LEI DA CONSERVAÇÃO DE ENERGIA

$$\alpha : I \subset \mathbb{R} \longrightarrow \mathbb{R}^2$$
$$t \longmapsto \alpha(t) = (X(t), Y(t))$$

$\alpha'(t) =$ VELOCIDADE

$\alpha''(t) =$ ACELERAÇÃO

$$\vec{F} = m \cdot a = m \cdot \alpha''(t)$$

$$W = \int_e \vec{F} \cdot d\vec{r} = \int_a^b m \cdot \underbrace{\alpha''(t) \cdot \alpha'(t)}\, dt$$

$$(\alpha' \cdot \alpha')' = \alpha'' \cdot \alpha' + \alpha' \cdot \alpha''$$

$$W = m \cdot \int_a^b \frac{(\alpha' \cdot \alpha')'}{2} \cdot dt = \frac{m}{2} \cdot \left(\alpha'^2 \Big|_a^b \right)$$

VARIAÇÃO DA ENERGIA CINÉTICA

$$W = \boxed{\frac{mv^2}{2}(b) - \frac{mv^2(a)}{2}} \qquad W = \Delta E_c$$

$\vec{F} = -\nabla P$ (CAMPO CONSERVATIVO)

$$W = \int_C \vec{F} \, d\vec{r} = \int_C -\nabla P \cdot d\vec{r} = -\left(P(\alpha(b)) - P(\alpha(a))\right)$$

$$W = P(\alpha(a)) - P(\alpha(a))$$

Logo, $E_c(\alpha(b)) - E_c(\alpha(a)) = P(\alpha(a)) - P(\alpha(b))$

$$\boxed{E_c(\alpha(a)) + P(\alpha(a)) = E_c(\alpha(b)) + P(\alpha(b))}$$

Ex: $\vec{F}(x,y) = (\underbrace{3 + 2xy}_{P}, \underbrace{x^2 + 3y^2}_{Q})$

$$\frac{dP}{dy} = 2x = \frac{dQ}{dx}$$

$$f(x,y) = x^2 y + 3x - y^3 + C$$

$$\boxed{\vec{\nabla} f = \vec{F}}$$

b) $\int_C \vec{F}\cdot d\vec{r}$ EM QUE C É $r(t)=(e^t\cdot\cos(t), e^t\cdot sen(t))$
$t\in[0,\pi]$.

USANDO A DEFINIÇÃO:

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{\pi} \vec{F}(\alpha(t))\cdot r'(t)\,dt =$$

$$= \int_0^{\pi} (3+2\cdot e^t\cdot\cos(t)\cdot e^t sen(t), (e^t\cos(t)+e^t(-sen(t)), e^t\cdot sen(t)+e^t\cdot\cos(t))\,dt$$

$= \ldots$

OUTRA MANEIRA: $\int_C \vec{F}\cdot d\vec{r} = \int \vec{\nabla} f\cdot d\vec{r} = f(r(\pi)) - f(r(0))$

$$= f(-e^{\pi},0) - f(1,0) = \boxed{-3e^{\pi}-3}$$

EX: $\vec{F}(x,y,z) = (\underbrace{y^2}_{f_x}, \underbrace{2xy+e^{3z}}_{f_y}, \underbrace{3ye^{3z}}_{f_z}) = \vec{\nabla} f$?

$$\frac{df}{dx} = y^2 \Rightarrow f(x,y,z) = \boxed{xy^2} + \boxed{I(y,z)}$$

$$\frac{df}{dy} = 2xy + e^{3z} \Rightarrow f(x,y,z) = \boxed{xy^2} + \boxed{y \cdot e^{3z}} + \overset{0}{\boxed{J(x,z)}}$$

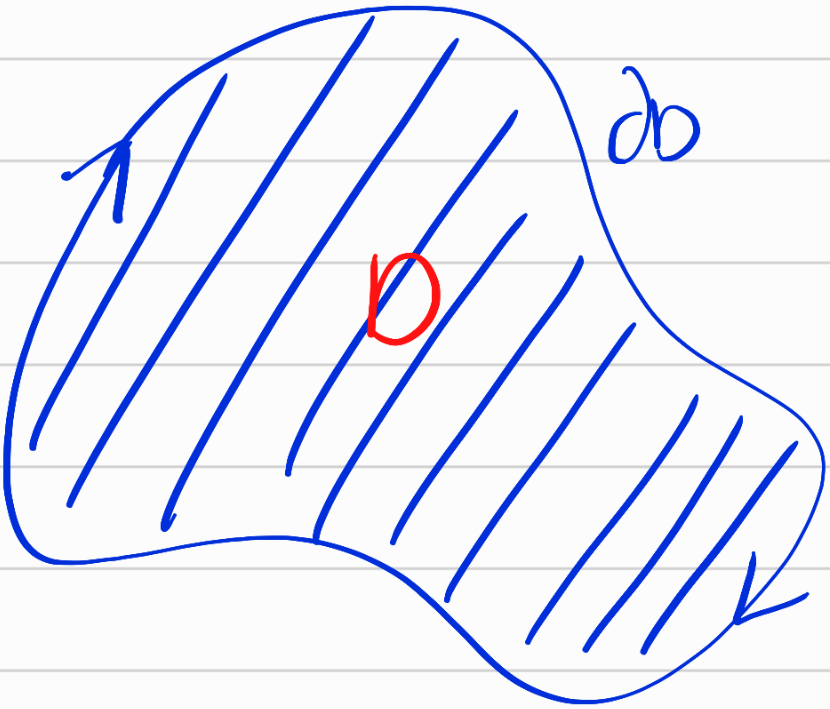
$$\frac{df}{dz} = 3ye^{3z} \Rightarrow f(x,y,z) = \boxed{ye^{3z}} + \boxed{K(x,y)}$$

$$\therefore f(x,y,z) = xy^2 + ye^{3z}$$

TEOREMA DE GREEN: SEJA C UMA CURVA PLANA SIMPLES, $\underbrace{\text{CONTÍNUA}}_{\text{SUAVE}}$ POR PARTES E ORIENTADA POSITIVAMENTE, E D É A REGIÃO DELIMITADA PELA CURVA $(C = \partial D)$. SE P e Q POSSUEM DERIVADA DE PRIMEIRA ORDEM CONTÍNUAS, ENTÃO:

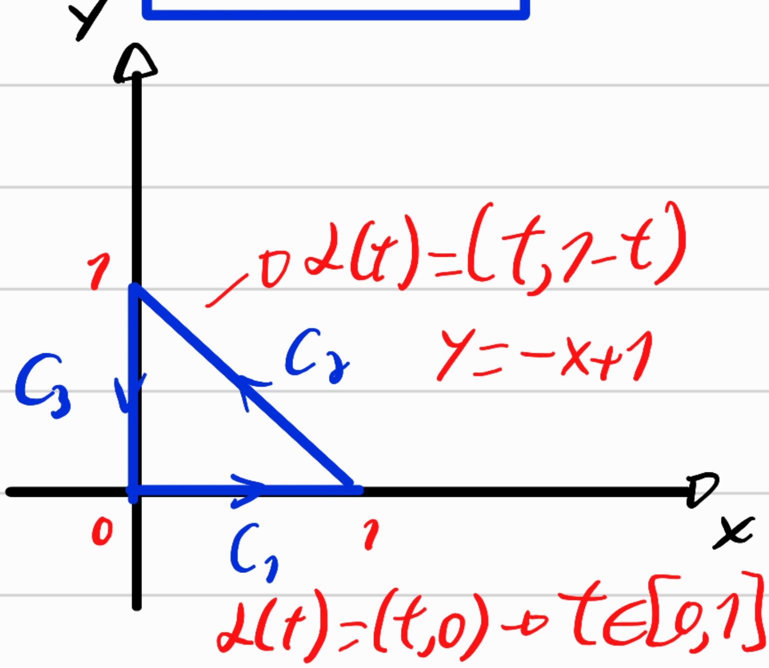
$$\oint_{\partial D} P\,dx + Q\,dx = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy$$

CURVA C



EX: $\int_C x^4\,dx + xy\,dy = ?$ EM QUE C É A CURVA

$\boxed{C_1 \cup C_2 \cup C_3}$



TRANSFORMANDO EM INT. DUPLA

$$\int_C \underbrace{x^4}_{P}\,dx + \underbrace{xy}_{Q}\,dy = \iint_D (y-0)\,dx\,dy = \int_0^1\int_0^{1-x} y\,dy\,dx$$

$$= \int_0^1 \frac{y^2}{2}\Big|_0^{1-x} dx = \int_0^1 \frac{(1-x)^2}{2}\,dx = \int_0^1 \frac{1-2x+x^2}{2}\,dx$$

$$= \frac{1}{2}\int_0^1 1-2x+x^2\,dx = \frac{1}{2}x\Big|_0^1 - x^2\Big|_0^1 + \frac{x^3}{3}\Big|_0^1 = \frac{1}{2}\cdot 1 - 1 + \frac{1}{3}$$

$$= \frac{1}{2} - 1 + \frac{1}{3} = \frac{1}{2} - \frac{2}{3} = \frac{3-2}{6} = \frac{1}{6}$$