ROTACIONAL

$$\vec{F}(x,y,z) = (P(x,y,z), Q(x,y,z), R(x,y,z))$$

Um campo em $\mathbb{R}^3$

$$\text{rot}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ d/dx & d/dy & d/dz \\ P & Q & R \end{vmatrix}$$

$$= \left(\frac{dR}{dy} - \frac{dQ}{dz}, \frac{dP}{dz} - \frac{dR}{dx}, \frac{dQ}{dx} - \frac{dP}{dy}\right)$$

$$= (R_y - Q_z, P_z - R_x, Q_x - P_y)$$

TEOREMA: $\text{rot}(\nabla f) = 0$

DEM: $\text{rot}\underbrace{\left(\underbrace{\frac{df}{dx}}_{P}, \underbrace{\frac{df}{dy}}_{Q}, \underbrace{\frac{df}{dz}}_{R}\right)}_{\nabla f} = (f_{zy} - f_{yz}, f_{xz} - f_{zx}, f_{yx} - f_{xy}) = \vec{0}$

Obs: SE $\vec{F}$ É $\underbrace{\text{CONSERVATIVO}}_{\vec{F}=\nabla f}$ ENTÃO $\text{rot}\,\vec{F}=0$

TEOREMA: $\vec{F}$ É UM CAMPO EM $\mathbb{R}^3$ CUJAS FUNÇÕES COM COMPONENTES SEJAM DE CLASSE $\mathcal{C}^2$ (DUAS VEZES DIFERENCIÁVEL COM SEGUNDA DERIVADA CONTÍNUA) E $\text{rot}\,\vec{F}=0$, ENTÃO $\vec{F}$ É CONSERVATIVO.

EX: $\vec{F}(x,y,z) = (\underbrace{\overbrace{y^2z^3}^{f_x}}_{P}, \underbrace{\overbrace{2xyz^3}^{f_y}}_{Q}, \underbrace{\overbrace{3xy^2z^2}^{f_z}}_{R})$ É CONSERVATIVO?

$$\text{rot}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ y^2z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix}$$

$$\text{rot}\,\vec{F} = (6xyz^2 - 6xyz^2, 3y^2z^2 - 3y^2z^2, 2yz^3 - 2yz^3)$$

$$\text{rot}\,\vec{F} = \vec{0} \quad \therefore \quad \vec{F} = \nabla f$$

b) $f = ?$

$f_x = y^2 z^3 \Rightarrow f = xy^2z^3 + h_1(y,z)$

$f_y = 2xyz^3 \Rightarrow f = xy^2z^3 + h_2(x,z)$

$f_z = 3xy^2z^2 \Rightarrow f = xy^2z^3 + h_3(x,y)$

$\therefore \boxed{f(x,y,z) = xy^2z^3}$

DIVERGENTE: $\vec{F} = (P,Q,R)$ UM CAMPO EM $\mathbb{R}^3$ E $\frac{dP}{dx}$, $\frac{dQ}{dy}$ E $\frac{dR}{dz}$ EXISTEM.

DEFINIMOS O DIVERGENTE DE $\vec{F}$ POR:

$$\text{div}\,\vec{F} = \frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$$

$$\boxed{\text{div}\,\vec{F} = \nabla \cdot \vec{F}}$$

TEOREMA: $\vec{F}=(P,Q,R)$, P, Q e R POSSUEM DERIVADAS PARCIAIS DE SEGUNDA ORDEM CONTÍNUAS, ENTÃO

$$\operatorname{div}(\operatorname{rot}\vec{F})=0$$

DEM: $\operatorname{div}(R_y-Q_z,\ P_z-R_x,\ Q_x-P_y)$

$$=R_{yx}-Q_{zx}+P_{zy}-R_{xy}+Q_{xz}-P_{yz}$$

$$=\cancel{R_{yx}}-\cancel{Q_{zx}}+\cancel{P_{zy}}-\cancel{R_{xy}}+\cancel{Q_{xz}}-\cancel{P_{yz}}$$

$$=0$$

EX: $\vec{F}=(xz,\ xyz,\ -y^2)$

$$\operatorname{div}\vec{F}=z+xz+0=z+xz=\boxed{z(1+x)}$$

$$\operatorname{div}(\nabla f)=\operatorname{div}(f_x,f_y,f_z)=f_{xx}+f_{yy}+f_{zz}$$

$$\nabla\cdot\nabla f=\nabla^2 f$$

∇^2 → OPERADOR DE LAPLACE

$$\vec{F}(x,y) = (P(x,y), Q(x,y)) \approx (P,Q,0)$$

$$\operatorname{rot}\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ P & Q & 0 \end{vmatrix} = \left(0-0,\, 0-0,\, \frac{dQ}{dx} - \frac{dP}{dy}\right)$$

TEORAMA DE GREEN REESCRITO:

$$\oint_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy$$

$$= \iint_D \operatorname{rot}(\vec{F}) \cdot \hat{k} \cdot dx\,dy$$

$C: r(t) = (x(t), y(t))$: CURVA

$$T(t) = \frac{r'(t)}{|r'(t)|} = \left(\frac{x'(t)}{|r'(t)|}, \frac{y'(t)}{|r'(t)|}\right)$$

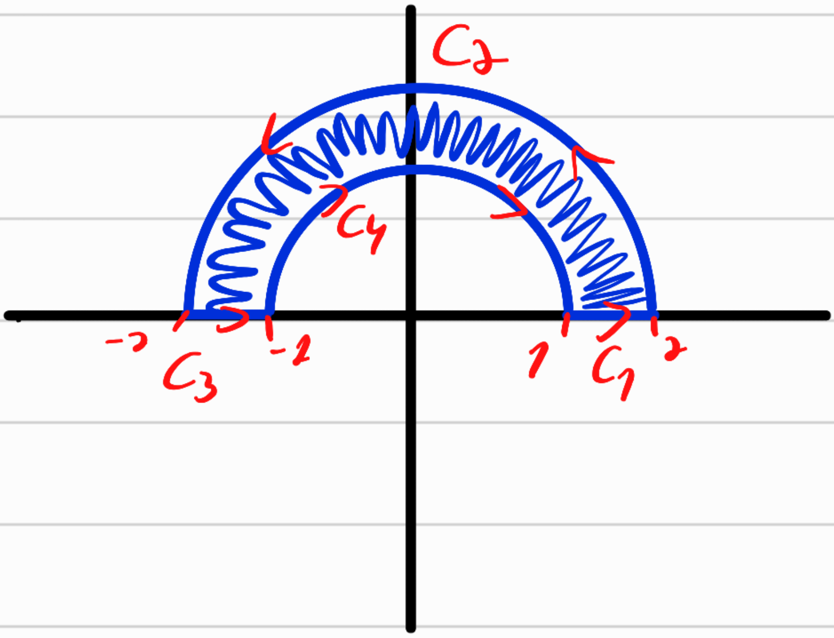
$$\vec{n}(t) = \left(\frac{y'(t)}{|r'(t)|}, \frac{-x'(t)}{|r'(t)|}\right)$$

$$\oint_C \vec{F} \cdot \vec{n} \cdot ds = \oint_c (P, Q) \cdot \frac{(y', -x')}{\cancel{|r'(t)|}\,\cancel{|r'(t)|}} \cdot \cancel{|r'(t)|}\, dt$$

$$= \oint_c P\,dy - Q\,dx \overset{\text{TEOR. GREEN}}{=} \iint_D \frac{dP}{dx} - \left(-\frac{dQ}{dy}\right) dx\,dy$$

$$= \boxed{\iint_D \operatorname{div} \vec{F}\, dx\,dy}$$

Ex: $\displaystyle\int_C \underbrace{x^2}_{P}dx + \underbrace{3xy}_{Q}dy = ?$



P. def $= \displaystyle\int_C = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$

T. GREEN

$$\iint_D \frac{dQ}{dx} - \frac{dP}{dy}\, dxdy$$

$$\int_C \underbrace{x^2}_{P}dx + \underbrace{3xy}_{Q}dy = \iint_D 3y - 0\ dxdy = \iint_D 3y\,dxdy$$

COORD. POLARES

$$\begin{cases} x = r\cos(\theta) \\ y = r\cdot\operatorname{sen}(\theta) \end{cases}$$

$$= \int_0^{\pi}\int_1^2 3\cdot r\cdot\operatorname{sen}(\theta)\cdot r\cdot dr\cdot d\theta = r^3\Big|_1^2 \cdot \left(-\cos(\theta)\Big|_0^{\pi}\right) = 7\cdot 2 = 14$$