Here are the full, or partial solutions.

Year 8 and below

Thirty-one people took an exam and each person achieved a different, whole number score, from 70 to 100. The average of the scores is calculated. Then one person's score is removed, but when the average is recalculated, it has not changed, the new mean is the same as the mean of the original 31 marks. Which score was taken out?

Solution

It might be easier to start with an example containing easier numbers. Suppose we have five examinees and their scores are 1, 2, 3, 4, 5.

The sum of the scores is 15 and the mean is $\frac{15}{5} = 3$.

If we take out the middle score, 3, the new sum of scores is 12 and the new mean is $\frac{13}{4} = 3$, the mean did not change when we removed the middle value.

Notice also that if we find the sum by adding in pairs, $1 + 5 = 6$, then $2 + 4 = 6$ until the middle value is left, the total is the same each time, there is a kind of symmetry around the middle value.

Now if we start with the problem given, we can add up the thirty-one scores in pairs: $70 + 100 = 170$, then $71 + 99 = 170$, then $72 + 98 = 170$ and so on until we reach $84 + 86 = 170$, that's fifteen pairs, leaving 85 as the middle number. The average of the thirty-one scores:

$$\frac{(170 \times 15) + 85}{31} = 85$$

$$(170 \times 15) + 85 = 31 \times 85$$

$$(170 \times 15) + 85 = 30 \times 85 + 85$$

$$170 \times 15 = 30 \times 85$$

The average of the thirty scores after removing the middle value, 85:

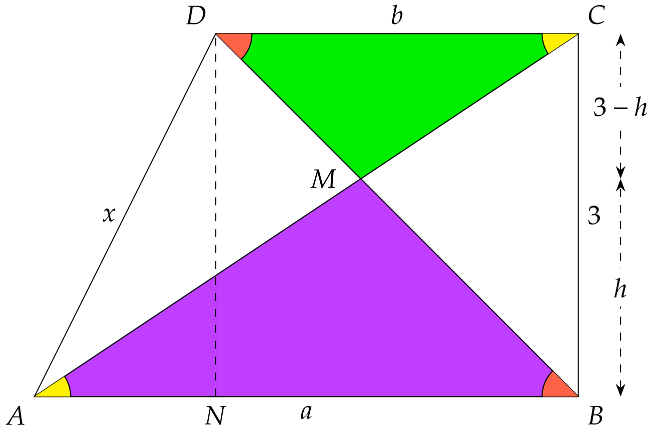
$$\frac{170 \times 15}{30} = 85$$

The averages are the same.

85 was the score removed.

Year 9 and above

The diagram shows a trapezium. The line of length 3 cm is perpendicular to the two parallel lines of the trapezium. The purple area is 6 cm^2 greater than the green area. Find the length x.



Solution

Surprisingly, we don't need to know much about trapeziums, other than that $AB \parallel DC$, to solve this interesting geometry puzzle. It's more important to remember similar triangles, triangle area, and good old Pythagoras. To help us proceed, we add some labels, the vertical height h of the point M where the two diagonals intersect, and we make a right-angled triangle $\triangle ADN$.
By Pythagoras:

$$x^2 = 3^2 + (a - b)^2 \tag{1}$$

Area P of the purple triangle, $\triangle ABM$:

$$P = \frac{1}{2}ah \tag{2}$$

Area G of the green triangle, $\triangle CDM$:

$$G = \frac{1}{2}b(3 - h) \tag{3}$$

but we know $P = G + 6$, use Equations 2 & 3:

$$\begin{aligned}\frac{1}{2}ah &= 6 + \frac{1}{2}b(3 - h)\\ ah &= 12 + 3b - bh\\ (a + b)h &= 12 + 3b \qquad (4)\end{aligned}$$

Now $\angle CAB = \angle ACD$ and $\angle ABD = \angle CDB$ so the purple triangle$\triangle ABM$ and the green triangle $\triangle CDM$ are similar. So equivalent distances in each triangle differ by the same proportion. For example DM is to BM as b is to a. Also CM is to AM as $(3 - h)$ is to h. Therefore,

$$a : b :: h : (3 - h)$$

that is,

$$\begin{aligned}\frac{a}{b} &= \frac{h}{3 - h}\\ 3a - ah &= bh\\ (a + b)h &= 3a \qquad (5)\end{aligned}$$

Now we set Equation 4 equal to Equation 5, eliminating h:

$$\begin{aligned}3a &= 12 + 3b\\ a - b &= 4 \qquad (6)\end{aligned}$$

now we can use Equation 6 to substitute 4 for $a - b$ in Equation 1.

$$\begin{aligned}x^2 &= 9 + 4^2\\ x^2 &= 25\\ x &= \pm 5\end{aligned}$$

but distance x must be positive, so

$$x = 5$$