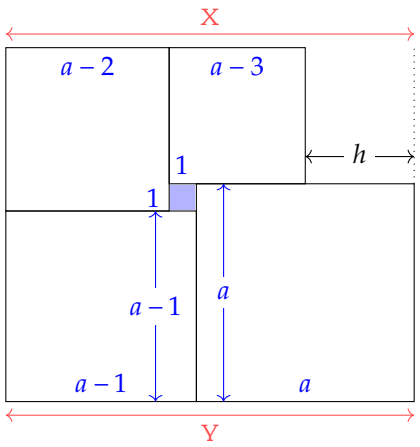
Here are the full, or partial solutions.

Year 8 and below

Five squares are positioned as shown. The smallest, shaded square has an area of 1 cm^2. What is the value of h?



Solution

The shaded square has area 1 cm^2 so it has sides of length 1 cm. Let the length of the side of the largest square be a. Then we can see that the next largest square must have side $a-1$ because the shaded square on top of it exactly reaches to the top of the largest square. By the same reasoning the third largest square has side $a-2$ and the smallest unshaded square has side $a-3$. Now we can find two expressions for the distance across the diagram left to right.

$$\begin{aligned} Y &= X \\ (a-1)+a &= (a-2)+(a-3)+h \\ 2a-1 &= 2a-5+h \\ -1 &= -5+h \\ h &= 4 \end{aligned}$$

Year 9 and above

The numbers $4x$, $2x-3$ and $4x-3$ are consecutive terms in a linear sequence. Find the value of x.

Solution

A linear sequence is like an arithmetic sequence except that the terms of the sequence need not be integers. There is a constant difference d between terms. If we suppose that $4x$ is the first term of the sequence we have:

n	1	2	3	4	⋯
n^{th} term	$4x$	$2x-3$	$4x-3$		

Now, d, is the difference between each successive term, so

$$d = (4x-3)-(2x-3) \quad \text{and} \quad d = (2x-3)-4x$$

So equating, $(4x-3)-(2x-3) = (2x-3)-4x$

$$2x = -2x-3$$

$$4x = -3 \implies x = -\frac{3}{4}$$

We have our solution but we should check by substituting in for x.

n	1	2	3	4	⋯
n^{th} term	$4x$	$2x-3$	$4x-3$		
n^{th} term	-3	$-\frac{9}{2}$	-6		

We can see that there is a constant difference of $d = -\frac{3}{2}$.