A always as honest verifier).