SUPERFÍCIE PARAMETRIZADA

$$\sigma: D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$
$$(u,v) \longmapsto \sigma(u,v) = (x(u,v), y(u,v), z(u,v))$$

$\vec{n}$: VETOR NORMAL DO PLANO TANGENTE

$\vec{\sigma}_u \times \vec{\sigma}_v \neq 0$: σ É SUAVE (REGULAR)

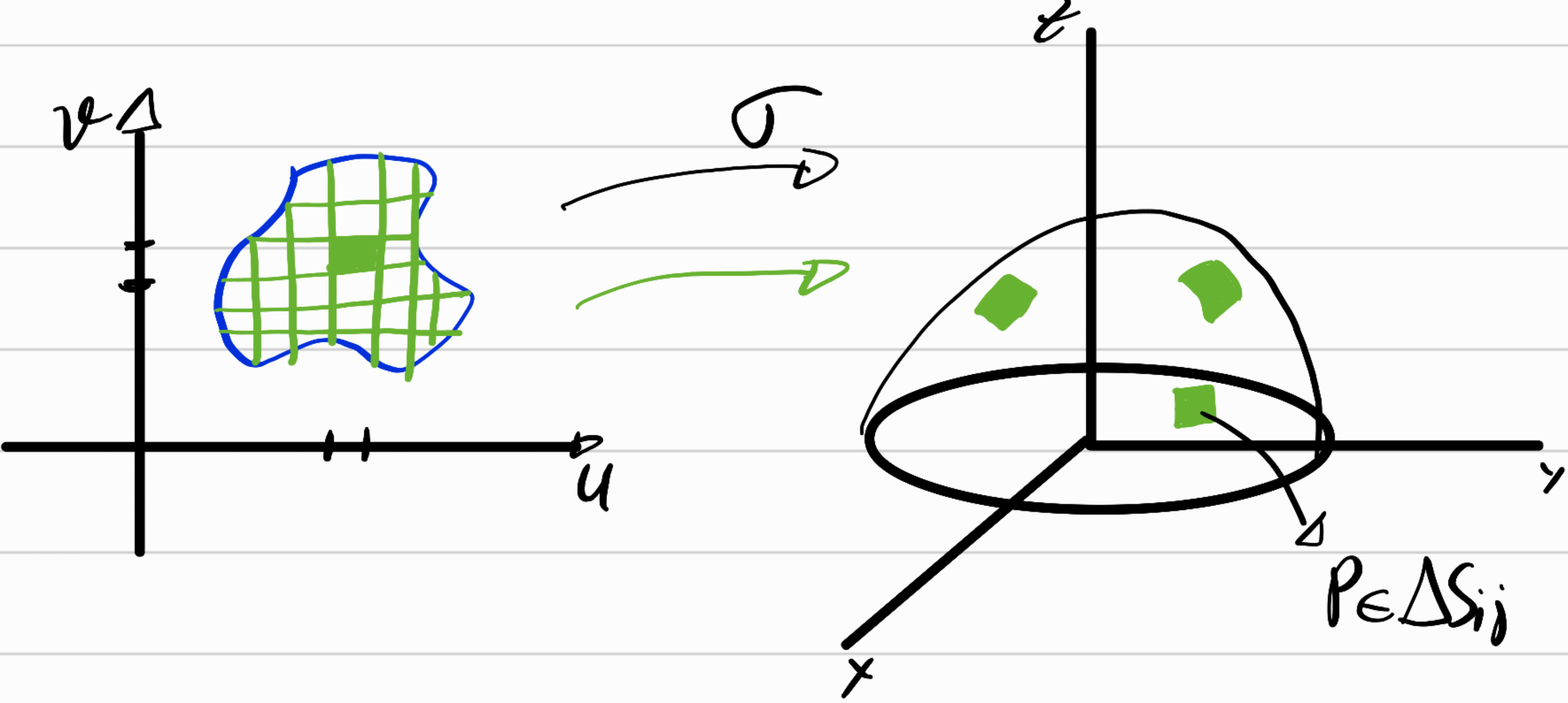
$$A(S) = \iint_{D_{u,v}} |\vec{\sigma}_u \times \vec{\sigma}_v| \, du \, dv$$: ÁREA DA SUPERFÍCIE S.

INTEGRAIS DE SUPERFÍCIE DE FUNÇÃO ESCALAR.

$f: B \subset \mathbb{R}^3 \longrightarrow \mathbb{R}$: FUNÇÃO REAL

$\sigma: D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$: UMA SUPERFÍCIE PARAMETRIZADA.

$S = \operatorname{Im} \sigma \subset B$



$$\sum_{i} \sum_{j} f(P) \cdot \Delta S_{ij} = \underbrace{\iint_{S} \underbrace{f(x,y,z)}_{\text{DENSIDADE}} \, \underbrace{dS}_{\text{ÁREA}}}_{\text{MASSA}}$$

$$= \iint_{D_{u,v}} f(\sigma(u,v)) \left| \vec{\sigma}_u \times \vec{\sigma}_v \right| du \, dv$$

$$S = S_1 \cup S_2 \Rightarrow \iint_{S} f \, dS = \iint_{S_1} f \, dS_1 + \iint_{S_2} f \, dS_2$$

Ex: $\iint_S x^2\, dS = ?$ em que S é a esfera $x^2+y^2+z^2=1$

$$\underset{(\varphi,\theta)}{\sigma(u,v)} = (\operatorname{sen}(u)\cos(v), \operatorname{sen}(u)\operatorname{sen}(v), \cos(u))$$

$$\sigma_u = (\cos(u)\cos(v), \cos(u)\operatorname{sen}(v), -\operatorname{sen}(u))$$

$$\sigma_v = (-\operatorname{sen}(u)\cdot\operatorname{sen}(v), \operatorname{sen}(u)\cos(v), 0)$$

$$\sigma_u \times \sigma_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos(u)\cos(v) & \cos(u)\operatorname{sen}(v) & -\operatorname{sen}(u) \\ -\operatorname{sen}(u)\operatorname{sen}(v) & \operatorname{sen}(u)\cos(v) & 0 \end{vmatrix}$$

$$\sigma_u \times \sigma_v = (\operatorname{sen}^2(u)\cos(v), \operatorname{sen}^2(u)\operatorname{sen}(v), \operatorname{sen}(u)\cos(u))$$

$$|\sigma_u \times \sigma_v| = \sqrt{\operatorname{sen}^4(u) + \operatorname{sen}^2(u)\cos^2(u)} = \operatorname{sen}(u)$$

$$\iint_S x^2\, dS = \int_0^{2\pi}\int_0^{\pi} \operatorname{sen}^2(u)\cos^2(v)\cdot\operatorname{sen}(u)\cdot du\, dv$$

$$= \int_0^{\pi} sen^3(u)\, du \cdot \int_0^{2\pi} \cos^2(v)\, dv = \left(-\frac{1}{3}(2+sen^2(u))\right)\Bigg|_0^{\pi} \cdot$$

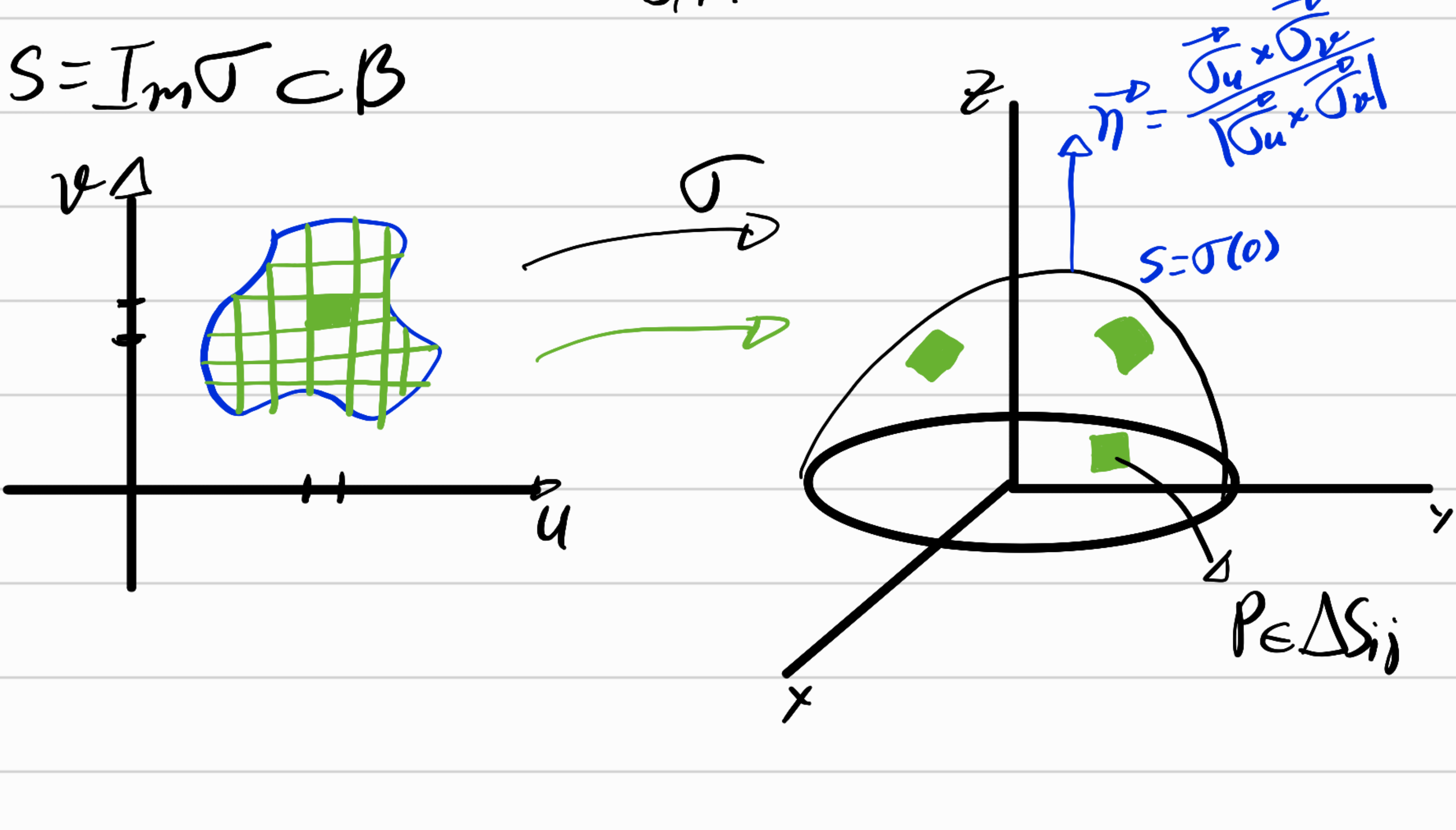
$$\left(\frac{1}{2}u + \frac{1}{4} sen(2u)\right)\Bigg|_0^{2\pi} = -\frac{2}{3}(-1-1)\cdot\pi = \frac{4\pi}{3}$$

INTEGRAIS DE SUPERFÍCIE DO CAMPO VETORIAL

$\vec{F} : B \subset \mathbb{R}^3 \longrightarrow \mathbb{R}^3$: FUNÇÃO REAL

$\sigma : D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$: UMA SUPERFÍCIE PARAMETRIZADA.

$S = Im\,\sigma \subset B$



$$\underbrace{\iint_S \vec{F}\, ds}_{} = \iint_S \vec{F}\cdot\vec{n}\cdot ds = \boxed{\iint_{D_{u,v}} \vec{F}(\sigma(u,v))\cdot(\vec{\sigma_u}\times\vec{\sigma_v})\, du\, dv}$$

FLUXO $\boxed{F}$

EX: CALCULE O FLUXO DO CAMPO $\vec{F}(x,y,z) = (z,y,x)$ SOBRE A ESFERA $x^2+y^2+z^2=1$

$$\sigma(u,v) = (\text{sen}(u)\cos(v), \text{sen}(u)\,\text{sen}(v), \cos(u))$$
(φ, θ)

$$\sigma_u = (\cos(u)\cos(v), \cos(u)\,\text{sen}(v), -\text{sen}(u))$$

$$\sigma_v = (-\text{sen}(u)\cdot\text{sen}(v), \text{sen}(u)\cos(v), 0)$$

$$\sigma_u\times\sigma_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos(u)\cos(v) & \cos(u)\,\text{sen}(v) & -\text{sen}(u) \\ -\text{sen}(u)\,\text{sen}(v) & \text{sen}(u)\cos(v) & 0 \end{vmatrix}$$

$$\vec{F}(\sigma(u,v)) = (\cos(u), \text{sen}(u)\,\text{sen}(v), \text{sen}(u)\cos(v))$$

$$\vec{F}(\sigma(u,v))\cdot(\vec{\sigma}_u \times \vec{\sigma}_v) = sen^2(u)\cos(v)\cos(u) + sen^3(u)\cdot sen^2(v) + sen^2(u)\cos(v)\cos(u)$$

$$Fluxo = \iint_S \vec{F}\, ds = \int_0^{2\pi}\int_0^{\pi} 2sen^2(u)\cdot\cos(u)\cos(v) + sen^3(u)sen^2(v)\, du\,dv$$

$$= \int_0^{\pi} 2sen^2(u)\cos(u)\,du \cdot \int_0^{2\pi}\cos(v)\,dv + \int_0^{\pi} sen^3(u)\,du \cdot \int_0^{2\pi} sen^2(v)\,dv$$

$$= \cancelto{0}{2\cdot\frac{sen^3(u)}{3}\Big|_0^{\pi}\cdot sen(u)\Big|_0^{2\pi}} + \left(-\frac{1}{3}\left(2+sen^2(u)\right)\cos(u)\Big|_0^{\pi}\right)\cdot\left(\frac{1}{2}v - \frac{1}{4}sen(2v)\Big|_0^{2\pi}\right)$$

$$Fluxo = \frac{4\pi}{3}$$

Teorema de Gauss (Divergência)

Seja E uma região sólida simples e $S=\partial E$ fronteira de E orientada positivamente (para fora). Seja $\vec{F}$ um campo vetorial cujas componentes possuem derivadas parciais contínuas em E. Então

$$\iint_S \vec{F}\,ds = \iiint_E \operatorname{div}\vec{F}\cdot dv$$

Ex: Calcule o fluxo de $\vec{F}=(z,y,x)$ sobre $x^2+y^2+z^2=1$.

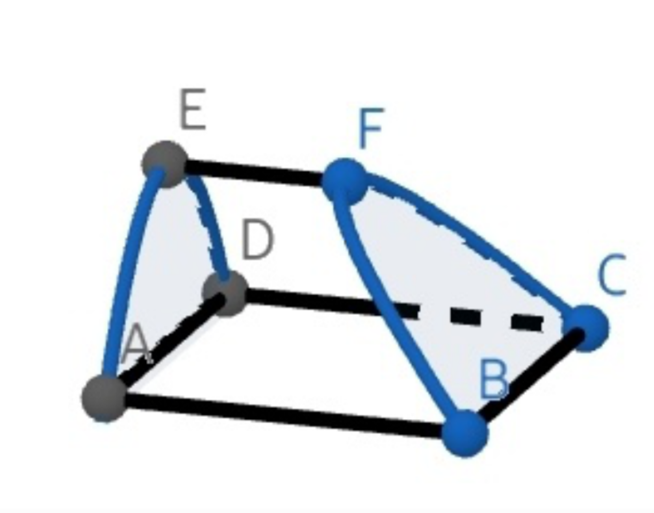
$$\text{Fluxo} = \iint_S \vec{F}\,ds \overset{\text{Gauss}}{=} \iiint_E \operatorname{div}\vec{F}\,dv = \iiint_E 1\cdot dv$$

$$\operatorname{div}\vec{F} = \underbrace{0}_{\frac{dz}{dx}} + \underbrace{1}_{\frac{dy}{dy}} + \underbrace{0}_{\frac{dx}{dz}}$$

$$\text{Fluxo} = \iiint_E 1\cdot dv = \text{Vol}(E) = \frac{4\pi}{3}$$

EX: CALCULE $\iint_S \vec{F} \cdot ds$ EM QUE $\vec{F} = (xy, y^2 + e^{xz^2}, \operatorname{sen}(xy))$

$$S \begin{cases} z = 1 - x^2 \\ y = 0 \\ z = 0 \\ y + z = 2 \end{cases}$$



$$\iint_S \vec{F} ds \overset{T.\ GAUSS}{=} \iiint_E \operatorname{div} \vec{F} dv = \int_{-2}^{2} \int_{0}^{1-x^2} \int_{0}^{2-z} 3y \, dy \, dz \, dx$$

TEOREMA DE STOKES

SEJA S UMA SUPERFÍCIE ORIENTADA, SUAVE POR PARTES, CUJA FRONTEIRA, $C = \partial S$, É UMA CURVA FECHADA, SIMPLES, SUAVE POR PARTES E ORIENTADA POSITIVAMENTE. SEJA $\vec{F}$ UM CAMPO CUJAS COMPONENTES SÃO $\mathcal{C}^1$ EM S. ENTÃO:

$$\boxed{\int_C \vec{F}\,d\vec{r} = \iint_S \underbrace{\text{rot}\,\vec{F}}_{\nabla\times\vec{F}}\,dS}$$

EX: $\vec{F} = (-y^2, x, z^2)$. CALCULE $\int_C \vec{F}\,d\vec{r}$ EM QUE C É A INTERSEÇÃO DO PLANO $y+z=2$ E O CILINDRO $x^2+y^2=1$.