REGRA DA CADEIA

CASO 1: $f(x,y)$ e $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$

$$f(x(t), y(t)) \Rightarrow \frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$

EX: $f(x,y) = x^2y + 3xy^4$ e

$x = sen(2t)$ e $y = cos(t)$

CALCULE $\frac{df}{dt}$ PARA $t=0$

$$\frac{df}{dt} = (2xy + 3y^4) \cdot (cos(2t) \cdot 2) + (x^2 + 12xy^3) \cdot (sen(t))$$

Ex: $PV = n \cdot R \cdot T$

$$PV = 8{,}31 \cdot T \rightarrow \boxed{P = 8{,}31 \cdot \frac{T}{V}} = P(T,V) = P(T(t),V(t))$$

$$\frac{dP}{dt} = ?$$

$T = 300\,K$ $\quad V = 100\,\ell$

$\frac{dT}{dt} = 0{,}1\ K/h$ $\quad \frac{dV}{dt} = 0{,}2\ \ell/h$

$$\frac{dP}{dt} = 8{,}31 \cdot \left(\frac{dP}{dT} \cdot \frac{dT}{dt} + \frac{dP}{dV} \cdot \frac{dV}{dt} \right)$$

$$\frac{dP}{dt} = 8{,}31 \cdot \left(\frac{1}{V} \cdot \frac{dT}{dt} + T \cdot \left(-\frac{1}{V^2} \right) \frac{dV}{dt} \right)$$

$$\frac{dP}{dt} = 8{,}31 \cdot \left(\frac{1}{100} \cdot 0{,}1 - \frac{300}{100^2} \cdot 0{,}2 \right)$$

$$\frac{dP}{dt} = -0{,}04155$$

CASO 2: $f(x,y)$ com $x = x(s,t)$
$y = y(s,t)$

$$\frac{df}{ds} = \frac{df}{dx}\cdot\frac{dx}{ds} + \frac{df}{dy}\cdot\frac{dy}{ds}$$

$$\frac{df}{dt} = \frac{df}{dx}\cdot\frac{dx}{dt} + \frac{df}{dy}\cdot\frac{dy}{dt}$$

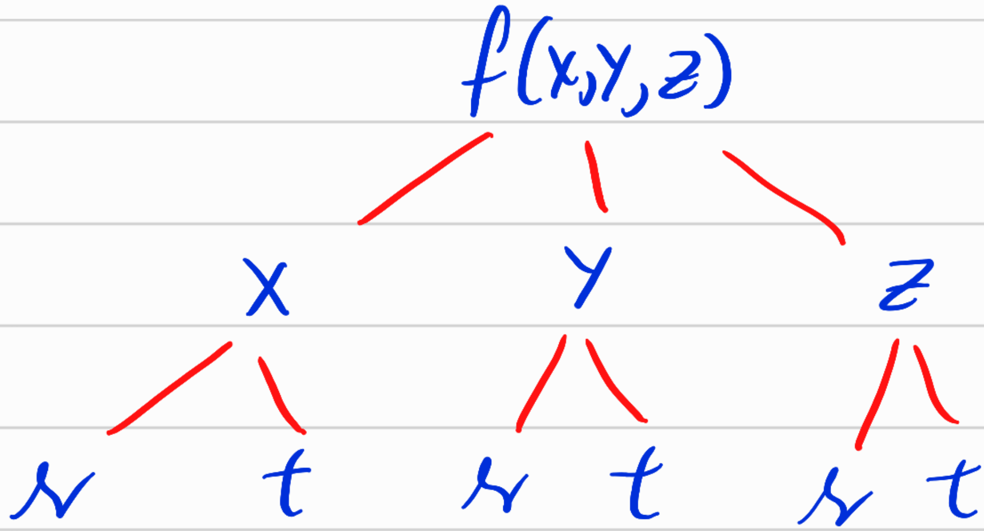
Ex: $f(x,y) = e^{x}\cdot sen(y)$ $x = st^2$
$y = s^2t$

$$\frac{df}{ds} = \frac{df}{dx}\cdot\frac{dx}{ds} + \frac{df}{dy}\cdot\frac{dy}{ds}$$

$$= e^{x}\cdot sen(y)\cdot t^2 + e^{x}\cdot cos(y)\cdot 2s(t)$$
$$= t\cdot e^{st^2}(t\cdot sen(s^2t) + 2s\,cos(s^2t))$$

$$\frac{df}{dt} = e^{x}\cdot sen(y)\cdot 2st + e^{x}cos(y)\cdot s^2$$

$$= s e^{st^2}\cdot(2t\cdot sen(s^2t) + s\cdot cos(s^2t))$$



$$\frac{df}{dr} = \frac{df}{dx}\cdot\frac{dx}{dr} + \frac{df}{dy}\cdot\frac{dy}{dr} + \frac{df}{dz}\cdot\frac{dz}{dr}$$

$$\frac{df}{dt} = \frac{df}{dx}\cdot\frac{dx}{dt} + \frac{df}{dy}\cdot\frac{dy}{dt} + \frac{df}{dz}\cdot\frac{dz}{dt}$$

CASO GERAL: $f(x_1, \dots, x_n)$ e $x_i = x_i(t_1, \dots, t_n)$
$i = 1, \dots, n$

$$\frac{df}{dt_i} = \frac{df}{dx_1}\cdot\frac{dx_1}{dt_i} + \frac{df}{dx_2}\cdot\frac{dx_2}{dt_i} + \dots + \frac{df}{dx_n}\cdot\frac{dx_n}{dt_i}$$

EX: $f(x,y,z,t) = w$

$x(u,v)$; $y(u,v)$; $z(u,v)$; $t(u,v)$

$$\frac{df}{du} = f_x \cdot X_u + f_y \cdot Y_u + f_z \cdot z_u + f_t \cdot t_u$$

$$\frac{df}{dv} = f_x \cdot X_v + f_y \cdot Y_v + f_z \cdot z_v + f_t \cdot t_v$$

Ex: $u = x^4 y + y^2 z^3$

$x = r s e^t$; $y = r s^2 e^{-t}$; $z = r^2 \cdot s \,\text{sen}(t)$

$\frac{du}{ds} = ?$ $r = 2$, $s = 1$, $t = 0$,

$$\frac{du}{ds} = \frac{du}{dx} \cdot \frac{dx}{ds} + \frac{du}{dy} \cdot \frac{dy}{ds} + \frac{du}{z} \cdot \frac{dz}{ds}$$

$$\frac{du}{ds} = (4x^3 y + y^2 z^3) \cdot (r e^t) +$$

$$+ (x^4 + 2yz^3) \cdot (2 r s e^{-t}) +$$

$$+ (3y^2 \cdot z^2) \cdot (r^2 \cdot \text{sen}(t))$$

$$X(r,s,t) = rse^{t}$$

$$X(2,1,0) = 2\cdot 1\cdot e^{0} = 2\cdot 1\cdot 1 = 2 \qquad \boxed{X=2}$$

$$Y(r,s,t) = rs^{2}e^{-t}$$

$$Y(2,1,0) = 2\cdot 1^{2}\cdot e^{-0} = 2\cdot 1\cdot 1 = 2 \qquad \boxed{Y=2}$$

$$Z(r,s,t) = r^{2}\cdot s\cdot \operatorname{sen}(t)$$

$$Z(2,1,0) = 2^{2}\cdot 1\cdot \operatorname{sen}(0) = 4\cdot 1\cdot 0 = 0 \qquad \boxed{Z=0}$$

$$\frac{du}{ds} = (4\cdot 2^{3}\cdot 2 + 2^{2}\cdot 0^{3})\cdot(2\cdot e^{0}) + (2^{4} + 2\cdot 2\cdot 0^{3})\cdot(2\cdot 2\cdot 1\cdot e^{-0})$$

$$+ (3\cdot 2^{2}\cdot 0^{2})\cdot(2^{2}\cdot \operatorname{sen}(0))$$

$$\frac{du}{ds} = 64\cdot 2 + 16\cdot 4 + 0\cdot 0$$

$$= 128 + 64 + 00$$

$$= \boxed{192}$$

EX: $g(r,t) = f(\underbrace{r^2 - t^2}_{x}, \underbrace{t^2 - r^2}_{y})$

MOSTRE QUE $t \cdot \frac{dg}{dr} + r \cdot \frac{dg}{dt} = 0$

?

$$\frac{dg}{dr} = \frac{df}{dx} \cdot \frac{dx}{dr} + \frac{df}{dy} \cdot \frac{dy}{dr}$$

$$= \frac{df}{dx} \cdot 2r + \frac{df}{dy} \cdot (-2r)$$

$$\frac{dg}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$

$$= \frac{df}{dx} \cdot (-2t) + \frac{df}{dy} \cdot (2t)$$

LOGO $t \cdot \frac{dg}{dr} = \cancel{2rt \cdot \frac{df}{dx}} - \cancel{2rt \frac{df}{dy}}$

$+$

$r \cdot \frac{dg}{dt} = \cancel{-2rt \cdot \frac{df}{dx}} + \cancel{2rt \frac{df}{dy}}$

$0 = 0$

Ex: $z = f(x, y)$, $x = r^2 + s^2$ $y = 2rs$

$$\frac{dz}{dr} = ? \quad \frac{d^2z}{dr^2} = ?$$

$$\frac{dz}{dr} = \frac{dz}{dx} \cdot \frac{dx}{dr} + \frac{dz}{dy} \cdot \frac{dy}{dr}$$

$$= \frac{dz}{dx} \cdot 2r + \frac{dz}{dy} \cdot 2s$$

$$\frac{d^2z}{dr^2} = \frac{d}{dr}\left(\frac{dz}{dr}\right)$$

$$= \frac{d}{dr}\left(\frac{dz}{dx}\right) 2r + \frac{dz}{dx} \cdot \frac{d}{dr} \cdot (2r) + 2s \cdot \frac{d}{dr}\left(\frac{dz}{dr}\right)$$

$g(x, y)$

$$= \left[\frac{d}{dx}\left(\frac{dz}{dx}\right) \cdot \frac{dx}{dr} + \frac{d}{dy}\left(\frac{dz}{dx}\right) \cdot \frac{dy}{dy}\right] 2r + 2 \cdot \frac{dz}{dx} +$$

$$+ 2s\left(\frac{d}{dx}\left(\frac{dz}{dy}\right)\frac{dx}{dr} + \frac{d}{dy}\left(d\right.\right.$$

FUNÇÃO IMPLICITA

$y = f(x)$ DADO POR $F(x, \overbrace{f(x)}^{y}) = 0$

PELA REGRA DA CADEIA:

$$\frac{dF}{dx} = \frac{dF}{dx} \cdot \cancelto{1}{\frac{dx}{dx}} + \frac{dF}{dy} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-F_x}{F_y}$$

TEOREMA DA FUNÇÃO IMPLICITA

$z = f(x,y)$ DADO POR

$F(x,y,z) = 0$ $\quad \frac{dz}{dx} = ? \longrightarrow \frac{dz}{dx} = \frac{-F_y}{F_z}$

$$\frac{dz}{dy} = \frac{-F_y}{F_z}$$

$$\frac{dF}{dx} \cdot \cancelto{1}{\frac{dx}{dx}} + \frac{dF}{dy} \cdot \cancelto{0}{\frac{dy}{dx}} + \frac{dF}{dz} \cdot \frac{dz}{dx} =$$

EX: DETERMINE $\frac{dz}{dx}$ e $\frac{dz}{dy}$, EM QUE

$$\underbrace{x^3+y^3+z^3-6xyz}_{f(x,y,z)}=0$$

$$\frac{dz}{dx}=\frac{-F_x}{F_z}=\frac{-(3x^2-6yz)}{3z^2-6xy}=\frac{-3x^2+6yz}{3z^2-6xy}$$

$$\frac{dz}{dy}=\frac{-F_y}{F_z}=\frac{-(3y^2-6xz)}{3z^2-6xy}$$