Choosing the number of edges with which to augment the MSF

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Let $G = (V, E)$ be a weighted graph. Let (V, E_*) be a maximum spanning forest of G and let E_n be the set of n edges in G of largest weight. The graph

$$G' = (V, E_* \cup E_n)$$

is an augmented MSF of G. Suppose that $n = |V|$, the order of G. Then G' has size

$$
\begin{aligned}
|E_* \cup E_n| &\approx |E_*| + |E_n| \\
&= (|V| - c) + |V| \\
&= 2|V| - c,
\end{aligned}
$$

where c denotes the number of components of G, and where the approximation in the first line arises due to the fact that E_* and E_n may intersect. It follows that the vertices in G' have average degree

$$
\begin{aligned}
\frac{1}{|V|} \sum_{v \in V} \deg(v) &= \frac{2|E_* \cup E_n|}{|V|} \\
&\approx \frac{2(2|V| - c)}{|V|},
\end{aligned}
$$

which converges to four as $|V| \to \infty$ (assuming that c is roughly constant in $|V|$).