Here are the full, or partial solutions.

A game begins with 100 people standing in a circle. One person holds a ball, call them person Number 1, and then number the other players consecutively anticlockwise: the person to the ball-holder's left is No. 100, to their right stands person No. 2.

The person holding the ball throws it over the head of the first person on their right, the person on the other side catches it. So at the start No. 1 throws to No. 3.

A person skipped by the ball, immediately leaves the circle for tea and chocolate biscuits.

So person No. 3 would be the next holder of the ball. No. 3 throws to No. 5 and No. 4 leaves the circle.

The process continues until there is one person left in the circle. If there are only two people left, the person holding the ball throws to themselves, eliminating the other person.

What is the number of the last person in the circle?

Solution

Solution by playing the full game: Each line of the table shows the people left in the circle after that round:

Round 1:	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99
Round 2:	1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, 93, 97
Round 3:	1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97
Round 4:	9, 25, 41, 57, 73, 89
Round 5:	9, 41, 73
Round 6:	9, 73
Round 7:	73

So person No. 73 is the last one in the circle.

Solution by reasoning: For an even number of people starting the game, half are out after the first round.
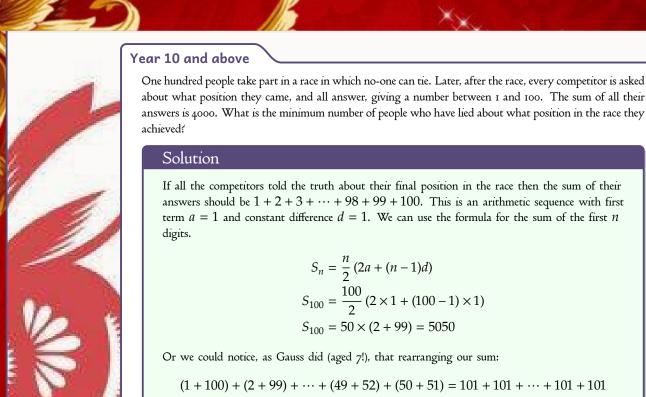
So if the number of people playing is a power of 2, the person holding the ball at the beginning will be the last person in the circle.

The closest power of 2, less than 100 is 64. So if we can see who is holding the ball once 36 people have been eliminated, we have found the answer.

The 36^t person to be eliminated must be No. 72, and No. 73 has just caught the ball.

Since there are now 64 people in the circle, the holder of the ball, No. 73, will be the last person in the circle.

One hundred people take part in a race in which no-one can tie. Later, after the race, every competitor is asked about what position they came, and all answer, giving a number between 1 and 100. The sum of all their answers is 4000. What is the minimum number of people who have lied about what position in the race they achieved?

Solution

If all the competitors told the truth about their final position in the race then the sum of their answers should be $1 + 2 + 3 + \cdots + 98 + 99 + 100$. This is an arithmetic sequence with first term $a = 1$ and constant difference $d = 1$. We can use the formula for the sum of the first n digits.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{100} = \frac{100}{2}(2 \times 1 + (100-1) \times 1)$$
$$S_{100} = 50 \times (2 + 99) = 5050$$

Or we could notice, as Gauss did (aged 7!), that rearranging our sum:

$$(1 + 100) + (2 + 99) + \cdots + (49 + 52) + (50 + 51) = 101 + 101 + \cdots + 101 + 101$$

(50 pairs of digits, each pair adding to 101)

$$= 101 \times 50$$
$$= 5050$$

The scores obtained from the runners is only 4000 though, there is $5050 - 4000 = 1050$ worth of fibbing going on!

Now, for the minimum number of people lying, we want the 'missing' 1050 to be accounted for by the smallest number of people, so we choose the people who came 100th, 99th and so on.

Each of these people has to report a made up position, to minimise the number of liars, the number they give should be as small as possible, so we'll assume they all say they came first! Let's see what's happening:

Person number	Real place	False place	Amount reduced	Total before	Total after
1	100	1	99	5050	4951
2	99	1	98	4951	4853
3	98	1	97	4853	4756
⋮	⋮	⋮	⋮	⋮	
11	90	1	89	4105	4016
			1034		

Eleven competitors lying has reduced the total to 4016, we need one more person to lie about their finishing place to make the total of scores 4000. If the person in 17th place says they came first, then the sum of scores is 4000 as required.

So twelve people is the minimum number who lied.