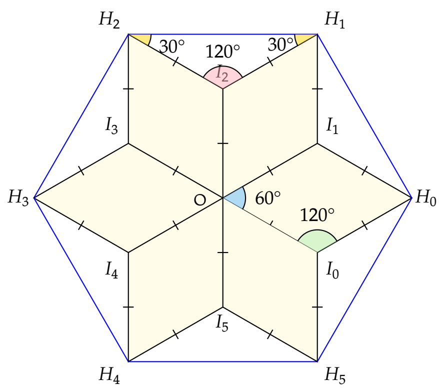
Here are the full, or partial solutions.

Year 8 and below

Six identical rhombuses, each of area 5 cm^2, form a star. The tips of the star are joined to form a regular hexagon, as shown in blue.
What is the area of the hexagon?

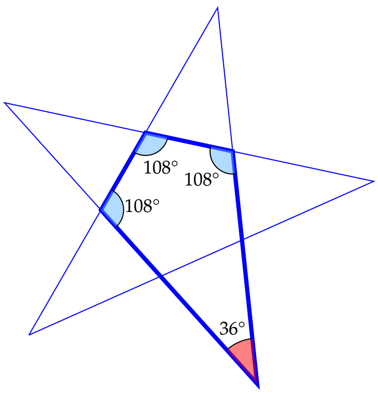


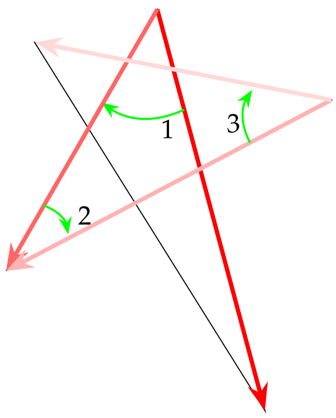
Solution

It's a good idea to start off by marking some angles on the diagram.
The hexagon is regular so all the internal angles are the same. The rhombuses (sides are all the same length) are identical, so that means the angles at the center, O, are all equal, so each rhombus has an acute angle of $60° \left(= \frac{360°}{6}\right)$, see the blue angle $I_0\widehat{O}I_1$.
Opposite angles in a rhombus are equal and the angles in a quadrilateral sum to $360°$ so the obtuse angles in each rhombus are $120°$, see the green angle $H_0\widehat{I_0}O$.
At I_n the angles in each of the rhombuses are each $120°$ so angle $H_0\widehat{I_1}H_1$ must also be $120°$. (See the pink angle for example.)
Each of the outer white triangles, for example, $H_1I_2H_2$ are isosceles because the sides of the rhombuses are equal. This means that all the angles like $I_2\widehat{H_2}H_1$ must be $30°$, see the yellow angles.
So each white triangle is exactly half a rhombus and has an area of 2.5 cm^2. There are six of them having a total area of $6 \times 2.5 = 15$ cm^2. Add on the area of the six rhombuses for a total area of 45 cm^2.

Year 9 and above

In a regular pentagram (5-pointed star), (i) show that the angle at each point is $36°$. Thus the sum of the angles in all five points is $180°$.
Now, (ii) what is the sum of the angles in all five points of an irregular pentagram, as illustrated in the second diagram?





Solution

(i) Consider the quadrilateral shown in bold on the regular pentagram. We know that its internal angles should sum to $360°$ and we know that three of its interior angles are the same, that is, three times the interior angle of a regular pentagon. Recall the sum of interior angles of an n-sided polygon is $180 \times (n-2)$, so for a pentagon,

$$180 \times 3 = 540°, \text{ and one angle is } \frac{540}{5} = 108°$$

Then the angle at the point

$$= 360 - (3 \times 108) = 36°$$

(ii) There is a nice graphical solution to this. Place an arrow along one side of the irregular pentagram. Rotate the arrow around its base, clockwise, so that it lies over the line indicated by the green arrow (1). Imagine that at the same time the red line shrings to the size of the line it lies over. Now, we rotate the red line again clockwise, this time around the arrow tip (see green arrow 2). The next rotation of the red line is shown with green arrow 3. If we repeat this two more times the red arrow will lie over the same line it started on, but, the arrow will be at the other end. In other words, we have turned the red line through all the angles of the pentagram and it has turned $180°$. So the sum of the angles in all five points is $180°$! Isn't that neat?