Here are the full, or partial solutions.

Year 9 and below

If

$$ax = by = cz = 5 \qquad \text{and} \qquad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 4$$

What is the value of $a + b + c$?

Solution

Notice that we need to find a relationship between a, b and c, so we could try eliminating x, y and z. Using the first set of equations, we can rewrite them as:

$$ax = 5 \qquad by = 5 \qquad cz = 5$$

$$\frac{1}{x} = \frac{a}{5} \qquad \frac{1}{y} = \frac{b}{5} \qquad \frac{1}{z} = \frac{c}{5}$$

Now we can substitute these into the second equation of the question:

$$\frac{a}{5} + \frac{b}{5} + \frac{c}{5} = 4$$

$$5\left(\frac{a}{5} + \frac{b}{5} + \frac{c}{5}\right) = 5 \times 4$$

$$a + b + c = 20$$

Year 10 and above

Alnitak and Mintaka are 300 km apart. They travel towards each other in a straight line. Each travels at their own constant speed.
If they both set off at 9:00 a.m. then they meet at noon.
If Alnitak sets off at 6:00 a.m. and Mintaka sets off at 10:00 a.m. they still meet at noon.
What speed does Alnitak travel at and what speed does Mintaka travel at?

Solution

Since their speeds are constant we can use the

$$\textit{Speed} = \frac{\textit{distance}}{\textit{time}}$$

formula.
Let s_a be Alnitak's speed and let s_m be Mintaka's speed.
We will let d_1 be the distance from Alnitak's starting point to the place they meet when they both depart at 9 a.m., and d_2 will be the distance from Alnitak's starting point to the place they meet when they depart at 6 a.m. and 10 a.m.
In the first case, distance d_1 is covered by Alnitak in 3 h. Then Mintaka travels $300 - d_1$ in 3 hours.
In the second case, distance d_2 is travelled by Alnitak in 6 h. Then Mintaka covers $300 - d_2$ in 2 hours.

Case 1: Both depart at 9 a.m.

$$s_a = \frac{d_1}{3}$$
$$s_m = \frac{300 - d_1}{3}$$

Eliminate d_1 using these two equations.

$$3s_a = 300 - 3s_m$$
$$3s_a + 3s_m = 300$$
$$s_a + s_m = 100$$

Case 2: 6 a.m. and 10 a.m. departures

$$s_a = \frac{d_2}{6}$$
$$s_m = \frac{300 - d_2}{2}$$

Eliminate d_2 using these two equations.

$$6s_a = 300 - 2s_m$$
$$6s_a + 2s_m = 300$$
$$3s_a + s_m = 150$$

Now we have a pair of simultaneous equations:

$$\left.\begin{aligned} s_a + s_m &= 100 \\ 3s_a + s_m &= 150 \end{aligned}\right\}$$

Solving, we find that
Alnitak's speed $s_a = 25\,\mathrm{km\,h^{-1}}$, and
Mintaka's speed $s_m = 75\,\mathrm{km\,h^{-1}}$.