

Here are the full, or partial solutions.

Year 8 and below

Insert the missing parentheses into each equation so that it becomes true when worked out according to the rules of order of operations (e.g. BIDMAS).

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 2$$

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 5$$

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 8$$

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 11$$

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 12$$

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 19$$

Solution

First, with no brackets:

$$9 + 12 \div 3 + 4 \div 2 + 1 \times 2 = 17$$

$$(9 + 12) \div (3 + 4) \div (2 + 1) \times 2 = 2$$

$$(((9 + 12) \div (3 + 4) \div 2) + 1) \times 2 = 5$$

$$(9 + 12) \div 3 + 4 \div (2 + 1 \times 2) = 8$$

$$(9 + 12) \div 3 + 4 \div 2 + 1 \times 2 = 11$$

$$9 + (12 \div 3 + 4) \div (2 + 1 \times 2) = 11$$

$$9 + 12 \div (3 + (4 \div (2 + 1 \times 2))) = 12$$

$$(((9 + (12 \div 3) + 4) \div 2) + 1) \times 2 = 19$$

Year 9 and above

Find two rational numbers, both less than ten, whose product is ninety-nine.

A rational number is a fraction $\frac{a}{b}$, where $b \neq 0$ and $a, b \in \mathbb{Z}$, that is, a and b are both integers.

Solution

We are looking for $\frac{a}{b}$ and $\frac{c}{d}$ such that

$$\frac{a}{b} \times \frac{c}{d} = 99$$

We can try starting with $c = 99$

$$\frac{a}{b} \times \frac{99}{d} = 99$$

Then we must have

$$\frac{a}{bd} = 1$$

and we must have

$$\frac{a}{b} < 10, \quad \frac{99}{d} < 10$$

Multiplying numerator and denominator by 999:

$$\frac{999}{b} \times \frac{99}{999} = 99$$

and then by 100:

$$\begin{aligned} \frac{999}{100} \times \frac{99 \times 100}{999} &= 99 \\ \frac{999}{100} \times \frac{9900}{999} &= 99 \end{aligned}$$

Where

$$\frac{999}{100} = 9.99 < 10 \quad \text{and} \quad \frac{9900}{999} = 9.\overline{909} < 10$$

An alternative way: Write 99 as a product of two of its factors,

$$9 \times 11 = 99$$

then multiply by a fraction and its reciprocal,

$$\frac{13}{11} \times 9 \times \frac{11}{13} \times 11 = 99$$

Notice that we multiply 9 by the smaller fraction and 11 by the larger.

But,

$$\frac{13}{7} \times 9 > 10$$

so we need to try smaller fractions:

$$\frac{11}{10} \times 9 \times \frac{10}{11} \times 11 = 99$$

but

$$\frac{10}{11} \times 11 \not< 10$$

trying again:

$$\frac{22}{21} \times 9 \times \frac{21}{22} \times 11 = 99$$

but still $(21 \div 22) \times 11 > 10$ so we try again,

$$\frac{21}{19} \times 9 \times \frac{19}{21} \times 11 = 999$$

At last!

$$\frac{189}{19} = 9.947 \dots < 10 \quad \text{and} \quad \frac{209}{21} = 9.952 \dots < 10$$