

Here are the full, or partial solutions.

Year 8 and below

The pages of a book are numbered; 1, 2, 3, ...
It takes 852 digits to number all the pages of the book.
What is the number of the last page of the book?

Solution

From page 1 to page 9 we need 9 digits.
From page 10 to page 99 we need $90 \times 2 = 180$ digits. 189 digits in total so far.
We have $852 - 189 = 663$ digits left, and we have up to page 99 so far.
From page 100 to page 999 we would need $900 \times 3 = 2700$ digits.
With 663 digits we can number $663 \div 3 = 221$ three-digit pages.
Starting with page 100 that means we would reach page 220.

Year 9 and above

How many set of three positive integers, a , b , c satisfy the following two conditions simultaneously?

$$6ab = c^2 \quad (1)$$

$$a < b < c \leq 35 \quad (2)$$

Solution

We see from the second condition that the three integers must be different.
From the first condition we can see that c must be a multiple of 6.
So c could be 6, 12, 18, 24 or 30.

Now we need a and b such that $a \times b = \frac{c^2}{6}$. Let's look at the possibilities.

No.	c	c^2	$\frac{c^2}{6}$	a	b	
1	6	36	6	2	3	Solution
2	12	144	24	2	12	Two digits the same, not a solution
3	"	"	24	3	8	Solution
4	"	"	24	4	6	Solution
5	18	324	56	4	14	Solution
6	"	"	56	7	8	Solution
7	24	576	96	6	16	Solution
8	"	"	96	8	12	Solution
9	30	900	150	6	25	Solution
10	"	"	150	10	15	Solution

So we have nine solutions, in triplets (a, b, c) they are
(2, 3, 6), (3, 8, 12), (4, 6, 12), (4, 14, 18), (7, 8, 18),
(6, 16, 24), (8, 12, 24), (6, 25, 30), (10, 15, 30)