



Here are the full, or partial solutions.

Year 8 and below

Some molten rhenium is being poured into three containers, A, B and C. Container A gets 1 litre plus one third of what is left. Container B then receives 6 litres plus one third of what remains, then Container C gets the rest which is 40 litres. What is the volume of rhenium poured into Container B?

Solution

Let R be the total volume of rhenium in litres.

Step	A	B	C	Remainder	
1	$1 + \frac{R-1}{3}$?	40	$R - \left(1 + \frac{R-1}{3}\right)$	Simplify the remainder,
				$\frac{2R-2}{3}$	simplify A and work out B in Step 2:
2	$\frac{R+2}{3}$	$6 + \frac{\frac{2R-2}{3} - 6}{3}$	40	0	Simplify:
3	$\frac{R+2}{3}$	$6 + \frac{2R-20}{9}$	40	0	Now we can equate the sum of A, B & C to R.

$$R = \frac{R+2}{3} + 6 + \frac{2R-20}{9} + 40$$

$$R = \frac{3R+6}{9} + \frac{2R-20}{9} + 46$$

$$R = \frac{5R-14}{9} + 46$$

$$9R - 5R = -14 + 414$$

$$4R = 400$$

$$R = 100 \text{ L}$$

Alternatively, we can attack the problem from the end, begin with Container C with 40 and work backwards:

We know that C is 40 and from the question, this is $\frac{2}{3}$ of the quantity that B got $\frac{1}{3}$ of. So $40 \times \frac{1}{2} = 20$.

But we know that B got this plus 6, so B got 26. Then for Container A: it has $\frac{1}{3}$ of 66, (Container B and Container C's volume) plus 1 more litre, that is 34 litres. There is less algebra this way but perhaps it is trickier to think about. So the total volume of rhenium is $34 + 26 + 40 = 100 \text{ L}$.

Year 8 and above

The diagram contains four squares. The smallest square has an area of 5 cm^2 . Find the area of the shaded triangle.

Solution

If you remember that the area of a triangle is half the base times the height, this problem becomes easier. Can you see that the dashed line EC is parallel to the side of the triangle JK ? This is because they are both the diagonals in squares so their slope must be the same.

If we move the vertex of the blue triangle at E along the dotted line to C , the base JK stays the same and so does its height, thus its area also stays the same. In other words, we have shown that $\triangle KEJ$ has the same area as the green $\triangle CJK$.

Now we can do the same trick again. Line BK is parallel to line JC , so if we now move vertex K of the green triangle along the dotted line BK to B , we end up with $\triangle BCJ$.

The triangle fills half of the square, which has area four times that of the smallest square. So the triangle has an area of 10 cm^2 .

