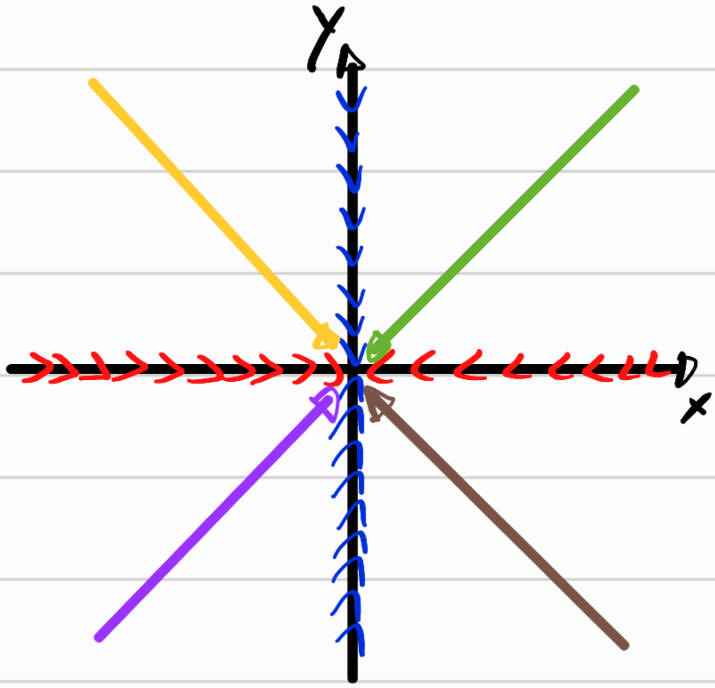
LIMITES COM 2 VARIÁVEIS

- $f: D \subset \mathbb{R}^2 \to \mathbb{R}$



$$\exists \lim_{x \to x_0} f(x) ?$$

$$\left.\begin{array}{l} \lim_{x \to x_0} f(x) = 1 \\ \lim_{x \to x_0} f(x) = 1 \\ \lim_{x \to x_0} f(x) = 2 \end{array}\right\} \lim_{x \to x_0} f(x,y) \nexists$$

Ex: $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$, $\nexists \lim_{(x,y) \to (0,0)} f(x)$

SOLUÇÃO:

$$D = \{(x,y) \in \mathbb{R}^2 : (x,y) \neq (0,0)\}$$

POR X: $\gamma(t)=(\underset{x}{t},\underset{y}{0})$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \lim_{t\to 0} \frac{t^2-0^2}{t^2+0^2}$$

$$= \lim_{t\to 0} \frac{t^2}{t^2} = 1$$

POR Y: $\gamma(t)=(0,t)$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \lim_{t\to 0} \frac{0^2-t^2}{0^2+t^2} = \lim_{t\to 0} \frac{-t^2}{t^2} = -1$$

$$\boxed{1 \neq -1} \Rightarrow \nexists \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

EX: $f(x,y)=\dfrac{2xy}{x^2+y^2}$

POR X: $\gamma(t)=(t,0)$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{t\to 0} \frac{2\cdot t\cdot 0}{t^2+0} = \lim_{t\to 0} \frac{0}{t^2}$$

$$= \lim_{t\to 0} 0 = 0$$

Por y: $\gamma(t) = (0, t)$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{t\to 0} \frac{2\cdot 0\cdot t}{0^2+t^2} = \lim_{t\to 0} \frac{0}{t^2} =$$

$$= \lim_{t\to 0} 0 = 0$$

POR UMA RETA $x = y$ $\quad \gamma(t) = (t, t)$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{t\to 0} \frac{2\cdot t^2}{t^2+t^2} = \frac{2t^2}{2t^2} = 1$$

O LIMITE NÃO EXISTE

Por $\gamma(t)=(t,-t)$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{t\to 0} \frac{2\cdot t\cdot(-t)}{t^2+(-t)^2}$$

$$= \lim_{t\to 0} \frac{-2t^2}{2t^2} = -1$$

Por $\gamma(t)=(t,mt)$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{t\to 0} \frac{2\cdot t\cdot(m t)}{t^2+(m t)^2}$$

$$= \lim_{t\to 0} \frac{2\cdot m\cdot t^2}{t^2+m^2\cdot t^2} = \lim_{t\to 0} \frac{2mt^2}{t^2\cdot(1+m)^2} =$$

$$= \lim_{t\to 0} \frac{2m}{1+m^2} = \frac{2m}{1+m^2}$$

TEOREMA DO SANDUÍCHE

$$f(x) \leq h(x) \leq g(x)$$

$$\lim_{x \to x_0} f(x) = L \qquad \lim_{x \to x_0} g(x) = L$$

$$\therefore$$

$$\lim_{x \to x_0} f(x) = L$$

f É LIMITADA SE $\exists\, M > 0$ TAL QUE
$|f(x)| \in M \quad \forall\, x \in D$

SEJAM $f, g \; D \subset \mathbb{R}^n \to \mathbb{R}$ FUNÇÕES DE V.V, $x_0 \in \mathbb{R}^n$.

Se f é LIMITADA e $\lim_{x \to x_0} g(x) = 0$

$$\Rightarrow \lim_{x \to x_0} f(x) \cdot g(x) = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \cdot y}{\sqrt{x^2+y^2}}$$