CAMPOS: $\vec{F}: A \subset \mathbb{R}^n \longrightarrow \mathbb{R}^n$, $n=2,3$

INTEGRAIS DE LINHA:

$f: D \subset \mathbb{R}^{2/3} \longrightarrow \mathbb{R}$

C: $\alpha: I \subset \mathbb{R} \longrightarrow \mathbb{R}^{2/3}$: CURVA PARAMETRIZADA

$$\int_C f \cdot dr = \int_a^b f(\alpha(t)) \cdot |\alpha'(t)| \, dt$$

$$\mathcal{T} = \int_C \vec{F} \cdot dr = \int_a^b \vec{F} \cdot \vec{T} \cdot dr = \int_a^b \vec{F}(\alpha(t)) \cdot \underbrace{\frac{\alpha'(t)}{\cancel{|\alpha'(t)|}}}_{} \cdot \cancel{|\alpha'(t)|} \, dt$$

VETOR TANGENTE

T UNITÁRIO

$$= \int_a^b \vec{F}(\alpha(t)) \cdot \alpha'(t) \, dt$$

$$\int_{C = C_1 \cup C_2 \cup \cdots \cup C_n} \vec{F} \cdot dr = \int_{C_1} \vec{F} \cdot dr + \int_{C_2} \vec{F} \, dr + \cdots + \int_{C_n} \vec{F} \cdot dr$$

CAMPOS CONSERVATIVO

$$\vec{F} = \nabla f$$

$$\int_C \vec{F} \cdot d\vec{r} = f(P_f) - f(P_i)$$

$$\oint_C \vec{F} \cdot d\vec{r} = 0$$ → MESMO PONTO FINAL E INICIAL, POIS $f(P_f) - f(P_i)$ VAI DAR ZERO, SE FOREM IGUAIS.

TEOREMA:

PARA SER CONSERVATIVO, $\dfrac{dQ}{dx} = \dfrac{dP}{dy}$

SE, E SOMENTE SE, $\vec{F} = (P, Q) = \nabla f = \left(\dfrac{df}{dx}, \dfrac{df}{dy}\right)$

TEOREMA: $\operatorname{rot} \vec{F} = 0 \Rightarrow \vec{F} = (P, Q, R) = \nabla f$

$$\operatorname{rot} \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ d/dx & d/dy & d/dz \\ P & Q & R \end{vmatrix}$$

$$\text{rot}\,\vec{F}=\left(\frac{dR}{dy}-\frac{dQ}{dz},\frac{dP}{dz}-\frac{dR}{dx},\frac{dQ}{dx}-\frac{dP}{dy}\right)$$

TEOREMA:

$$\text{div}\,\vec{F}=\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}$$

TEOREMA DE GREEN: $\vec{F}=(P,Q)$

$$\oint_{\partial D=C}\vec{F}\cdot d\vec{r}=\iint_D\left(\frac{dQ}{dx}-\frac{dP}{dy}\right)dx\,dy$$

INTEGRAIS DE SUPERFICIE:

$S:\sigma:D\subset\mathbb{R}^2\longrightarrow\mathbb{R}^3$: SUPERFICIE PARAMETRIZADA.

$$A(S)=\iint_D\|\sigma_u\times\sigma_v\|\,du\,dv=\iint_S 1\cdot dS$$

$$\iint_S f \cdot dS = \iint f(\sigma(u,v)) \cdot \|\sigma_u \times \sigma_v\| \, du \cdot dv$$

$$\text{FLUXO} = \iint_S \vec{F} \cdot dS = \iint_S \vec{F} \cdot \vec{n} \cdot dS = \iint_D \vec{F}(\sigma(u,v)) \cdot (\sigma_u \times \sigma_u) \, du dv$$

TEOREMA DE GAUSS (TEOREMA DO DIVERGENTE):

$$\iint_{S=\partial E} \vec{F} \cdot dS = \iiint_E \operatorname{div} \vec{F} \cdot dV$$

$$= \iiint_E \left(\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz} \right) dV$$

TEOREMA DE STOKES:

$$\iint_S \operatorname{rot} \vec{F} \, dS = \int_{C=\partial S} \vec{F} \cdot dS$$

16.8 ⑦ $\vec{F}(x,y,z) = (\underbrace{x+y^2}_{P}, \underbrace{y+z^2}_{Q}, \underbrace{z+x^2}_{R})$

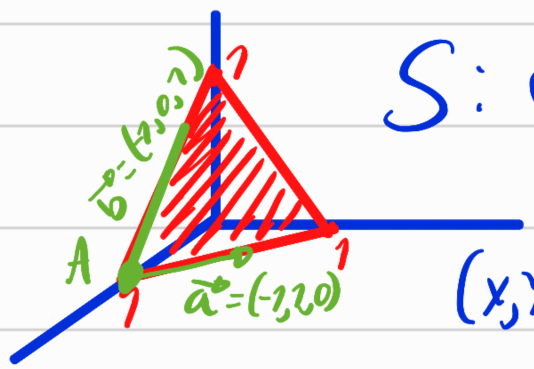
C: TRIÂNGULO COM VÉRTICES $(1,0,0), (0,1,0), (0,0,1)$

$$\oint_C \vec{F} \cdot d\vec{r} = ?$$

USANDO TEOREMA DE STOKES.

$$\iint_S \operatorname{rot} \vec{F}\, ds = \oint_{C=\partial S} \vec{F} \cdot ds$$

$$\operatorname{rot} \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = (-2z, -2x, -2y)$$



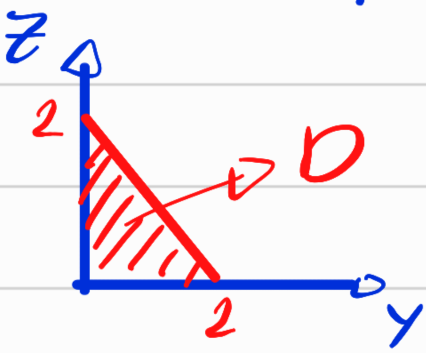
$S: \sigma(u,v) = (1-u-v, u, v)$

$(x,y,z) = (1,0,0) + u(-1,1,0) + v(-1,0,1)$

$$\sigma_u = (-1,1,0)$$

$$\sigma_v = (-1,0,1)$$

$$\sigma_u \times \sigma_v = \begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = (1,1,1)$$



$$\iint_S \operatorname{rot} \vec{F}\, dS = \iint_D \left(-2v, -2(1-u-v), -2u\right) \cdot (1,1,1)\, du\, dv$$

$$= \int_0^1 \int_0^{1-u} -2v + (-2+2u+2v) + (-2u)\, dv\, du$$

$$= \int_0^1 \int_0^{1-u} -2\, dv\, du = -2 \int_0^1 \int_0^{1-u} dv\, du$$

$$= -2 \cdot \int_0^1 1-u\,du = -2 \cdot \int_0^1 du - \int_0^1 u\,du =$$

$$= -2 \cdot 1 - \frac{1}{2} = -2 - \frac{1}{2} = -\frac{5}{2}$$

5 $\vec{F} = (xye^2, xy^2z^3, -ye^z)$

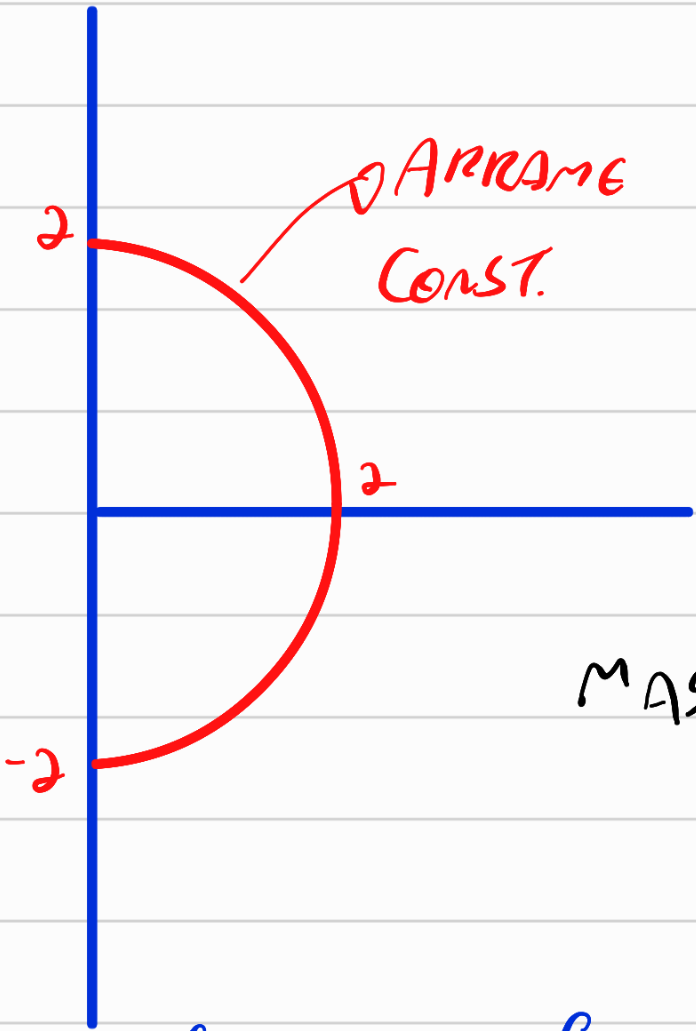
S: SUPERFÍCIE DA CAIXA $(x,y,z) \in [0,1] \times [0,2] \times [0,1]$

FLUXO = ??

$$\text{FLUXO} = \iint_S \vec{F}\,ds \overset{T.\,GAUSS}{=} \iiint_E \operatorname{div} \vec{F} \cdot dV =$$

$$= \int_0^1 \int_0^2 \int_0^1 (ye^2 + 2xyz^3 - ye^z)\,dz\,dy\,dx$$

16.2 33 $\quad x^2+y^2=4 \quad x\geq 0$



ARRAME COM DENSIDADE CONST.

MASSA = ?

$$m=\int_C \rho\cdot ds=\rho\cdot\underbrace{\int_C ds}_{2\pi}=\underline{2\pi\rho}$$

$$\alpha(t)=(2\cos(t),2\,sen(t)),\quad t\in[-\pi/2,\pi/2]$$

$$\alpha'(t)=(-2\,sen(t),2\cos(t))\Rightarrow |\alpha'(t)|=2$$

$$e\int_{-\pi/2}^{\pi/2} 2\,dt = e\cdot 2t\Big|_{-\pi/2}^{\pi/2} = 2e\cdot\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \underline{\underline{2\pi e}}$$

$$\vec{F} = (x^2, y^2)$$

$$C:\ y = 2x^2 \text{ de } (-1,2), (2,8)$$

$$W = \int_C \vec{F}\,ds = f(2,8) - f(-1,2)$$

$$W = \frac{8}{3} + \frac{512}{3} - \left(-\frac{1}{3} + \frac{8}{3}\right) = \frac{513}{3}$$

$$\alpha'(t) = (1, 4t)$$

$$\int_C \vec{F}\,d\vec{r} = \int_{-1}^{2} (t^2, 4t^4)\cdot(1,4t)\,dt = \int_{-2}^{2} t^2 + 16t^5\,dt =$$

$$\frac{t^3}{3} + \cancel{16}^{\,8}\cdot\frac{t^6}{\cancel{6}}\Big|_{-2}^{2} =$$

$$\frac{8}{3}+\frac{8\cdot 64}{3}-\left(-\frac{1}{3}+\frac{8}{3}\right)=\frac{573}{3}$$