Outline

- Review derivation of OLS estimator and variance
- Interactions!
  - Rules of interactions
  - Data session

Rules of Interactions

$$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

1. X and Z are the "constituent terms" (or "constitutive terms").

2. XZ (or X*Z or X x Z) is the "interaction term" or "multiplicative term."

3. ***ALWAYS*** include both constituent terms in the regression model. Never exclude X or Z on their own (see Brambor et al. 2006 on noncompliance with this simple rule).

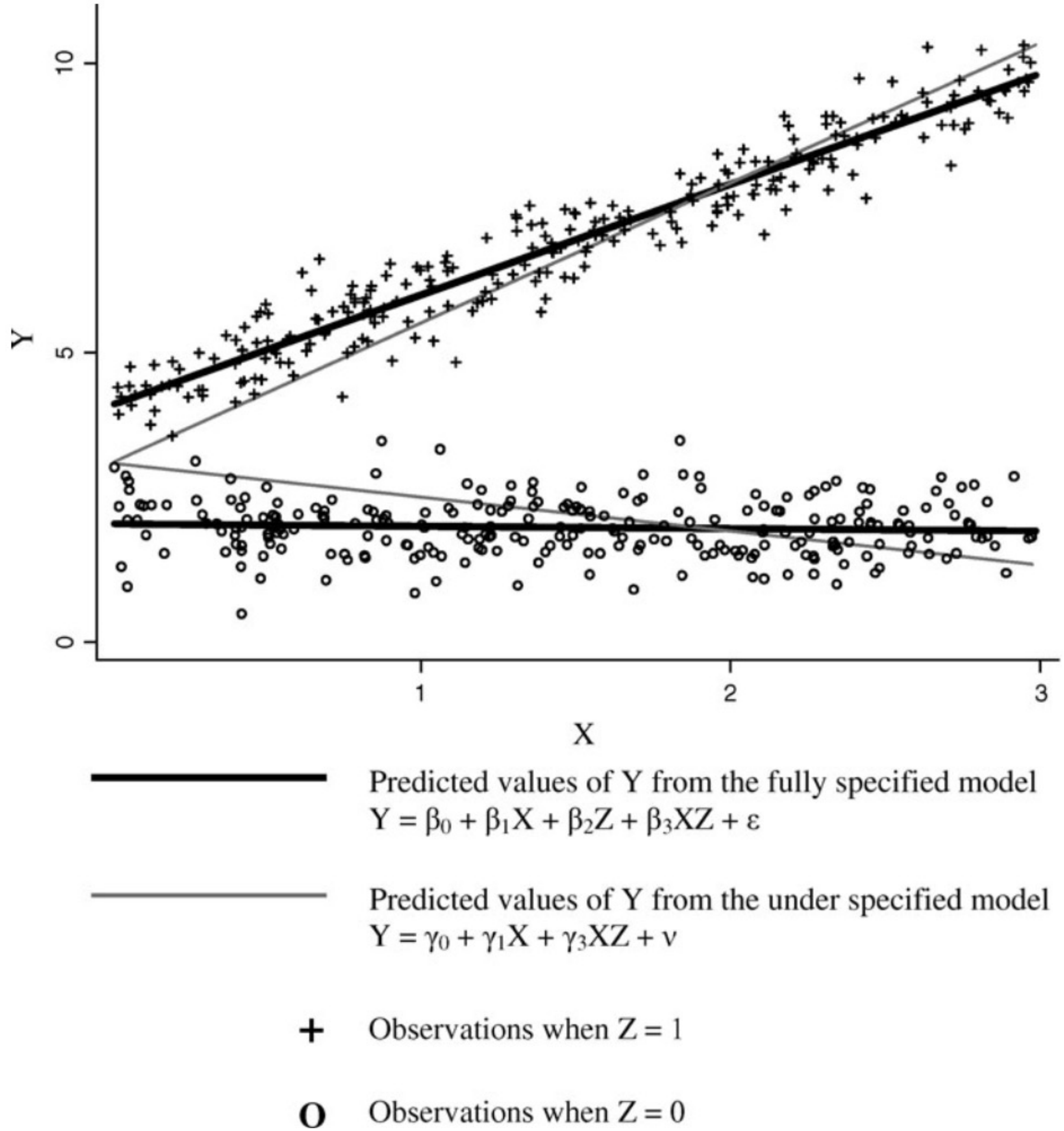


Fig. 2 An illustration of the consequences of omitting a constitutive term.

Rules of Interactions

$$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

1. X and Z are the "constituent terms" (or "constitutive terms").

2. XZ (or X*Z or X x Z) is the "interaction term" or "multiplicative term."

3. ***ALWAYS*** include both constituent terms in the regression model. Never exclude X or Z on their own (see Brambor et al. 2006 on noncompliance with this simple rule).

4. When you include an interaction, you're modeling the conditional effects of *both* X and Z: Marginal effect of X conditional on Z; and the marginal effect of Z conditional on X.

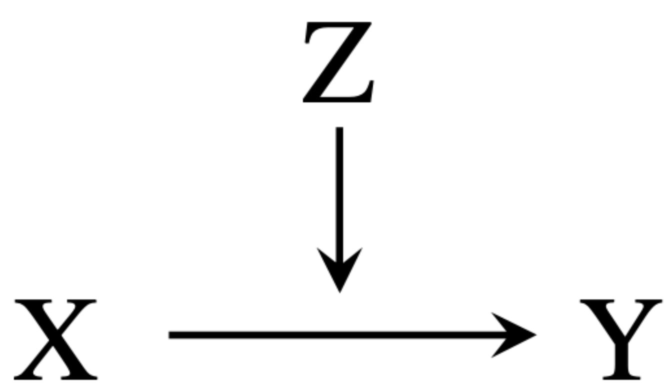
   - Z *moderates* (or *modifies*) the effect of X on Y
   - X *moderates* (or *modifies*) the effect of Z on Y

Moderate versus Mediate!

FIGURE 1: Z *Mediates* the Relationship Between X and Y

$$X \longrightarrow Z \longrightarrow Y$$

FIGURE 2: Z *Moderates* the Relationship Between X and Y



Rules of Interactions

$$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

5. Marginal effect of X on Y:

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

   Marginal effect of Z on Y:

$$\frac{\partial Y}{\partial Z} = \beta_2 + \beta_3 X$$

6. Interpreting β_1 *and* β_2 (constituent terms):

   β_1: Marginal effect of X when Z=0.

   β_2: Marginal effect of Z when X=0.

Important: β_1 *and* β_2 are NOT "main effects" or "average effects." They're *conditional* effects (effects conditional on ONE value of the moderating variable).

Rules of Interactions

$$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

7. Standard errors of conditional marginal effects (var. eq.):

$$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = \text{var}(\hat{\beta}_1) + Z^2\text{var}(\hat{\beta}_3) + 2Z\text{cov}(\hat{\beta}_1\hat{\beta}_3)$$

8. Mean-centering X and Z? What's the "benefit?"

β_1: Marginal effect of X when Z=0.
β_2: Marginal effect of Z when X=0.

Mean-centering much ado about nothing.

Rules of Interactions

9. Graph your results!

   Two ways of graphing:

   1. "Brambor et al. graph"; graph the marginal effect of X on Y as a function of the moderator, Z.

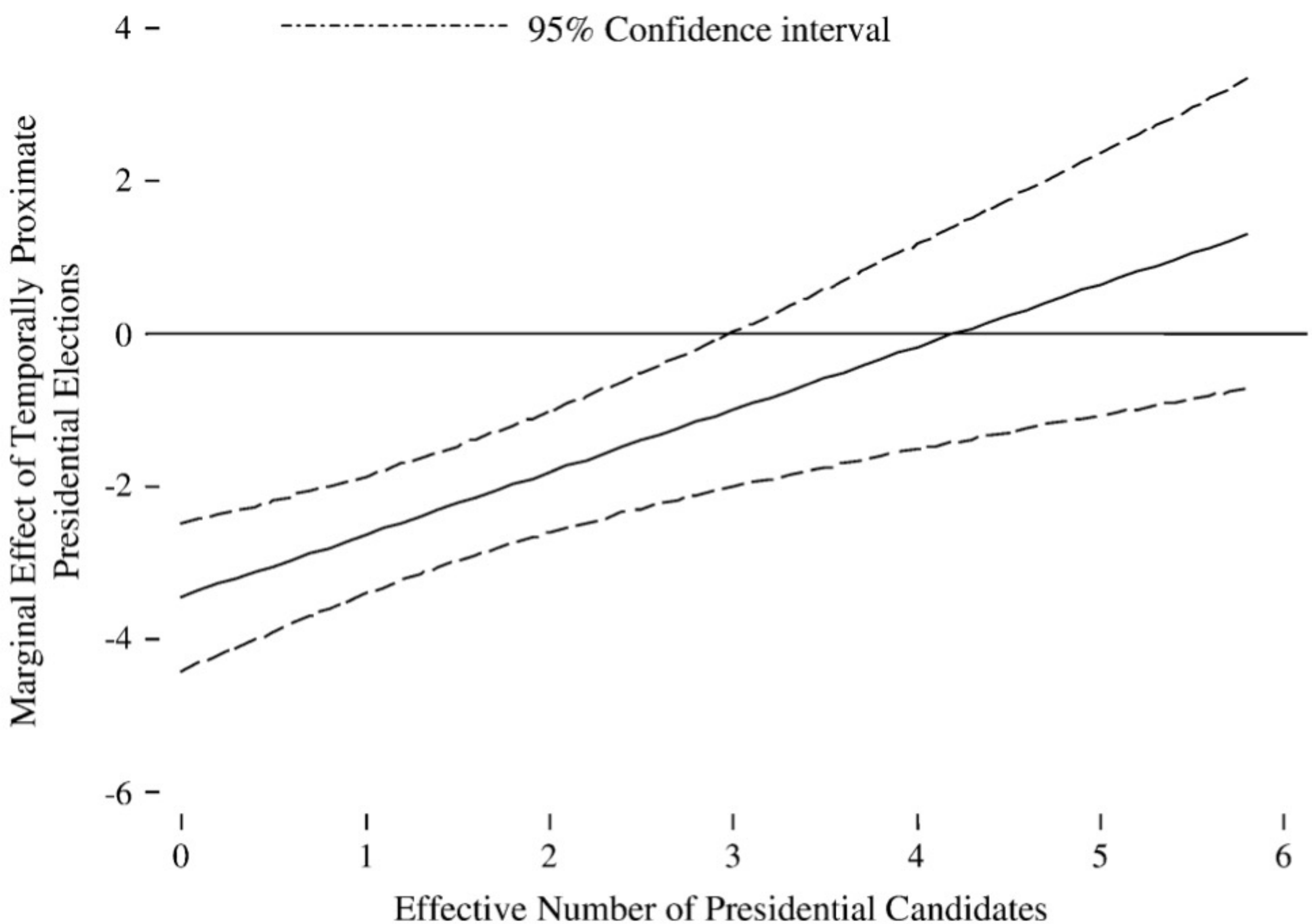


Fig. 3 The marginal effect of temporally proximate presidential elections on the effective number of electoral parties.

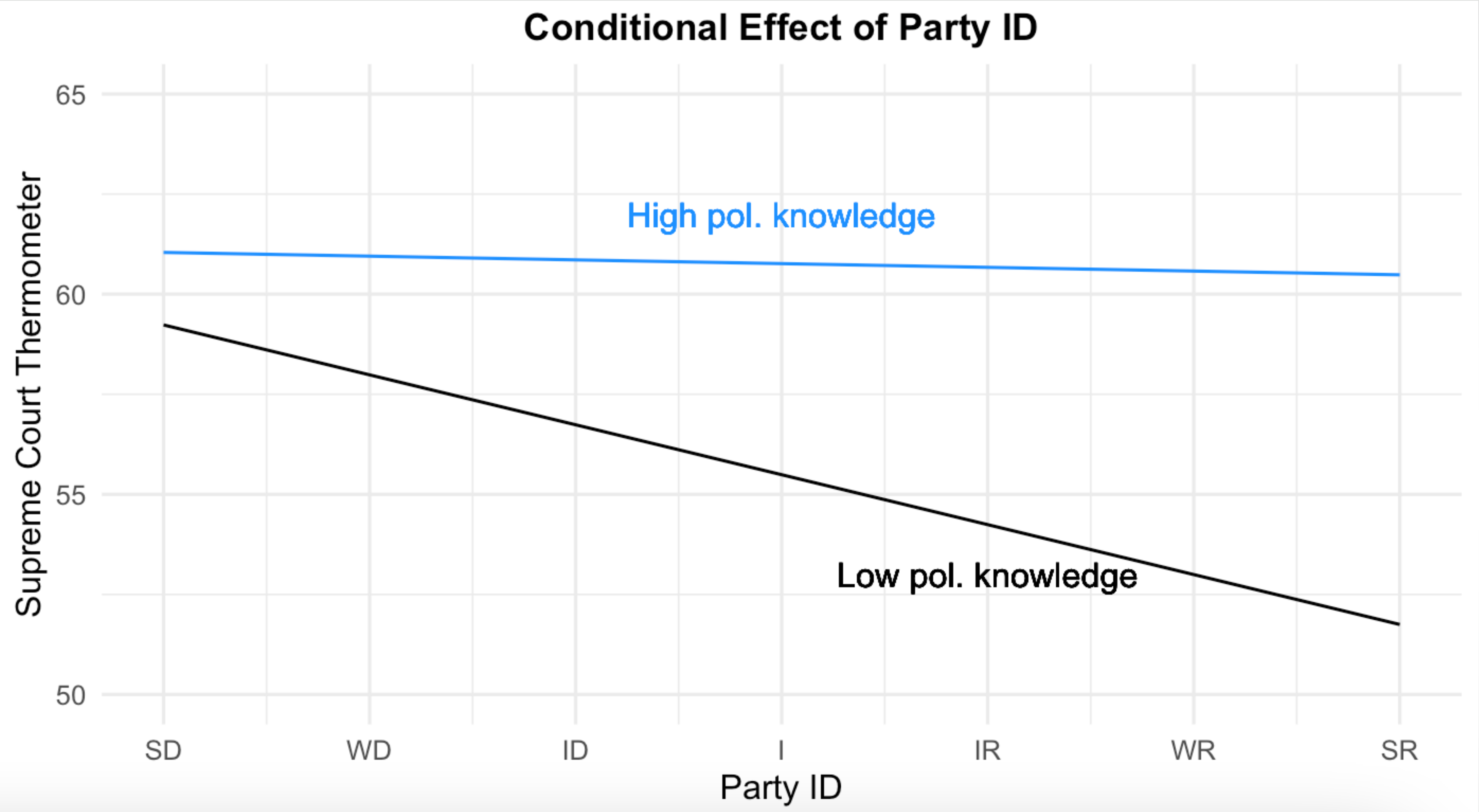
Rules of Interactions

9. Graph your results!

   Two ways of graphing:

   1. "<u>Brambor et al. graph</u>": Graph the marginal effect of X on Y as a function of the moderator, Z.

   2. <u>Differing slopes graph</u>: Graph predicted values of Y (i.e., $\hat{Y}$) against X for multiple (typically 2) values of the moderator, Z. Graph two slopes for low and high values of the moderator, Z, to show how Z moderates the effect of X on Y.



Conditional Effect of Party ID
High pol. knowledge
Low pol. knowledge
Supreme Court Thermometer
65
60
55
50
SD
WD
ID
I
IR
WR
SR
Party ID