INT. DUPLA

ÁREA DE D $= \iint_D 1 \cdot dA$

VOLUME DE $G_f = \iint_D f(x,y)\, dA$

ABAIXO DE $f(x,y)$
E ACIMA DE D

SE $f(x,y)$ FOR A DENSIDADE ρ, A INTEGRAL DUPLA SERÁ A MASSA DO CORPO D.

$\rho = \frac{m}{A}$

$A \cdot \rho = m$

$m = \iint \rho \cdot dA$

$$m = \iint_D \rho(x,y)\, dA$$

MUDANÇA DE VARIÁVEIS

$$\iint_E f(x,y)\,dx\,dy = \iint_{T(E)} f(x(u,v), y(u,v)) \cdot |JAC|\,du\,dv$$

COORD. POLARES: $\begin{cases} x = r\cos(\theta) \\ y = r \cdot \operatorname{sen}(\theta) \end{cases}$

r: distância do ponto P até a origem

θ: ângulo do eixo x positivo

$$\left|\frac{d(x,y)}{d(r,\theta)}\right| = r$$

ÁREA DE SUPERFÍCIE DE G_f $= \displaystyle\iint_{D_f} \sqrt{1 + \left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2}\,dx\,dy$

INTEGRAL TRIPLA

VOLUME DE E $= \iiint_E 1 \cdot dv$

$$= \iiint_E f(x,y,z)\, dv$$

$f(x,y,z) = \rho(x,y,z)$

$V = \dfrac{m}{\rho}$

$\rho \cdot V = m$

$m = \rho(x,y,z) \cdot V$

$m\ E = \iiint_E \rho(x,y,z) \cdot dv$

COORD. CILINDRICAS

$$\begin{cases} X = r\cos(\theta) \\ Y = r\cdot sen(\theta) \\ Z = \end{cases} \qquad \left|\frac{d(x,y,z)}{d(r,\theta,z)}\right| = r$$

COORD CILINDRICAS

$$X = \rho\cdot\cos(\theta)\cdot sen(\varphi)$$
$$Y = \rho\cdot sen(\theta)\cdot sen(\varphi)$$
$$Z = \rho\cdot\cos(\varphi)$$

EX 36:

$$V = \iiint_C 1\cdot dv$$

$$= \int_0^{2\pi}\int_0^{a}\int_0^{\pi/2} 1\cdot\rho^2\cdot sen(\varphi)\,d\varphi\,d\rho\,d\theta$$

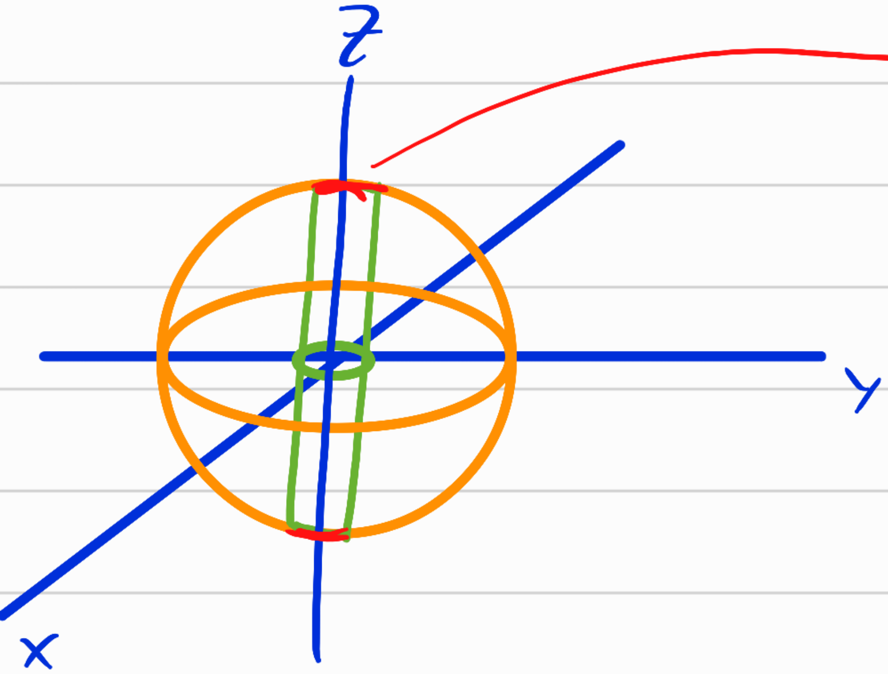
$$= 2\pi \cdot \frac{\rho^3}{3}\Big|_0^a \cdot (-\cos(\varphi))\Big|_0^{\pi/12} = 2\pi \cdot \frac{a^3}{3} \cdot (-\cos(\pi/12) + \cos(0))$$

$$= \frac{2\pi a^3}{3} \cdot \frac{7}{3} = \frac{7a^3\pi}{3}$$

22) $V = ?$

$$E = \begin{cases} x^2 + y^2 = 1 \\ x^2 + y^2 + z^2 = 4 \end{cases}$$

$V = 2 \cdot$ VOLUME ABAIXO DO GRÁFICO $z = \sqrt{4 - x^2 - y^2}$



$$V = \iiint_E 1 \cdot dV = \int_0^{2\pi}\int_0^{1}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} 1 \cdot r \cdot dz \cdot dr \cdot d\theta$$

$$= 2\pi \cdot \int_0^2 r \cdot z \Big|_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dr = 2\pi \int_0^1 r \cdot \left(2\sqrt{4-r^2}\right) dr$$

$$= 4\pi \cdot \int_0^1 \sqrt{4-r^2} \cdot r \cdot dr = -2\pi \frac{(4-r^2)^{1/2+1}}{\underbrace{1/2+1}_{3/2}} \Bigg|_0^1 = -\frac{4\pi}{3}\left(3\sqrt{3}-8\right)$$

(25) $\begin{cases} z = x^2 + y^2 \\ z = 36 - 3x^2 - 3y^2 \end{cases}$ $\begin{cases} x^2 + y^2 = 36 - 3x^2 - 3y^2 \\ \end{cases}$