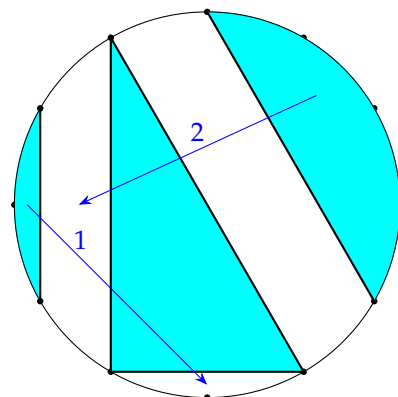
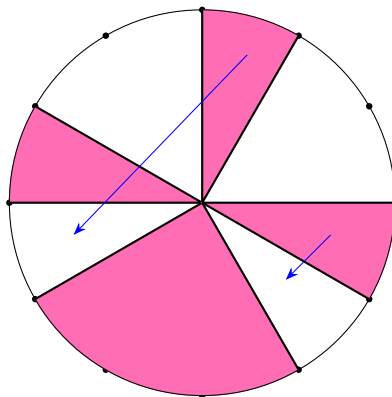
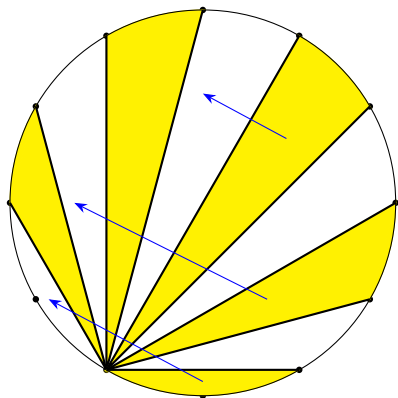




Here are the full, or partial solutions.

Year 8 and below



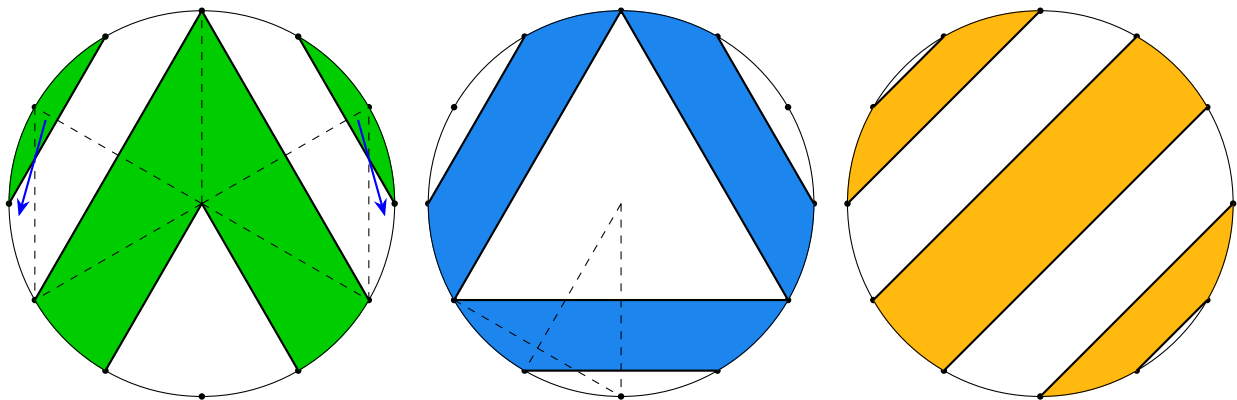
Each picture is a circle with twelve points spread equally around the circumference, like the hour marks on a clock face. Find the proportion of each circle that is shaded. Three questions in one!

Solution

The first one is not so difficult. Notice that each of the yellow shaded parts has an unshaded 'partner' of exactly the same shape. Placing the yellow shaded areas to the right of the diameter into their congruent white-shaded areas on the left of the diameter, we see that half of the circle is shaded yellow.

The second, similarly, two of the pink-shaded 'hours' fit into two of the white-shaded hours, filling one half of the circle. The third circle, is a bit trickier, but again, we can move around the cyan-shaded segments. First, notice that the segment on the left hand side fits exactly into the congruent segment below the cyan triangle. Now we can put the segment at the upper right of the circle into the congruent white segment on the left of the triangle. The answer, again, is half the circle is shaded.

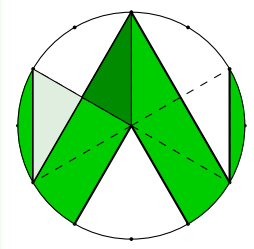
Year 9 and above



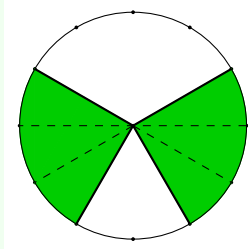
These are harder versions of the three circles in the Year 8 and below box, give those a try to warm up! As with those questions, each of these three pictures is a circle with twelve points spread equally around the circumference. The only other point used is the centre of the first circle (with the green shading). Find the proportion of each circle that is shaded.

Solution

The Green One: We need to do a bit of dissecting here before we can move shaded areas into congruent unshaded areas. We draw in some radii and move the two segments downwards by one sector.

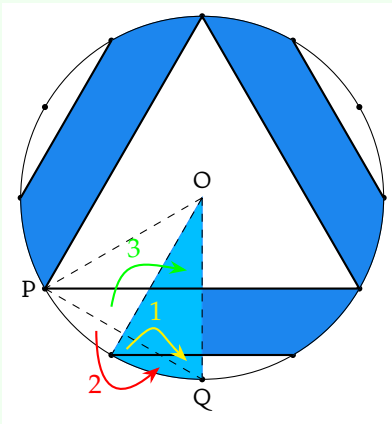


Next you can see that, by symmetry, the dark green shaded triangle is congruent to the pale grey-green triangle. So we can 'move' the green triangle into the 'empty' space. We can do exactly the same on the left-hand side of the circle.



You can see that six hour sectors are shaded, so we have shown that half of the circle was shaded green in the question.

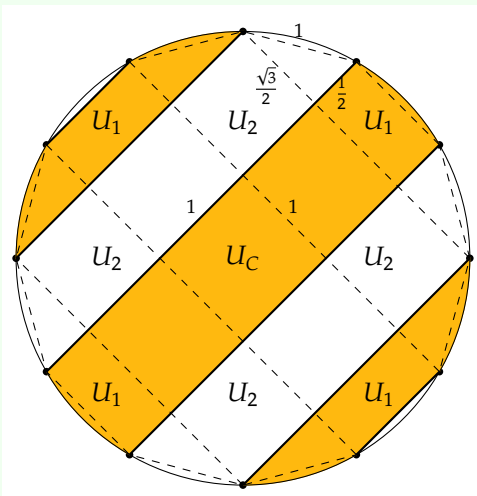
The Blue One: Again, we need to try a bit of cutting. First notice that there is rotational symmetry. We can show that half of one of the blue bands is the same area as one 'hour' sector. Take a look at the dashed lines added to the middle puzzle above.



By symmetry, the small triangles marked by the yellow arrow (1) are congruent, so we move the blue shaded area as indicated. Now we can move the half-segment as indicated by the red arrow (2). Lastly, we can move the larger triangle as indicated by the green arrow (3). The source triangle and destination triangle are congruent by symmetry in the equilateral triangle $\triangle OPQ$. So now we have shown that the area of half of one of the original blue bands is the same as the area of one 30° sector. If we carry out the same procedure on the rest of diagram, we end up with six shaded sectors, we have shown that half the circle is shaded.

Solution

The Orange One: A bit harder, we have to do a little calculating for this one, unless you can find a better solution by dissection.



Notice that six of the small segments are shaded and six are not. So half of the twelve segments are shaded. Now examine the eight, congruent triangles. Four are shaded, four are not. Of the four rectangles surrounding the central 1 by 1 square, two are shaded, two are not.

So far, of the shapes making up the whole circle, those not labelled U, 50% have been shaded.

Now let's look at the remaining parts labelled U. (For unbalanced). If we say the length of each of the chords forming the segments

is 1 then the sides of the triangle must be $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$ because each triangle has angles 90° , 30° and 60° . Why?

Let's find the area of the unshaded 'U' areas.

$$U_2: \quad 4 \times \left(\frac{\sqrt{3}}{2} \right)^2 = 3$$

Now let's find the area of the shaded 'U' areas.

$$U_1 + U_c: \quad 4 \times \left(1 \times \frac{1}{2} \right) + 1 \times 1 = 3$$

So we have shown, once again, that the shaded area and the unshaded area is equal and so the circle is fifty percent orange.