

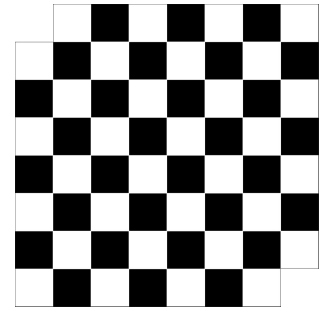


Year 8 and below

You have a chess board from which two diagonally opposite squares have been removed. You also have thirty-one dominoes, each of which can cover two squares of the chess board. Can the dominoes be arranged so that they cover all sixty-two squares of the chess board? If so, how? If not, why not?

Solution

You can try proving that the thirty-one dominoes will cover the sixty-two squares but it is easier to find a solution if you start by trying to show the puzzle cannot be solved. Notice that two diagonally opposite squares are always the same colour, and that a domino must always cover two squares of different colours. Therefore the problem cannot be solved because there will be too few black squares.



Year 9 and above

Find the integer solutions to this pair of equations.

$$ab + c = 2020 \quad (1)$$

$$a + bc = 2021 \quad (2)$$

Solution

Subtract the first equation from the second.

$$\begin{aligned} a + bc - ab - c &= 1 \\ a(1 - b) - c(1 - b) &= 1 \\ (a - c)(1 - b) &= 1 \end{aligned}$$

Notice that for the last line to be true, either (Case 1),

$$a - c = 1, \text{ and } 1 - b = 1$$

or (Case 2),

$$a - c = -1, \text{ and } 1 - b = -1$$

Case 1

Since $1 - b = 1$ we have $b = 0$. Substituting this into the original two equations (1) & (2), we have one set of solutions:

$$\begin{aligned} a &= 2021 \\ b &= 0 \\ c &= 2020 \end{aligned}$$

Case 2

Since $1 - b = -1$ we have $b = 2$. Substituting this into the original two equations we get a pair of simultaneous equations:

$$\begin{aligned} 2a + c &= 2020 \\ a + 2c &= 2021 \end{aligned}$$

Solving by elimination or substitution we obtain the second set of solutions:

$$\begin{aligned} a &= 673 \\ b &= 2 \\ c &= 674 \end{aligned}$$