UMA FUNÇÃO DE DUAS VARIÁVEIS É UMA REGRA QUE ASSOCIA A CADA PAR ORDENADO DE NÚMEROS REAIS (x,y) DE UM DOMÍNIO D UM ÚNICO VALOR REAL, DENOTADO POR $f(x,y)$. O CONJUNTO D É CHAMADO DOMÍNIO DE f E SUA IMAGEM É O CONJUNTO DE TODOS OS VALORES POSSÍVEIS DE f, OU SEJA, $\{f(x,y) : (x,y) \in D\}$

ESCREVEMOS $z = f(x,y)$

EX: O DOMÍNIO DA FUNÇÃO f

$$f(x,y) = \sqrt{9 - x^2 - y^2}$$

É O CONJUNTO

$$D = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9$$

EX: O DOMÍNIO DA FUNÇÃO h:

$$h(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

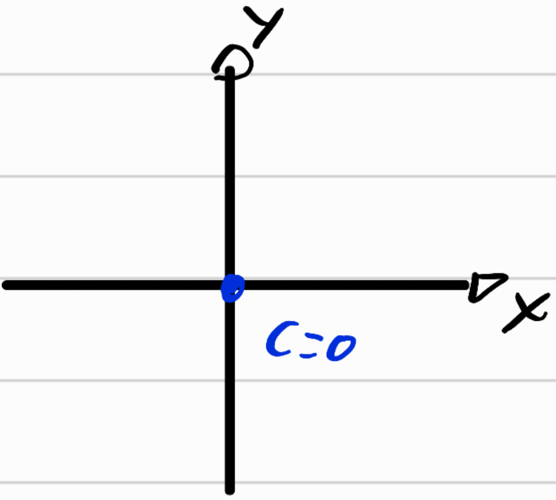
É O CONJUNTO

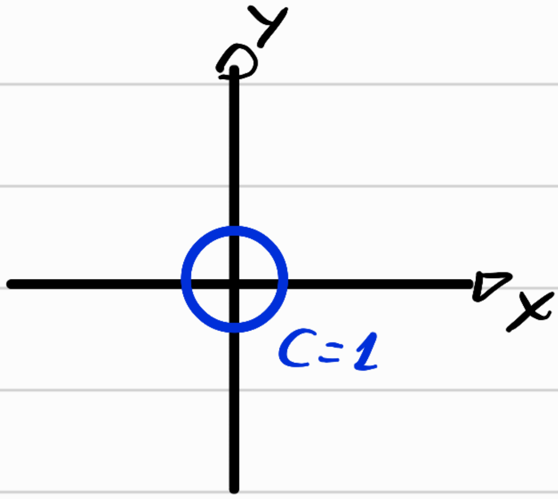
$$D = \{(x,y) \in \mathbb{R}^2 : x + y + 1 \geq 0,\ x \neq 1\}$$

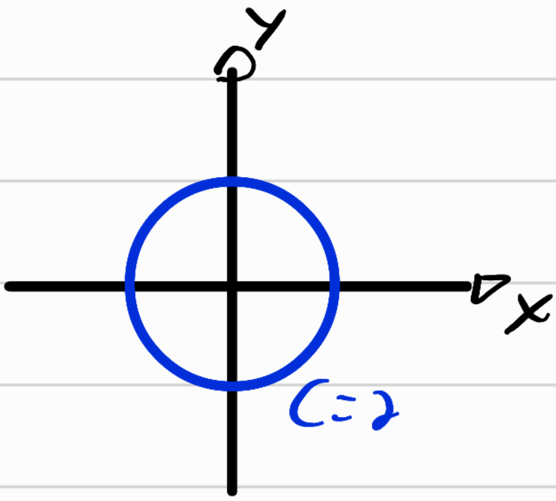
$c \in \operatorname{Im}(f) = [0, +\infty)$, $f(x,y) = c$

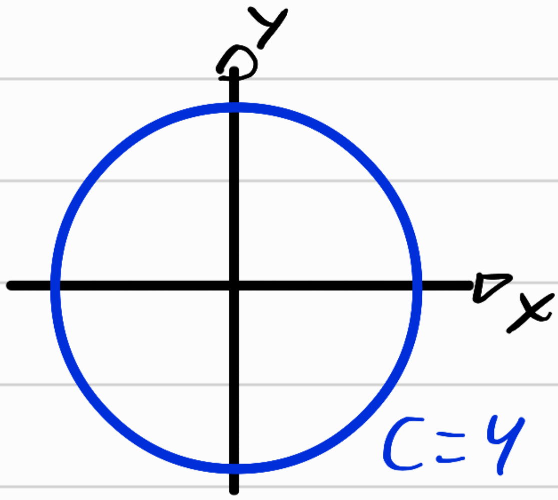
$$\iff x^2 + y^2 = c$$
$$\iff x^2 + y^2 = (\sqrt{c})^2$$

- $c=0$ $\quad \{(x,y) \in D \mid x^2+y^2=0\}$
- $c=1$ $\quad \{(x,y) \in D \mid x^2+y^2=1\}$
- $c=2$ $\quad \{(x,y) \in D \mid x^2+y^2=(\sqrt{2})^2\}$
- $c=4$ $\quad \{(x,y) \in D \mid x^2+y^2=(\sqrt{4})^2\}$









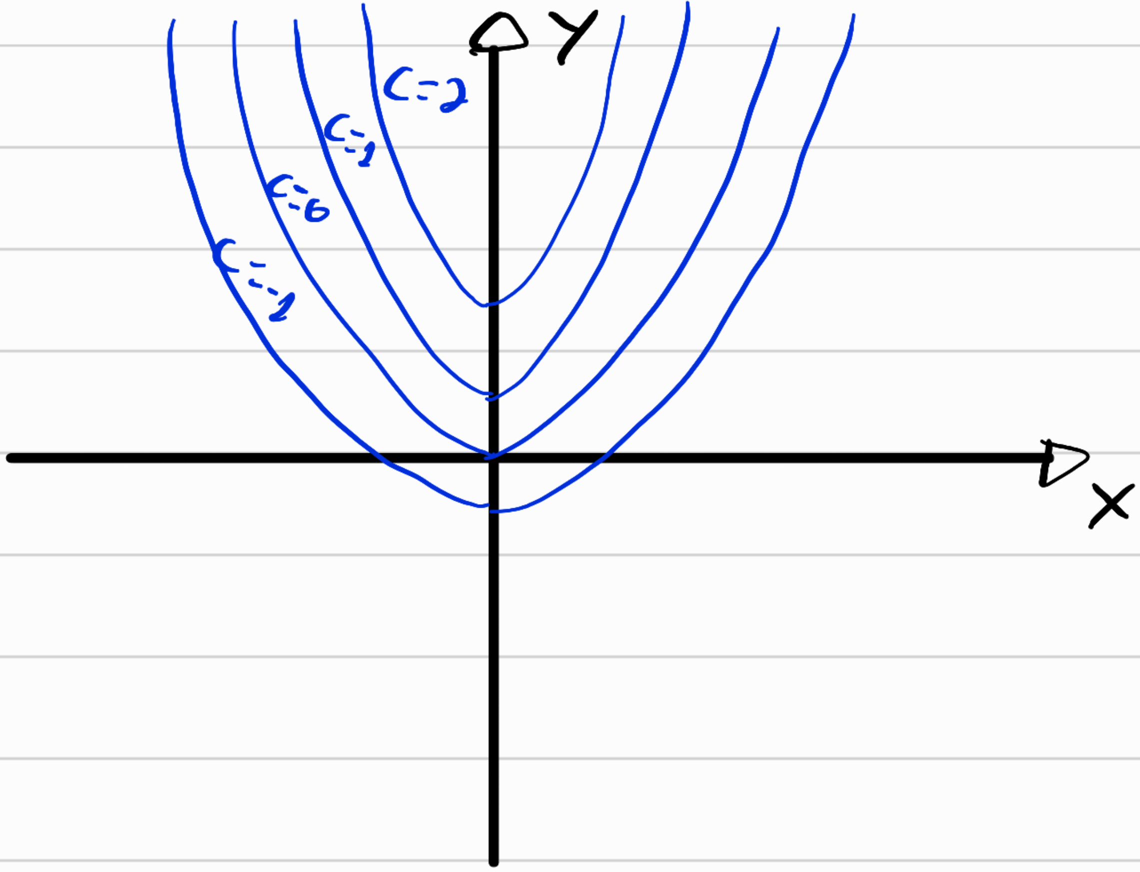
$f(x,y) = y - x^2$ $\quad$ $y=0$ $f(x,0) = -x^2 \leq 0$

$\text{Domi}(f) = \mathbb{R}^2$ $\quad$ $x=0$ $f(0,y) = y$

$\text{Im}(f) = \mathbb{R}$ $\quad$ $f(x,y) \in \mathbb{R}$

- CURVAS DE NÍVEL

$$c \in \text{Im}(f) = \mathbb{R}, \quad f(x,y) = c$$
$$\Longleftrightarrow \quad y - x^2 = c$$
$$\Longleftrightarrow \quad y = x^2 + c$$



$f: D \subset \mathbb{R}^3 \to \mathbb{R}$

$c \in Im(f)$

$\{(x,y,z) \in D \mid f(x,y,z) = c\}$

↳ "SUPERFÍCIE DE NÍVEL"

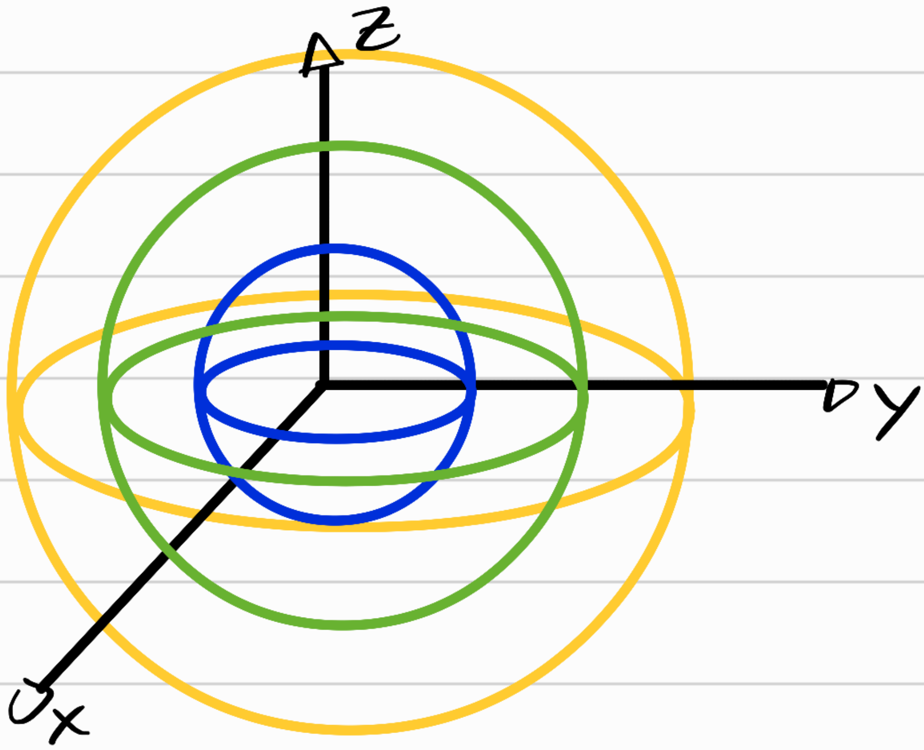
(4) $f(x,y,z) = x^2 + y^2 + z^2$

- $Dom(f) = \mathbb{R}^3$
- $Im(f) = [0, +\infty)$
- SUPERFÍCIE DE NÍVEL: $c \in Im(f) = [0, +\infty)$

$f(x,y,z)=c$

$\iff x^2+y^2+z^2=c$

$\iff x^2+y^2+z^2=(\sqrt{c})^2$



PROCEDIMENTOS PARA ESBOÇAR O GRÁFICO DE UMA FUNÇÃO

1. DOMI(f)
2. $Im(f)$
3. CURVAS DE NÍVEL
4. Oyz
5. Oxz

Ex:

① $f(x,y) = x^2 + y^2$

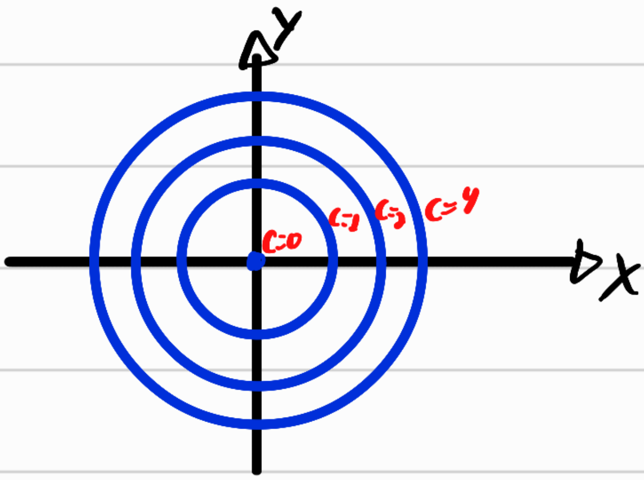
$Domi(f) \in \mathbb{R}^2$ ①

$Im(f) \in [0, +\infty[$ ②

③ $f(x,y) = c$

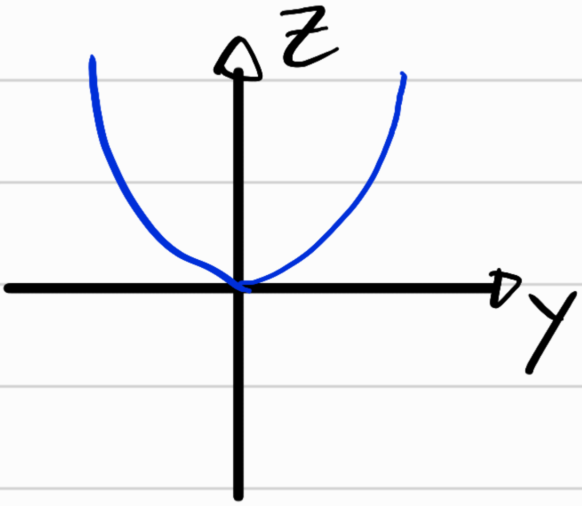
$c = x^2 + y^2$

$$\begin{cases} c = 0 & x^2 + y^2 = 0 \\ c = 1 & x^2 + y^2 = 1 \\ c = 2 & x^2 + y^2 = 2 \\ c = 4 & x^2 + y^2 = 4 \end{cases}$$



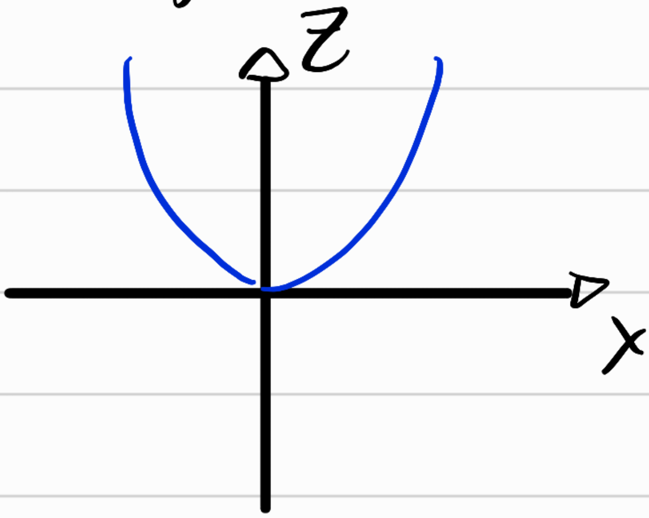
④ O_{yz} $(x=0)$

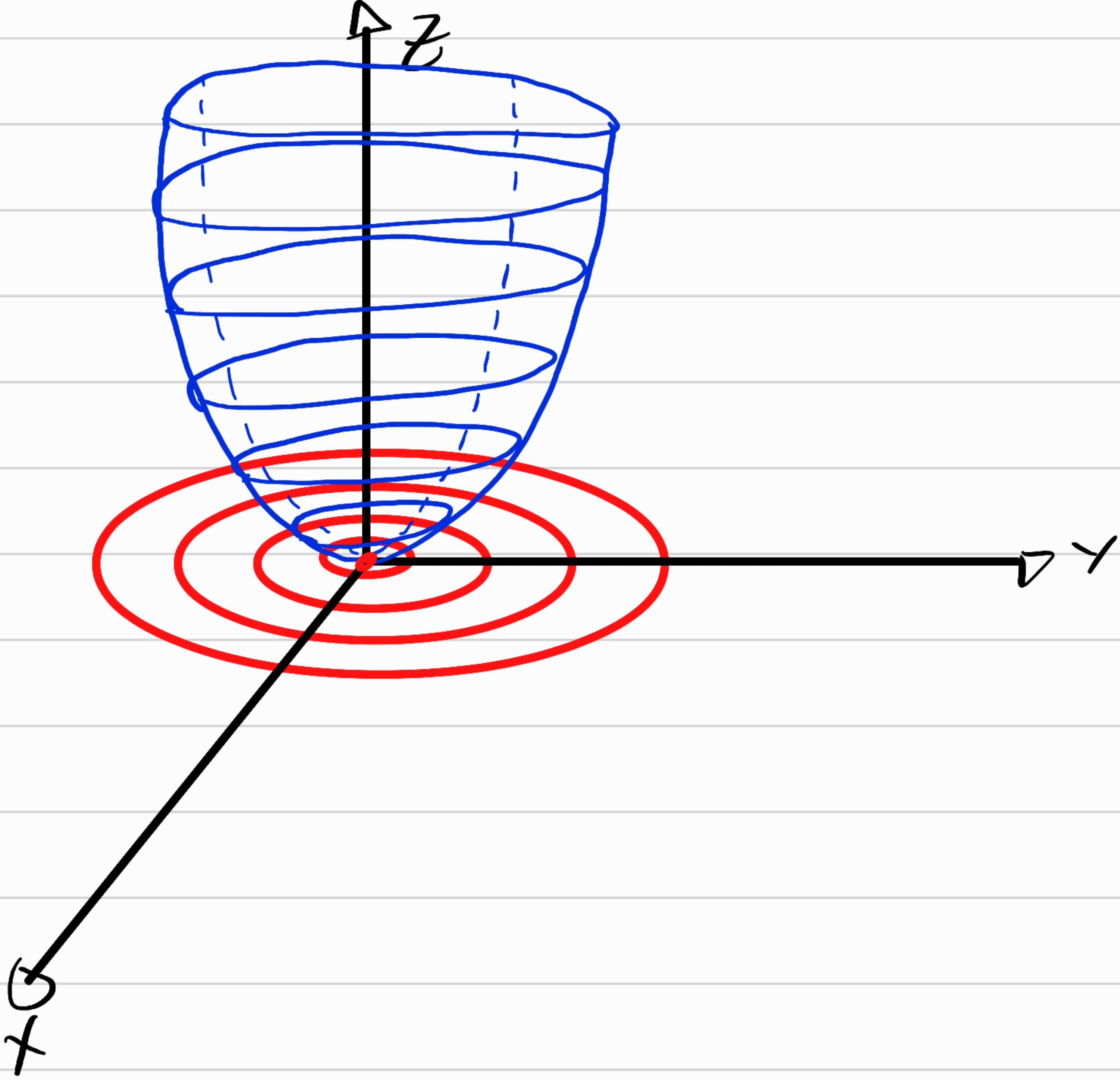
$$f(0,y)=0^2+y^2=y^2$$



⑤ O_{xz} $(y=0)$

$$f(x,0)=x^2+0^2=x^2$$





EX: ② $f(x,y) = \sqrt{x^2+y^2}$

① $Dom(f) = \mathbb{R}^2$

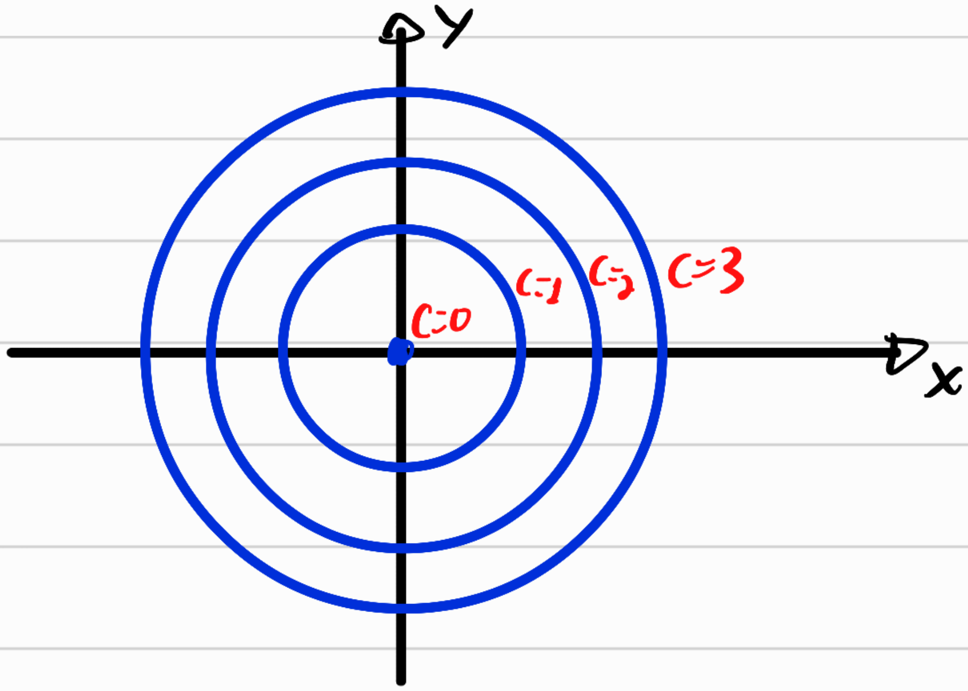
② $Im(f) = [0, +\infty[$

③ CURVAS DE NÍVEL

$c \in Im(f) = [0, +\infty[, \ f(x,y) = c$

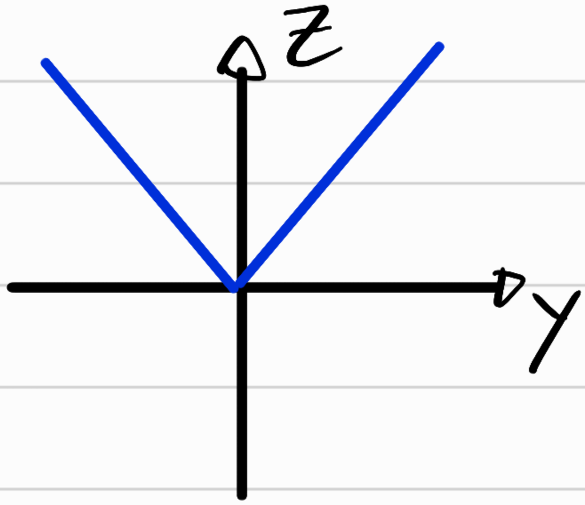
$\Leftrightarrow \sqrt{x^2+y^2} = c$

$x^2+y^2 = c^2$



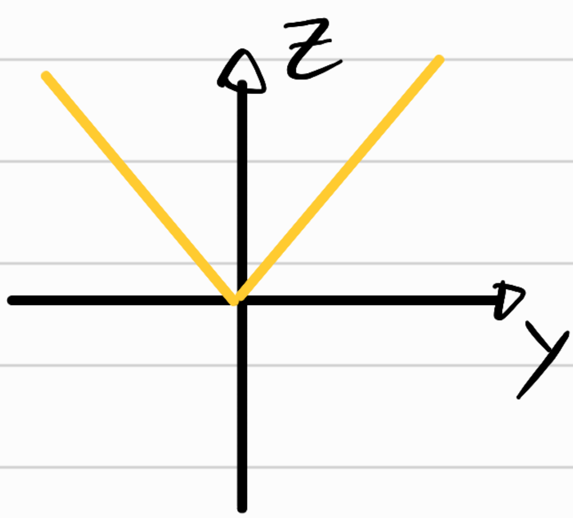
Ⓘⓥ O_{yz} $(x=0)$

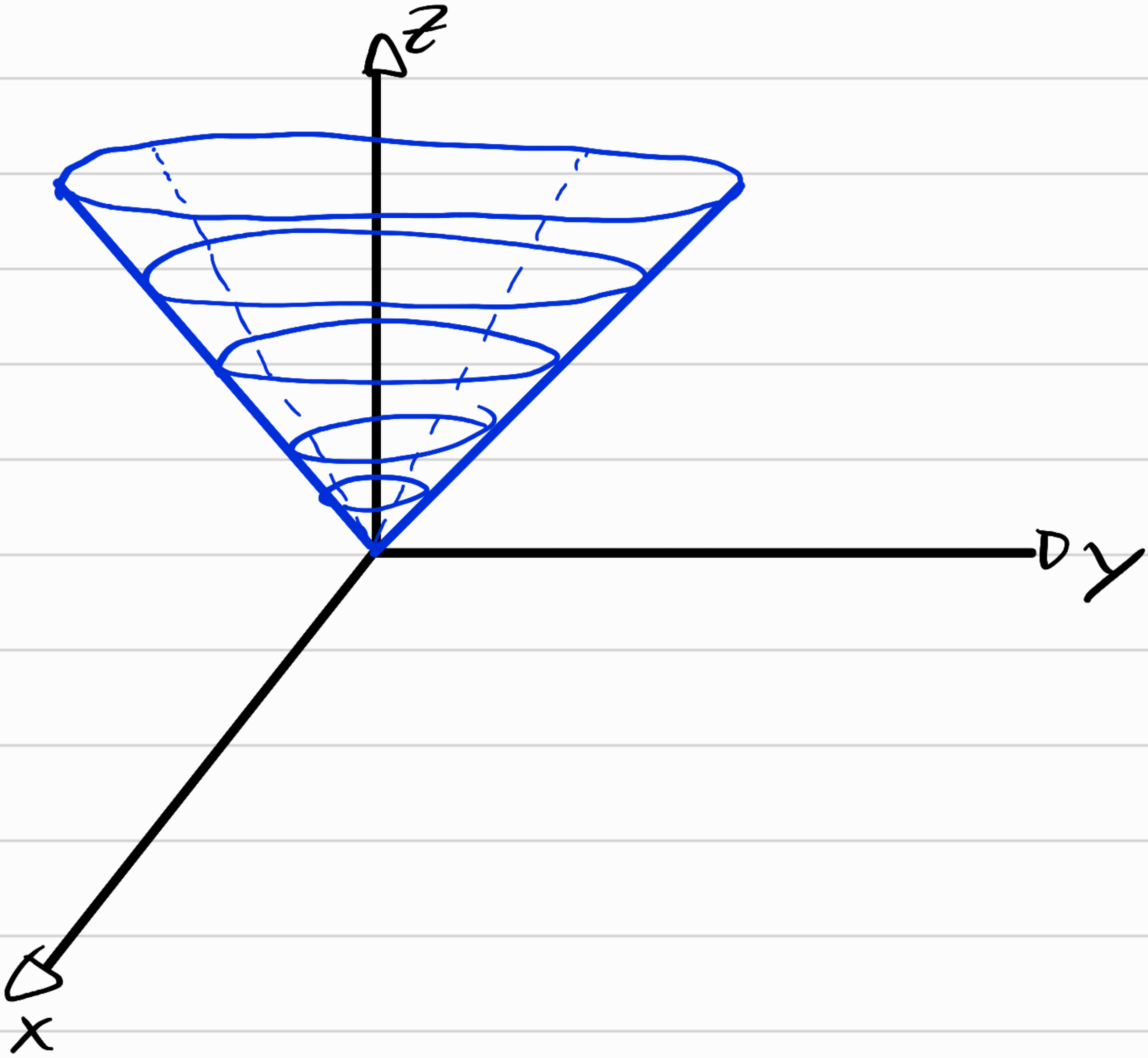
$$f(0,y) = \sqrt{0^2+y^2} = \sqrt{y^2} = |y|$$



Ⓥ O_{xz} $(y=0)$

$$f(x,0) = \sqrt{x^2+0^2} = \sqrt{x^2} = |x|$$





EX 3: ESBOÇE O GRÁFICO DE

$$f(x,y) = y^2 - x^2$$

Ⓘ $Domi(f) = \mathbb{R}^2$

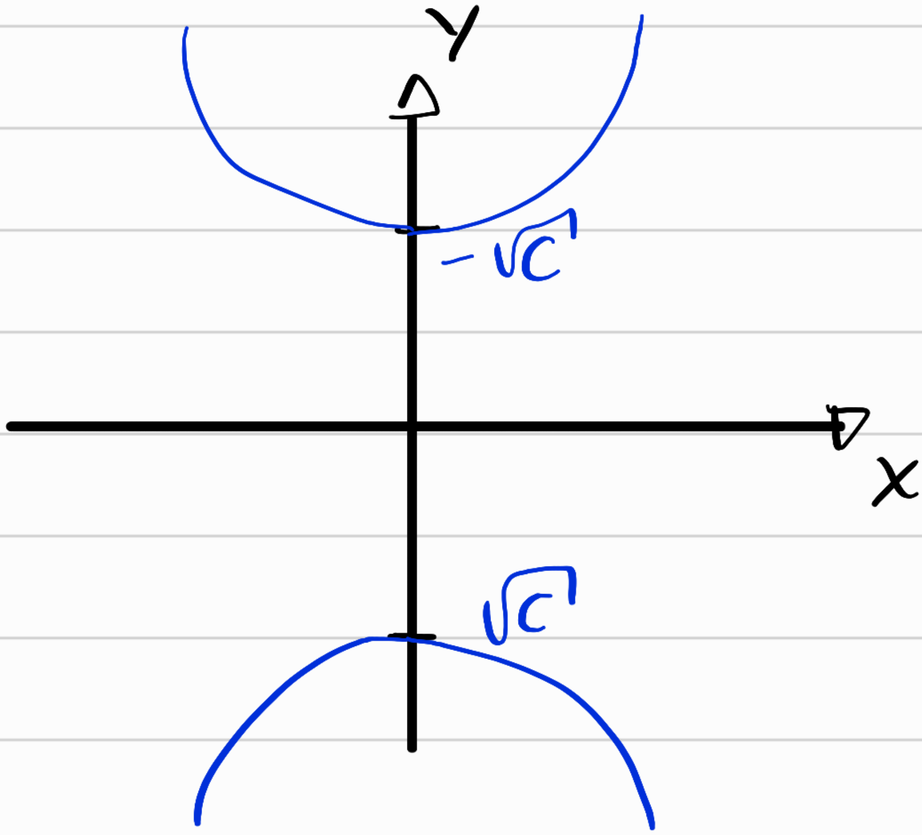
Ⓘ Ⓘ $Im(f) = \mathbb{R}$

Ⓘ Ⓘ Ⓘ $f(x,y) = c \Rightarrow y^2 - x^2 = c$

$c > 0$

$y^2 = x^2 + c$

$y = \pm\sqrt{x^2+c}$



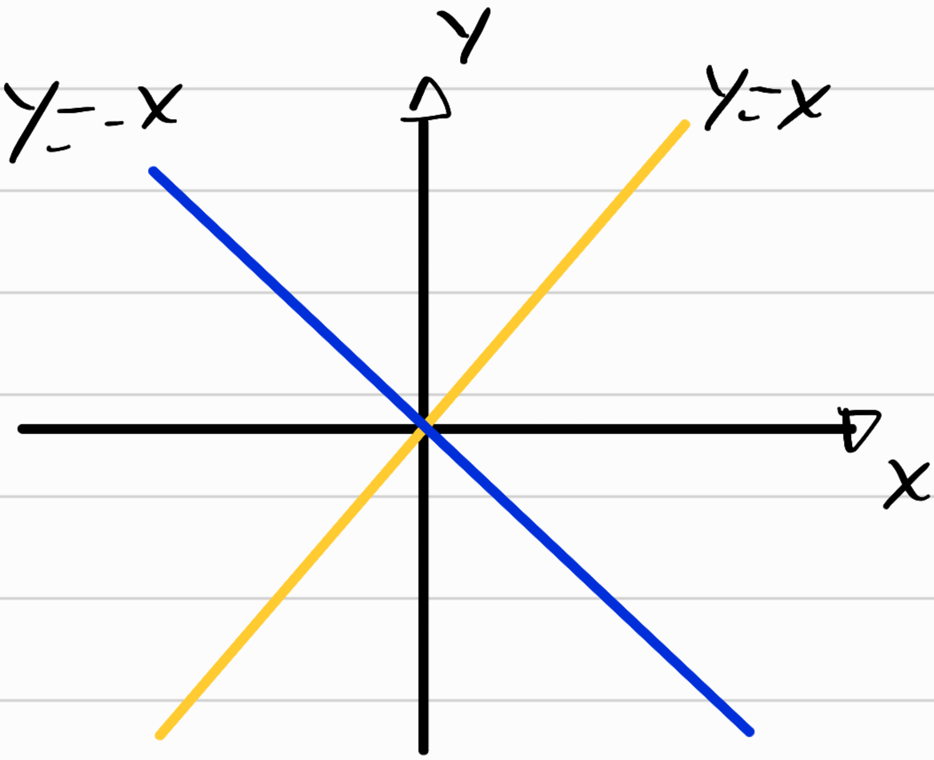
$c = 0$

$y^2 - x^2 = 0$

$y^2 = x^2$

$y = \pm\sqrt{x^2}$

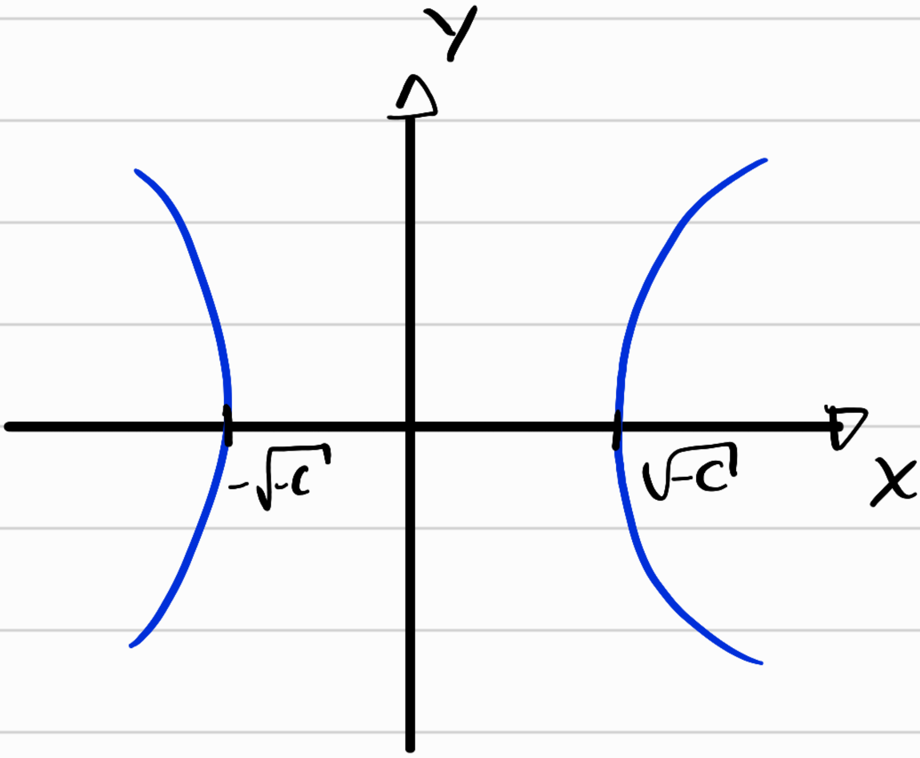
$y = \pm|x|$



$c < 0 \quad y^2 - x^2 = c$

$x^2 = y^2 - c$

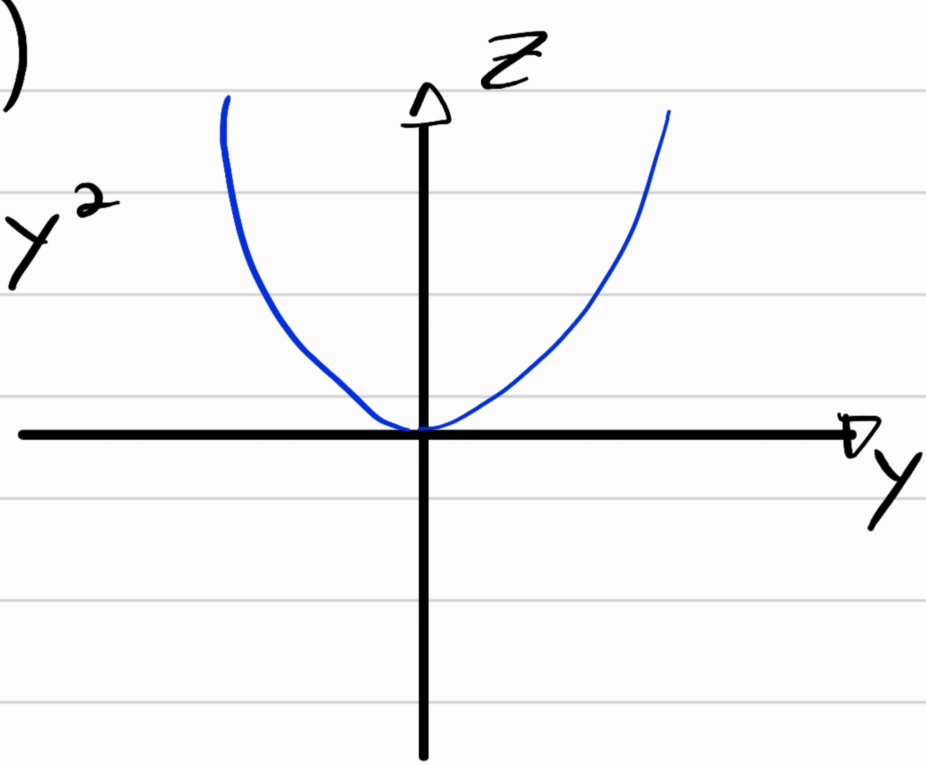
$x = \pm\sqrt{y^2 - c}$



$y = 0, \quad x = \pm\sqrt{-c}$

IV O_{yz} $(x=0)$

$f(0,y) = y^2 - 0^2 = y^2$



VI O_{xz} $(y=0)$

$f(x,0)=0^2-y^2=-y^2$