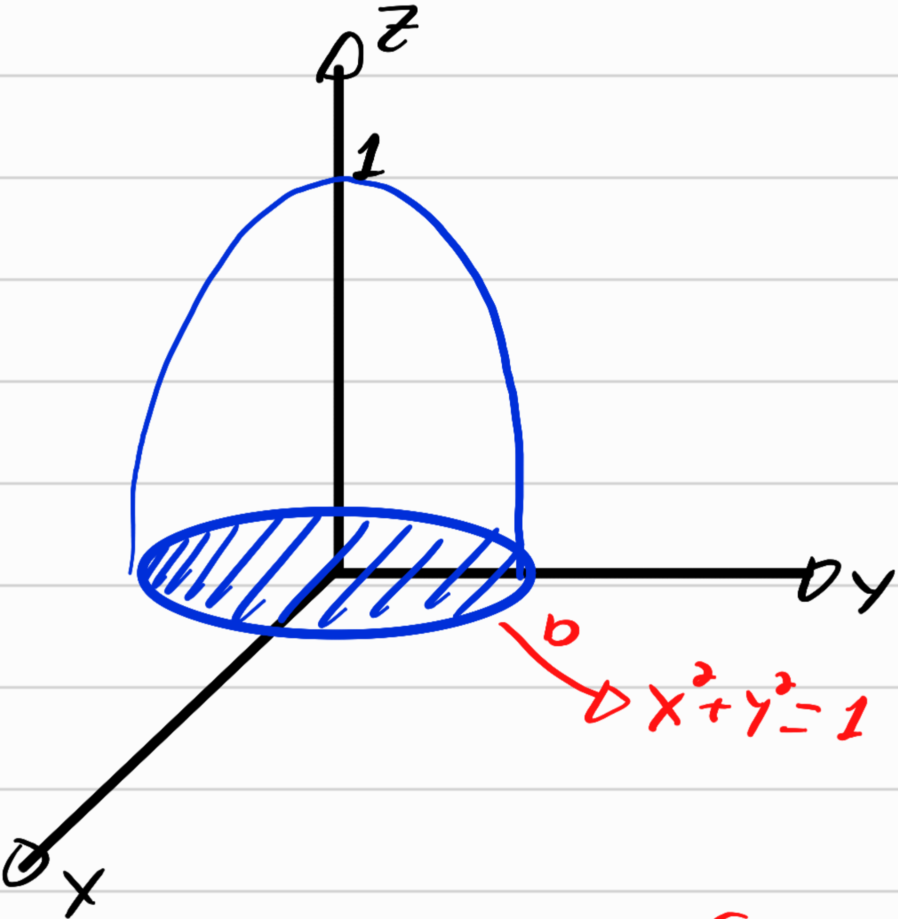
EX: DETERMINE O VOLUME DO SÓLIDO LIMITADO PELO PLANO $Z=0$ E O PARABOLOIDE $Z=1-X^2-Y^2$.



SEMPRE QUE HOUVER UMA ARENCIA CIRCULAR, USAR COORDENADAS POLARES.

$$V=\iint z\,dA$$

$$V=\iint_D 1-x^2-y^2\,dA$$

$$\begin{cases} x=r\cdot\cos(\theta) \\ y=r\cdot\operatorname{sen}(\theta) \end{cases}$$

$$V=\int_0^1\int_0^{2\pi} 1-(r\cdot\cos(\theta))^2-(r\cdot\operatorname{sen}(\theta))^2\cdot \overset{JAC}{r}\,d\theta\,dr$$

$$V=\int_0^1\int_0^{2\pi} 1-r^2\cdot\cos^2(\theta)-r^2\operatorname{sen}^2(\theta)\cdot r\,d\theta\,dr$$

$$V=\int_0^1\int_0^{2\pi} 1-\left(r^2\cdot\cos^2(\theta)+r^2\operatorname{sen}^2(\theta)\right)\cdot r\,d\theta\,dr$$

$$V=\int_0^1\int_0^{2\pi} 1-\left(r^2\cdot(\cos^2(\theta)+\operatorname{sen}^2(\theta)\right))\cdot r\,d\theta\,dr$$

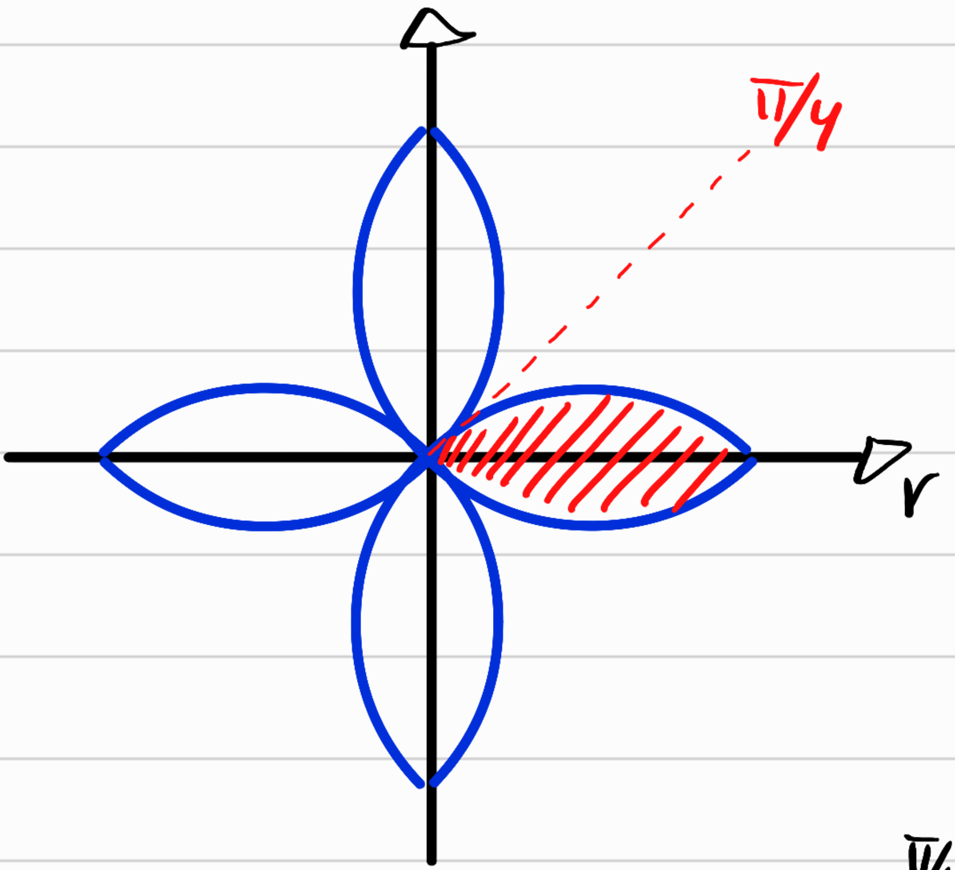
$$V=\int_0^1\int_0^{2\pi} 1-\left(r^2\cdot 1\right)\cdot r\,d\theta\,dr$$

$$V=\int_0^1\int_0^{2\pi} (1-r^2)\cdot r\,d\theta\,dr$$

$$V=\int_0^1\int_0^{2\pi} r-r^3\,d\theta\,dr=\int_0^{2\pi} \frac{r^2}{2}-\frac{r^4}{4}\Big|_0^1\,d\theta$$

$$V = \int_0^{2\pi} \frac{1^2}{2} - \frac{1^4}{4}\, d\theta = \int_0^{2\pi} \frac{1}{4}\, d\theta = \frac{2\pi}{4} = \frac{\pi}{2}$$

Ex: $r = \cos(2\theta)$



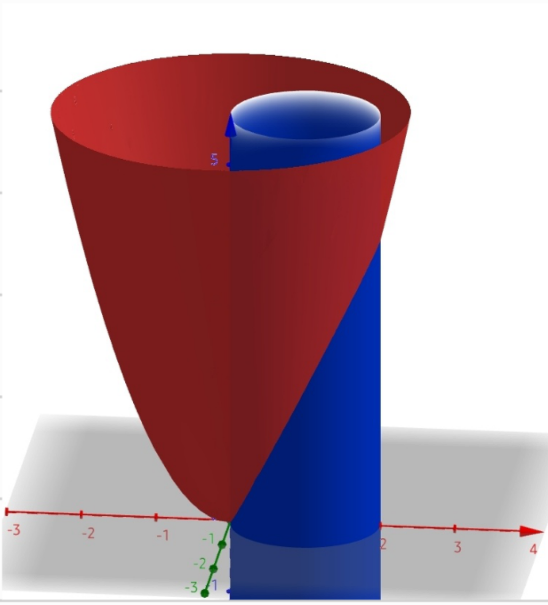
$$\text{AREA} = \iint_D 1 \cdot dA = \iint_R 1 \cdot r \cdot d\theta\, dr = \int_0^{\pi/4}\int_0^{\cos(2\theta)} 1 \cdot r \cdot d\theta\, dr$$

$$= \int_0^{\pi/4} \frac{r^2}{2}\Big|_0^{\cos(2\theta)} d\theta = \frac{1}{2}\int_0^{\pi/4} \cos^2(2\theta)\, d\theta = \frac{1}{2}\int_0^{\pi/4} \frac{1}{2} + \frac{1}{2}\cos(4\theta)\, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/4} 1+\cos(4\theta)\,d\theta = \frac{1}{4}\cdot\left(\theta+\frac{sen(4\theta)}{4}\right)\Bigg|_0^{\pi/4}$$

$$= \frac{1}{4}\cdot\left[\left(\frac{\pi}{4}+0\right)-(0+0)\right] = \frac{\pi}{16} =$$

$$A_T = \frac{\pi}{8}$$



Ex: $z = x^2+y^2$

$z = 0$

$x^2+y^2 = 2x$

$x^2-2x+1+y^2 = 1$

$(x-1)^2+(y-0)^2 = 1$

$$V = \iint_C x^2+y^2\,dx\,dy = \int_{-\pi/2}^{\pi/2}\int_0^{2\cos(\theta)} r^2 r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{r^4}{4}\Bigg|_0^{2\cos(\theta)} d\theta =$$