State Representation Learning for Control: An Overview

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Abstract

Representation learning algorithms are designed to learn abstract features that characterize data. State representation learning (SRL) focuses on a particular kind of representation learning where learned features are in low dimension, evolve through time, and are influenced by actions of an agent. As the representation learned captures the variation in the environment generated by agents, this kind of representation is particularly suitable for robotics and control scenarios. In particular, the low dimension helps to overcome the curse of dimensionality, provides easier interpretation and utilization by humans and can help improve performance and speed in policy learning algorithms such as reinforcement learning.

This survey aims at covering the state-of-the-art on state representation learning in the most recent years. It reviews different SRL methods that involve interaction with the environment, their implementations and their applications in robotics control tasks (simulated or real). In particular, it highlights how generic learning objectives are differently exploited in the reviewed algorithms. Finally, it discusses evaluation methods to assess the representation learned and summarizes current and future lines of research.

Keywords: State Representation Learning, Low Dimensional Embedding Learning, Learning Disentangled Representations, Disentanglement of control factors, Robotics, Reinforcement Learning

1 Introduction

Robotics control and artificial intelligence (AI) in a broad perspective heavily rely on the availability of compact and expressive representations of the sensor data. Designing such representations has long been performed manually by the designer, but deep learning now provides a general framework to learn such representations from data. This is particularly interesting for robotics where multiple sensors (such as cameras) can provide very high dimensional data, while the robot objective can often be expressed in a much lower dimensional space (such as the 3D position of an object in a manipulation task). This low dimensional representation, frequently called the *state* of the system, has the crucial role of encoding essential information (for a given task) while discarding the many irrelevant aspects of the original data.

Such state representation is at the basis of the classical reinforcement learning (RL) framework [Sutton, 1998] in which an agent interacts with its environment by choosing an action as a function of the environment state in order to maximize an expected (discounted) reward. Following this framework, we call *observation* the raw information provided by one or several of the robot sensors, and we call *state* a compact depiction of this observation that retains the information necessary for the robot to choose its actions.

While deep reinforcement learning algorithms have shown that it is possible to learn controllers directly from observations [Mnih et al., 2015], reinforcement learning (or other control algorithms) can take advantage of low dimensional and informative representations, instead of raw data, to solve tasks more efficiently [Munk et al., 2016]. Such efficiency is critical in robotic applications where experimenting an action is a costly operation. In robotics, as well as in machine learning, finding and defining interesting states (or features) for control tasks usually requires a considerable amount of manual engineering. It is therefore interesting to learn these features with as little supervision as possible. The goal is thus to avoid direct supervision using a *true* state, but instead use information about the actions performed by the agent, their consequences in the observation space, and rewards (even if sparse, and when available). Along with this information, one can also set generic constraints on what a good state representation should be [Jonschkowski and Brock, 2015, Lesort et al., 2017].

Feature learning in general is a wide domain which aims at decomposing data into different features that can faithfully characterize it. It has been a particular motivation for deep learning to automatically learn a large range of specific feature detectors in high dimensional problems. State representation learning is a particular case of feature learning in which the features to learn are low dimensional, evolve through time, and are influenced by actions or interactions. SRL is generally framed in a control setup constrained to favor small dimensions to characterize an instance of an environment or an object, often with a semantic meaning that correlates with some physical feature. The physical feature can be a position, distance, angle or an orientation. The objective of SRL is to take advantage of time steps, actions and optionally rewards to transform observations into states: a set of reduced vector of most representative features that is sufficient for efficient policy learning [Shelhamer et al., 2017]. It is also worth distinguishing between feature learning on a process that is only observed, and learning the state representation of a process in which the learning agent possesses embodiment and acts. The latter opens up a broader set of research questions and gives rise to exploiting more possibilities such as active learning, artificial curiosity or balancing between exploration and exploitation.

As stated above, learning in this context should be performed without explicit supervision. In this article we therefore focus on SRL where *learning* does not have the *pattern recognition* regression or classification sense, but rather the sense of the process of model building [Lake et al., 2016]. Building such models can then exploit a large set of objectives or constraints, possibly taking inspiration from human learning. As an example, infants expect inertial objects to follow principles of persistence, continuity, cohesion and solidity before appearance-based elements such as color, texture and perceptual goodness. At the same time, these principles help guide later learnings such as object' rigidity, softness and liquids properties. Later, adults will reconstruct perceptual scenes using internal representations of the objects and their physically relevant properties (mass, elasticity, friction, gravity, collision, etc.) [Lake et al., 2016]. In the same way, the SRL literature may make use of knowledge about the physics of the world, interactions and rewards whenever possible as a semi-supervision or self-supervision that aids the challenge of learning state representations without explicit supervision.

Recently, several different approaches have been proposed to learn such state representation. In

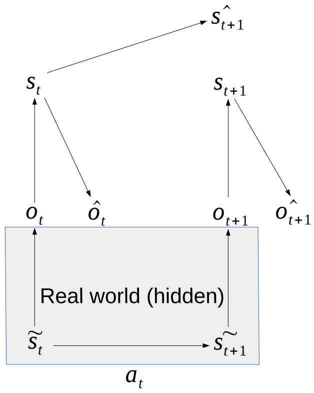


Figure 1: Notation illustration: $\tilde{s}_t$ represents the true state, s_t the learned state computed from the observation o_t, and $\hat{s}_{t+1}$ the estimation or prediction of the learned state, e.g., through a forward model. The same notation applies for each observation o and action a.

this review paper, our objective is to present and analyze those different approaches, highlight their commonalities and differences, and to propose further research directions. We extend a previously published review [Böhmer et al., 2015] with the most recent and rapidly evolving literature of the past years and focus on approaches that learn low dimensional[1] Markovian representations without direct supervision, i.e., exploiting sequences of observations, actions, rewards and generic learning objectives. The works selected in this survey mostly evaluate their algorithms in simulations where agents interact with an environment. More marginally, some SRL algorithms are tested on real settings such as robotics tasks, e.g., manipulation or exploration.

In the remainder of the paper, we first introduce the formal framework and notation, then present the objectives that can be used to learn state representations, and discuss the implementation aspects of these approaches before summarizing some current and future lines of research.

2 Formalism and definitions

2.1 SRL Formalism

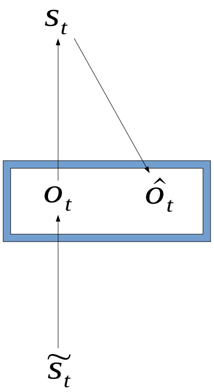
The nomenclature we use defines, as in the reinforcement learning literature, an environment $\mathcal{E}$ where an agent performs actions $a_t \in \mathcal{A}$ at time step t and $\mathcal{A}$ is the action space (continuous or discrete). Each action makes the agent transition from a true state $\tilde{s}_t$ to $\tilde{s}_{t+1}$. We call the true state space $\tilde{\mathcal{S}}$. The agent obtains an observation at time step t, $o_t \in \mathcal{O}$, from its sensors, which makes it possible to learn by interacting with $\mathcal{E}$ through actions. Often, on a given state $\tilde{s}_t$, by performing an action, the agent may receive a reward r_t. The reward is given at $\tilde{s}_t$ by a reward function that the agent tries to maximize. The reward function is designed in order to lead the agent to a certain behavior that solves a task. By $\hat{o}_t$, we denote the reconstruction of o_t; by $\hat{a}_t$, the estimate of a_t, and by $\hat{r}_t$, the estimate of r_t. The notation organization is shown in Fig. 1.

The SRL task is to learn a representation $s_t \in \mathcal{S}$ of dimension K with characteristics similar to those of $\tilde{s}_t$. More formally, SRL usually finds a mapping ϕ of the history of observation to the current state $s_t = \phi(o_{1:t})$. Note that actions $a_{1:t}$ and rewards $r_{1:t}$ can also be added to the parameters of ϕ [Jonschkowski and Brock, 2015]. In this paper, we are specifically interested in the particular setting in which this mapping is learned through proxy objectives without any exploitation of the true state $\tilde{s}_t$, by using only observations, actions and rewards, or generic objectives that lead to good representations. This family of approaches is called unsupervised or self-supervised.

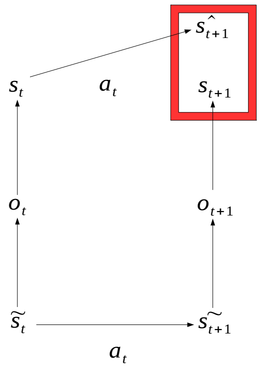
Based on the previously defined notations, we can briefly summarize the common strategies used in state representation learning that are detailed in the next sections. In the following, θ represents the parameters optimized by minimizing the model's loss function. This model will generally be implemented with a neural network.

[1] *Low dimensional* means that the learned state dimension is significantly smaller than the observation space dimensionality.

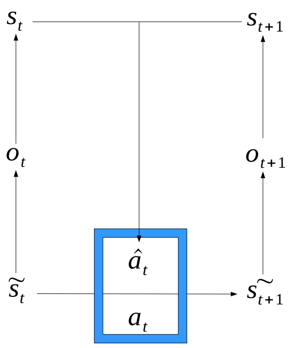
Figure 2: Schema of the different transitions and projections between different spaces. $\tilde{s}_t$ represents the true state, s_t the learned state, and $\hat{s}_t$ the estimation or prediction of the learned state. The same notation applies for each observation o and action a. Each schema shows its respective highlighted components that are compared and optimized in the loss functions.



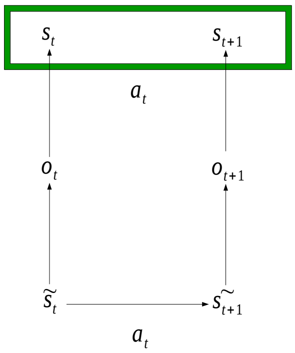
(a) Auto-Encoder



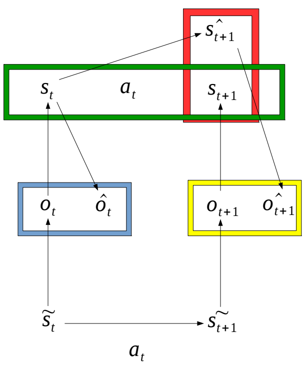
(b) Forward Model



(c) Inverse Model



(d) Model with prior



(e) Example of hybrid model containing combinations of previously described blocks in a-d.

- **Reconstructing the observation**: learning the function ϕ (Eq. 1) so that it is possible to reconstruct the observation with a decoder ϕ^{-1} (Eq. 2) by minimizing the reconstruction error (between the original observation and its predicted reconstruction) under different constraints (e.g., dimensionality constraints, local denoising criterion [Vincent et al., 2010], sparse encoding constraints [Vincent et al., 2008], etc.) (Fig. 2a).

$$s_t = \phi(o_t; \theta_\phi) \tag{1}$$

$$\hat{o}_t = \phi^{-1}(s_t; \theta_{\phi^{-1}}) \tag{2}$$

  where θ_ϕ and $\theta_{\phi^{-1}}$ are the parameters learned for the encoder and decoder, respectively.

- **Learning a forward model**: A forward model predicts s_{t+1} from o_t (or s_t) (Fig. 2b). In other words, given a state and action taken at time t, a forward model predicts the next state at $t + 1$.

$$\hat{s}_{t+1} = f(s_t, a_t; \theta_{fwd}) \tag{3}$$

  Learning such a model makes it possible to impose structural constraints on the model for state representation learning. For example, the forward model can be constrained to be linear, imposing that the system in the learned state space follows simple linear dynamics.

- **Learning an inverse model**: An inverse model predicts action a_t given observations o_t and o_{t+1} (or states s_t and s_{t+1}):

$$\hat{a}_t = g(s_t, s_{t+1}; \theta_{inv}) \tag{4}$$

  Learning such model enforces that the state encodes enough information to recover the action that modified the state (Fig. 2c).

- **Using prior knowledge to constrain the state space**: A last approach is to handle SRL by using specific constraints or prior knowledge about the functioning, dynamics or physics of the world (besides the constraints of forward and inverse models) such as the temporal continuity or the causality principles that generally reflect the interaction of the agent with objects or in the environment [Jonschkowski and Brock, 2015]. *Priors* are defined as objective or loss functions $\mathcal{L}$, applied on a set of states $s_1, ..., s_n$ (Fig. 2d), to be minimized (or maximized) under certain condition c. An example of condition can be enforcing locality or time proximity within the set of states.

$$Loss = \mathcal{L}_{prior}(s_1, ..., s_n; \theta_\phi | c) \tag{5}$$

All these approaches are detailed in Section 3.

2.2 State representation characteristics

Besides the general idea that the state representation has the role of encoding essential information (for a given task) while discarding irrelevant aspects of the original data, let us detail what the characteristics of a good state representation are.

In a reinforcement learning framework, the authors of [Böhmer et al., 2015] defines a good state representation as a representation that is:

- Markovian, i.e. it summarizes all the necessary information to be able to choose an action within the policy, by looking only at the current state.
- Able to represent the true value of the current state well enough for policy improvement.
- Able to generalize the learned value-function to unseen states with similar futures.
- Low dimensional for efficient estimation.

Note that these are some characteristics expected of the state representation, but they cannot be used for learning this representation. Instead, they can later be verified by assessing the task performance for a controller based on the learned state. Note also that multiple state representations can verify these properties for a given problem and that therefore, there is no unique solution to the state representation learning problem. We detail this problem when discussing the evaluation of the learned state space in Section 4.3.

State representation learning can also be linked with the idea of learning disentangled representations that clearly separate the different factors of variation with different semantics. Following [Achille and Soatto, 2017], a good representation must be sufficient, as efficient as possible (i.e., easy to work with, e.g., factorizing the data-generating factors), and minimal (from all possible representations, take the most efficient one). The *minimal* assumption is comparable to the simplicity prior [Jonschkowski and Brock, 2015]. It assumes that only a small number of world properties are relevant, and that there exists a low dimensional state representation of a higher level input observation. Related to *Occam's razor*, this prior favors state representations that exclude irrelevant information to encourage a lower dimensionality. The *efficiency* aspect of the representation means that there should be no overlapping between dimensions of the learned state features. Unfortunately, independence of features alone is not enough to assure a good quality of representations and guarantee a disentanglement of factors of variation. Higher level abstractions can, however, allow to improve this disentanglement and permit easier generalization and transfer. Cues to disentangle the underlying factors can include spatial and temporal scales, marginal independence of variables, and controllable factors [Thomas et al., 2017].

2.3 State representation learning applications

The main interest of SRL is to produce a low dimensional state space in which learning a control policy will be more efficient. Indeed, deep reinforcement learning in the observation space has shown spectacular results in control policy learning [Mnih et al., 2015, Lillicrap et al., 2015, Mnih et al., 2016] but is known to be computationally difficult and requires a large amount of data. Separation of representation learning and policy learning is a way to lighten the complete process. This approach was used in most of the reviewed papers [Mattner et al., 2012, Watter et al., 2015, van Hoof et al., 2016, Munk et al., 2016, Curran et al., 2016, Wahlström et al., 2015, Shelhamer et al., 2017, Oh et al., 2017] to make reinforcement learning faster in time and/or lighter in computation. The learned policies are applied to real robots and/or simulation settings as detailed in Section 4.4.

SRL can be particularly relevant with multimodal observations that are produced by several complementary sensors with high dimensionality as is, for example, the case of autonomous vehicles. Low dimensional representations are then key to make an algorithm able to take decisions from hidden factors extracted from these complementary sensors.This is for instance the case of representation learning from different temporal signals in [Duan, 2017, Bohg et al., 2017]. Audio and images are blended in [Yang et al., 2017] while RGB and depth are combined in [Duan, 2017].

SRL can also be used in a transfer learning setting by taking advantage of a state space learned on a given task to rapidly learn a related task. This is for example the case in [Jonschkowski and Brock, 2015] where a state space related to a robot position is learned in a given navigation task and reused to quickly learn another navigation task. RL is also used as pretraining for transfer to other applications afterwards such as reinforcement learning. For concrete examples on SRL application scenarios see Section 4.4.

Another case where SRL could be useful is in the application of Evolution Strategies (ES) for robot control learning [Stulp and Sigaud, 2013]. Evolution strategies are a family of black box optimization algorithms that do not rely on gradient descent and can be an alternative to RL techniques (such as Q-learning and policy gradients) but are less adapted to high-dimensional problems. ES optimization methods could take a clear advantage of a lower dimension input to explore much faster the parameter space than using raw data.

3 Learning objectives

In this section, we review what objectives can be used to learn a relevant state representation. A schema detailing the core elements involved in each model's loss function is in Fig. 2, which highlights the main approaches to be described here.This section touches upon machine learning tools used in SLR such as auto-encoders or siamese networks. A more detailed description of these is later addressed in Section 4.

3.1 Reconstructing the observation

A first idea that can be exploited is the fact that a true state, along with some noise, was used to generate the observation. Under the hypothesis that the noise is not too large, compressing the observation should retain the important information contained in the true state. While this idea is very often exploited with dimensionality reduction algorithms [Fodor, 2002] such as Principal Component Analysis (PCA), we focus here on the recent approaches specifically dealing with state representation.

The PCA algorithm is a linear transformation able to compress and decompress observations with minimal reconstruction error. Classical approaches such as PCA have been exploited to reduce the dimensionality of the state space during learning [Curran et al., 2016]. By projecting images into a 3- or 4-dimensional space, it is possible to produce a state that is used by a reinforcement learning algorithm and that reduces the convergence time in Super Mario games [Karakovskiy, 2012] and different simulations such as Swimmers or Mountain Car.

Auto-encoders are models that learn to reproduce their input under constraints on their internal representations such as dimensionality constraints (Fig. 2a). Their architecture can therefore be used to learn a particular representation in low dimensions s_t by reconstructing o_t.

Simple auto-encoders can be used to learn 2D representation of a real pole from raw images [Mattner et al., 2012] (see Section 4.4 on evaluation scenarios). After training, the encoding vector from the AE is used to learn a controller to balance the pole. An auto-encoder whose internal representation is constrained to represent a 2D position in order to learn a spatial state representation that serves as input to a controller is also presented in [Finn et al., 2015]. The proposed model learns a state representation from raw pixels of a PR2 robot's hand.

These models, based on auto-encoders that reconstruct the observation at the same time step, can however learn only if the factors of variations are only linked to the state, or if very prominent features exists. In order to relax this assumption, it is possible to reconstruct observations from other time steps or to use constraints on the evolution of the state (as will be more detailed in Section 3.2) to focus reconstruction on features relevant to the system dynamics.

An auto-encoder with a siamese encoder can for example project sequences of images into a state representation space $\mathcal{S}$ with constraints on the fact that the transition between s_t and s_{t+1} should be linear [Goroshin et al., 2015]. They use observations at several time steps in order to take time into account in the representation, and predict future observations through a single decoder that reconstructs $\hat{o}_{t+1}$. This makes the model able to learn representations that are related to time, so that random features of the environment can be filtered out.

The idea of using an auto-encoder to learn a projection into a state space where transitions are assumed to be linear has also been used by [Watter et al., 2015]. The model presented, "Embed To Control" (E2C), consists of a deep generative model that learns to generate image trajectories from a linear latent space.

Extending [Watter et al., 2015], state encoders and decoders can be trained in a way that respects the transition dynamics of the controlled system in the state space at the same time as it can reconstruct their observations [van Hoof et al., 2016]. They compared different types of auto-encoders to learn visual and tactile state representations and use this representation afterwards to learn manipulation task policies for a robot. Sharing the same idea, Deep Variational Bayes Filter (DVBF) are an extension of Kalman filters which learn to reconstruct the observation based on a nonlinear state space using variational inference [Karl et al., 2016]. The reconstruction from a non linear state space based on a model inspired by a Deep Dynamical Model (DDM) [Wahlström et al., 2015] and E2C [Watter et al., 2015] is proposed in [Assael et al., 2015]. It is argued that the model is adapted for better training efficiency and it can learn tasks with complex non-linear dynamics [Assael et al., 2015]. Their result shows improvements over the PILCO model [Deisenroth and Rasmussen, 2011], which learns a state representation by only minimizing the reconstruction error without constraining the latent space.

3.2 Learning a forward model

Last subsection discussed how compressing the information contained in a single observation results being a useful task to learn representations. Next, we will show how temporal dynamics of the system can also help the same purpose. Therefore we present approaches that rely on learning a *forward* model to learn a state space. The general idea is to force it to efficiently encode the information necessary to make the prediction of next state (Fig. 2b).

In the case of the forward models we study here, the model is used as a proxy for learning s_t. The model firstly makes a projection from the observation space to the state space to obtain s_t and applies a transition to predict $\hat{s}_{t+1}$. The error is computed by comparing the estimated next state $\hat{s}_{t+1}$ with the value of s_{t+1} derived from the next observation o_{t+1} at the next time step.

Forward models can benefit from the observation reconstruction objective presented in Section 3.1. As an example, the works presented in Section 3.1 [Goroshin et al., 2015, van Hoof et al., 2016, Watter et al., 2015, Assael et al., 2015, Karl et al., 2016] belong to the auto-encoder category of models. However, they all predict future observations to learn representations and therefore, they as well belong to the family of forward models.

The method these works use to combine forward models and auto-encoders consists of mapping o_t to s_t, and then compute the transition, with the help of a_t, to obtain $\hat{s}_{t+1}$. $\hat{s}_{t+1}$ is then remapped onto the pixel space in form of a vector $\hat{o}_{t+1}$. The error is then computed pixel-wise between $\hat{o}_{t+1}$ and o_{t+1}. One common assumption is that the forward model in the learned state space is linear [Goroshin et al., 2015, van Hoof et al., 2016]. The transition is then just a linear combination of s_t and a_t as in Eq. 6. W, U and V are either fixed or learned parameters [van Hoof et al., 2016].

$$\hat{s}_{t+1} = W * \hat{s}_t + U * a_t + V \tag{6}$$

In a similar way, the *Embed to Control* model (E2C) uses Eq. 6 to compute the mean μ of a distribution and learn supplementary parameters for the variance σ of the distribution [Watter et al., 2015]. Then, $\hat{s}_{t+1}$ is computed with Eq. 7:

$$\hat{s}_{t+1} \sim \mathcal{N}(\mu = W * \hat{s}_t + U * a_t + V, \sigma) \tag{7}$$

Using distributions to compute $\hat{s}_{t+1}$ allows to use the KL-divergence to train the forward model. This method is also used in [Karl et al., 2016] and [Krishnan et al., 2015]. However, the transition model in [Krishnan et al., 2015] considers the KL-divergence between $P(s_{t+1})$ and $P(\hat{s}_{t+1})$ and does not use the loss of the reconstruction based on o_{t+1} and $\hat{o}_{t+1}$.

The use of a_t is nearly mandatory in forward models because s_t does not contain enough information to predict s_{t+1}. However, some approaches get rid of the need for actions by assuming that the transition from s_{t-1} to s_t allows to deduce the transition from s_t to s_{t+1} [Goroshin et al., 2015]. Other way to circumvent this is letting the model use several past states to predict s_{t+1}.

Another use of a forward model, connected to an intrinsic curiosity model (ICM) which helps agents explore and discover the environment out of curiosity when extrinsic rewards are sparse or not present at all, is proposed in [Pathak et al., 2017]. In this model, the reward signal is computed from the forward model's loss function $\mathcal{L}_{fwd}$ ($\hat{\phi}$ is the forward function learned by the model):

$$\mathcal{L}_{fwd}(\phi(o_{t+1}), \hat{\phi}(o_t, a_t)) = \frac{1}{2} \parallel \hat{\phi}(o_t, a_t) - \phi(o_{t+1}) \parallel_2^2 \tag{8}$$

It is argued that there is no incentive in this model for s_t to learn encoding any environmental features that cannot influence or are not influenced by the agent's actions. The learned exploration strategy of the agent is therefore robust to uncontrollable aspects of the environment such as the presence of distractor objects, changes in illumination, or other sources of noise in the environment [Pathak et al., 2017].

Related to the exploitation of a forward model is the idea of learning a controllable representation through the *controllability* prior [Jonschkowski et al., 2017]. Controllable objects are relevant for state representation learning as the elements that can be controlled by robots are likely relevant for their task. If a robot acts by applying forces, controllable things could be those whose accelerations correlate with the actions of the robot. Accordingly, a loss function can be defined to minimize the covariance between an action dimension i and the accelerations in the state dimension i. The following formula from [Jonschkowski et al., 2017] makes it explicit:

$$Controllability_i = e^{-cov(a_{t,i}, s_{t+1,i})}, \tag{9}$$

where $cov(a_{t,i}, s_{t+1,i})$ is the covariance between the state $s_{t+1,i}$ at dimension i and time t and $a_{t,i}$ (the action at dimension i that led to such state). Note that here the learned state is assumed to represent an acceleration. Related with this prior also is the notion of *empowerment*, defined as an information-theoretic capacity of an agent's actuation channel to influence its own evolution. The concept of empowerment is related to *accountability* or *agency*, i.e., recognizing when an agent is responsible for originating the change of state in the environment [Klyubin et al., 2005].

3.3 Learning an inverse model

The forward model approach can be turned around and, instead of learning to predict next state (given previous state and action), use current and next states to predict the action between them. The inverse model framework is used in SRL by firstly performing a projection of o_t and o_{t+1} onto learned states s_t and s_{t+1}, and secondly, by predicting the action $\hat{a}_t$ that would explain the transition of s_t into s_{t+1} (Fig. 2c). As before, learning this model can impose constraints on the state representation to be able to efficiently predict actions.

An example using an inverse models to learn state representation is the Intrinsic Curiosity Module (ICM) [Pathak et al., 2017]. It integrates both an inverse and forward model where the *surprise* element in the action prediction is used as reward for the action selection. This is a way to bypass the hard problem of predicting original observations (e.g. images), since actions have much lower dimension. In the inverse model, the parameters θ_{inv} are trained to optimize:

$$\mathcal{L}_{inv} = min_{\theta_{inv}} \mathcal{L}_{inv}(\hat{a}_t, a_t) \tag{10}$$

where, $\mathcal{L}_{inv}$ is the loss function that measures the discrepancy between the predicted and actual actions.

A different kind of inverse model is used in [Shelhamer et al., 2017], where the policy gradient used to learn a state representation is augmented with auxiliary gradients from what is called *self-supervised* tasks. In this case, in lack of external supervision, the prediction error resulting from interactions with the environment acts as a self-supervision. They learned a inverse dynamics model to retrieve from o_t and o_{t+1} the action a_t performed between the two successive time step.

Note that connections among forward and inverse models are important, for example forward models can regularize inverse dynamics models [Agrawal et al., 2016]. The latter can provide supervision to construct informative features that the forward model can predict and, in this way, regularize the feature space for the inverse model. In practice, this is implemented by decomposing the joint loss function as a sum of the inverse model loss plus the forward model loss [Agrawal et al., 2016]. Another approach including forward and inverse models, as well as a reconstruction of the observation including multimodal input is [Duan, 2017].

3.4 Using feature adversarial learning

Adversarial networks [Goodfellow et al., 2014] can also be used for unsupervised learning of state representations. The use of the Generative Adversarial Network (GAN) framework to learn state representations is proposed in [Chen et al., 2016]. They present a model named InfoGAN that achieves the disentanglement of latent variables on 3D poses of objects. As described in [Chen et al., 2016], the goal is to learn a generator distribution $P_G(o)$ that matches the real distribution $P_{data}(o)$. Instead of trying to explicitly assign a probability to every o in the data distribution, GANs learn a generator network G that samples from the generator distribution P_G by transforming a noise variable $z \sim P_{noise}(z)$ into a sample $G(z)$. The noise variable has two components. A first one, z_G, randomly sampled from a Gaussian distribution, and a second one with smaller dimension, z_U, sampled from a uniform distribution. The latter is used during training so that the $G(z)$ has a high mutual information with z_U. Then, the sample from z_U has a high correlation with $G(z)$ and can thus be considered as a state representation. This generator is trained by playing against an adversarial discriminator network D that aims at distinguishing between samples from the true distribution P_{data} and the generator distribution P_G. The authors succeed to learn states corresponding to object orientations from sequences of images.

Another example of SRL with Generative Adversarial Network is presented by [Donahue et al., 2016]. BiGAN is an extension of regular GANs to learn the double mapping from image space to latent space, and from latent space to image space. It allows the learned feature representation to be useful for auxiliary supervised discrimination tasks, and competitive with unsupervised and self-supervised feature learning. The BiGAN has also been experimented in [Shelhamer et al., 2017] to learn state representations before being used for reinforcement learning, and compared to their own approach (Section 3.2).

3.5 Exploiting rewards

As opposed to RL, the use of a reward value in SRL is not compulsory. However, it can be used as supplementary information to help differentiating states and to learn task related representations. Rewards are helpful information to disentangle meaningful information from a noisy or distracting one, and to tie the representation to a particular task. However, in a multi-task setting, this approach can be used to learn a generic state representation that is relevant to different tasks.

A *predictable reward prior* which estimates $\hat{r}_{t+1}$ given a certain state s_t and an action a_t is implemented in [Munk et al., 2016]. A similar RL approach that integrates learned state representations in its architecture that are later used for learning a policy is [Oh et al., 2017]. In the same way as [Munk et al., 2016], [Oh et al., 2017] also learns to predict future rewards, but in the latter case, in order to learn the policy, the reward is used.

A dimensionality reduction model called *reward weighted principal component analysis* (rw-PCA), as another way of using rewards for state representation was proposed in [Parisi, 2017]. *rwPCA* uses data collected by an RL algorithm and operates a dimensionality reduction strategy which takes reward into account to keep the information into a compressed form. The compressed data is afterwards used to learn a policy.

On the same idea of constructing a task-related representation, [Jonschkowski and Brock, 2015] and [Lesort et al., 2017] use rewards as supplementary information to impose constraints on the state space topology. One of these constraints makes the space more suited to discriminate between states with different rewards. The state space is then particularly adapted to solve a given task. This constraint is called *causality prior* in [Jonschkowski and Brock, 2015] and [Lesort et al., 2017]. It assumes that if we have two different rewards after performing the same action, then the two states should be differentiated and far away in the representation space (Equation 11).

$$\mathcal{L}_{Caus}(D, \hat{\phi}) = \mathbf{E}[e^{-\|\hat{s}_{t_2} - \hat{s}_{t_1}\|^2} \mid a_{t_1} = a_{t_2}, r_{t_1+1} \neq r_{t_2+1}] \tag{11}$$

The rewards can also be self-produced by auxiliary functions to pre-train a model for future reinforcement learning [Shelhamer et al., 2017] or to perform curiosity-driven exploration [Pathak et al., 2017].

3.6 Other objective functions

In this section, we present other approaches assuming various specific constraints for state representation learning. The learning process can be constrained by prior knowledge (either initially provided by the designer or acquired via learning) to allow the agent to leverage existing common sense, intuitive physics, physical laws, mental states of others, as well as other abstract regularities such as compositionality and causality [Lake et al., 2016]. This kind of a priori knowledge is called *prior* [Bengio et al., 2012], [Jonschkowski and Brock, 2015], [Lesort et al., 2017],[Jonschkowski et al., 2017], and is defined through cost functions. These loss functions are applied in the state space in order to impose the required constraints to construct the model projecting the observations in the state space. In the following, $\Delta s_t = s_{t+1} - s_t$ is the difference in between states at times t and $t+1$, and D is a set of observations.

- **Slowness Principle**
  The slowness principle assumes that interesting features fluctuate slowly and continuously through time and that a radical change inside the environment has low probability [Wiskott and Sejnowski, 2002, Kompella et al., 2011].

  $$\mathcal{L}_{Slowness}(D, \phi) = \mathbb{E}[\| \Delta\ s_t \|^2] \tag{12}$$

  This assumption can have other naming depending on the unit of s_t, e.g., prior of time coherence (time) [Jonschkowski and Brock, 2015, Lesort et al., 2017] or inertia (velocity) [Jonschkowski et al., 2017].

- **Variability**
  The assumption of this prior is that positions of relevant objects vary, and learning state representations should then focus on moving objects [Jonschkowski et al., 2017].

  $$\mathcal{L}_{Variability}(D, \phi) = \mathbb{E}[e^{-\|s_{t1} - s_{t2}\|}] \tag{13}$$

$e^{-distance}$ is used as a similarity measure that is 1 if the distance among states is 0 and goes to 0 with increasing distance between states. Note that this prior is counter-balancing the slowness prior introduced above as the slowness alone would lead to constant values.

- **Proportionality**
The proportionality prior introduced in [Jonschkowski and Brock, 2015] assumes that for the same action in different states, the reactions to this action will have proportional amplitude or effect. The representation then vary in the same amount for two equal actions in different situations.

$$\mathcal{L}_{Prop}(D, \phi) = \mathbb{E}[(\| \Delta s_{t_2} \| - \| \Delta s_{t_1} \|)^2 | a_{t_1} = a_{t_2}] \tag{14}$$

- **Repeatability**
This prior states that two identical actions applied at similar states should provide similar state variations, not only in magnitude but also in direction [Jonschkowski and Brock, 2015].

$$\mathcal{L}_{Rep}(D, \phi) = \mathbb{E}[e^{-\|s_{t_2} - s_{t_1}\|^2} \| \Delta s_{t_2} - \Delta s_{t_1} \|^2 | \, a_{t_1} = a_{t_2}] \tag{15}$$

- **Dynamic verification**

Dynamic verification consists in identifying the corrupted observation o_{t_c} in a history of k observations o_t where $t \in [\![0, K]\!]$. Observations are first encoded into states and the sequence is classified by a learned function f to output the corrupted time step. Negative samples are produced by incorporating observations from a wrong time step into the sequence of images [Shelhamer et al., 2017]. This discriminative approach forces SRL to encode the dynamics in the states.

- **Selectivity**
States can be learned by using the idea that factors such as objects correspond to 'independently controllable' aspects of the world that can be discovered by interacting with the environment [Thomas et al., 2017]. Knowing the dimension K of the state space, the aim is to train K policies π_k with $k \in [\![1, K]\!]$. The goal is that the policy π_k causes a change in $s_t^{(k)}$ only, and not in any other feature. To quantify the change in $s_t^{(k)}$ when actions are taken according to π_k, the selectivity of a feature k is:

$$\mathcal{L}_{sel}(D, \phi, k) = \mathbb{E}\Bigg[\frac{\| s_{t+1}^{(k)} - s_t^{(k)} \|}{\sum_{k'} \| s_{t+1}^{(k')} - s_t^{(k')} \|} | s_{t+1} \sim P_{s_t, s_{t+1}}^a\Bigg] \tag{16}$$

where $P_{s_t, s_{t+1}}^a$ is the environment transition distribution from s_t to s_{t+1} under action a. The selectivity of $s_t^{(k)}$ is maximal when only that single feature changes as a result of some action. Maximizing the selectivity improves the disentanglement of controllable factors in order to learn a good state representation.

3.7 Using hybrid objectives

Reconstruction of data in the observation space, forward models, inverse models, exploitation of rewards, and other objective functions presented in the previous sections are different approaches to tackle the state representation learning challenge. However, these approaches are not incompatible, and works often take advantage of several objective functions at the same time.

For instance, interactively *learning to poke by poking* [Agrawal et al., 2016] is an example of empirical learning of intuitive physics using o_t and o_g as current and goal images, respectively, in order to predict the poke action. The latter is composed by the location, angle and length of the action that sets the object in the state of goal image o_g . Simulations shows that using the inverse model or jointly the inverse and forward models beat a blob model at pushing objects and that when the training data available is reduced, the joint model outperforms the inverse model with a performance comparable to using a considerably larger amount of data.

Table 1: Characteristization of different SRL models

Model	**Actions/Next state constraints**	**Forward model**	**Inverse model**	**Reconstruct observation**	**Predicts next observation**	**Uses rewards**
AE [Mattner et al., 2012]	no	no	no	yes	no	no
Priors [Jonschkowski and Brock, 2015]	yes	no	no	no	no	yes
PVE [Jonschkowski et al., 2017]	yes	no	no	no	no	no
E2C [Watter et al., 2015]	yes	yes	no	yes	yes	no
ML-DDPG [Munk et al., 2016]	yes	yes	no	no	no	yes
VAE/DAE [van Hoof et al., 2016]	yes	yes	no	yes	yes	no
AE [Finn et al., 2015]	yes	no	no	yes	no	no
DVBF [Karl et al., 2016]	yes	yes	no	yes	yes	no
[Goroshin et al., 2015]	yes	yes	no	yes	yes	no
ICM [Pathak et al., 2017]	yes	yes	yes	no	no	no
[Shelhamer et al., 2017]	yes	no	yes	no	no	no
VPN [Oh et al., 2017]	no	yes	no	no	no	yes
DDM [Assael et al., 2015]	yes	yes	no	yes	yes	no
AE [Wahlström et al., 2015]	yes	yes	no	yes	yes	no
[Thomas et al., 2017]	yes	no	no	yes	no	no
PCA [Curran et al., 2015]	no	no	no	yes	no	no
PCA [Curran et al., 2016]	no	no	no	yes	no	no
rwPCA [Parisi, 2017]	no	no	no	yes	no	yes
[Magrans de Abril and Kanai, 2018]	yes	yes	no	no	no	yes
InfoGAN [Chen et al., 2016]	no	no	no	yes	no	no
BiGAN [Donahue et al., 2016]	no	no	no	yes	no	no
[Duan, 2017]	yes	yes	yes	yes	yes	no

The authors in [Finn et al., 2015, Goroshin et al., 2015] use the reconstruction of the observation and the slowness principle in their SRL approach. [Goroshin et al., 2015, van Hoof et al., 2016, Watter et al., 2015, Assael et al., 2015, Karl et al., 2016] combine the reconstruction of observation and forward models. [Jonschkowski and Brock, 2015, Lesort et al., 2017] take advantage of rewards with a causality prior (Eq. 11) and several other objective functions such as the slowness principle, proportionality, and repeatability to learn state representations. We illustrate, as an example, the combination of objective functions from [Watter et al., 2015] in Figure 2e.

Table 1 summarizes all the reviewed models. It shows which model or combination of models has been used for each SRL approach presented. The model of a particular approach indicates which proxies or surrogate functions have been used for learning: reconstruction of observation, prediction of the future (forward model) and/or retrieving actions (inverse model), and what kind of information is used: action and/or rewards.

4 Building blocks of State Representation Learning

In this section, we cover various implementation aspects relevant to state representation learning and its evaluation. We refer to specific surrogate models, loss function specification tools or strategies that help constraining the information bottleneck and generalizing when learning low-dimensional state representations,

4.1 Learning tools

We first detail a set of models that through an auxiliary objective function, help learning a state representation. One or several of these learning tools can be integrated in broader SRL approaches as was previously described.

4.1.1 Auto-encoders

Auto-encoders (AE) are a common tool used to learn state representations that are widely used for dimensionality reduction [Hinton et al., 2006, Wang et al., 2012, Wang et al., 2016]. Their objective is to output a reproduction of the input. Its architecture is composed by an encoder and a decoder. The encoder projects the input to a latent space representation (often in lower dimension than the input), which is re-projected to the output afterwards by the decoder. In our problem setting, o_t is the input, s_t the latent representation, and $\hat{o}_t$ is the output. The dimensionality of the latent representation can be chosen depending on the dimension of the state representation we want to learn and enforcing it in such case. The AE will then automatically learn a compact representation by minimizing the reconstruction error between input and output. The usual loss function $\mathcal{L}$ to measure the reconstruction error is the mean squared error (MSE) between input and output, computed pixel-wise. However, it can be any norm.

$$Loss = \mathcal{L}(x, \hat{x}) \tag{17}$$

Auto-encoders are used in different SRL settings [Finn et al., 2015, Mattner et al., 2012]; PCA can also be considered as a particular case of auto-encoder [Curran et al., 2016].

4.1.2 Denoising auto-encoders (DAE)

The main issue of auto-encoders is the risk of finding an easy but not satisfying solution to minimize the pixel reconstruction error. This occurs when the decoder reconstructs a kind of *average* looking dataset. To make the training more robust to this kind of mean optimization solution, denoising auto-encoders (DAE) [Vincent et al., 2008, Vincent et al., 2010] can be used. This architecture adds noise to the input and makes the "average" image a more corrupted solution than the original AE. The DAE architecture is used in [van Hoof et al., 2016] to learn visual and tactile state representations. The authors compared state representations learned by a DAE and a variational auto-encoder (VAE) by using the learned states in a reinforcement learning setting. They found that, in most cases, DAE state representation models gather less rewards than those with VAE state representations.

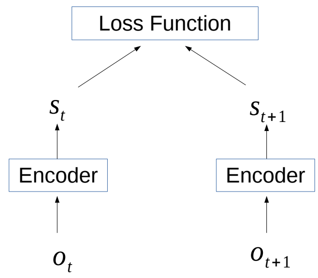


Figure 3: Siamese encoder networks with tied (shared) weights

4.1.3 Variational auto-encoders (VAE)

The SRL literature has also benefited from the variational inference used in variational auto-encoders (VAE) [Kingma and Welling, 2013, Jimenez Rezende et al., 2014] to learn a mapping from observations to state representations. A VAE interprets $\mathcal{S}$ as a set sampled from distribution $P(s_t|o_t)$. It then approximates $P(s_t|o_t)$ with a model $q_\theta(s_t|o_t)$ called the approximate posterior or recognition model. θ represents the parameters of the model, which often is a neural network. The VAE also provides a generator which approximates $P(o_t|s_t)$ with a model p_ϕ. ϕ are the parameters of the generator. Both models p and q are then trained by optimizing the error between o_t and $\hat{o}_t$ and the KL divergence between $q_\theta(s_t|o_t)$ and the normal distribution $\mathcal{N}(\mu = 0, \sigma = \mathbb{I})$ (where μ is the mean of the distribution, σ its covariance matrix and $\mathbb{I}$ the identity matrix).

VAE-related models that do not use exactly the original VAE, but variations of it, are [Watter et al., 2015, Assael et al., 2015, Krishnan et al., 2015, van Hoof et al., 2016, Karl et al., 2016].

4.1.4 Siamese networks

Siamese networks [Chopra et al., 2005] consist of two identical networks or more that share their parameters, i.e., have the exact same weights. The objective of the siamese architecture is not to classify input data, but to differentiate between the inputs (same versus different class or condition for example). This kind or architecture is useful to impose constraints in the latent space of a neural network. For example it can be used to learn similarity metrics or time dependencies such as it is the case in time-contrastive networks [Sermanet et al., 2017].

In the context of SRL, siamese networks can be employed to implement some priors previously presented in Section 3.6. For example, two siamese networks can be used to compute a similarity loss and optimize the slowness principle (or temporality prior) between s_t and s_{t+1} as in [Lesort et al., 2017]. In [Goroshin et al., 2015] they use three siamese networks to compute three consecutive states at the same time that are fed into another model that predicts the next state.

4.2 Observation/action spaces

This section presents a summary of the dimensions of o_t, $\hat{s}_t$ and a_t used in the reviewed papers. We also point at the continuity/discreteness of actions and states modeled, considering it as different dimensions of complexity to be learned. In general, the higher the dimensionality of o_t and a_t (as well as the smaller we want the dimension of $\hat{s}_t$ to be), the harder is the task of learning a state representations. In practice, and due to a lack of a solid theory, the dimension of $\hat{s}_t$ is rarely minimal (as it is hard to prove what is a minimal but sufficient representation). The literature normally presents results with presumably higher dimensionality of states learned than theoretically needed. The dimensionality of the state may seem obvious when we are learning a state that should (according to the task) correlate with a clear dimension (position, distance, angle) in the environment. However, deciding the dimensionality of the state space is often not trivial when we are learning more abstract states with no clear dimension associated to it. For instance, visually representing the state associated to an Atari game scene in a complex situation is not as easy to interpret nor assessing in comparison to the dimensionality of states associated to a position of an arm, its angle or velocity.

The effects and capacity of dimensionality in SRL can be observed in different application scenarios. Table 2 details the dimensionality of the observation, state and action spaces, as well as the application in which these are evaluated. The cardinality and continuity or discreetness of the action space is also shown. These can be some of the best proxies to assess the complexity of the problem tackled, together with the number of dimensions of the observation space (i.e., pixels or joint positions in many cases). Since there is not a well established procedure to select the size of the minimal state representation to be learned, often models learn a state space that, in most scenarios, is larger than the dimension we may want the data to show correlation with.

4.3 Evaluating learned state representations

This section provides a review of validation metrics and embedding quality evaluation techniques used across the literature. These are summarized in Table 3.

The common way of evaluating the quality of the learned state space is by letting the real robot use the states to perform a task, and by assessing whether the representation is general enough to be transferable to accomplish other tasks. The performance of an algorithm applied on the real target task can be done using reinforcement learning [Jonschkowski and Brock, 2015, Jonschkowski et al., 2017, Munk et al., 2016, van Hoof et al., 2016, Finn et al., 2015, Pathak et al., 2017, Shelhamer et al., 2017, Oh et al., 2017, Parisi, 2017, Assael et al., 2015]. However, this approach is often very costly and inefficient in terms of time, computation and data. Also, various state-of-the-art RL algorithms may be applied to learn a policy and may result in very different performances for a given state representation. The uncertainty inherent to RL therefore makes RL algorithms sufficient but not practical nor appropriate to be a necessary condition to validate a particular state representation. In consequence, it would be desirable to have an intermediate manner to assess the representation that is independent of the algorithm applied to complete the task and there are, indeed, several more direct ways to assess the learned state space. For example, visual assessment of the representation's quality can be done using a Nearest-Neighbors approach as in [Sermanet et al., 2017, Pinto et al., 2016]. The idea is to look at the nearest neighbors in the learned state space, and for each neighbor, retrieve their corresponding observation. Visual inspection can then reveal if these two observations indeed correspond to nearest neighbors in the ground truth state space $\tilde{s}$ we intend to learn.

While the nearest neighbor coherence can be assessed visually, KNN-MSE is a quantitative metric derived from this qualitative information [Lesort et al., 2017]. Using the ground truth state value for every observation, KNN-MSE measures the distance between the value of an observation and the value of the nearest neighbor observations retrieved in the learned state space. A low distance means that a neighbor in the ground truth is still a neighbor in the learned representation, and thus, local coherence is conserved.

For an observation o, KNN-MSE is computed using its associated learned state $s = \phi(o)$ as follows:

$$\text{KNN-MSE}(s) = \frac{1}{k} \sum_{s' \in KNN(s,k)} ||\tilde{s} - \tilde{s}'||^2 \tag{18}$$

where $\text{KNN}(s, k)$ returns the k nearest neighbors of s (chosen with the Euclidean distance) in the learned state space $\mathcal{S}$, $\tilde{s}$ is the ground truth associated to s, and $\tilde{s}'$ is the ground truth associated to s'.

One of the characteristics that a good representation should possess is to produce a disentangled representation of variation factors. The evaluation of these characteristics can be done using the selectivity prior (see Section 3.6 and Eq. 16) from [Thomas et al., 2017]. This prior cares about the independence among variations of the representation under each action. However, it is applicable mainly if actions are known to be independent.

Another way to quantitatively compare the degree of disentanglement reached by a model is using the disentanglement metric score [Higgins et al., 2016]. It assumes that the data is generated by a process in which the generative factors are known, interpretable, and that some are conditionally independent. The metric measures both the independence and interpretability, because a representation consisting of independent latent variables is not necessarily disentangled. In order to measure the disentanglement, it uses a simple low-capacity and low VC-dimension linear classifier's accuracy. The classifier's goal is to predict the index y of the generative factor that was

Table 2: The effects of dimensionality on State Representation Learning

Reference	Observation Dimension	State Dimension	Action Dimension	Environment	Data
Priors [Jonschkowski and Brock, 2015]	16*16*3	2	25 discrete	Slot cars, mobile robot localization	Raw images
PVE[Jonschkowski et al., 2017]	Unavailable	5	Discrete	Inverted pendulum, ball in cup, cart-pole	Raw images
E2C [Watter et al., 2015]	40*40*3	8	Discrete	Agent with obstacle	Raw images
E2C [Watter et al., 2015]	48*48*3	8	Discrete	Inverted pendulum	Raw images
E2C [Watter et al., 2015]	80*80*3	8	Discrete	Cart-pole	Raw images
E2C [Watter et al., 2015]	128*128*3	8	Discrete	3 link arm	Raw images
[van Hoof et al., 2016]	20*20*3	3	Continuous	Pendulum swing-up	Raw images
[van Hoof et al., 2016]	228	3	Continuous	Real-robot manipulation task	Tactile data
ML-DDPG [Munk et al., 2016]	18 or 24	6	2 discrete	2 link arm	Joint position
ML-DDPG [Munk et al., 2016]	192 or 308	96	36 discrete	Octopus	Joint position
[Finn et al., 2015]	240*240*3	32	Continuous	Robotics manipulation tasks	Raw images
DVBF [Karl et al., 2016]	16*16*3	3	Unavailable	Pendulum	Raw images
DVBF [Karl et al., 2016]	16*16*3	2	Unavailable	Bouncing ball	Raw images
DVBF [Karl et al., 2016]	16*16*3	12	Unavailable	2 bouncing balls	Raw images
[Goroshin et al., 2015]	3 frames of 32*32	2	2 discrete	NORB dataset	Raw images
ICM [Pathak et al., 2017]	42*42*3	3	4 discrete	3D VizDoom navigation game	Raw images
ICM [Pathak et al., 2017]	42*42*3	2	14 discrete	Mario Bros	Raw images
[Shelhamer et al., 2017]	Unavailable	Un.	Unavailable	Atari	Raw images
VPN [Oh et al., 2017]	3*10*10	Un.	4 discrete	2D navigation	Raw images
VPN [Oh et al., 2017]	4*84*84	Un.	4 discrete	Atari	Raw images
[Curran et al., 2016]	Unavailable	4	5 discrete	Mountain car 3D	Unavailable
[Curran et al., 2016]	Unavailable	12	243 discrete	6 link swimmer	Unavailable
rwPCA [Parisi, 2017]	21*21*3	Auto.	2 continuous	Picking a coin and putting it on a goal	Raw images
[Parisi, 2017]	20	Auto.	2 continuous	Hit a ball with a ping-pong paddle	Position of objects
[Magrans de Abril and Kanai, 2018]	40	2	Continuous	Explore a 3 room simulated 2D map	Position of agent
[Duan, 2017]	240*240*4	512	4 continuous	Poking cube	Raw images + depth

Table 3: Evaluation methods for state representation learning

Metric	Evaluation target	Context/Observations
Task performance [Jonschkowski and Brock, 2015, Jonschkowski et al., 2017, Munk et al., 2016, van Hoof et al., 2016, Finn et al., 2015, Pathak et al., 2017, Shelhamer et al., 2017, Oh et al., 2017, Parisi, 2017, Assael et al., 2015]	Quality of the state space for a given task	Reinforcement learning
Disentanglement metric score [Higgins et al., 2016]	Data-generating latent factor disentanglement	Transfer learning, object recognition. Assumes generative factors are known and interpretable
Distortion [Indyk, 2001]	Preservation of local and global geometry coherence	Unsupervised representation learning
NIEQA (Normalization Independent Embedding Quality Assessment) [Zhang et al., 2012]	Local & global neighborhood embedding quality assessment	Manifold learning, not limited to isometric embeddings
KNN-MSE [Lesort et al., 2017]	Task related representation learning	Unsupervised SLR for robotics

kept fixed for a given absolute linear difference between the inferred latent representations. This ensures that, on average, the difference among embeddings from the same fixed latent factor is lower than the difference among embeddings belonging to a different factor. The average accuracy of this classifier to predict all latent factors is reported as the disentanglement metric score.

Other metrics from the area of manifold learning can be used, such as distortion [Indyk, 2001] and NIEQA [Zhang et al., 2012]; both share the same principle as two quantitative measures of the global quality of a representation: the representation space should, as much as possible, be an undistorted version of the original space.

Distortion [Indyk, 2001] gives insight of the quality of a representation by measuring how the local and global geometry coherence of the representation changes with respect to the ground truth. It was designed in the *embeddings* context as a natural and versatile paradigm for solving problems over metric spaces.

NIEQA (Normalization Independent Embedding Quality Assessment) [Zhang et al., 2012] is a more complex evaluation than distortion that measures the local geometry quality and the global topology quality of a representation. NIEQA local part checks if the representation is locally equivalent to an Euclidean subspace that preserves the structure of local neighborhoods. NIEQA objectives are aligned with KNN-MSE [Lesort et al., 2017], as a measure to assess the quality of the representation, especially locally. The global NIEQA measure is also based on the idea of preserving original structure in the representation space, but instead of looking at the neighbors, it samples "representative" points in the whole state space. Then, it considers the preservation of the geodesic distance between those points in the state space.

Regarding the assessment of observation's predictions quality, using mean square error is a common approach, but other measures can be used such as Multi-Scale Similarity Metrics (MSSIM) [Wang et al., 2003] to assess the realistic characteristics of GAN-generated images [Odena et al., 2017], or, adversarial training methods [Donahue et al., 2016, Mathieu et al., 2015]. More concretely, the *Peak Signal to Noise Ratio, Structural Similarity Index Measure*, and *image sharpness* are examples of metrics for next frame prediction assessment used in [Mathieu et al., 2015]. The authors propose image gradient difference, a multi-scale architecture and adversarial training methods as better loss functions than MSE. A potential study could assess if better quality on the observation reconstruction can lead to better quality of states.

4.4 Evaluation scenarios

Datasets used to validate state representation learning include varied, but mainly simulated, environments because they are easier to reproduce and generate. Unlike in image recognition challenges where MNIST digits or ImageNet datasets prevail, in state representation learning, a varied set of regular video games or visuomotor tasks in robotics can be found as a test suite for robotics control. Examples of simulated environments include, among others:

- Pendulum (Inverted or classical): The goal is to represent the state of the pendulum [Watter et al., 2015, Jonschkowski et al., 2017, van Hoof et al., 2016, Mattner et al., 2012]. The pendulum starts in a random position, and the objective is to swing it up so it stays upright (there is no specified reward threshold at which the task is considered solved).

- Cart-Pole: consists of an inverted pendulum attached to a cart which moves along a frictionless track; the system is controlled applying +1 or -1 force to the cart, and a reward of +1 is provided for every time step that the pole remains upright. The episode ends when the pole is more than 15 degrees from vertical, or the cart moves more than 2.4 units from the center[2] ([Watter et al., 2015], [Jonschkowski et al., 2017]).

- Atari games [Bellemare et al., 2013]: mostly low (2D) dimensional simulated environments with different agents and goals. In these games, states can be represented through different variables (time in achieving a task, amount of bonus, keeping alive, etc.) [Shelhamer et al., 2017, Oh et al., 2017].

- More advanced test games include *VizDoom*, where the levels passed, reward accumulated and exploration levels are used as evaluation metrics [Pathak et al., 2017]. Likewise, Mario Benchmark [Karakovskiy, 2012] is a platform designed for reinforcement learning based on the "Super Mario Bros" video game. The task is to collect as much points as possible by collecting coins, kill enemies, eat mushrooms or fireflowers and completing as many levels of difficulty as possible without getting hurt or killed. Possible actions are to move left/right/none, jump/no jump, run/fire and no run/fire. This test suite is for example experimented in [Curran et al., 2016, Pathak et al., 2017].

- Other evaluation benchmarks tested in the reviewed works in this survey include simulated octopus arms [Engel et al., 2006, Munk et al., 2016], labyrinths [Thomas et al., 2017], navigation grids [Magrans de Abril and Kanai, 2018, Oh et al., 2017], driving cars [Jonschkowski and Brock, 2015], or *mountain car* scenarios [Curran et al., 2016]. Another example is the *bouncing ball*, where the goal is to learn a representation of one bouncing ball position in 2D (x,y) [Karl et al., 2016].

- In the robotics domain we can find benchmarks on robot manipulation skills [Finn et al., 2015, van Hoof et al., 2016] such as Baxter pushing a button [Lesort et al., 2017], grasping [Finn et al., 2015], stabilizing [van Hoof et al., 2016], poking [Agrawal et al., 2016, Duan, 2017] objects or balancing a real pendulum [Mattner et al., 2012]. Nevertheless, some approaches achieve to learn in real environment scenarios, for instance, with mobile robots that explore an arena [Jonschkowski and Brock, 2015].

Many of the latter simulated scenarios are part of Universe and OpenAI Gym [Brockman et al., 2016] or DeepMind Labs [Beattie et al., 2016]. These and other benchmarking tasks used in the most prominent state representation learning literature are summarized in Table 2.

More generally, the reinforcement learning literature provides benchmarks on similar robotic tasks such as pushing, sorting, grasping objects [Finn et al., 2016, Finn et al., 2017], pouring into a glass [Sermanet et al., 2017], and other tasks involving humanoid and static robots. *DeepMind Control* is a simulation platform of control tasks to test the continuous version of learning algorithms with MuJoCo physics engine [Tassa et al., 2018]. Reinforcement learning also tackles more complex problems such as the road-crossing chicken game [Machado et al., 2017] or Montezuma's Revenge Atari game, which happens to be the one with the worse performance when it comes to human-level control by AI agents [Mnih et al., 2015]. These challenging games and environments could be tackled by SRL algorithms; they require ahead thinking in order to learn states, actions and longer term planning. Future work may benefit from SRL approaches and applying them to

[2] https://github.com/openai/gym/wiki/Leaderboard#pendulum-v0

learn an intermediate state representation, since it could help boosting performance and create unified environments.

5 Discussion and future trends

In this section, we first discuss the implications of SRL for autonomous agents and the assessment, comparison and reproducibility of the representation learned. Finally, we explore the consequences of SRL on the interpretability of machine learning algorithms.

5.1 SRL models for autonomous agents

The role of environment exploration is an important dimension to investigate in SRL. If the space is not sufficiently explored by the agent, acquisition of varied observations and exposure to actions that lead to optimal performance can be hindered, making learning a relevant state space impossible. In most of the reviewed papers, exploration is assumed to be efficient and not studied in details, but for an autonomous agent, this question has to be solved.

One way to incorporate exploration in SRL is to integrate curiosity or intrinsic motivations [Oudeyer et al., 2007] in the algorithm that collects data. The overall idea of this approach is to complement the extrinsic reward by an intrinsic reward that favors states where SRL makes the most progress. This is done for example in the Intrinsic Curiosity Module (ICM) [Pathak et al., 2017] by defining an intrinsic reward linked to the forward model error which encourages exploration. This approach is improved in [Magrans de Abril and Kanai, 2018] by balancing this exploratory behavior with an homeostatic drive to also favor actions that lead to familiar state-action pairs. The reverse question of how the learned state space can influence the performance of intrinsic motivation approaches [Péré, 2018] is also relevant. The automatic exploration designed to maximize the quality of a learned state representation is a field to be further explored in order to build high quality representations.

Another problem to ultimately perform SRL autonomously (i.e., without parameter tuning) is the choice of the state representation dimension, which is done manually in all the reviewed approaches. The challenge of deciding the dimensionality can be related to a bias-variance trade-off, as the dimensionality of the representation constrains the capacity of the model. Unfortunately, an automatic process may often choose a larger dimension than necessary. Indeed, increasing the states dimension increases the capacity of the model, which, as a result, will be better at reducing the training error, but also leads to overfitting. A model that chooses its own capacity is consequently prone to over estimate the dimension needed.

To avoid choosing manually the state dimension, it is possible to choose automatically a number of features from a larger set such that they have a certain variance and are orthogonal [Parisi, 2017]. It can be done by using PCA to produce a set of features in which the most significant ones are selected with respect to the chosen variance. PCA can also be modified to select reward related components [Parisi, 2017]. Although the variance has to be fixed a priori, it is claimed that this is usually easier than choosing the state dimension. Extending this technique to other state representation approach could be an interesting research direction.

5.2 Assessment, comparison and reproducibility in SRL

The assessment challenge of SRL is two-sided. First, there is no easy nor certified way for validating a learned representation. Secondly, the evaluation difficulties makes a fair comparison between approaches difficult. As mentioned in Section 4.3, the most common method to evaluate the quality of representations is to check if the state representation learned can be used by an RL algorithm to solve a task. However, this assessment is uncertain and unstable. This is why having good performance in RL cannot be a necessary condition to guarantee quality in the states learned. This uncertainty gives rise to the second problem. If there is uncertainty in the assessment method, then the comparison among approaches cannot be objective. Moreover, comparing approaches from published results is particularly hard because of the high variability of the environments and data dimensions. To illustrate the diversity on the different works reviewed and their different settings, Table 2 exhibits the environments, the data dimensions involved, and the nature of the data.

The variability in the approaches points out at another main issue in SRL, which is the problem of results reproducibility. Apart from the algorithm' source code, which could be unavailable, there is no set of baseline experiments to easily reproduce approaches. Reproducibility guidelines with proper experimental techniques and reporting procedures, as pointed in [Henderson et al., 2017] for RL, should therefore be defined for SRL.

5.3 Providing interpretable systems

The interpretability of results in machine learning is a challenging problem that needs proper definition [Lipton, 2016]. We define interpretability in the SRL context as the capacity for a human to be able to link a variation in the representation to a variation in the environment, and be able to know why the representation was sensitive to this variation. As SRL is designed to be able to give this level of interpretability, it could help improving the understanding we have about learning algorithms' output. Indeed, the higher the dimension, the less interpretable the result is for humans. Therefore, the dimensionality reduction induced by SRL could be highly beneficial to improve our understanding capacity and for creating visually compelling results. Unfortunately, assessment methods used on learned states such as reinforcement learning gives generally better results when the dimension increases including overlapping factors. One reason for this is that the assessment method does not take into account the interpretability characteristic of a representation in the same sense as us. More generally, representations in neural networks are often composed by overlapping dimensions and thus, they are not very prone to be understandable and controllable by humans. Since SRL often learns a disentangled representation, it makes them easier to be interpreted.

The objective of representation learning providing more interpretable explanations to algorithmic decisions is highly considered. In 2018, European Union regulations on algorithmic decision-making include a "right to explanation", "right to opt-out" and "non discrimination" of models. Artificial intelligence research is thus granted with an opportunity to further provide meaningful explanations to why algorithms work the way they do [Goodman and Flaxman, 2016].

In any case, monitoring the degree to which AI systems show the same thinking flows as humans is invaluable and crucial; not only to explain how human cognition works, but also to help AI make better and more fair decisions [Lake et al., 2016].

6 Conclusion

We reviewed State Representation Learning algorithms that are designed to find a way to compress high-dimensional observations data into a low and meaningful dimensional space for controlled systems. These models only require observations made by the system, the actions performed and optionally the reward of an associated task. We presented the various constraints that can be used in this objective: such representation should contain enough information to be able to reproduce the input observation; it should contain enough information about the dynamics of the environment to predict the future state given an action, or reversely to predict actions given two states; it should contain information related to a task reward; it should follow various constraints based on a priori knowledge directly on the representation space, e.g., using physics laws or common sense priors.

A general trend is to add as many learning objectives as possible depending on the available data. As an example, one could use a reconstruction objective for linking the state space to the observations, combined with a predictive objective (forward model) to capture dynamics, and a reward based objective to apprehend the causality of actions performed. The prior learning objective can also be added to force the state space to be coherent and understandable for humans. While many models integrate several of these objectives, no proposed model currently includes all of them together.

As SRL is designed to automatically learn representations from a set of unlabeled observations, it could be used in future work to learn from evolving environments and could be a step towards continual or lifelong learning. Another area to explore in the future is the integration of exploration strategies for data collection specifically designed to be able to improve the state representation learned.

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