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This project is an investigation of the properties of the circular lake analytic element. We previously investigated individual lakes in the third homework, but the methods used in that assignment are not sufficient for modelling multiple lakes nearby each other.

The first step in this project was to model a single lake of given potential in a field of uniform flow. This was accomplished using the same methods as homework three. A system of two equations and two unknowns was setup by evaluating the complex potential at the reference point and at the edge of the lake. This system of equations was then solved for Q, the lake discharge required to achieve the given lake head, and C the constant. A detailed mathematical derivation is contained in the attached 'code and analysis' file. This method of modelling a single lake successfully matches the potential at the edge of the lake to the given potential.

The next step in the project was to add a second lake to the model and see how the interaction of the two lakes affected the value of the complex potential along the boundary of each lake. For this portion of the project the complex potential was calculated using the methods of 8.16 in *Analytical Groundwater Mechanics.* The complex potential for each lake is represented by equation 8.322, except that only one value of n is considered, n = 1. Alpha and Q for each lake are unknown. Alpha is equal to the negative complex conjugate of a, where a is a coefficient of a term in the taylor expansion of omega_other(the complex potential of everything besides the lake) along the boundary of the lake. Omega_other can be expanded as a taylor series in this way because it is holomorphic about the lake boundary. To solve for the a value, and subsequently the alpha value at each lake 8.329 was used. Phi other in 8.329 did not include the other lake, as was instructed in the problem statement. This calculation was done using the method solve_lakes_fulit to call the Cauchy_intergal_Phi function to find each lake's a term, then calculates Q for each lake using 8.338. The constant is calculated by subtracting the complex potential at the reference point (calculated using the new alpha and Q values) from the user specified reference potential.

This process was performed 4 times as the lakes were moved successively closer. As the lakes got closer together, the complex potential along each lake boundary deviated from the specified potential along the lake boundary. Figures and values from this analysis are in the attached document.

In order to make the potential along the lake boundary to match with the given lake potential more terms in the Omega_other taylor expansion are required. For the second part of the project, 20 terms of the taylor expansion were calculated (n = 1, 2, …., 20). Using more terms in the taylor expansion cause the calculated complex potential to exactly match the user specified potential along the edge of each lake. For the second part of the project, phi_other included the other lake.