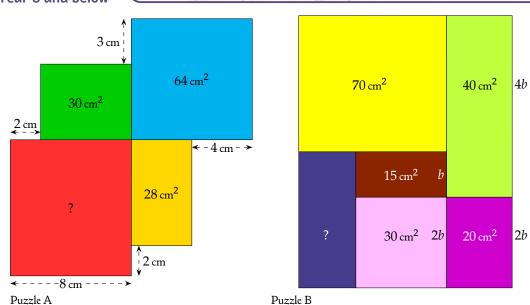


Christmas Maths Challenge

Here are the full, or partial solutions.

Year 8 and below



Two puzzles! Remember you have to show how you got the answer. You cannot justify your answer by saying "Because it looks that way!" Have fun!

Solution

Puzzle A

First we can see that the green-red boundary must be $8-2=6\,\mathrm{cm}$.

Then, since the green rectangle is 30 cm², the green-blue boundary has to be 5 cm

Then, the height of the blue rectangle is 8 cm and so its width must be 8 cm too.

Now, the width of the yellow rectangle must be $8-4=4\,\mathrm{cm}$ and so its height must be $7\,\mathrm{cm}$.

From this we get the height of the red rectangle is 7 + 2 = 9 cm.

So the area of the red rectangle is $8 \times 9 = 72 \text{ cm}^2$.

Puzzle E

It is tempting to say the height of the pink rectangle plus the height of the dark red rectangle is the same as the yellow rectangle, which makes the unknown rectangle's area easy to find. But if that is true, we must show it to be true.

We let the height of the dark red rectangle be b.

Then the height of the pink rectangle must be 2b as it has the same width but is double the area of the dark red rectangle.

Now label the height of the purple rectangle 2b as it is the same height as the pink rectangle, and the height of the light green rectangle 2d as it has the same width as the purple rectangle but is double the area.

But now we have shown that the height of the yellow rectangle is 3b, that is, we know that the yellow rectangle has the same area as the dark blue, dark red and pink rectangles put together. Therefore the area of the dark blue rectangle must be $70 - (15 + 30) = 25 \text{ cm}^2$.



tive integers. Find the area covered by the three squares.

Solution

Let's call the length of the side of the smallest square a.

We can then write some of the other dimensions in terms of a, and find the dimensions of the rectangular overlap of the larger two squares.

Triangles $\triangle ABC$ and $\triangle ADE$ are congruent so ED = 2.

Applying Pythagoras' Thm. to $\triangle BDF$ we have,

$$4^{2} + (2a + 2)^{2} = (4\sqrt{10})^{2}$$
$$4a^{2} + 8a + 4 = 160$$
$$4a^{2} + 8a + 20 - 160 = 0$$
$$a^{2} + 2a - 35 = 0$$
$$(a + 7)(a - 5) = 0$$

Distance must be positive, so we reject the a=-7 solution, and take a=5. Now the total area is,

$$5^2 + (5+1)^2 + (5+2)^2 = 25 + 36 + 49$$

= 110

minus the overlapping rectangle,

$$110 - 2 \times 1 = 108 \,\mathrm{cm}^2$$

