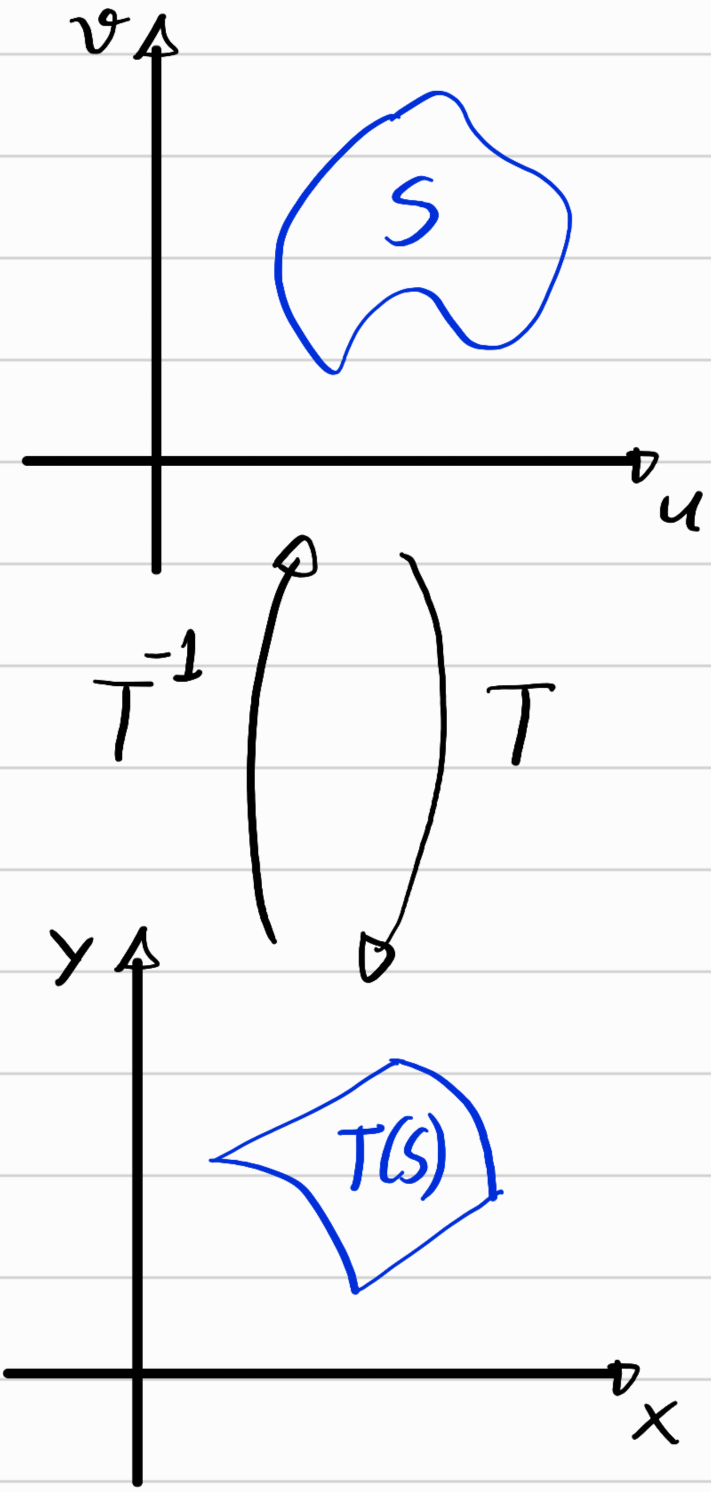
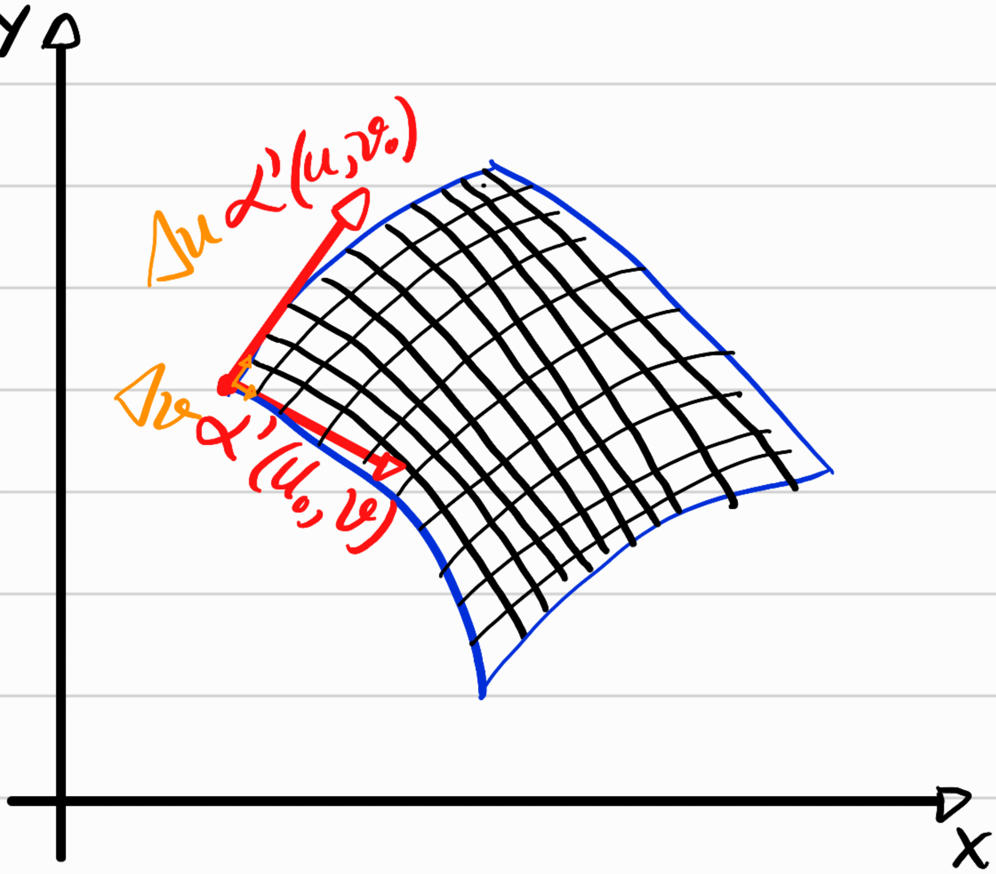
MUDANÇA DE VARIÁVEIS NA INTEGRAL DUPLA

TRANFORMAÇÕES EM $\mathbb{R}^2$

$T \in \mathcal{C}^1$ (T É CONTÍNUA COM PARCIAIS CONTÍNUA)





$$\Delta A = \left|\Delta u \cdot \frac{d\alpha}{du} \times \Delta v \cdot \frac{d\alpha}{dv}\right| = \left|\frac{d\alpha}{du} \times \frac{d\alpha}{dv}\right| \Delta u \cdot \Delta v$$

$$\alpha(u,v) = (X(u,v), Y(u,v))$$

$$\frac{d\alpha}{du} = \left(\frac{dx}{du}, \frac{dy}{du}\right)$$

$$\frac{d\alpha}{dv} = \left(\frac{dx}{dv}, \frac{dy}{dv}\right)$$

$$\left|\frac{d\sigma}{du} \times \frac{d\sigma}{dv}\right| = \left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{dx}{du} & \frac{dy}{du} & 0 \\ \frac{dx}{dv} & \frac{dy}{dv} & 0 \end{matrix}\right| \begin{matrix} \hat{i} & \hat{j} \\ \frac{dx}{du} & \frac{dy}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} \end{matrix}$$

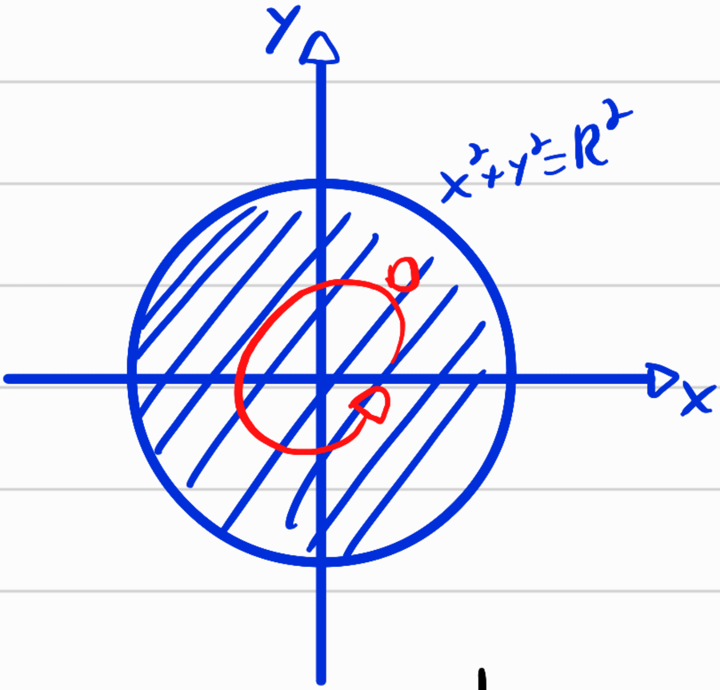
$$\left\| \begin{vmatrix} \frac{dx}{du} & \frac{dy}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} \end{vmatrix} \cdot \hat{k} \right\| = \left|\frac{d(x,y)}{d(u,v)}\right| : \text{JACOBIANO}$$

TEOREMA: $S = T(R)$, $T \in \mathcal{C}^1$

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(x(u,v), y(u,v)) \cdot \left|\frac{d(x,y)}{d(u,v)}\right| du\,dv$$

TRANSFORMAÇÕES EM COORDENADAS POLARES

$$\begin{cases} x = r \cdot \cos(\theta) \\ y = r \cdot \operatorname{sen}(\theta) \end{cases}$$



$0 \leq \theta \leq 2\pi$

$0 \leq r \leq R$

$$\begin{cases} x = r \cdot \cos(\theta) \\ y = r \cdot sen(\theta) \end{cases}$$

$$\left|\frac{d(x,y)}{d(r,\theta)}\right| = \begin{vmatrix} \frac{dx}{du} & \frac{dy}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} \cos(\theta) & -r sen(\theta) \\ sen(\theta) & r\cos(\theta) \end{vmatrix}$$

$$= r\cos^2(\theta) - (-r\, sen(\theta))$$

$$= r\cos^2(\theta) + r\, sen(\theta)$$

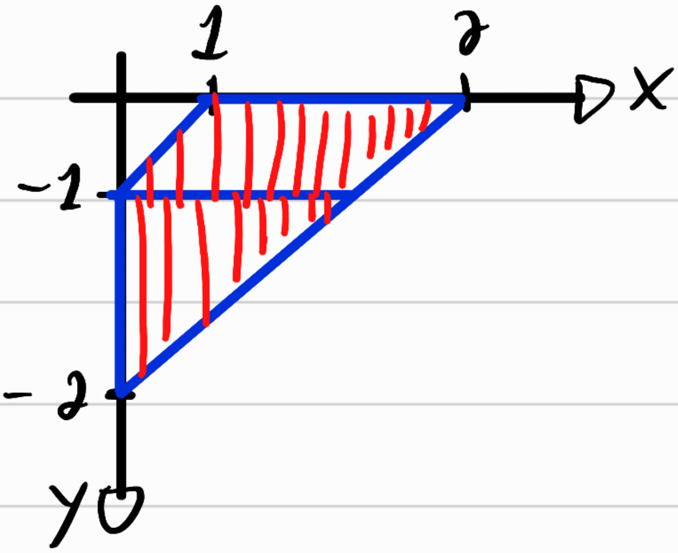
$$= r \cdot (\cos^2(\theta) + sen(\theta))$$

$$= r \cdot (1)$$

$$= r$$

$$\iint_D f(x,y)\, dx\, dy = \iint_R f(\overset{x}{r\cos(\theta)}, \overset{y}{r sen(\theta)})\, r \cdot dr \cdot d\theta$$

Ex: $\iint_R e^{\frac{x+y}{x-y}}\,dx\,dy$



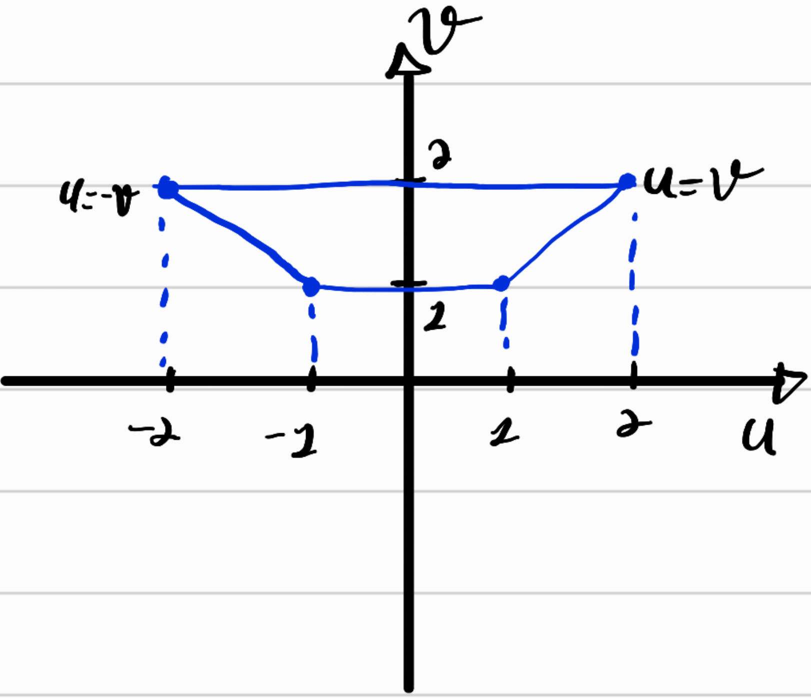
$\begin{cases} u = x+y \\ v = x-y \end{cases}$

SOMA $\rightarrow x = \dfrac{u+v}{2}$

SUBTRAI $\rightarrow y = \dfrac{u-v}{2}$

$$\left|\frac{d(x,y)}{d(u,v)}\right| = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2} \Rightarrow \left|-\frac{1}{2}\right| = \frac{1}{2}$$

$$\int_1^2 \iint e^{\frac{u}{v}} \cdot \frac{1}{2}\, du \cdot dv$$



x	y
0	-1
1	0
2	0
0	-2

$$\left.\begin{aligned} u &= 0+(-1) = -1 \\ v &= 0-(-1) = 1 \end{aligned}\right\} (-1,1)$$

$u=-v$

$$\left.\begin{aligned} u &= 1+0 = 1 \\ v &= 1-0 = 1 \end{aligned}\right\} (1,1)$$

$$\left.\begin{aligned} u &= 2+0 = 2 \\ u &= 2-0 = 2 \end{aligned}\right\} (2,2)$$

$$\left.\begin{aligned} u &= 0+(-2) = -2 \\ u &= 0-(-2) = 2 \end{aligned}\right\} (-2,2)$$

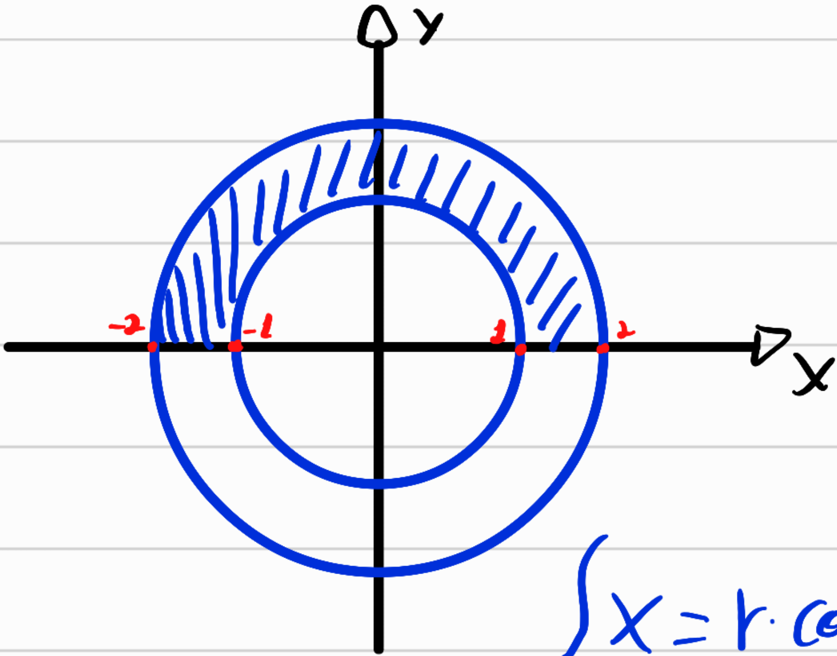
$$\int_{1}^{2}\int_{-v}^{v} e^{\frac{u}{v}} \cdot \frac{1}{2}\, du \cdot dv = \frac{1}{2}\int_{1}^{2} e^{\frac{u}{v}} \Big|_{-v}^{v} dv$$

$$= \frac{1}{2} \cdot \int v \cdot (e - e^{-1})\, dv$$

$$= \frac{1}{2} \cdot (e - e^{-1}) \cdot \frac{v^2}{2} \Big|_{1}^{2} = \frac{1}{2}(e - e^{-1})\left(2 - \frac{1}{2}\right)$$

$$= \frac{3}{4}(e - e^{-1})$$

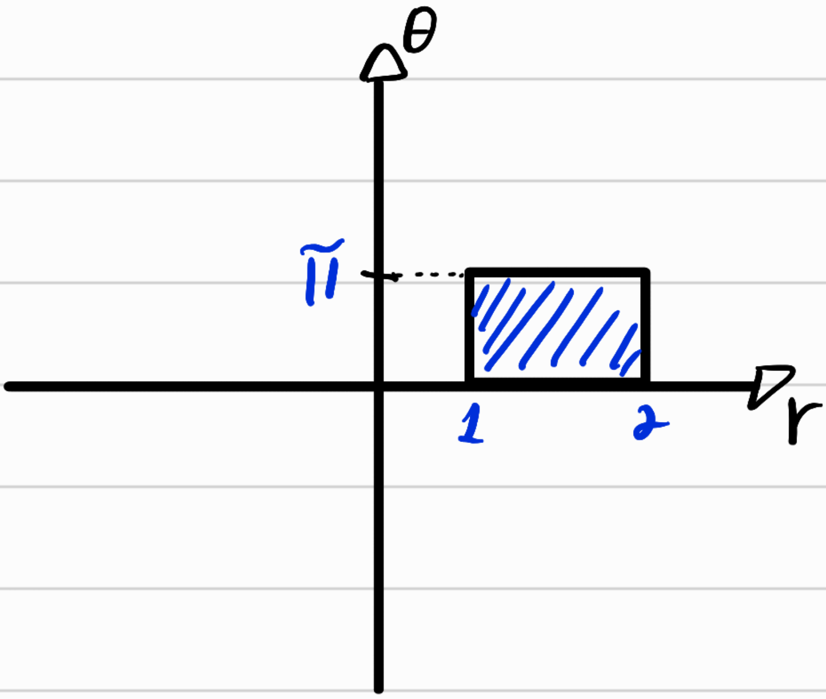
Ex: $\iint_R 3x + 4y^2\, dx\, dy$ ONDE R É A REGIÃO DO SEMIPLANO SUPERIOR POR $x^2+y^2=1$ E $x^2+y^2=4$.



$0 \leq \theta \leq \pi$

$1 \leq r \leq 2$

$$\begin{cases} x = r \cdot \cos(\theta) \\ y = r \cdot sen(\theta) \end{cases}$$



$$\int_1^2 \int_0^{\pi} (3 \cdot r\cos(\theta) + 4 \cdot (r \cdot sen(\theta))^2 \cdot r \cdot d\theta \, dr$$

$$\int_1^2\int_0^{\pi} 3\cdot r\cos(\theta)+4r^3\cdot sen^2(\theta)\,d\theta\,dr$$

$$\int_1^2 3r^2 sen(\theta)+4r^3\left(\frac{1}{2}\theta-\frac{1}{2}sen(2\theta)\right)\Bigg|_0^{\pi} dr$$

$$\int_1^2 \cancel{3r^2\left(sen(\pi)-sen(0)\right)}^{\,0}+4r^3\left(\frac{1}{2}\pi-\cancel{\frac{1}{4}sen(\pi)}^{\,0}-\left(\cancel{\frac{1}{2}0}^{\,0}-\cancel{\frac{1}{4}sen(2\cdot 0)}^{\,0}\right)\right)dr$$

$$\int_1^2 2\pi r^3\,dr=\cancel{2}\pi\frac{r^4}{\cancel{4}}\Bigg|_1^2=\frac{\pi}{2}\left(2^4-1^4\right)=\frac{15\pi}{2}$$