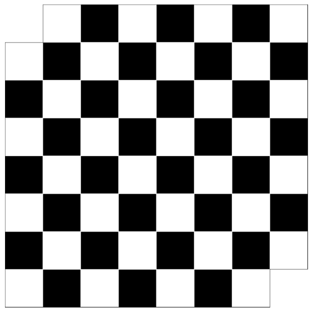
Here are the full, or partial solutions.

Year 8 and below

You have a chess board from which two diagonally opposite squares have been removed. You also have thirty-one dominoes, each of which can cover two squares of the chess board. Can the dominoes be arranged so that they cover all sixty-two squares of the chess board? If so, how? If not, why not?



Solution

You can try proving that the thirty-one dominoes will cover the sixty-two squares but it is easier to find a solution if you start by trying to show the puzzle cannot be solved.
Notice that two diagonally opposite squares are always the same colour, and that a domino must always cover two squares of different colours.
Therefore the problem cannot be solved because there are too few black squares.

Year 9 and above

Find the integer solutions to this pair of equations.

$$ab + c = 2020 \qquad (1)$$
$$a + bc = 2021 \qquad (2)$$

Solution

Subtract the first equation from the second.

$$a + bc - ab - c = 1$$
$$a(1-b) - c(1-b) = 1$$
$$(a-c)(1-b) = 1$$

Notice that for the last line to be true, either (Case 1),

$$a - c = 1, \text{ and } 1 - b = 1$$

or (Case 2),

$$a - c = -1, \text{ and } 1 - b = -1$$

Case 1

Since $1 - b = 1$ we have $b = 0$. Substituting this into the original two equations (1) & (2), we have one set of solutions:

$$a = 2021$$
$$b = 0$$
$$c = 2020$$

Case 2

Since $1 - b = -1$ we have $b = 2$. Substituting this into the original two equations we get a pair of simultaneous equations:

$$2a + c = 2020$$
$$a + 2c = 2021$$

Solving by elimination or substitution we obtain the second set of solutions:

$$a = 673$$
$$b = 2$$
$$c = 674$$