INTEGRAIS TRIPLAS

$$\iiint_B f(x,y,z)\,dv = \lim_{m,n,l\to+\infty} \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} f(x^*,y^*,z^*)\underbrace{\Delta x_i \Delta y_j \Delta z_k}_{\Delta V}$$

APLICAÇÕES

- VOLUME: $\iiint_B 1 \cdot dv = V(B)$

- MASSA: $\iiint_B \underbrace{\rho(x,y,z)}_{\text{DENSIDADE DE } \underline{B}}\,dv = m(B)$

$$\frac{m}{V} = \rho$$

$$m = \rho \cdot V$$

- CENTRO DE MASSA: $(\bar{x},\bar{y},\bar{z})$

$$\bar{x} = \frac{1}{m(B)} \cdot \iiint_B x \cdot \rho(x,y,z)\,dv$$

$$\bar{y} = \frac{1}{m(B)} \cdot \iiint_B y \cdot \rho(x,y,z)\, dv$$

$$\bar{z} = \frac{1}{m(B)} \cdot \iiint_B z \cdot \rho(x,y,z)\, dv$$

- MOMENTO DE INÉRCIA:

$$I_x = \iiint_B (y^2+z^2) \cdot \rho(x,y,z)\, dv$$

$$I_y = \iiint_B (x^2+z^2)\, \rho(x,y,z)\, dv$$

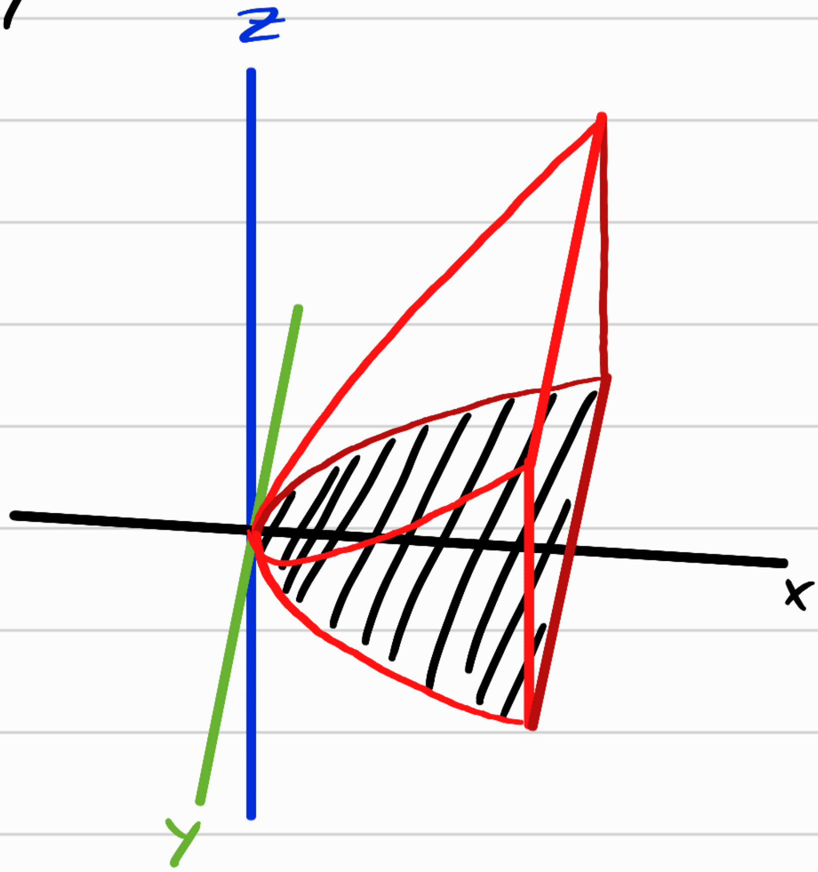
$$I_z = \iiint_B (x^2+y^2) \cdot \rho(x,y,z)\, dv$$

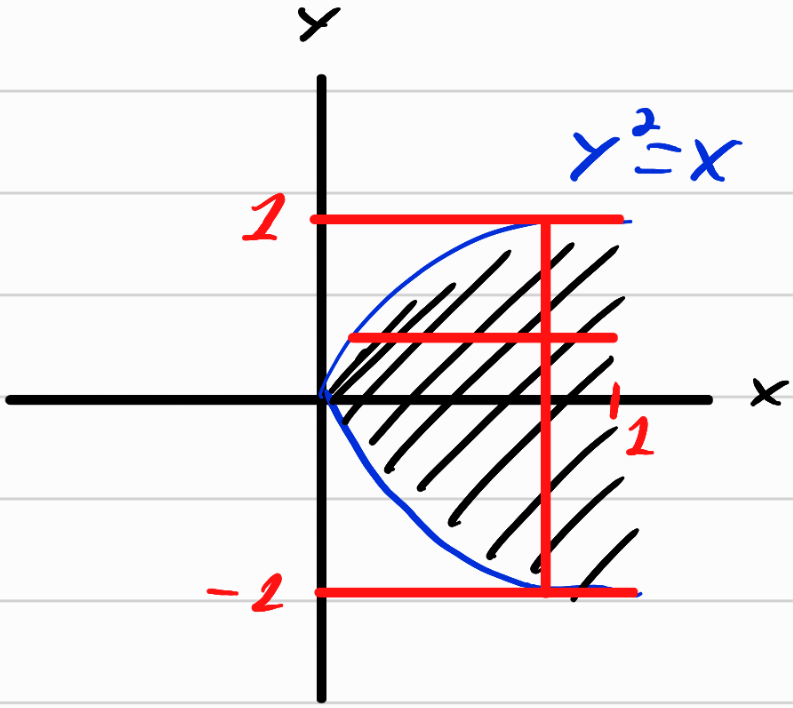
- CARGA: $Q_T = \iiint_B \underbrace{\sigma(x,y,z)}_{\text{DENSIDADE DE CARGA}}\, dv$

EX: CALCULE O CENTRO DE MASSA DE E

$\rho(x,y,z) = \text{CONST}$

$$E \Rightarrow \begin{cases} x = y^2 \\ x = z \\ z = 0 \\ x = 1 \end{cases}$$





CENTRO DE MASSA: $(\bar{x}, \bar{y}, \bar{z})$

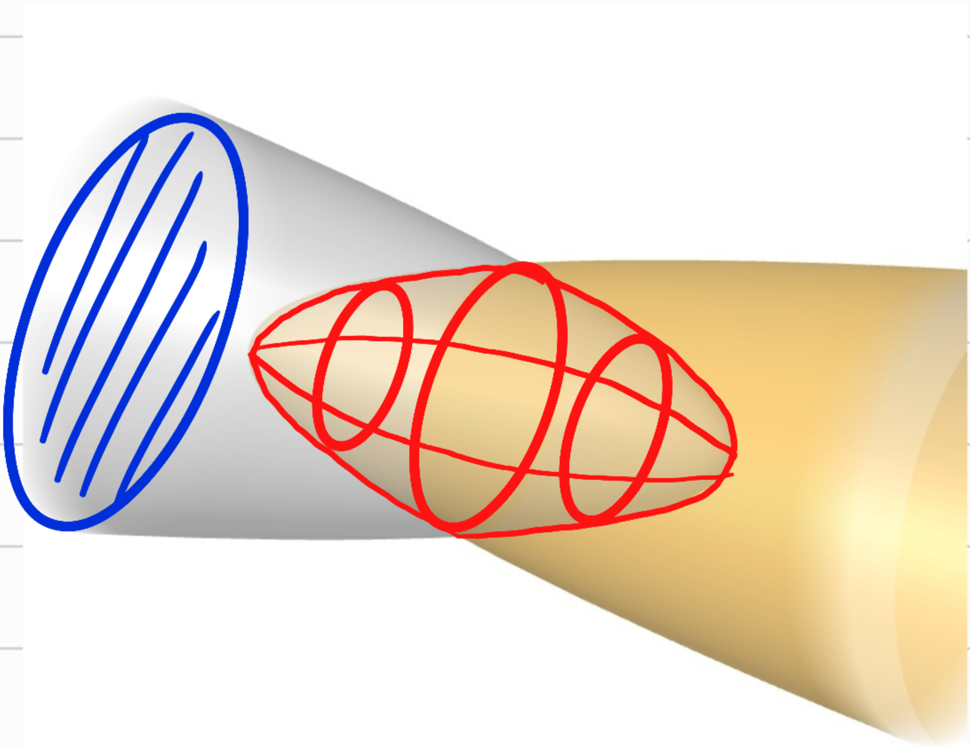
$$m(E) = \iiint_E K \cdot dV = \int_{-1}^{1}\int_{y^2}^{1}\int_{0}^{x} K \, dz\, dx\, dy = \ldots = \frac{4K}{5}$$

$$\bar{x} = \frac{1}{m(E)} \cdot \iiint_E x \cdot K \, dV$$

$$\bar{y} = \frac{1}{m(E)} \cdot \iiint_E y \cdot K \, dV$$

$$\bar{z} = \frac{1}{m(E)} \cdot \iiint_E z \cdot K \, dV$$

$\boxed{15.7}$ Ex 20: VOLUME DE E

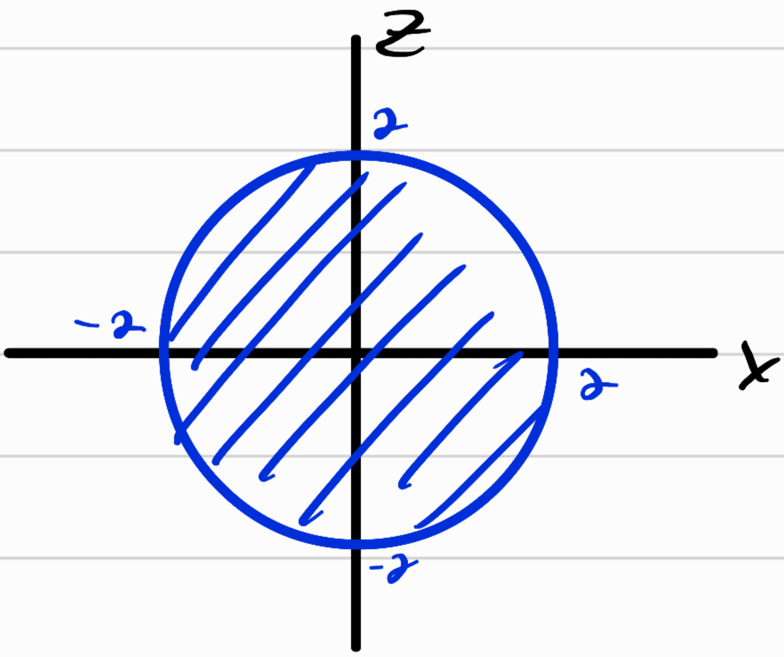


$$E = \begin{cases} y = x^2 + z^2 \\ y = 8 - x^2 - z^2 \end{cases}$$

$$V(E)=\iiint_E 1\cdot dV=\iint\int_{x^2+z^2}^{8-x^2-z^2} 1\cdot dy$$

$$8-x^2-z^2=x^2+z^2$$

$$2x^2+2z^2=8$$
$$x^2+z^2=4$$



$$V(E)=\iiint_E 1\cdot dV=\iint_D\int_{x^2+z^2}^{8-x^2-z^2} 1\cdot dy\, dA$$

$$V(E) = \iint_D [8 - x^2 - z^2] - [x^2 + z^2]\, dA$$

COORD. POL. $\begin{cases} x = r\cos(\theta) \\ z = r\cdot sen(\theta) \end{cases}$

$$\int_0^{2\pi} \int_0^2 (8 - 2r^2) \cdot r \cdot dr \cdot d\theta$$

$$2\pi \left(4r^2 - \frac{r^2}{2} \right)\Big|_0^2 = 2\pi(16 - 8) = 16\pi$$

$$V(E) = 16\pi$$

COORDENADAS CILINDRICAS

$$\begin{cases} x = r\cdot \cos(\theta) \\ y = r\cdot sen(\theta) \\ z = z \end{cases}$$

$$dx\,dy\,dz = \left|\frac{d(x,y,z)}{d(r,\theta,z)}\right| \cdot dr\,d\theta\,dz$$

$$\frac{d(x,y,z)}{d(r,\theta,z)} = \begin{vmatrix} dx/dr & dx/d\theta & dx/dz \\ dy/dr & dy/d\theta & dy/dz \\ dz/dr & dz/d\theta & dz/dz \end{vmatrix} = r$$

EX: MASSA DE E?

$$E\begin{cases} z = 1 - x^2 - y^2 \\ x^2 + y^2 = 1 \\ z = 4 \end{cases}$$

$$\rho(x,y,z) \propto K \underbrace{d((x,y,z),(0,0,z))}_{\sqrt{x^2+y^2}}$$

$$\rho(x,y,z) = K \cdot \sqrt{x^2+y^2}$$

$$m(E) = \iiint_E \rho(x,y,z)\, dv = \iiint_E K \cdot \sqrt{x^2+y^2}\, dv$$

CILÍNDRICAS:

$$\iiint$$

$$E\begin{cases} z = 1 - x^2 - y^2 = 1 - r^2 \\ x^2 + y^2 = 1 \\ z = 4 \end{cases}$$