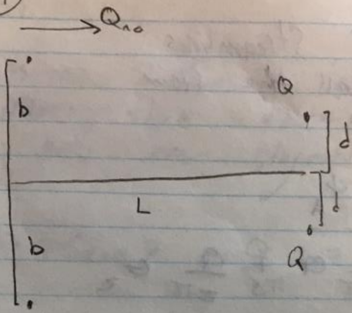
1



1) Want d s.t. there is one Stagnation point, (W=0 at one point)

$$\Omega = -Q_{xo} + \frac{Q}{2\pi}\ln(z-id) + \frac{Q}{2\pi}\ln(z+id) + c$$

$$W = -\frac{d\Omega}{dz} = Q_{xo} - \frac{Q}{2\pi}\frac{1}{z_s-id} - \frac{Q}{2\pi}\frac{1}{z_s+id} = 0$$

$$\Downarrow$$

$$z_s^2 - \frac{Q}{\pi Q_{xo}} z_s + d^2 = 0$$

$$\Downarrow$$

$$z_1 = z_2 = \frac{Q}{2\pi Q_{xo}} \pm \sqrt{\left(\frac{Q}{2\pi Q_{xo}}\right)^2 - d^2}$$

$$\Rightarrow \boxed{d = \frac{Q}{2\pi d_{xo}}}$$

1) cont
want the outer streamlines to capture all the flow between them

$$0 = \psi_B - \psi_A$$

$$\psi = Q_{xo} y + \frac{Q}{2\pi} \theta + \frac{Q}{2\pi} \theta_2$$

$$\psi_B = Q_{xo}(-b) - \frac{Q}{2\pi}\left(\pi - \arctan\frac{b+d}{L} + \pi - \arctan\frac{b-d}{L}\right)$$

$$\psi_A = \left(Q_{xo}(b)\right) + \frac{Q}{2\pi}\left(\pi - \arctan\frac{b-d}{L} + \pi - \arctan\frac{b-d}{L}\right)$$

$$0 = \psi_B - \psi_A = -2Q_{xo}b - \frac{Q}{\pi}\left(2\pi - \arctan\frac{b+d}{L} - \arctan\frac{b-d}{L}\right)$$

$$\Rightarrow Q = \left(-2Q_{xo}\, b\, \pi\right)\left(2\pi - \arctan\frac{b+d}{L} - \arctan\frac{b-d}{L}\right)^{-1}$$

2)

Assume "d from the latter design" means that there exists only one stagnation point

$$\Omega = Q_{x0} z + \frac{Q}{2\pi} \ln z + \frac{Q}{2\pi} \ln(z - 2d)$$

$$W = Q_{x0} - \frac{Q}{2\pi}\frac{1}{z_s} - \frac{Q}{2\pi}\frac{1}{z_s - 2d} = 0$$

$$W = Q_{x0} z_s (z_s - 2d) - \frac{Q}{2\pi}(z_s - 2d) - \frac{Q}{2\pi} z_s$$

$$= Q_{x0} z_s^2 - 2d\, Q_{x0} z_s - \frac{Q}{2\pi} z_s + \frac{Q}{2\pi} 2d - \frac{Q}{2\pi} z_s$$

$$= Q_{x0} z_s^2 - \left(2d\, Q_{x0} + 2\frac{Q}{2\pi}\right) z_s + \frac{Q}{2\pi} 2d = 0$$

need "$b^2 - 4ac$" $= 0$

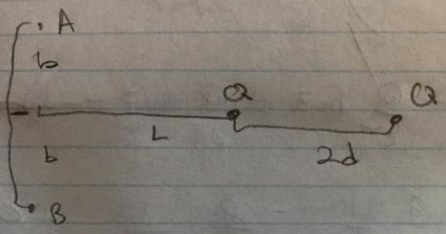
$$\left(2d\, Q_{x0} + 2\frac{Q}{2\pi}\right)^2 - 4\left(Q_{x0} \frac{Q}{2\pi} 2d\right) = 0$$

from the book 8.148

$$\frac{Q}{\pi Q_{x0}} = \Delta d$$ so $2d$ from the first line is $\frac{Q}{\pi Q_{x0}}$

2) cont.

Want $0 = \Psi_B - \Psi_A$



$$\Psi_B = Q_{xo}(-b) - \frac{Q}{2\pi}\left(\pi - \arctan\left(\frac{b}{L}\right) + \pi - \arctan\left(\frac{b}{L+2d}\right)\right)$$

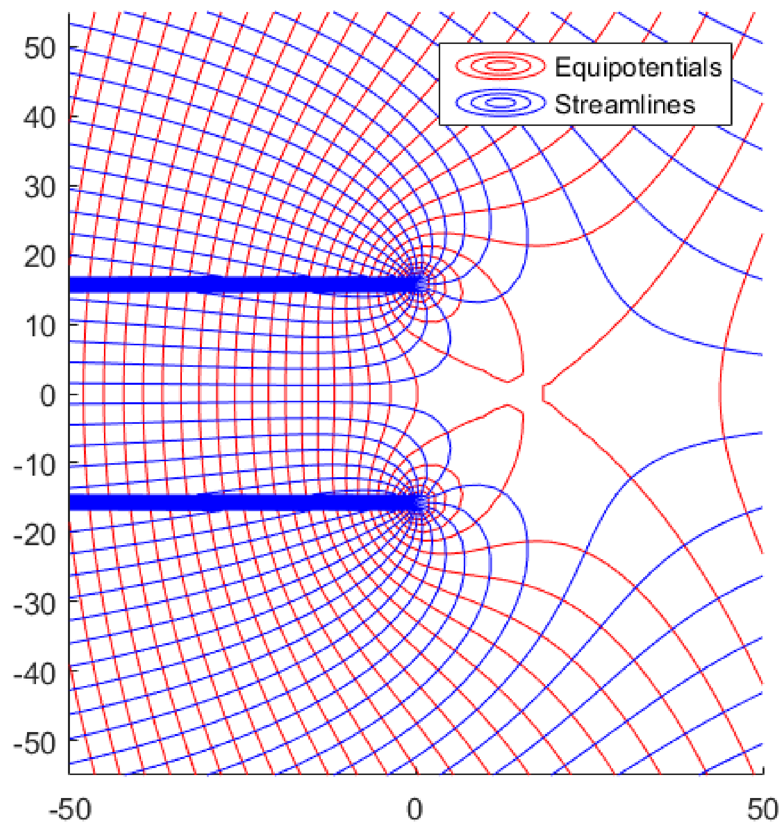
$$\Psi_A = Q_{xo}(b) + \frac{Q}{2\pi}\left(\pi - \arctan\left(\frac{b}{L}\right) + \pi - \arctan\left(\frac{b}{L+2d}\right)\right)$$

$$0 = \Psi_B - \Psi_A = -2 Q_{xo} b - \frac{Q}{\pi}\left(2\pi - \arctan\frac{b}{L} - \arctan\frac{b}{L+2d}\right)$$

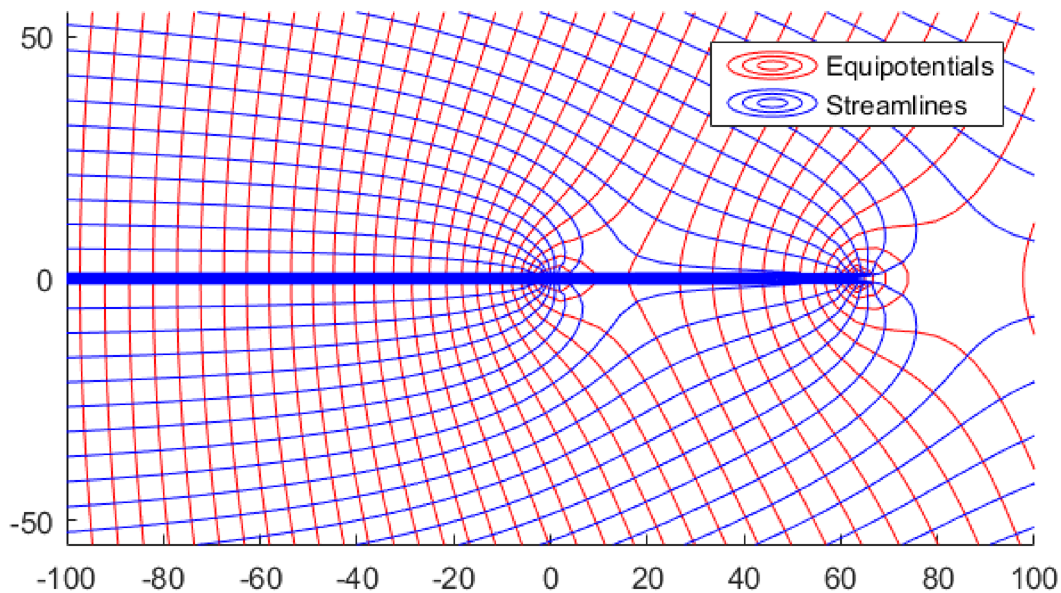
$$\boxed{Q = -2(Q_{xo}) b \pi \left(2\pi - \arctan\frac{b}{L} - \arctan\frac{b}{L+2d}\right)^2}$$

3) The in-line variation is superior because there is no possibility of contamination escaping the system. In the system with wells aligned on the y axis, contamination could escape between the wells.

4)



Equipotentials
Streamlines
50
40
30
20
10
0
-10
-20
-30
-40
-50
-50
0
50



Equipotentials
Streamlines
50
0
-50
-100
-80
-60
-40
-20
0
20
40
60
80
100