COORD. CILÍNDRICA

$$\begin{cases} X = r \cdot \cos(\theta) \\ Y = r \cdot \operatorname{sen}(\theta) \\ z = z \end{cases}$$

$$\left| \frac{d(x,y,z)}{d(r,\theta,z)} \right| = r$$

$$\iiint_E f(x,y,z)\, dx\, dy\, dz = \iiint_C f(r,\theta,z) \cdot r \cdot dr \cdot d\theta \cdot dz$$

Ex:

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$$

$$=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}(x^2+y^2)\cdot z\Big|_{\sqrt{x^2+y^2}}^{2}\,dy\,dx$$

$$\cancel{=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\left[(x^2+y^2)\left(2-\sqrt{x^2+y^2}\right)\right]dy\,dx}$$

$$=\int_{0}^{2\pi}\int_{0}^{2}\int_{r}^{2}r^2\cdot r\cdot dz\cdot dr\cdot d\theta$$

$$\sqrt{(r\cos(\theta))^2+(r\cdot sen(\theta))^2}$$

$$\sqrt{r^2\cdot\cos^2(\theta)+r^2\cdot Sen^2(\theta)}$$

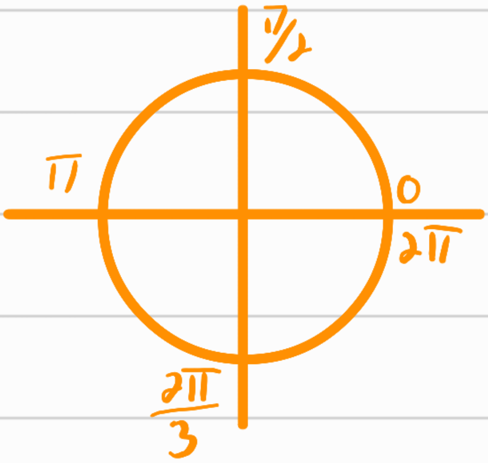
$$\sqrt{r^2\cdot(\cos^2(\theta)+Sen^2(\theta))}$$

$$\sqrt{r^2\cdot 1}=\sqrt{r^2}=r$$

O RAIO VAI DA MENOR DISTÂNCIA DO CENTRO (0) ATÉ O RAIO DA CIRCUNFERÊNCIA (2)

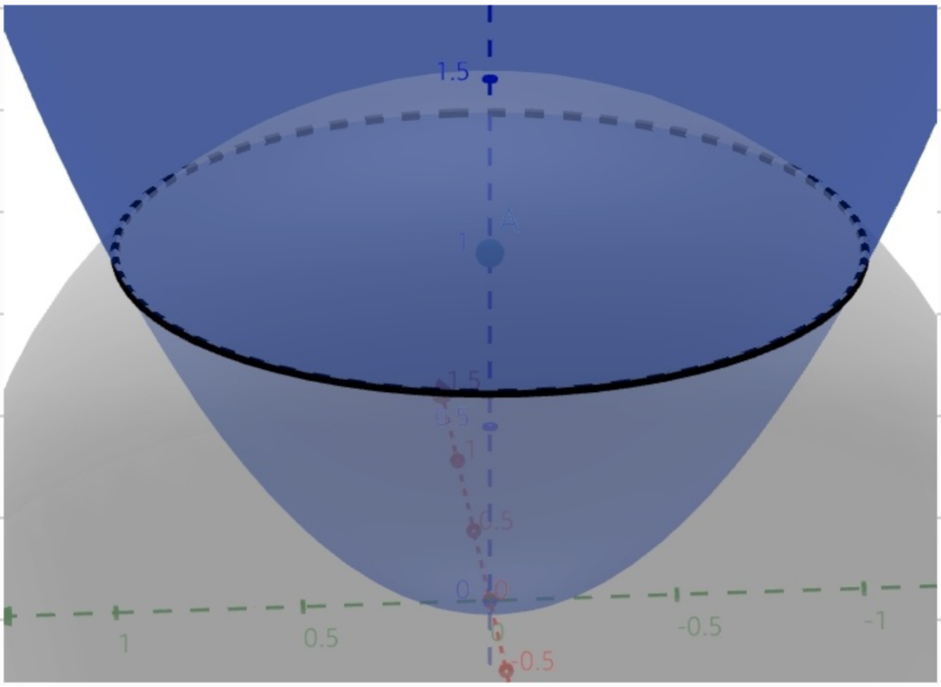
$$\int_0^2$$

O ÂNGULO É A VOLTA NA CIRCUNFERÊNCIA, OU SEJA, 2π

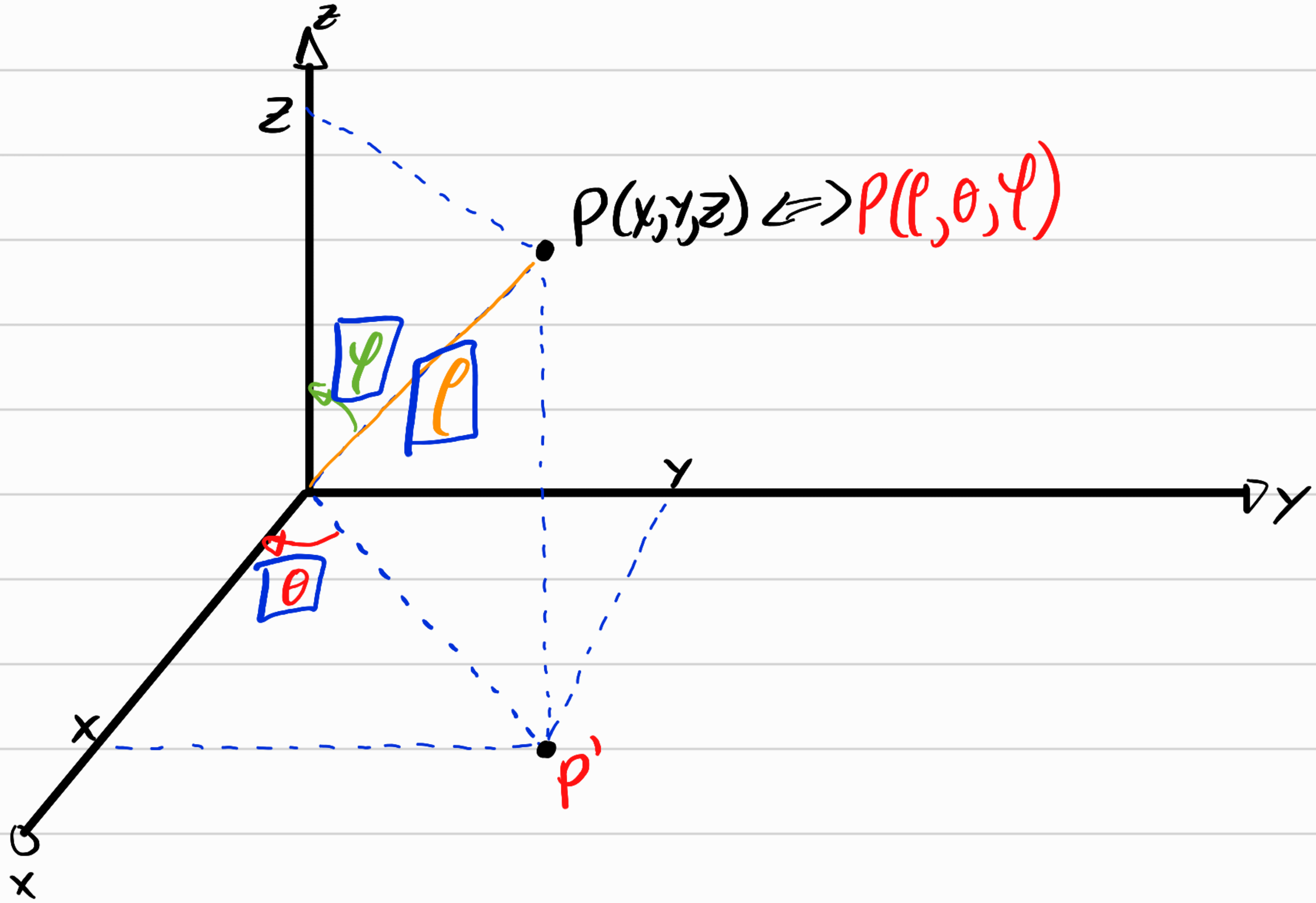


EX 24: Volume de E?

$$E = \begin{cases} z = x^2 + y^2 \\ x^2 + y^2 + z^2 = 2 \end{cases}$$



COORDENADAS ESFÉFICAS



ρ = DISTÂNCIA DE P À ORIGEM $= \sqrt{x^2+y^2+z^2}$

θ = ÂNGULO ENTRE A RETA $\overline{OP'}$ E O EIXO POSITIVO X

φ = ÂNGULO ENPRE A RETA $\overline{OP}$ E O EIXO Z

$$\begin{cases} X = \rho \cdot sen(\varphi) \cdot cos(\theta) \\ Y = \rho \cdot sen(\varphi) \cdot sen(\theta) \\ Z = \rho \cdot cos(\varphi) \end{cases}$$

$$\frac{d(x,y,z)}{d(\rho,\theta,\varphi)} = \begin{vmatrix} dx/d\rho & dx/d\theta & dx/d\varphi \\ dy/d\rho & dy/d\theta & dy/d\varphi \\ dz/d\rho & dz/d\theta & dz/d\varphi \end{vmatrix}$$

$$\begin{vmatrix} sen(\varphi)cos(\theta) & -\rho sen(\varphi)cos(\theta) & \rho cos(\varphi)cos(\theta) \\ sen(\varphi)sen(\theta) & \rho sen(\varphi)cos(\theta) & \rho cos(\varphi)sen(\theta) \\ cos(\varphi) & 0 & -\rho sen(\varphi) \end{vmatrix} =$$

$$= \boxed{-\rho^2 sen(\varphi)} \rightarrow |JAC| = |-\rho^2 sen(\varphi)|$$

$$|JAC| = \rho^2 sen(\varphi)$$

$$\iiint f(x,y,z)\,\underbrace{dx\,dy\,dz}$$

COORD. ESFÉR. $\Rightarrow$

$$\iiint f(x,y,z)\,\overbrace{\rho^2 \operatorname{sen}(\varphi)\, d\rho\, d\theta\, d\varphi}$$

$z = \rho \cdot \cos(\varphi)$

$y = \rho \cdot \operatorname{sen}(\varphi) \cdot \operatorname{sen}(\theta)$

$x = \rho \cdot \operatorname{sen}(\varphi) \cdot \cos(\theta)$

EX:

$$\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$$

$B = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \leq 1\}$

COORD ESF.

$$= \int_0^{2\pi}\int_0^{1}\int_0^{\pi} e^{\boxed{(\rho^2)^{3/2}}} \cdot \underbrace{\rho^2 \cdot \operatorname{sen}(\varphi)\, d\varphi\, d\rho}_{du}\, d\theta$$

$\rho^3 = u$

$$= 2\pi\left(-\cos(\varphi)\right)\Big|_0^{\pi} \cdot e^{\rho^3}\Big|_0^{1} = 2\pi(1+1)\cdot(e-1) = 4\pi(e-1)$$

Ex: Volume E=?

$$E = \begin{cases} z = \sqrt{x^2+y^2} \\ x^2+y^2+z^2 = z \end{cases}$$