Maths Challenge 5

Welcome to this week's Maths Challenge!
Everyone is encouraged to take part.
There are 3 to 5 merit points for correct solutions, depending on the difficulty of the problem and how impressed the marker is with your solution.
There are house points for students who submit 7 or more correct solutions in one term.
Solutions must be explained in detail, responses with just the answer may be ignored.
Drop your solution in the box in the staffroom by Tuesday.

Year 9 and below

This is a version of a very well known logic puzzle. Try to figure it out by yourself though!
You are alone and lost, following a small path through a dark and eldritch forest. Shadows loom, strange rustles menace and the day is failing. You come to a fork in the path: there are two ancient oak trees and a peculiar sign grows in front of you and tells you that travelling down one path will lead you out of the forest, but walking down the other dooms you to a dreadful end. You may ask one question of one of the two wyrd oaks, they can only answer by pointing a gnarled branch. You are also told that one of the trees always lies while the other always tells the truth, but you have no way of telling which is which. What question should you ask to escape the deep dire wood?

Year 10 and above

If

$$\sin^3\theta + \cos^3\theta = \frac{11}{16}$$

find the value of

$$\sin\theta + \cos\theta$$

Hint: Factorise the first equation. Let $x = \sin\theta + \cos\theta$. You should obtain a cubic polynomial in x. $x = \frac{1}{2}$ is a root.