CONTINUAÇÃO: DERIVADAS PARCIAIS

DEFINIÇÃO:

Seja $f: U \subset \mathbb{R}^n \to \mathbb{R}$, U ABERTO DE $\mathbb{R}^n$, $a \in U$, $a = (a_1, a_2, \ldots, a_n) \in U$.

Seja $i \in \{1, 2, \ldots, n\}$ DEFINIMOS A i-ÉSIMA DERIVADA PARCIAL DE f NO PONTO $a \in U$ COMO

$$\frac{df}{dx}(a) = \lim_{t \to 0} \frac{f(a + te_i) - f(a)}{t}$$

QUANDO O LIMITE EXISTE.

EX: $f(x,y) = 2xy - 4y$

(x_0, y_0)

$f_x(x_0, y_0) = 2y_0$

$f_y(x_0, y_0) = 2x_0 - 4$

$$f_x(x_0,y_0)=\lim_{t\to 0}\frac{f(x_0+t,\,y_0)-f(x_0,y_0)}{t}$$

$$=\lim_{t\to 0}\frac{2\cdot(x_0+t)\cdot y_0-4y_0-(2\cdot x_0y_0-4y_0)}{t}$$

$$=\lim_{t\to 0}\frac{\cancel{2x_0y_0}+2ty_0-\cancel{4y_0}-\cancel{2x_0y_0}+\cancel{4y_0}}{t}$$

$$=\lim_{t\to 0}\frac{2\cancel{t}y_0}{\cancel{t}}=\lim_{t\to 0}2y_0=\boxed{2y_0}\,//$$

$$f_y(x_0,y_0)=\lim_{t\to 0}\frac{f(x_0,\,y_0+t)-f(x_0,y_0)}{t}$$

$$=\lim_{t\to 0}\frac{2\cdot x_0(y_0+t)-4(y_0+t)-(2x_0y_0-4y_0)}{t}$$

$$=\lim_{t\to 0}\frac{\cancel{2x_0y_0}+2x_0t-\cancel{4y_0}-4t-\cancel{2x_0y_0}+\cancel{4y_0}}{t}$$

$$= \lim_{t \to 0} \frac{2x_0 t - 4t}{t} = \lim_{t \to 0} \frac{\cancel{t}(2x_0 - 4)}{\cancel{t}}$$

$$= \lim_{t \to 0} 2x_0 - 4 = \boxed{2x_0 - 4}$$

Ex: Se $f(x,y) = x^3 + x^2y^3 - 2y^2$. Determine $f_x(2,1)$ e $f_y(2,1)$

$$f_x(x,y) = 3x^2 + 2xy^3$$
$$f_x(2,1) = 3 \cdot 2^2 + 2 \cdot 2 \cdot 1^3 = 12 + 4 = 16$$

$$f_y(x,y) = 3y^2x^2 - 4y$$
$$f_y(2,1) = 3 \cdot 1^2 2^2 - 4 \cdot 1 = 12 - 4 = 8$$

Ex: $f(x,y) = \operatorname{sen}(x^2 - y^3)$, $f_x(1,1) = ?$ $f_y(1,1) = ?$

$$f_x(x,y) = \cos(x^2 - y^3) \cdot 2x$$
$$f_x(1,1) = \cos(1^2 - 1^3) \cdot 2 \cdot 1 = \cos(0) \cdot 2 = 1 \cdot 2 = 2$$

$$f_y(x,y) = \cos(x^2 - y^3) \cdot (-3y^2)$$

$$f_y(1,1) = \cos(1^2 - 1^3)\cdot(-3\cdot 1^2)$$
$$= \cos(0)\cdot(-3) = 1\cdot(-3) = -3$$

Ex: $f(x,y) = \dfrac{x}{e^{x^2+y^2}}$, $f_x(x,y) = ?$ $f_y(x,y) = ?$

$$f_x(x,y) = x\cdot e^{-(x^2+y^2)}$$

$$f_x(x,y) = x'\cdot e^{-(x^2+y^2)} + x\cdot\left[e^{-(x^2+y^2)}\right]'$$

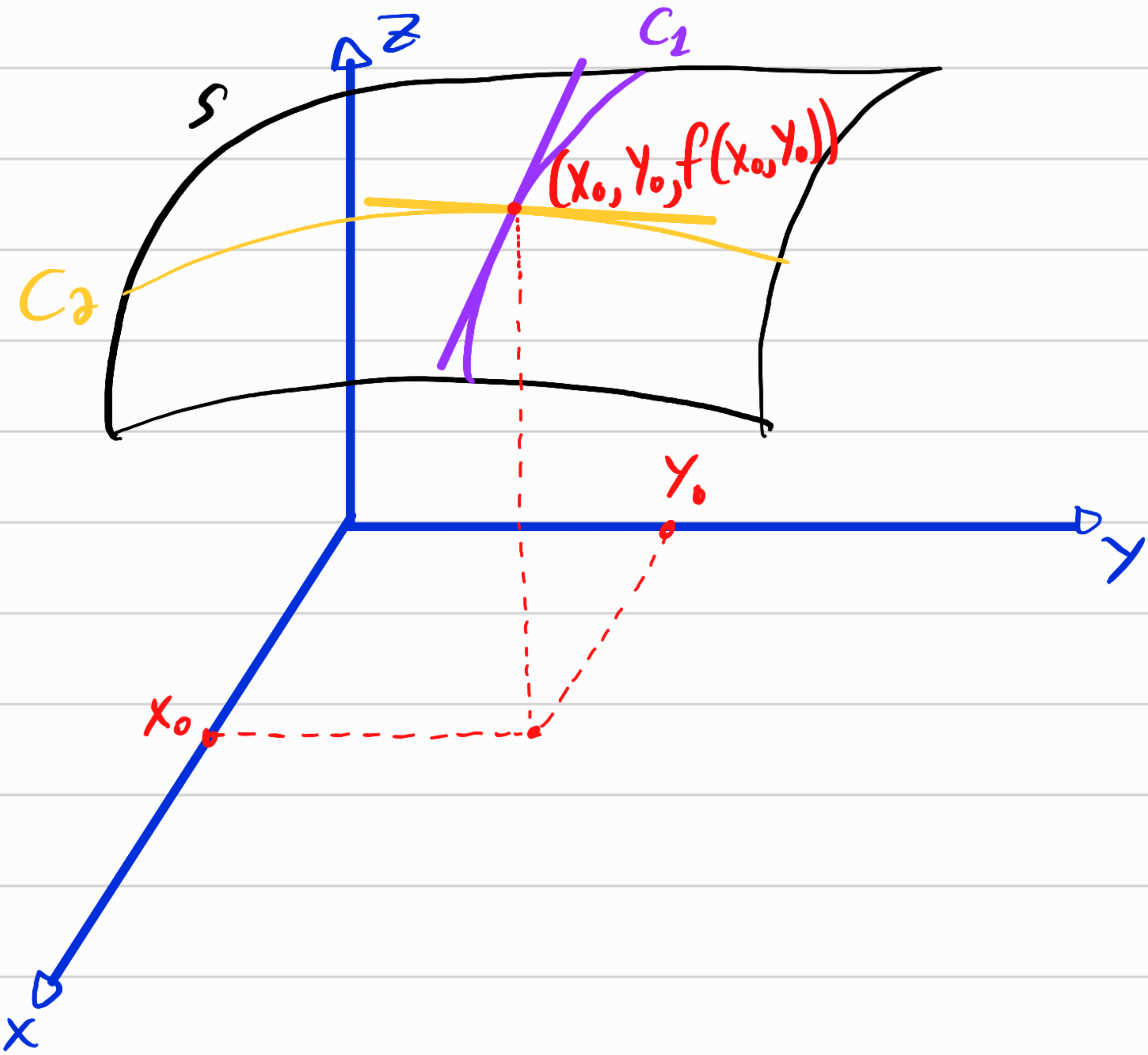
$$f_x(x,y) = e^{-(x^2+y^2)} + x\cdot(-2x)\,e^{-(x^2+y^2)}$$

$$f_x(x,y) = (1+x)\cdot e^{-(x^2+y^2)}$$

$$= \frac{1-2x^2}{e^{x^2+y^2}}$$

$$f_y(x,y) = x(-2y)\cdot e^{-(x^2+y^2)} = \frac{-2xy}{e^{x^2+y^2}}$$

$g'(x)$ É O COEFICIENTE ANGULAR ($m \to a + mb$) DA RETA TANGENTE AO GRÁFICO DE f.



y_0 Fixo

$$\{(x, y_0, z) : x, z \in \mathbb{R}\}$$

x_0 Fixo

$$\{(x_0, y, z) : y, z \in \mathbb{R}\}$$

$f_x(x_0, y_0) \to$ É O COEFICIENTE ANGULAR DA RETA TANGENTE À CURVA C