1 Algebra

Absolute Value Inequalities:
$|f(x)| < a \Rightarrow -a < f(x) < a$
$|f(x)| > a \Rightarrow f(x) > a$ or $f(x) < -a$

2 Important probability distributions

Bernoulli

Parameter $p \in [0,1]$. Discrete, describes the success or failure in a single trial.

$$p_X(k) = \begin{cases} p, & \text{if k = 1} \\ (1-p), & \text{if k = 0} \end{cases}$$

$E[X] = p$
$Var(X) = p(1-p)$

Exponential

Parameter λ. Continuous

$$f_x(x) = \begin{cases} \lambda exp(-\lambda x), & \text{if x >= 0} \\ 0, & \text{o.w.} \end{cases}$$

$$F_x(x) = \begin{cases} 1 - exp(-\lambda x), & \text{if x >= 0} \\ 0, & \text{o.w.} \end{cases}$$

$E[X] = \frac{1}{\lambda}$
$Var(X) = \frac{1}{\lambda^2}$

Normal (Gaussian)

Parameters μ and $\sigma^2 > 0$. Continuous

$f(x) = \frac{1}{\sqrt{(2\pi\sigma)}} exp(-\frac{(x-\mu)^2}{2\sigma^2})$
$E[X] = \mu$
$Var(X) = \sigma^2$
Useful properties:

Poisson

Uniform

3 Expectation and Variance

Expectation

Variance

Covariance

Variance and expectation of mean of n iid random variables

Let $X_1, ..., X_n \overset{iid}{\sim} P_\mu$, where $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$ for all $i = 1, 2, ..., n$ and $\overline{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

Variance of the Mean:

$Var(\overline{X_n}) = (\frac{\sigma^2}{n})^2 Var(X_1 + X_2, ..., X_n) = \frac{\sigma^2}{n}$.

Expectation of the mean:

$E[\overline{X_n}] = \frac{1}{n}E[X_1 + X_2, ..., X_n] = \mu$.

4 Law of large Numbers

5 Central Limit theorem

6 Statistical models

7 Estimators

8 Confidence intervals

Onesided

Twosided

Delta Method

9 Hypothesis tests

Onesided

Twosided

P-Value