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On the Normativity of Logic

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Both Frege and Peirce maintained that logic is one of three *normative sciences* (the others being ethics and aesthetics). Hence, we find Frege writing in a manuscript of 1897, "Like ethics, logic can also be called a normative science. How must I think in order to reach the goal, truth? … So if we call [logical laws] laws of thought or, better, laws of judgement, we must not forget we are concerned here with laws which, like the principles of morals or the laws of the state, prescribe how we are to act, and do not, like the laws of nature, define the actual course of events." And we find Peirce writing in his 1903 classification of the sciences, "Normative science has three widely separated divisions: i. Esthetics; ii. Ethics; iii. Logic. ... Logic is the theory of self-controlled, or deliberate, thought; and as such, must appeal to ethics for its principles" (CP 1.191). Logic has a normative subject matter. Logic is *about* the norms of thought, the norms of right reasoning, the norms of dialectic, and the norms of inquiry. Put in a more general form, logic aspires to tell agents how to act in order to achieve their (broadly) cognitive aims. In virtue of its normative subject matter, logic is distinct from mathematics (which studies the abstract structure of whatever we might imagine there to be), linguistics (which studies the structure of language), and psychology (which studies reasoning as it is actually carried out). In this paper, I develop an explicitly normativist account, according to which logic *directs* agents to means that are optimal relative to their ends and *evaluates* agents with respect to their cognitive performance. I hope to illuminate the nature of logic and advance debates about the normativity of logic by taking seriously the idea that logic is analogous to ethics in the ways suggested by Frege and Peirce.

Here is how I proceed. In Section 1, I begin with two related challenges to the claim that logic is normative for reasoning: one from Harman (1984 and 1986) and one from Russell

(2020). In Section 2, I introduce the standard approach to defending the normativity of logic by way of bridge principles from facts about logical consequence to normative constraints on beliefs (or other doxastic states). In Section 3, I reflect on how bridge principles are supposed to secure the normativity of logic, and I argue that the usual treatment of bridge principles actually invites the view that logic is *not* normative. I consider an alternative approach inspired by Corcoran (1973) and Shapiro (1998 and 2001). In Section 4, I consider a third challenge to the claim that logic is normative for reasoning, due to Tajer (2022). Then in Section 5, I develop my positive account of the nature of logic, and I offer explicit replies to Harman, Russell, and Tajer. In Section 6, I offer some concluding thoughts.

1. *Two Challenges to the Normativity of Logic*

Historically, many philosophers, logicians, and mathematicians have characterized logic as some kind of normative study. These include at least Boethius (522), Bacon (1245), Bacon (1620), Descartes (1644), Leibniz (1670, 1696), Kant (1800), Whately (1826), Mill (1843), Boole (1854), De Morgan (1860), Venn (1876), Frege (1893, 1897), Peirce, (1898, 1902, 1903a, 1903b), Ramsey (1926), Stebbing (1934, 1943), Strawson (1952), and Church (1956). But since the 1980s, the status of logic as a normative study has become controversial. In this section, I review two related challenges to the claim that logic is normative and specifically that it is normative *for reasoning*. The first challenge, from Harman (1984 and 1986), is that logic doesn't impose appropriate requirements on reasoning. The second challenge, from Russell (2020), is that since the subject matter of logic contains no normative elements, logic is only normative to the degree that ordinary descriptive sciences such as physics and biology are normative, which is to say that logic is not normative at all. I take up a third challenge, due to Tajer (2022), that the logical norms are not autonomous but are reducible to more general epistemic norms in Section

Commented [LJM1]: De Topicis Differentiis

Commented [LJM2]: The Art and Science of Logic

Commented [LJM3]: The Great Instauration & especially, in his Letter to the King

Commented [LJM4]: Letter of the Author to the Translator of the Principles

Commented [LJM5]: Preface to an Edition of Nizolius and Letter to Gabriel Wagner on the Value of Logic, respectively

Commented [LJM6]: Logik

Commented [LJM7]: Elements of Logic

Commented [LJM8]: A System of Logic

Commented [LJM9]: An Investigation of the Laws of Thought, especially pp. 407-408.

Commented [LJM10]: Syllabus of a Proposed System of Logic

Commented [LJM11]: The Logic of Chance

Commented [LJM12]: Grundgesetze and a ms on Logic, respectively

Commented [LJM13]: A full catalog of Peirce's writings on normative science would be difficult to assemble. More than 90 occurrences in his collected papers. See especially (not explicitly given in the list here: 1.575 ff; 2.7 (critical analysis of logical theories); 2.82; 5.14ff (1903 lectures on pragmatism); 5.126 (lecture V); 5.513 (1905, consequences of critical commonsensism); 5.566

Commented [LJM14]: The First Rule of Logic, EP 2, 47-48

Commented [LJM15]: A Detailed Classification of the Sciences, CP 1.281

Commented [LJM16]: An Outline Classification of the Sciences, CP 1.191

Commented [LJM17]: Lectures on Pragmatism, CP 5.14ff, especially 5.120-150

Commented [LJM18]: Truth and Probability

Commented [LJM19]: Logic in Practice; and A Modern Elementary Logic

Commented [LJM20]: Introduction to Logical Theory

Commented [LJM21]: Introduction to Mathematical Logic, Vol 1

4. All three challenges have in common the idea, which I reject, that logic is the study of purely structural relations of entailment or logical consequence.

1.1 *Harman*

Harman challenges the historical tradition in a 1984 essay on logic and reasoning and in an influential 1986 book, *Change in View*. In building his challenge, he first distinguishes between *inference* or *reasoning*, understood as a dynamic mental activity of controlled change in view, and *implication* or *entailment*, understood as a static structural relation that holds between (sets of) formulas in a formal system. Harman identifies the discipline of logic with the study of entailment. And he claims that if one distinguishes between reasoning and entailment, one will be inclined to be skeptical that deduction and induction are "different species of the same sort of thing" (1986, 6). According to Harman, there is deductive entailment but no deductive reasoning, and similarly, there is inductive reasoning but no inductive entailment. Consequently, there is no such thing as inductive logic. Or at least, we should be suspicious of the possibility of inductive logic. Summing up his position at the end of Chapter 1 of his 1986 book, he writes: "Reasoning in the sense of reasoned change in view should never be identified with proof or argument; inference is not implication. Logic is the theory of implication, not directly the theory of reasoning. Although we can say there is inductive reasoning, it is by no means obvious that there is any such thing as inductive argument or inductive logic" (1986, 10).

Harman denies that entailment facts have any special role to play in reasoning, and he suggests that logic is a purely descriptive study of such entailment facts—differing from other ordinary, descriptive sciences only in being more abstract and more widely applicable.

> This is an extreme view that no one seems to hold in an unqualified way, which is surprising, since the view seems to be quite viable. Frege may seem to take the extreme view when he says the laws of logic are laws of truth and since he attacks

> “psychologism”; but he also says the laws of logic “prescribe universally the way in which one ought to think if one is to think at all”, which is to reject the extreme view. Similarly, Quine may seem to advocate the extreme view when he says logic is a science of truths. But he also sees a special connection between logic and inference when he says one needs logic to get to certain conclusions from certain premises. As far as I have been able to determine, other philosophers who may seem at one place to put forward the extreme view that logic is a science, a body of truths, go on some place else to say that logic has a special role to play in reasoning. I am not sure why I cannot find anyone who has unequivocably [sic] endorsed the extreme view. (1984, 109-110)

Given his understanding of logic as a descriptive study of entailment facts or truths, Harman identifies four problems for the claim that logic is normative for reasoning: [1] Correct reasoning doesn’t always proceed from premisses to their logical consequences; [2] In order to avoid cognitive clutter, our beliefs shouldn’t be closed under logical consequence; [3] Sometimes, the right thing to do cognitively is to have beliefs that we know are jointly inconsistent; and [4] No cognitive norm can require our beliefs to be closed under logical consequence, since any such norm would be excessively demanding.[1]

How do Harman’s four problems support the claim that logic is not normative for reasoning? On my reading, each problem corresponds to an argument that the classical deductive entailment relation is not normative for reasoning. For example, according to Harman, if the claim that Γ entails φ, denoted $\Gamma \vDash \varphi$, is normative, then it may be understood as saying, “If you believe Γ, then you ought to believe φ.” But sometimes, what follows from our current beliefs is an absurdity. If entailment is normative for reasoning in the way considered by Harman, then sometimes one ought to believe an absurdity. But one never ought to believe an absurdity. So, entailment is not normative for reasoning. Harman’s other problems similarly provide reason for

[1] Given his emphatic concerns with reasoning as reasoned *change* in view, it strikes me as very odd that the last three of Harman’s challenges to the normativity of logic for reasoning are only *indirectly* about reasoning, if they are about reasoning at all. Good reasoning avoids *producing* cognitive clutter. Good reasoning sometimes *produces* jointly inconsistent beliefs. Good reasoning may permissibly (and perhaps even as a matter of obligation) *produce* a belief state that is not closed under logical consequence. The constraints are explicitly on products, not on any process as such. These challenges would be worth exactly as much, for example, if we were contemplating creatures with belief states but no ability to reason at all.

thinking that (classical, deductive) entailment is not normative for reasoning. The argument against the normativity of logic then proceeds as follows:

[H1] Logic is the study of entailment.

[H2] If logic is the study of entailment, then logic is normative for reasoning iff entailment is normative for reasoning.

[H3] Entailment is not normative for reasoning.

[H4] Logic is not normative for reasoning.

In Section 2, I will consider the standard response from normativists about logic, first articulated by MacFarlane (2004). I suggest an alternative approach at the end of Section 3 and complete my reply to Harman at the end of Section 5. But here is a preview. On my reading, the standard normativist response rejects [H3] by proposing a suitably strong bridge principle connecting entailment facts to doxastic norms. My own approach will be to reject [H1]. Logic is directly the study of norms for reasoning. Entailment facts matter when we aim to justify the norms or to explain why we ought to follow them. But entailment facts are not the object of study in logic.

1.2 *Russell*

Russell agrees with Harman in thinking that logic is not normative. Much of her essay involves setting out and responding to arguments in favor of the normativity of logic. But she also offers an interesting positive argument (on pages 383-385) for her descriptive view. Russell considers three varieties or grades of "entanglement" that a discipline might have with the normative. The weakest variety is the kind of entanglement that physics, biology, psychology, and other descriptive sciences have with the normative. We are obligated to believe the claims in those sciences insofar as the sciences make *true* claims. And we ought to believe what the sciences tell us because there are "common normative commitments concerning truth and falsity" that enjoin us to believe what is true and not what is false. According to Russell, logic is just like every other descriptive science. Logic is distinguished from other descriptive sciences by having a distinct object of study. For the purpose of my reconstruction here, let *truth-relations* denote both relations of truth-bearing and relations of truth-preservation or entailment. Then for Russell, logic is the study of truth-relations in languages. Since truth-relations in a language may be adequately described without appeal to anything normative, logic has no normative content. Hence, logic isn't normative for reasoning or for anything else. Put explicitly, Russell's main line of positive argument goes like this:

[R1] Logic is the study of truth-relations in languages.

[R2] If [R1], then logic has no normative content.

[R3] If logic has no normative content, then it isn't normative.

[R4] Logic isn't normative.

I read Russell as defending [R2] by way of a compressed story of how a formal logician might characterize the truth-relations in a simple sentential language: describe the grammar for formulas of the language, describe how to assign truth-values to formulas in the language, and define entailment or logical consequence in terms of possible truth-assignments. She then claims (plausibly, I think) that if there isn't any normativity in her simple example, adding more complication (e.g. to handle predicates or quantification or tense) won't introduce any. With respect to [R3], Russell supposes that one cannot derive an ought from an is, and she writes, "I feel entitled to make this assumption in the present discussion because proponents of the normativity of logic usually make the same assumption in the argument from normative consequences” (383, n. 12).

In Section 2, I will discuss what I expect proponents of bridge principles to say in response to Russell’s argument. I suggest an alternative at the end of Section 3 and complete my reply to Russell at the end of Section 5. But here is a preview. On my reading, the standard normativist response rejects [R2] by proposing a suitably strong bridge principle connecting entailment facts to doxastic norms. My own approach will be to reject [R1]. Logic is directly the study of norms for reasoning. Truth relations, including entailment facts, matter when we aim to justify the norms or to explain why we ought to follow them. But truth relations are not the object of study in logic.

2. *Bridge Principles*

In response to Harman, MacFarlane (2004 and 2017), Field (2009 and 2015), and Steinberger (2019a, 2019b, and 2019c) have proposed and discussed various bridge principles that might carry us from descriptive entailment facts to explicitly normative claims about our doxastic states. Recall Harman’s suggestion that if the claim that Γ entails φ, denoted $\Gamma \vDash \varphi$, is normative, then it may be understood as saying, “If you believe Γ, then you ought to believe φ.” Let *Bel* be an epistemic operator for belief such that Bel(S, φ) denotes that S believes that φ, and where Γ is a set, Bel(S, Γ) denotes that S believes all of the members of Γ. Let *O* be a deontic operator such that O(φ) denotes that φ ought to be the case. Then, we may transform Harman’s suggestion into an explicit bridge principle as follows:

[Bridge] If $\Gamma \vDash \varphi$, then Bel(S, Γ) only if O(Bel(S, φ)).

MacFarlane (2004) considers three dimensions along which one might constrain a bridge principle. First, there are three different ways a deontic operator might appear in the consequent

Commented [LJM22]: Steinberger, F. (2019a). Logical pluralism and logical normativity. *Philosophers’ Imprint* 19(12), 1-19.

Commented [LJM23]: Steinberger, F. (2019b). Three ways in which logic might be normative. *The Journal of Philosophy* 116(1), 5-31.

Commented [LJM24]: Steinberger, F. (2019c). Consequence and normative guidance. *Philosophy and Phenomenological Research* 98(2), 306-328.

of the bridge principle. The operator might have narrow scope over the consequent of the right-nested conditional, as it does in [Bridge]. It might have scope over both the antecedent and the consequent of that conditional. Or it might have wide scope over the whole conditional, as follows:

[Wide] If $\Gamma \vDash \varphi$, then O(Bel(S, Γ) only if Bel(S, φ)).

MacFarlane understood the wide scope deontic operator to have the sense of Broome's (1999) "normative requirement." Hence, [Wide] should be read as saying that when $\Gamma \vDash \varphi$, one ought to see to it that they believe Γ only if they believe φ.

Second, MacFarlane considers three different deontic operators: obligation, permission, and reason. Hence, one might have a bridge principle saying that if Γ entails φ, then one is permitted to believe Γ only if one is permitted to believe φ. Or one might have a bridge principle saying that if Γ entails φ, then one has reason to believe Γ only if one has reason to believe φ.

Third, MacFarlane notes that bridge principles could have either positive or negative polarity in the sense that the consequent of the right-nested conditional might involve a positive doxastic attitude or it might involve the absence of a negative doxastic attitude. Both [Bridge] and [Wide] have positive polarity. Now, let *Dis* be an epistemic operator for disbelief such that Dis(S, φ) denotes that S disbelieves that φ. The following bridge principle has negative polarity:

[Negative] If $\Gamma \vDash \varphi$, then O(Bel(S, Γ) only if $\neg$Dis(S, φ)).

The [Negative] principle says that when $\Gamma \vDash \varphi$, one ought to see to it that if they believe Γ, they don't *dis*believe φ.[2] MacFarlane did not consider bridge principles having more than one kind of deontic operator, such as a principle saying that when $\Gamma \vDash \varphi$, one has reason to believe φ if one

[2] MacFarlane explicitly formulates his principles in terms of disbelieving φ, rather than in terms of believing $\neg\varphi$ (2004, 8-9 and footnote 8). He is right to do so, since it is a point of controversy whether disbelieving φ is the same as believing $\neg\varphi$. Dialetheists, for example, deny that disbelieving φ is the same as believing $\neg\varphi$.

ought to believe Γ. Hence, MacFarlane's considerations yield $3 \times 3 \times 2 = 18$ different bridge principles. He designated distinct principles by expressions indicating the scope (C, B, or W), the type of operator (o, p, or r), and the polarity (+ or -).[3] Hence, for example, Co+ designates Harman's [Bridge], Wo+ designates [Wide], and Wo- designates [Negative].

MacFarlane rejects the C-type bridge principles on the basis of the following considerations. According to MacFarlane, "Any reasonable logic will contain $A \vDash A$ as a theorem."[4] But if a Co or Cp bridge principle is correct, then the fact that one believes φ thereby confers some normative goodness to one's attitude respecting φ. And that is absurd. The Cr bridge principles either fall to the same argument or collapse into the Br bridge principles. However, the B-type bridge principles are too weak, since they prohibit logic from giving any guidance to reasoners whose epistemic starting point is not already sufficiently normatively good. The B-type bridge principles imply that "logic is only normative for those whose beliefs are already in order ... To the unfortunate others, logical norms simply do not apply" (2004, 10). MacFarlane rejects Wo+ on the ground that it is excessively demanding, ultimately opting for a combination of Wo- and Wr+.

In Section 1.1, I noted four challenges that Harman raised against the claim that logic is normative for reasoning: [1] Correct reasoning doesn't always proceed from premisses to their logical consequences; [2] In order to avoid cognitive clutter, our beliefs shouldn't be closed under logical consequence; [3] Sometimes, the right thing to do cognitively is to have beliefs that we know are jointly inconsistent; and [4] No cognitive norm can require our beliefs to be closed under logical consequence, since any such norm would be excessively demanding. With

[3] MacFarlane also considers bridge principles that build in an assumption that one knows the relevant entailment fact, but that issue is not crucial to my project, so I set it aside.

[4] However, those substructural logicians who reject the reflexivity of logical consequence disagree. See French (2016).

MacFarlane's bridge principles in mind, Field (2009) answers Harman's challenges roughly as follows: [1] We should give the deontic operator wide scope, and we should make the normative requirement conditional on the *obviousness* of the logical consequence; [2] We should understand logical consequence to regulate *implicit* beliefs, so that cognitive clutter isn't an issue; [3] Pace Harman, in cases like the preface paradox, the right thing to do isn't to have beliefs we know to be jointly inconsistent, it's to focus on *degrees of belief* rather than full belief; and [4] We should understand logical consequence to impose computable *constraints* on degrees of belief. Putting these ideas together, Field suggests the following bridge principle (slightly modified for style here), where degree of belief that φ is denoted $\mathrm{Cr}(\varphi)$:

[Field] If it's obvious that $A_1, \ldots, A_n$ together entail B, then one ought to see to it that in any circumstance where $A_1, \ldots, A_n$, and B are in question, one's degrees of belief in $A_1, \ldots, A_n$, and B are related as follows: $\mathrm{Cr}(B) \geq \mathrm{Cr}(A_1) + \ldots + \mathrm{Cr}(A_n) - (n - 1)$.

The requirement that the entailment is obvious deserves some commentary. For Field, the condition is normative: "an obvious entailment is one that an agent *ought* to see" (262). The ought might be taken in a subjective sense or in an objective sense. For example, we might criticize a reasoner for failing to notice something that she could have seen given her ability and information had she been paying proper attention. Or we might criticize a reasoner for failing to notice something that *we* notice, regardless of whether she could have seen it.

Since a normative claim, such as that one ought to believe some proposition, might be understood in more than one way, we need to say how the normative claims in the bridge principles are to be understood in order to make the principles precise. Steinberger (2019b) suggests understanding the normative claims in the bridge principles in terms of their functional role, and he identifies three functions we might understand a normative claim as serving:

A normative claim is **directive** iff it provides first-personal guidance.

> A normative claim is **evaluative** iff it provides an objective, third-personal standard.
>
> A normative claim is **appraisive** iff it provides a third-personal basis for assignment of praise or blame.

Steinberger's categorization is helpful, but it seems to me to be incomplete: the directive function could be divided into evaluative and appraisive sub-types. We would then have four normative roles that are described by the values they take along two dimensions: internal/external and ideal/non-ideal. The distinction between internal and external depends on whether the logical requirement is already accepted by the reasoner being criticized or is instead imposed by an assessor. The distinction between ideal and non-ideal depends on what capacities we suppose the reasoner to have or to be responsible to. Steinberger does not say whether his directives are supposed to be directives for ideal or non-ideal agents. I'll assume that he means for directives to provide guidance for non-ideal agents, so I'll use Steinberger's term "directive" as a label for norms that function as internal standards for non-ideal agents, and I'll introduce the term "imperative" as a label for norms that function as internal standards for ideal agents:

> A normative claim is **imperative** iff it provides an internal standard for an ideal agent.
>
> A normative claim is **directive** iff it provides an internal standard for a non-ideal agent.
>
> A normative claim is **evaluative** iff it provides an external standard for an ideal agent.
>
> A normative claim is **appraisive** iff it provides an external standard for a non-ideal agent.

One might further complicate things by adding dimensions or by treating some dimensions as having more than two values. I suspect that at least the ideal/non-ideal dimension is actually a continuum of values representing something like the cognitive power of the agent to whom the norm is applied.

Steinberger claims that Harman's challenge is really about whether there is any interesting connection between facts of entailment and **directives**. According to Steinberger, the specific bridge principle one endorses depends on the normative role at issue. Specifically, a bridge principle stated in terms of wide scope *obligation* is well-suited to the normative role of evaluation, but the role of directive is better filled by a principle stated in terms of *reasons*. Hence, bridge principles poised to answer Harman's challenge should be stated in terms of reasons, rather than in terms of obligations. Bearing the normative role in mind, Steinberger proposed the following bridge principle (modified slightly for style):

[Direct] If S believes that $A_1, \ldots, A_n \vDash C$ and S considers C or has subjective reasons to consider C, then S has reasons to see to it that she believes C if she believes all of $A_1, \ldots, A_n$.

Steinberger's [Direct] principle is similar to MacFarlane's Wr+, but it has some additional epistemic conditions. After stating the principle, Steinberger worries that [Direct] is overly intellectualized. He then modifies the principle, replacing "S believes that $A_1, \ldots, A_n \vDash C$" with the more complicated epistemic condition that "according to S's best estimation at the time, S takes it to be the case that $A_1, \ldots, A_n \vDash C$." I leave it to the reader to decide whether the resulting bridge principle is, in fact, less intellectualized.

3. *Bridges and Models*

All of the standard bridge principles have the form of a conditional. The antecedent of each bridge principle is either an entailment fact, such as that $\Gamma \vDash \varphi$, or a fact about an agent's doxastic attitude with respect to an entailment fact, such as that the agent believes that $\Gamma \vDash \varphi$. The consequent of each bridge principle has some explicitly normative content. But the presence of normative content in the consequent of the bridge principles is obviously not enough on its own to secure the normativity of logic. After all, one can postulate a bridge principle to a normative claim from any arbitrary fact. We routinely traffic in conditional oughts where the condition is a plain, descriptive fact. If you're cold, then you should put on a sweater. If you want to be stronger, then you should work out. If you know that you can't convince a skeptic, then you shouldn't try. It seems to me that the form of the usual bridge principles is apt to mislead. Skeptics of the normativity of logic might think, along with Harman and Russell, that logic is *about* the entailment facts, that those entailment facts are entirely descriptive, and that the bridge principles are not themselves part of logic. For such skeptics, the form of the bridge principles actually makes it seem more obvious that logic is not normative. Even the name "bridge principle" suggests, wrongly, that there is some gap between the *is* of entailment and the *ought* of reasoning.

How, then, are bridge principles supposed to show that logic is normative? MacFarlane (2017) argues that what is needed is a sufficiently strong conceptual connection between *validity* and norms for reasoning. He sums up his point as follows:

> To show that logic is normative in an interesting sense, one needs to make an analytical claim about the concept of (intertheoretic) validity. It is not enough just to argue for some bridge principles connecting validity and cognitive norms; these principles must be partially constitutive of the concept of validity. (16)

So, bridge principles are supposed to show that logic is normative by showing that cognitive norms—norms governing our doxastic attitudes—are baked into the concept of validity, which is to say that $\Gamma \vDash \varphi$ *isn't* purely descriptive. Fully understanding the claim that Γ entails φ requires appreciating some normative content, such as that we are not permitted to believe Γ while disbelieving φ. But if there is normative content built into the concept of entailment, then [H3] in Harman's argument is false and so is [R2] in Russell's argument.

I now want to recommend an alternative approach to thinking about the normativity of logic. To get the approach in view, it will be helpful to think about what MacFarlane means by the concept of *inter-theoretic* validity. To get a grip on the idea of inter-theoretic validity, it's worth contrasting it with the idea of a *pre-theoretic* concept of validity. MacFarlane expresses skepticism that people have any such concept, pointing to "the difficulties one faces in getting these notions [of validity, logical consistency, and following logically from] across to undergraduates in their first exposure to logic." If we think of validity in a technical way as something like necessary truth-preservation in a mathematically well-defined formal language, then perhaps MacFarlane is right (though I disagree that there is any real difficulty in getting undergraduates to understand the basic ideas even on first exposure). However, it seems perfectly obvious to me that we *do* have pre-theoretic notions of *good argument* and *good reasoning*. We learn to evaluate our own reasoning and the reasoning of others long before we have any explicit, mathematical account of why the reasoning is good or of what features all good reasoning must have (if there are any such features). Medieval logicians called our implicit, pre-theoretic standard of reasoning our *logica utens*. One of the things that has historically been required of the discipline of logic is that it provide an account of good reasoning. An account of good reasoning might start from an attempt to explicate our *logica utens*, but it shouldn't stop there, since our implicit, pre-theoretic standard of reasoning might not be optimally adapted to achieve our cognitive aims. The standard articulated by an explicit logical theory is our *logica docens*. By contrast with the idea of a pre-theoretic concept of validity, the idea of inter-theoretic validity doesn't assume that we have any implicit standard of reasoning or any grasp on validity prior to having an explicit account. But the concept of inter-theoretic validity *does* let us make sense of disagreements with respect to which of several competing formal systems is *correct* in some external sense.

Now, it seems to me that putting things the way I have just done is very natural from a modeling point of view, such as expressed by Corcoran (1973) or by Shapiro (1998 and 2001). First, replace "$\Gamma \vDash \varphi$" with "the argument from Γ to φ is a good argument" or "reasoning that carries you from Γ to φ is good reasoning" in the usual bridge principles, and then understand the mathematical machinery associated with the entailment expression that originally appeared in the bridge principle as a *mathematical model* of the norms of good reasoning. Instead of building a bridge from entailments to norms, we should think of the entailment expression in the antecedent of a bridge principle as part of a mathematical model of the norms for reasoning, just as $PV = nRT$ is a mathematical model of the behavior of a gas and $L = L_0 + kmg$ is a mathematical model of the length of a spring stretched by a mass under gravitation. From this perspective, to say that if $\Gamma \vDash \varphi$, then one ought to see to it that one believes Γ only if one believes φ, is at best an abbreviated way of saying that *if* $\Gamma \vDash \varphi$ *is an adequate model of good reasoning*, then the normative consequence follows from the fact that $\Gamma \vDash \varphi$. But this is just because an adequate model will be a good representation of its object. Since the object is normative, an adequate model will also be normative.[5] From a modeling perspective, the apparent order of dependence in the bridge principles is backwards, and the bridge principles only work when the model is adequate.

A model is a representation, where to be a representation requires that there is a representing-relation that holds between the representation (signifier) and its object (signified).

[5] I think a similar idea is expressed in some passages in Field (2009 and 2015), but he is much less explicit about the modeling perspective. For example, after arguing against the claim that logic is "the science of what forms of argument necessarily preserve truth," Field wrote: "If validity isn't defined in terms of necessary truth-preservation (whether general or restricted to 'when it matters'), how is it to be understood? In my view, the best approach is to take [validity] as a primitive notion that governs our inferential or epistemic practices" (2009, 267). Thinking of validity as primitive, we can go on to produce a model of validity and we can draw out how validity is related to various other objects of investigation, but we need not give any conceptual *analysis*. Moreover, insofar as the primitive notion of validity *governs* our inferential or epistemic practices, any model of validity that we produce must be a model of something normative.

In virtue of being a representation, a model supports surrogative reasoning about its object. But a model is not *identical* to its object. Models have properties that their objects lack and lack properties that their objects have. As Shapiro (1998, 138) notes, "There is almost always a gap between a model and what it is a model of. Typically, one can make a model more 'realistic' (i.e. more correct) at the cost of making it more cumbersome and difficult to study and use—as happens, for example, when volumes are added and friction is considered in models of physical systems." Now, my contention, which I elaborate in Section 5, is that logic is the normative science of reasoning, so the norms of reasoning are the objects of study in logic. An adequate model in logic will capture some—but not all—of the characteristics of the norms of reasoning. The logical model will stand in the representing-relation to rules for reasoning. The modeling perspective leaves open what the right metaphysical story is for the norms or rules of right reasoning. One might be a realist or an anti-realist with respect to normativity and still talk meaningfully about the mathematical machinery as a model of the norms. In the realist case, the object of representation is independent of what we think, and hence, the fact that we have a representation is in some sense more obvious. In the anti-realist case, the object of representation is not independent of what we think. But we might still be mistaken with respect to our models. For example, we might be attempting to model the norms of a pre-existing social practice of criticizing or evaluating arguments: to give instructions for playing a social game. But on an anti-realist view, there may be feedback from the modeling to the practice itself. As Shapiro (1998, 138) puts it, "Unlike the case with what may be called 'descriptive models', it seems plausible here that a good mathematical model, if generally accepted, or generally accepted by 'experts', can come to affect, or even to constitute what counts as 'correct reasoning'."

My position has two separable components. First, I claim we ought to adopt the modelers' perspective, rather than thinking in terms of bridge principles. Descriptivists can also adopt a modeling perspective, though they won't want to accept any bridge principles as properly part of *logic* or as constitutive of the meaning of "validity" or "entailment" or related terms. Instead, a descriptivist might say that the mathematical machinery typical in logic provides descriptive models of language (syntax and semantics) or of the abstract structure of the world or of the structure of mathematics itself. In addition to advocating for a modeling perspective, I claim that logic is constitutively the normative science of reasoning (broadly construed). I defer defending the second claim until Section 5. But I want to observe here that on my view, the norms are the *objects* of the mathematical models in logic. The models codify rules that we ought to follow in reasoning. In the language of the Medieval logicians, the point of the models is to guide us in discovering and judging. In the language of Francis Bacon, the point is "to contrive and prepare helps and guards for the understanding" (1620). In the language of Descartes, the point is to "teach the right conduct of the reason with the view of discovering the truths of which we are ignorant" (1644). All models are representations, and representation is an asymmetric relation. So, the object of a model grounds the model, and the adequacy of the model depends (in part) on its object. Hence, the norms ground the mathematical models used in logic, and the adequacy of a mathematical structure as a model in logic depends on the norms being modeled. The fact that a mathematical model for the norms of right reasoning doesn't capture everything we want to say about the norms is unsurprising to anyone who thinks like a modeler. As George Box famously said, "All models are false, but some are useful." From the perspective of the modeler, Harman's complaints about classical deductive logic look like a man objecting to Harry Beck's London tube map on the grounds that it doesn't provide good estimates of the distances between stops. Thinking in terms of bridge principles seems to me to obscure all of this and makes it more difficult to see what the dispute between normativists and descriptivists is really about—the scope and object of logic as a discipline and the purpose or intended use of the mathematical models constructed in logic.

Another virtue of the modeling perspective is that it helps to explain how we ended up in the dispute over the normativity of logic in the first place. In the nineteenth century, when mathematical tools were explicitly applied to the study of logic and when it was still customary for writers to defend their use of mathematics in the study of logic (as in Venn 1866 and 1881), the relationship of the mathematics to the object of study was relatively clear. But afterward, with a powerful mathematical model in hand, researchers seem to have forgotten what the model was *for*. So, we find Corcoran (1973, 30) writing that "hardly any of the current logicians feel pressure to decide the relation between the logical and the mental, to give an account of propositions, to explicate the ground of logical consequence, etc." According to Corcoran, with a model in hand, logicians felt less "pressure to give an account of the ontological status of the subject." And subsequently, the mathematical structure itself—the model—gets identified with the object of study. This explains, I think, Harman's use of the term "logic." Moreover, as with other mathematical models, the models in logic can be applied to more than one object of investigation. And insofar as we are working with the model and investigating the model, thinking of the model as an object of investigation in its own right, it is possible for researchers to move forward together without noticing that they are engaged in fundamentally different enterprises.

So, it seems to me that in logic, we begin with an idea of what good reasoning is like and then we construct models of the norms of good reasoning—refining and improving those models over time. Modeling in logic is essentially the same as modeling in any science. We begin with some idea of our object of investigation, we construct models of the object, and then we improve our models. The difference between logic and ordinary descriptive sciences is not in the modeling activity but in the objects of study. In fact, the modeling perspective—including the claim that what distinguishes two different disciplines is the object of study—is (or can be) a point of agreement between normativists and descriptivists. The *disagreement* between normativists and descriptivists is about whether the objects of study in logic are normative.

4. *On the Autonomy of Logical Norms*

As I noted in Section 1.2, Russell thinks we have an epistemic obligation to believe the truths of logic. But according to Russell, our obligation is not a peculiarly *logical* obligation. Instead, we ought to believe the truths of logic because there are "common normative commitments concerning truth and falsity" that enjoin us to believe what is true and not what is false. Tajer (2022) challenges the normativity of logic in a related way by showing that at least in some cases, the logical norms represented in the bridge principles are reducible to more general epistemic norms.

Tajer states eight such general epistemic norms in terms of a standard deontic operator O, indicating that its argument ought to be the case, a weak epistemic operator B, indicating that its argument is believed, and a similar operator R, indicating that there is a reason for its argument.[6] So, for example, OBφ says that φ ought to be believed, and RBφ says that there is a reason to believe φ. I will need four of Tajer's norms for my discussion. They are:

$T_{\rightarrow}$: $\varphi \rightarrow OB\varphi$ $F_{\rightarrow}$: $\neg\varphi \rightarrow O\neg B\varphi$

Tr: $\varphi \rightarrow RB\varphi$ Fr: $\neg\varphi \rightarrow R\neg B\varphi$

[6] Importantly for his project, the epistemic logic uses non-normal worlds so as not to impose closure under logical consequence, which would trivialize the results.

Tajer thinks of these as truth norms, though the truth predicate does not appear in them. The idea is that one has reason to believe (Tr) or ought to believe ($T_{\rightarrow}$) what is the case, and one has reason not to believe (Fr) and ought not to believe ($F_{\rightarrow}$) what is not the case.

Tajer proves nine results, which he calls Facts, about the relationship between the general epistemic norms and various standard bridge principles. I will focus on three results (his Facts 2, 3, and 8), which in the language of this paper bear on [Wide], [Negative], and [Direct].[7] Granting the assumptions built into his choices of deontic and epistemic logics, what Tajer shows is that $T_{\rightarrow}$ and $F_{\rightarrow}$ together imply [Wide], $F_{\rightarrow}$ implies [Negative], and Tr and Fr together imply [Direct]. Tajer suggests that his reduction results present a serious problem for normativists about logic. Since the standard bridge principles can be derived from general norms for belief, "the principles … expressing the *normative role of logic* are not autonomous. Apparently, there is no normative role of *logic*, only a normative role of *truth*" (2681).

Recall that the bridge principles are supposed to codify the normativity of logic. But if the bridge principles can be derived from epistemic norms that are also accepted by normativists about logic and that are agreed to be more basic or more general than the norms captured by the bridge principles, then logic has no *intrinsic* normativity. And if logic has no intrinsic normativity, then logic itself *isn't* normative. Tajer neatly sums up the point: "If logical normativity is not autonomous, then logic is normative in the same way in which physics or geology are normative. Investigating the normative role of logic would be like investigating the normative role of any discipline whatsoever" (2681). Put explicitly, Tajer's argument goes like this:

[7] These are MacFarlane's Wo+, Wo-, and Wr+.

[T1] Bridge principles can be derived from general epistemic norms.

[T2] If [T1], then logic has no independent normative content.

[T3] If logic has no independent normative content, then it isn't normative.

[T4] Logic isn't normative.

One might object to some assumptions that are built into Tajer's choices of deontic and epistemic logics, but I will not. I take it, then, that premiss [T1] follows for specific bridge principles according to Tajer's proofs. Tajer does not engage with more complicated principles involving credence, which would be a natural next step in this line of inquiry. Perhaps more complicated bridge principles resist reduction to any plausible, more general epistemic norms. But such a finding would be surprising, at least to me. So, I think it is better to meet Tajer's challenge in a different way.

I expect that normativists who appeal to bridge principles will reject [T3]. If one thinks that normative commitments constitute the entailment facts or if one thinks that the meaning of entailment claims has to be cashed out in terms of certain normative practices, one seems to be in position to reject [T3]. Normativists who appeal to bridge principles might say that Tajer's results show how and why the logical norms might be *justified*, but those results do not in any way undermine the claim that the logical norms are really *norms*. On this kind of view, one might think of a specific logical calculus as making the justifying normative commitments formally precise and usable. I wonder if something like this is what Frege had in mind by saying that logic consists in the laws of truth and that logic is normative for belief in virtue of truth being the cognitive goal of the sciences.

Perhaps a bridge principle approach to rejecting Tajer's argument can be made to work. I am skeptical. Tajer's argument seems stronger than either Harman's or Russell's. But whether a bridge principle approach can be made to work or not, I think there is a better way for normativists about logic to go. I reject [T2]. Since logic is directly the study of norms for reasoning, the so-called epistemic norms that Russell and Tajer appeal to are properly logical norms. What Tajer shows, then, is how to reduce some logical norms to other logical norms. In the next section, I finally come to my main point and complete my replies to Harman, Russell, and Tajer.

5. *What is Logic?*

In a remarkable lecture delivered in February 1898, titled, "The First Rule of Logic," Peirce argued that all scientific methods of inquiry have "the vital power of self-correction and of growth," and therefore, "there is but one thing needful for learning the truth, and that is a hearty and active desire to learn what is true." He went on to say (EP 2, 47-48):

> If you really want to learn the truth, you will, by however devious a path, be surely led into the way of truth, at last. No matter how erroneous your ideas of the method may be at first, you will be forced at length to correct them so long as your activity is moved by that sincere desire. ...
>
> Upon this first, and in one sense this sole, rule of reason, that in order to learn you must desire to learn and in so desiring not be satisfied with what you already incline to think, there follows one corollary which itself deserves to be inscribed upon every wall of the city of philosophy,
>
> **Do not block the way of inquiry.**

A person trained up in mathematical logic as it is presented in philosophy departments today and reading Peirce for the first time might find it odd to see Peirce's exhortation to *desire to learn* described as a rule of logic at all, let alone the *first* rule of logic. But it is exactly in line with Peirce's conception of logic as the study of norms or rules for *reasoning* or *inquiry*, where reasoning is understood as a deliberate, controlled process of replacing doubt and ignorance with stable and reliably action-guiding belief, which Peirce called *knowledge*.[8] I do not pretend to have a comprehensive grasp of the subtleties of Peirce's philosophy of logic, and I do not intend to give an interpretation of Peirce in this section. My goal is to articulate an account of logic that is inspired by Peirce, and I invite everyone to join me in conceptualizing logic in the way I do. One consequence of adopting my account of logic is the establishment of the normativity of logic in a way that is different—and I think more satisfying—than using bridge principles.

Before continuing, let me be clear about how I use some terms. I distinguish between three senses of the term *logic*: the discipline of study, the objects of study in the discipline (i.e. its subject matter), and the models of the objects of study (i.e. specific linguistic or mathematical structures). I understand logic, as a discipline, to be constitutively the study of the norms of right reasoning, just as ethics is constitutively the study of the norms of right action. Logic is the discipline that studies how we *ought* to reason, that *evaluates* reasoning, that categorizes reasoning into *good and bad*, and that gives *advice* about how to reason. Logic also provides an account or explanation for the correctness of particular judgments of logical goodness, in the same way that ethics provides an account or explanation for the correctness of particular judgments of ethical goodness. Just as a consequentialist and a deontologist might agree that killing an innocent person is wrong while disagreeing as to *why*, a classical logician and an intuitionist might agree that inferring a conditional from its consequent is correct while disagreeing as to *why*. Any discipline that is not ultimately aiming to characterize the norms of right reasoning is not logic, even if it is superficially similar to logic.[9]

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[8] I would add a non-accidentality requirement to Peirce's implicit theory of knowledge.

[9] We sometimes divide ethics into three levels: applied ethics, normative ethics, and meta-ethics. And normative ethics is often just called "ethics" simpliciter, since it is the most important of the three. The parallel I have been

One's account of the norms might be quite simple and tidy. For example, one might suppose that there is some peculiar *logical good* (perhaps Truth) and some peculiar *ethical good* (perhaps Flourishing), and then one might argue that there is just one norm in logic and just one norm in ethics: to maximize the relevant good. Alternatively, one might suppose that there are a number of competing norms (either with respect to the same good or with respect to various competing goods) or one might suppose that the norms of right reasoning and of right action are prior to any notion of the good. The shape of the ensuing arguments should be familiar to professional philosophers, so I will not belabor the point. But summing up, I observe that on my account, logic is normative in the same way that ethics is normative. Both have norms as their subject matter, and both have some aspiration to correct or instruct us with respect to things we do naturally and do more or less well.

The norms of right reasoning themselves are also sometimes called *logic*. When we say we are studying logic, we do not mean that we are studying the *discipline*. Instead, we mean we are studying the subject matter—what the discipline investigates. The same thing is true of ethics. When we say that someone who reasons badly has no logic or is not reasoning logically, we are saying substantially the same thing as when we say that someone who behaves badly has no ethics or is not behaving ethically.

At least since the time of Leibniz, the discipline of logic has made progress by constructing mathematical models of the norms of right reasoning. Hence, studying the logical norms involves studying some characteristic mathematical structures such as Boolean algebras and axiomatic set theories, just as studying quantum mechanics involves studying some characteristic mathematical structures, such as Hermite polynomials and linear operators on

drawing between logic and ethics suggests a similar structure for logic. What I have been describing as logic corresponds to normative ethics and so might be called normative logic.

complex vector spaces.[10] But the mathematical models are not *identical* to the objects of study in the discipline of logic. Hence, *a* logic is a specific model of the norms of right reasoning. As noted by Burgess (1992) and Shapiro (2001), there are (at least) two distinct ways in which we might think of a model as normative. We might understand the model as *describing* the existing norms for a community, where members are typically competent followers of the norms. A logical model is normative (insofar as it accurately captures the actual norms) for the reasoning performances of members of the community but is not a challenge to their competence. Alternatively, we might understand the model as *prescribing* norms for a community that either has no relevant norms in place or that (according to us) should revise its rules and practices. A logical model is normative in the second sense (insofar as it captures what the norms *ought to be*) for the reasoning competence of members of the community, essentially saying that this is what competence looks like.[11]

Many, perhaps most, philosophers today think of logic as the discipline that studies deductive consequence. One reason is the link that was forged by logicist philosophers and mathematicians at the end of the nineteenth and beginning of the twentieth centuries. I read Shapiro (2001) as broadly under this influence in his provocative discussion of revisionist accounts of correct reasoning *in mathematics*.[12] He writes (163):

[10] In the main text, I have restricted attention to deductive logic, but from my perspective, we could add several other mathematical structures, such as sigma fields (for probability theory) and Baire spaces (for formal learning theory), which are characteristic in studying non-deductive reasoning.
[11] Some philosophers have maintained that the discipline of logic is normative in the sense of aiming to characterize what is required in order to count as thinking at all (see Conant 2020; Mezzadri 2015a, 2015b; and Steinberger 2017 for recent discussions). In other words, logic, in the sense of the norms themselves, is constitutive for thought. I do not see this view as fundamentally competing with my own account. Instead, I see those who think of logic as constitutive for thought as addressing an important *part* of logic but not the whole. Since logic is the study of the norms of right reasoning, it must include—as a proper part—the requirements that have to be satisfied in order to count as reasoning at all, just as any complete account of how to play a game *well* must include an account of the rules of the game.
[12] An alternative reading is that Shapiro takes logic to be the general study of correct reasoning but is restricting attention in the target essay to the special case of mathematics.

> There are norms of reasoning that are implicit in, or somehow underlie, ordinary mathematical practice. A descriptive account is an attempt to uncover or describe those very norms, by constructing idealized mathematical models of them. In contrast, a normative logic is an attempt to say what the norms should be, never mind what the norms are. The normative logician claims that the norms that are implicit in practice are flawed, and he proposes better ones.

Or as he writes a bit later in that essay, "Most logicians are resistant to such revisionism. We are out to model *mathematics*, not some supposedly improved substitute for mathematics" (166). But historically, logic was not identified with the theory of deduction. Leibniz, Boole, De Morgan, Venn, Peirce, and Ramsey all explicitly counted probability as a part of logic. Even Aristotle in his logical writings reflects on inductive generalization and explanation. Logic is not synonymous with syllogistic. Hence, Shapiro's restriction of logic to an account of correct reasoning *in mathematics* strikes me as artificial. The logician is out to model *correct reasoning*, which includes but is not exhausted by correct reasoning in mathematics.

Many philosophers have divided reasoning into two broad categories, usually labeled deductive and inductive today. For example, Peirce (1878) divides reasonings into "Explicative, analytic, or deductive" on the one hand and "Amplifiative, synthetic, or (loosely speaking) inductive" on the other. Ramsey (1926) quotes Peirce approvingly. Carnap (1952, 3) writes:

> Any reasoning or inference in science belongs to one of two kinds: either it yields certainty in the sense that the conclusion is necessarily true, provided the premises are true, or it does not. The first kind is that of deductive inference including all transformations or calculations in pure mathematics (arithmetic, algebra, analysis, etc.). The second kind will here be called '*inductive inference*'.

Contemporary philosophers, such as John Norton (2003, 2005, 2014, 2021) explicitly and Graham Priest (2006) implicitly, also divide inferences into deductive and inductive.[13] The

[13] Plausibly, the division of reasoning into deductive and inductive is substantially the same as Leibniz's division of truths into those of reason and those of fact and Hume's division of reasonings into those concerning relations of ideas and those concerning matters of fact.

division is serviceable and clearly tracks an interesting feature of reasoning, which explains its long endurance. But I think we can do better.

The division of reasoning into deductive and inductive corresponds to a simple two-phase model of inquiry. Both Descartes' hypothetico-deductive method and Bacon's inductive ledger model are two-phase models. In his *Illustrations of the Logic of Science*, Peirce explicitly introduced a three-phase model, which he continued to defend throughout his career and elaborated in a long paper "On the Logic of Drawing History from Ancient Documents, Especially from Testimonies" of 1901. Later in the twentieth century, Feynman (1965, 156) described the process of looking for new physical laws as having the same three-phase structure, though using different language. In Peirce's language, inquiry proceeds by abduction (hypothesis), deduction, and induction. In Feynman's language, we look for new physical laws by guessing, computing the consequences of our guesses, and then comparing the computed consequences with nature. But later in his lecture, Feynman says that the three-phase model tends to "put experiment into a rather weak position" (157), and his discussion suggests some elaborations on Peirce's three-phase model of inquiry.

I suggest a five-phase model of inquiry, though I recognize that as a model, it is bound to be inaccurate in various ways. On my model, inquiry begins with wonder (or surprise or an itch of curiosity). It then has a two-part abductive phase in which hypotheses are first generated and then selected for pursuit. The selection of pursuit-worthy hypotheses is followed by a deductive phase and finally an inductive testing phase. Each phase of inquiry has a governing logic—a collection of norms for carrying out that phase well. So, logic as a discipline includes the study of norms for being surprised by experience (and hence, motivated to conduct inquiry), norms for generating guesses or building models for better understanding experience, norms for selecting from possible guesses some single guess or some collection of guesses to seriously investigate (which activity is costly in terms of time and energy), norms for unfolding the commitments embedded in accepting a theory as true, and norms for testing our commitments. The character of the constitutive aim of each phase of inquiry gives the corresponding logical rules their normative force. Suppose an agent S says, "I want *x* under constraints *y, z, …*," where the constraints determine what it means for S to optimally obtain *x*. Further suppose that S recognizes that following rules codified by L would allow them to obtain *x* optimally relative to their constraints. It wouldn't make sense (apart from weakness of the will) for S to then refuse to follow the rule.

Logical disputes, when they are genuine, are almost always ultimately disputes about the aims appropriate to the relevant phase of inquiry, since the question of whether a well-specified aim is optimally satisfied by a well-specified technique almost always results in consensus. Otherwise, trying to solve debates by sitting down to calculate would be a non-starter. In the case of deduction, we might dispute whether we think the aim of unfolding commitments is to see what has to be the case if some hypothesis is true (which is the classical story) or to see what can be constructed on the basis of the hypothesis (which is the intuitionist story). As has been observed many times in the debate about logical pluralism, no one is confused about whether a given inference is *classically valid* or *intuitionistically valid*.

Now consider the case of induction. As I observed in Section 1.1, Harman was skeptical that there is any inductive logic. In his 1986 book, he writes (5), "Rules of inductive argument would be rules of 'inductive logic' as opposed to deductive logic. It happens, however, that there is no well-developed enterprise of inductive logic in the way that there is for deductive logic." Harman was amusingly wrong when he wrote those words, and the claim is uproariously wrong today. The core problem is that there is a well-developed collection of norms for the inductive, testing phase of inquiry. The entire discipline of statistics, and especially the inferential part of statistics, is devoted to the logic of inductive testing (see Romeijn 2011 for a similar view). The problem for inductive logic, insofar as there is a problem at all, is that there are *too many* well-developed enterprises of inductive logic that disagree as to the aim of inductive testing! To a philosopher who thinks there are no serious debates as to the one true deductive logic, it might seem that there is a serious contrast between deductive logic, which has a *canonical* development, and inductive logic, which does not.

The central goal of inferential statistics is to characterize (some features of) a future data-stream or of the data-generating mechanism on the basis of observed data drawn from the stream or produced by the mechanism. There are (broadly-speaking) two traditional approaches to inferential problems in statistics.[14] I will here describe the two approaches with respect to the problem of estimation. Following a simple recipe in Kruschke (2013), the *Bayesian* approach has four steps: [1] Build a probability model, M_{data}, of the data-generating process; [2] Build a probability model, M_{bel}, of your credences for the parameters of M_{data}; [3] Collect data; [4] Apply Bayes' Rule of Conditionalization to update M_{bel}.[15] From the updated M_{bel}, one may read off a point estimate (best guess) for the parameter and an interval that carries with it whatever degree of belief one requires. Such intervals are called *credible intervals*. The simplest Frequentist approach to estimation assumes that we are sampling at random from the data-generating mechanism, under a predesignated stopping rule. We can think of our sample as being drawn from a collection of *possible* samples, and the stopping rule lets us characterize a distribution

[14] As I.J. Good (1971) points out, these are better understood as rather large families of approach, but I want to keep the discussion somewhat manageable.
[15] Bayes' Rule says to update one's credence with respect to a proposition *h* after seeing evidence *e* by setting the new credence of *h* equal to the old conditional credence of *h* given *e*.

with respect to the possible samples, called the *sampling distribution*. We may then read off a point estimate from the sample itself by taking the maximum likelihood and use the sampling distribution to construct intervals representing the reliability of the sampling-and-estimation procedure. Such intervals are called *confidence intervals*.[16]

The Bayesian approach has an internalist, evidentialist aim in view. Bayesians emphasize consistency, which is an internal characteristic of our doxastic states, and they update their beliefs according to a rule that is only sensitive to their actual evidence. The Frequentist approach has an externalist, pragmatist aim in view. Frequentists emphasize getting the truth, which is an external characteristic of our doxastic states, and they update their beliefs by following the recommendations of provably reliable methods.

Let's step back and take stock. I claim that the discipline of logic is constitutively the normative study of right reasoning. Setting aside the structure of inquiry, we might think of the logical norms as the output of a function that yields optimal means to a specified cognitive aim (such as getting the truth) given the aim and some constraints (such as using only mechanical procedures) as the function's arguments.[17] The discipline of logic studies that function, and we construct models of the norms in part to understand ourselves and our world and in part to give

[16] Another dimension along which we might consider debates about the inductive testing phase of inquiry has to do with whether we should be concerned with a quantitative notion of *partial belief* or rather with a qualitative notion of *full belief*. Qualitative accounts maintain that belief is all-or-nothing. With respect to any proposition φ, one either believes that φ or one does not believe that φ. Quantitative accounts maintain that belief comes in degrees. With respect to any proposition φ, one can have more or less conviction or confidence that φ is the case. Some philosophers are strict partisans of one or the other account of belief. But many think that both accounts get something right. If you think that we have both full beliefs and partial beliefs, then you face a challenge of harmonizing the two accounts. One obvious and initially attractive way to harmonize the two approaches is by setting a threshold and letting an agent have a full belief that φ iff the agent has a partial belief that φ with degree greater than the threshold. Turning this idea into an account of belief revision generates the so-called *Lockean* account of belief revision. An alternative qualitative account is the so-called *AGM* theory of belief revision. For discussions of belief representation and revision, see Easwaran (2016), Genin (2020), Huber (2013a, 2013b, and 2020), Lin (2020), and Shear & Fitelson (2019).

[17] Separating the aim and constraints is really a notational choice, since one could always embed the constraints in the aim itself.

ourselves and others guidance under the assumption that our models are reasonably good accounts of the norms. So, I come to the following view. Suppose I have some cognitive aim *x* and am subject to constraints {*y*, *z*, …}. Further suppose I believe that I will achieve x under the constraints if I follow the rules codified in L, and I believe there is no real alternative that I would prefer—by which I mean that there is no alternative I would prefer that isn't simply a notational variant of L from my point of view. Then I will (or would under ideal conditions) agree to follow L or to be *bound by* L. Thus, L codifies logical norms I accept.

I am now in position to answer Harman, Russell, and Tajer. My reply is immediate and trivial. The discipline of logic is constitutively a normative science. Logic studies the norms of right reasoning, which it characterizes by constructing models that attempt to capture the norms. Hence, I reject [H1] in Harman's argument. Logic is not the study of entailment, it is the study of norms of right reasoning. Elaborating a bit, Harman's challenge is confused. If he means to be suggesting that logic the discipline is not normative, he should be answered in the same way as if he had suggested that ethics is not normative: logic is normative because that's what "logic" means. If he means to be suggesting that *a* logic—meaning a model of the norms—is not normative, then we need two kinds of clarification. Does he mean that there is no normative goal to the modeling work? If so, then he is wrong in virtue of the fact that the model is a logical model. Alternatively, he might mean that the model isn't fit for its intended work. But in that case, we need to know both *which* model we're specifically talking about and *what* normative work the model is supposed to do. I submit that deductive logics are meant for unfolding our commitments, and although there is plenty of room for dispute about details, all of the deductive logics succeed, broadly, in doing that normative work.

According to Harman, reasoning is reasoned change of belief. If by "reasoned" we mean something like deliberate and controlled, then Harman is already in the neighborhood of what Peirce and I would call *inquiry*. If we further require that reasoning has an aim of producing in the reasoner beliefs that are stable and reliably guide our actions, then reasoning and inquiry are just the same thing. Now, Harman maintains that reasoned change of view is governed by norms of explanatory coherence and conservatism. But then, it seems to me that he has already accepted that logic is normative for reasoning by accepting that there are norms governing reasoned change of view. Harman writes (1986, 5), "If we clearly distinguish reasoned change in view from argument, we cannot suppose that the existence of inductive reasoning by itself … shows there is an inductive logic." But that is exactly what it *does* show. Whenever there is a cognitive activity that admits of evaluation, there is a corresponding logic. A logic need not have the form of a collection of claims about entailment relations.[18]

For the same reasons, I reject [R1] in Russell's argument. Logic is not the study of truth-relations in languages. Logic is the study of the norms of right reasoning. Truth relations, including entailment facts, plausibly have direct relevance when we are thinking about the norms of deductive logic. They also plausibly matter when we aim to justify norms of reasoning, especially non-deductive norms. Similarly, truth relations plausibly matter when we aim to explain why we ought to follow specific norms for reasoning. But truth relations are not the object of study in logic. To think that they are is to confuse a model for the object: it is to confuse a map for the territory. Hence, I find Russell baffling when she writes (2020, 375):

> I also hope that [normativists about logic] will be untroubled by the strengthening of their claim to one which says that singleton sets of E-sentences have normative consequences, since the usual arguments and examples … suggest that this is wholeheartedly accepted

[18] Of course, *one thing* that one studies in logic is the way truth is preserved by argument. But surely that's not the *only* thing one studies in logic. One might, for example, want to know how truth is *promoted* or *expanded*, either by argument or by reasoning or inquiry.

> by defenders of the normativity of logic. While the view has indeed undergone development and become more subtle in response to the famous objections of Harman (1986) and others (e.g. Celani 2015), what has evolved has been the sort of normative claim that is said to be entailed by an E-sentence, while the overall position that an E-sentence is what does the entailing persists throughout. This is left intact by the explicit argument above, leaving it neutral between the different views of what the normative consequences look like.

I am troubled. If an E-sentence is supposed to be a claim about a formal language (or a natural language) in the way suggested in the opening of Russell's essay, then I reject the claim that an E-sentence *entails* a normative claim. As I said in Section 3, to think of a normative claim as following from an E-sentence is to get the dependency backwards. The bridge principle approach invites such a reversal, which explains why Russell thinks normativists should be untroubled by the claim that a singleton set whose member is an E-sentence has normative consequences.[19] But E-sentences do not entail normative claims. Rather, E-sentences are *models of normative claims*. To give a mathematical account of logical consequence is to give a precise characterization of *good reasoning*, *correct inference*, *permissible thought*, or the like.

My reply to Tajer is that he has narrowed the scope of logic as a discipline, and relatedly, he has narrowed the class of logical norms. Since logic is the study of the norms of right reasoning, the so-called epistemic norms that Russell and Tajer appeal to are themselves *logical* norms. Hence, I reject [T2] in Tajer's argument. Bridge principles can be derived from general epistemic norms, just as he says. But that is just to say that specific logical norms can be derived from more general logical norms. That result is interesting, but it is no threat to the normativity of logic.

Why adopt my conception of logic? In closing this section, I want to sketch four arguments. Then in the concluding section, I will tie up some loose ends and consider how a

[19] If E-sentences entail normative claims, shouldn't it be the E-sentence itself and not the singleton set containing the E-sentence that has normative significance?

number of issues in the philosophy of logic appear from the point of view I am recommending. I take those considerations together with the responses to Harman, Russell, and Tajer to constitute a large, fifth argument from something like conceptual integrity and fruitfulness. Roughly, one should adopt my conception of logic because it offers a coherent and illuminating account of logic and philosophy.

First, there is an argument from history and the ethics of terminology. According to the most ancient accounts of logic and accepted usage for hundreds of years thereafter, "logic" names a normative discipline intimately related to *reasoning* as opposed to overt practical *behavior*. The specific cognitive activity that logic is supposed to be normative *for* may, possibly, have shifted over time. Reflecting on a theme in medieval texts on logic, Novaes (2015) suggests that logic is normative for dialectic, as opposed to being normative for individual thought. But it seems to me that there is rather a lot in common among the activities of argumentation, inference, dialectic, inquiry, reasoning, and thought. Unless there are strong reasons for revising terminology, we should follow the oldest uses. Hence, we should use "logic" to refer to a normative discipline intimately related to reasoning.

Second, there is an argument from the importance of the subject matter. We want to have a normative science of reasoning—a discipline dedicated to studying the norms of right reasoning. Harman, Russell, and Tajer all agree that there *are* norms of reasoning. They just don't want to apply the label "logic" to those norms or to the discipline that studies them. But they *should* apply "logic" as I do for three related reasons. As observed in my first argument, there is historical precedence for using "logic" as the name of the discipline that studies the norms of right reasoning. In part owing to the history, the term "logic" is widely *understood* as referring to a discipline that aspires to tell us how we ought to reason. So, using the term "logic" to pick out the normative science of reasoning is well-motivated from a public relations perspective. Moreover, there isn't any *better* label with wide public appeal for the discipline, whatever it is, that studies the norms of right reasoning. Hence, we should use the label "logic" for the normative study of reasoning, as I do.

Third, there is an argument from demarcation. The basic idea is that logic is a distinctive sub-discipline of philosophy, narrowly construed. If so, then logic is normative. For if logic were not normative, then it would be indistinguishable from mathematics or linguistics or psychology or metaphysics, depending on which way one understands it to be descriptive. Russell considers and dismisses a demarcation argument, and I agree with her that *on its own* it isn't a compelling argument. However, it seems to me to hang together very well with the other arguments, so that all of them gain cogency as a result.

Fourth, there is an explanatory argument from the centrality of logic to philosophy. When explaining what we philosophers do and what value we add to the academy, we frequently say that we focus on good reasoning and the critical evaluation of arguments. We say that we teach students to reason well. Moreover, we justify our claims in part by requiring the study of logic at both the undergraduate level and at the graduate level (at least, in most philosophy programs). Logic is central to philosophy. It is central to the practice of philosophy and central to our self-conception as philosophers. But what explains the centrality of logic to philosophy? If logic were the narrow study of deductive entailment, its centrality to philosophy would be, at best, non-obvious. But if logic is the normative science of reasoning, then its centrality to philosophy *is* obvious. We ought, tentatively, to adopt our best explanations of otherwise puzzling facts. So, we ought, tentatively, to say that logic is the normative science of reasoning.

6. *Concluding Remarks*

So far, I have described three challenges to the claim that logic is normative for reasoning, I have discussed the usual approach to defending the normativity of logic by appeal to bridge principles, I have advocated for an alternative account that treats specific logical systems as models of norms of reasoning, I have explained how my account works and how it underwrites replies to the challenges from Harman, Russell, and Tajer, and I have given arguments independent of the normativity debate in favor of conceptualizing logic as the normative study of reasoning. In this section, I want to tie up a half-dozen loose ends and consider how a number of issues in the philosophy of logic appear from the point of view I am recommending.

6.1 *Exceptionalism*

Is logic exceptional? I take the *science* seriously in the phrase "normative science." I agree with anti-exceptionalists that "Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories" (Hjortland 2017, 632). Insofar as our mathematical models of the norms of right reasoning serve a practical purpose, they admit evaluation. A model may be better or worse relative to the practical point of the modeling task. This seems to make logic continuous with the rest of human inquiry: logic is not exceptional.

One might worry that if logic is really a *normative* science, then it *is* exceptional in virtue of having a peculiar kind of subject matter. At a first pass, it's unclear to me why the descriptive-normative dimension should be any more interesting than the organic-inorganic dimension in chemistry or the social-natural dimension in the descriptive sciences. The really interesting consequence is that ethics and aesthetics—the other two traditional normative sciences—are not exceptional either.

6.2 *Truth, Dialectic, and Inquiry*

Novaes (2015) directs our attention to the question of what logic is supposed to be normative *for*. According to Kant and Frege, logic is normative—and perhaps *constitutive*—for thought. But an older tradition understands logic to be normative for dialogue or dialectic. What are the norms governing our practice of discussion and argumentation? Since dialogue is closely related to assertion, one might try to derive norms for dialogue from norms for assertion.

To take one example, Price (1988) argues that truth is normative "in a way not explained by the deflationary theory" (241). He proposes a distinctive truth norm,

[Truth] If it is not true that φ, then it is wrong to assert that φ.

and Price (2003) explains the value of the truth norm as providing the friction required to start a dialogue in response to a disagreement.

The main idea in Novaes is that deductive logic is normative for dialectic, where dialectic is a specialized practice that is related to ordinary dialogue. (By analogy, think of dialectic as sport fencing and dialogue as "live" sword fighting "in the wild.") Novaes might be right if we understand dialectic in a particularly narrow way. But if we are thinking of dialectic as itself a kind of model whose rules are supposed to be applicable to ordinary dialogue, then the absence of broadly inductive norms seems to be an especially large defect.

However, there is another sense in which emphasizing dialectic is very helpful. Dialectic is often thought of as *competitive* or *combative*. But it could be thought of as *cooperative* instead. If we understand dialectic as cooperative, and especially if we understand dialectic as having a shared, communal aim, then two things emerge. First, reasoning looks like a degenerate case of dialectic; and second, dialectic looks suspiciously like communal inquiry. Shifting focus to communal activities means that the relevant cognitive aims are likely to be *group* aims, rather than *individual* aims.

6.3 *Formality*

I understand logic to be the discipline that aims to characterize *normatively correct reasoning*. However, as Ramsey (1926) observes, there is a peculiarity in the idea that logic is normative. He writes (80), “We may agree that in some sense it is the business of logic to tell us what we ought to think; but the interpretation of this statement raises considerable difficulties. It may be said that we ought to think what is true, but in that sense we are told what to think by the whole of science and not merely by logic.” Suppose we were given a list of all the truths in our world. Presumably, we ought to believe all and only the propositions on the list. But the list would not constitute a logic. Nor would producing the list be the aim of logic. Logic aims at characterizing correct *reasoning*. What we want is a method or rule that could be applied *whatever the facts might be*. We want to be reliably guided to the truth (or whatever our cognitive aim happens to be) provided it is possible to get the truth (or other cognitive aim) at all. In order to give us the kind of security we want, we need standards and norms that are insensitive to the contingent facts. In this sense, logic aspires to be *formal*. Put differently, we want standards and norms that *necessarily* deliver on their aims (or that necessarily deliver on their aims in worlds where those aims are achievable at all).

6.4 *Pluralism*

One upshot of my view is that the question of logical pluralism arises at two distinct levels. First and most obviously, one might be a pluralist or not within each phase of inquiry. Second, one might be a pluralist or not with respect to the whole process of inquiry. For example, one might consider versions of inquiry that differ with respect to their aim. We might conduction an inquiry in order to get a true answer to our question or in order to know the answer or in order to achieve understanding.

6.5 *Logic's Relation to Mathematics*

If logic is the normative science of reasoning, then logic covers significantly more territory than the theory of deduction. Why is it, then, that logicians have focused so much on deduction? Why, for example, do we see Priest (2000) introducing logic as "the study of what counts as a good reason for what, and why" and then proceeding to develop only a deductive logic? My conjecture is that deduction came to be identified with logic as a whole owing to the influence of logicist philosophers of mathematics at the beginning of the twentieth century. So, in this section, I want to reflect briefly on logicism and the relationship between mathematics and logic. I begin with two logicist theses:[20]

[Conceptual] Mathematical concepts are reducible to logical concepts.

[Doctrinal] Mathematical knowledge is based on logical knowledge.

The conceptual thesis is a claim about ontology after a semantic ascent. How should we think about our mathematical conversation and specifically about our talk of mathematical objects? The conceptual thesis says that the objects of mathematics are (at most) just elaborate constructions out of logical objects. Given some language of logical concepts and the instructions for reduction, we have a language of mathematical concepts. Ultimately, then, the truths (or at least, the theorems) of mathematics are (analytic) truths of logic. The doctrinal thesis

[20] I first saw these clearly discussed in Haack (1993). I have adopted the labels [Conceptual] and [Doctrinal] because the two theses seem to me to correspond to the two sides of epistemology in Quine's (1969) "Epistemology Naturalized." For recent discussions of logicism, including contemporary defenses, see Boolos (2020), Leitgeb et al. (2024), and Tennant (2023).

is a claim about mathematical epistemology. Our apparent mathematical knowledge is extremely puzzling. On the one hand, mathematical claims seem maximally secure. They are (or at least seem) self-evident and a priori (or at least as close to those things as one will ever find). On the other hand, mathematical objects are abstract in a way that makes our access to them mysterious at best. Logicism explains the epistemic status of mathematics by way of the epistemic status of logic and the conceptual reduction of mathematical objects to logical ones.

The first thing for me to say is that there is no immediate incompatibility between my conception of logic and logicism. If mathematics can be reduced to and founded on *deductive* logic, then it can be reduced to and founded on logic simpliciter. In fact, my conception opens space for the reducing logic to be entirely de novo, so long as that logic has the relevant security and reducing power. Of course, we need to be clear with respect to what the reducing logic *is*, since we will get very different answers to the question of whether mathematics is reducible to logic for different proposed logics. Moreover, we should be especially careful here about the relevant sense of "logic." If we mean that mathematics is reducible to a mathematical model of the logical norms, then it seems to me we have only succeeded in reducing mathematics to other mathematics. If we mean that mathematics is reducible to the norms of reasoning, then mathematics is normative if logicism is true. That strikes me as a very strange result! Perhaps it is true nonetheless, but in my view, mathematics is a descriptive study of pure structure. So, it seems to me that logicism is either unhelpful (in that it doesn't cash out mathematics in terms of anything else) or false (in that it wrongly says that mathematics is normative).

The doctrinal thesis is also ambiguous with respect to the difference between *logica utens* and *logica docens*. If we understand the thesis in terms of *logica utens*, then the claim seems unobjectionable but also uninteresting. The reasoning that we actually deploy in mathematics is quintessentially secure. That was one of our starting points. But then mathematics doesn't really need any *help* from logic, at least not as a field of study. If we understand the thesis in terms of *logica docens*, then it's not at all clear that it's true. For one thing, mathematics developed independent of logic for most of its history. But more importantly, it's not clear that the logical "foundations" are as secure as mathematics itself. There is probably more to be said in favor of logical theory here, but it seems to me that we're frequently explaining the clear with the obscure when we "reduce" mathematics to an explicit logical theory.

In a different way, the snake eats its own tail. Logic studies the norms of reasoning, including the norms of reasoning in mathematics, and sometimes, our reasoning is out of alignment with the norms such that our reasoning requires revision or correction. Mistakes have happened and been corrected in the history of mathematics, and perhaps some sweeping revision is called for. But although logic does study the norms of reasoning in mathematics, we apply mathematics to the study of those norms. How do we know that the mathematics we apply is applied correctly? That is a fundamental problem that I am not presently ready to answer, though I think Huber (2017) is heading in the right direction when he argues [1] that deduction and induction are justified relative to different cognitive ends, and [2] that "we have an inductive justification of classical deductive logic relative to the cognitive end of reasoning in an actually truth-preserving way" (528).

6.6 *Is it a Verbal Dispute?*

Right at the beginning of Chapter 1 of the excellent textbook *Epistemology: A Contemporary Introduction*, Goldman (2015, 3) writes:

> Epistemology is the study of knowledge and related phenomena such as thought, reasoning, and the pursuit of understanding. It is less a study of customary thinking processes … than a study of better versus worse ways to think, reason, and form

> opinions. Moral theory reflects on what is right and wrong in the sphere of action, while epistemology reflects on what is rational or irrational, justified or unjustified, in the sphere of the intellect.

Goldman's description of epistemology sounds suspiciously like my account of logic. Is the debate about the normativity of logic just a verbal dispute (at least from my point of view)? Well, it seems to me that every dispute is a verbal dispute under some description. But I think the disagreement is not merely verbal in any interesting sense. More plausibly, the debate is part of a project of conceptual engineering or metalinguistic negotiation. We are trying to sort out how best to think and talk about a constellation of related fields and problems, and it is not at all obvious that any one-to-one substitution or even rather more elaborate translation scheme will let us move easily between descriptivist views like Russell's and normativist views like mine.

References