Volatility-Managed Volatility Trading

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Abstract

We develop volatility risk premium timing strategies that trade two assets: a volatility asset and a risk-free asset. We first analyze a benchmark portfolio that sells a constant weight of volatility assets each month. Then, we show that a volatility-managed portfolio, which reduces selling volatility assets during periods of heightened volatility, considerably enhances long-run performance. Our findings are robust across variance swaps, VIX futures, and S&P 500 straddles, and even in the presence of transaction costs. An ex-post study indicates that timing portfolios yield positive alpha and reduce exposure relative to constant-weight portfolios, mostly during volatility-spike periods rather than stable periods. Our findings help differentiate asset pricing theories on risk-return relations in the volatility asset market.

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1 Introduction

It's well known that sellers of volatility assets earn positive premiums (see e.g., Carr and Madan (1998), Coval and Shumway (2001), Bakshi and Kapadia (2003), Ang, Hodrick, Xing, and Zhang (2006), Carr and Wu (2009), Bollerslev and Todorov (2011), Kelly, Pástor, and Veronesi (2016)). We in this paper ask the following questions: How do long-run investors best harvest volatility risk premiums (VRP)? Is there a simple, efficient way for long-run investors to time VRP?

To answer these questions, we consider portfolios similar to Moreira and Muir (2017) and make a modification by replacing the stock market index with one of three volatility assets: a one-month variance swap contract, a one-month constant-maturity VIX futures portfolio, and a one-month constant-maturity S&P 500 ATM straddle portfolio. For each asset, we adhere to the longest period of available price data: 1990-2023 for variance swaps, 2004-2023 for VIX futures, and 1996-2022 for straddles. We source our data from standard resources such as CBOE and OptionMetrics. Each month t, a portfolio weight of

$$w_t^i = -cf_t, \tag{1.1}$$

is assigned to the volatility asset, and the remaining capital is allocated to risk-free assets. Here, c represents a positive scaling constant that is chosen to match the portfolio's unconditional return standard deviation with that of the S&P 500 index. The term f_t represents a timing factor that is expected to demonstrate a positive relationship with the conditional one-month VRP. In other words, we sell more (less) volatility assets when conditional VRP is higher (lower).

As noted in Moreira and Muir (2017), portfolio defined in equation (1.1) has a desirable property that researchers cannot manipulate the portfolio's Sharpe ratio (sometimes called "information ratio") or factor-model alpha t-statistics because the scaling constant

c proportionally affects the mean and standard deviation of portfolio returns.

Our benchmark portfolio puts a constant weight on the variance asset each month, i.e., $f_t = const$. We discover that by setting the constant weight close to -7% for variance swaps, -22% for VIX futures, and -60% for straddles, our portfolio achieves a similar level of return standard deviation as S&P 500. Although the benchmark portfolio returns exhibit larger negative skewness, kurtosis, and maximum drawdowns than S&P 500, these statistics remain at reasonable and acceptable levels. The benchmark portfolios exhibit significantly better long-run performance relative to S&P 500, as indicated by higher Sharpe ratios, excess returns, and factor-model alphas.

Next, we show that the benchmark portfolio's performance can be significantly enhanced by carefully selecting f_t. In the same spirit of Moreira and Muir (2017), we set the weight proportional to the inverse of stock market volatility. Specifically, we consider three commonly used volatility measures in the literature: realized volatility (RV), option-implied volatility (VIX), and a GARCH(1,1) conditional one-month volatility forecast:

$$\begin{aligned} f_t &= 1/RV_t, \\ f_t &= 1/VIX_t, \\ f_t &= 1/GARCH(1,1)_t. \end{aligned} \tag{1.2}$$

The choice of setting the weight equal to the inverse of volatility is motivated by recent literature (Bekaert and Hoerova (2014), Cheng (2019), Cheng (2020), Aït-Sahalia, Karaman, and Mancini (2020), Lochstoer and Muir (2022), Yang (2022)), which emphasizes that in the data an increase in market volatility is typically associated with a statistically significant decrease in the conditional one-month VRP. Cheng (2019) specifically refers to this negative relation as a "low-premium response puzzle" as established asset pricing theories with time-varying volatility, volatility-of-volatility, disaster risk, and jumps (Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), Wachter (2013), Dew-

Becker, Giglio, Le, and Rodriguez (2017)) all predict a positive relation.

We first use OLS predictive regressions to confirm that this negative relation robustly exists both in-sample and out-of-sample during the longest periods 1990-2023. We then follow Moreira and Muir (2017) to use the monthly time-series of three volatility measures to sort the following month's variance swap returns into five portfolios. We confirm that, as we move from the "lowest volatility" portfolio to "highest volatility" portfolio, overall, average variance swap returns tend to weaken, standard deviation tends to rise, and Sharpe ratio tends to fall in magnitude.

We find that all three volatility-managed portfolios defined above improve performance. Specifically, for variance swaps, we find that our timing portfolio, when compared to constant-weight portfolios, enhances the Sharpe ratio from 1.54 to a maximum of 1.76. Additionally, we observe an increase in excess return and Carhart (1997) 4-factor alpha, as well as a reduction in negative skewness, kurtosis, and maximum drawdowns.

While our findings are based on using VIX-squared as an approximation for variance swap prices (see, e.g., Carr and Madan (1998)), we demonstrate the robustness of our findings by utilizing actual quoted OTC variance swap price data obtained from Dew-Becker, Giglio, Le, and Rodriguez (2017). Furthermore, we show that our findings are not driven by specific historical periods, as the improved performance holds true even when considering the recent 15-year period.

For VIX futures, we consistently find that our timing portfolio, compared to constant-weight portfolios, leads to an improvement in the Sharpe ratio from 0.61 to a maximum of 0.78. Furthermore, we observe an increase in excess return and Carhart (1997) 4-factor alpha (as well as alpha's t-statistic from 1.1 to a maximum of 2.0), as well as a reduction in negative skewness, kurtosis, and maximum drawdowns.

In our previous consideration of variance swaps and VIX futures, we made the simplifying assumption that there is no transation cost such as bid-ask spreads and margin requirements. We show that introducing bid-ask spreads may reduce the performance of volatility-managed portfolios and constant-weight portfolios, but is unlikely to affect the relative performance between the two. This is because volatility assets all have limited maturities. We need to rebalance whether the portfolio has constant or time-varying weights. Thus, constant-weight portfolios have no advantage over timing portfolios in terms of transaction costs, a significant difference than equity premium timing in Moreira and Muir (2017). In the case of straddles, we account for margin requirements. Specifically, we assume that the margin requirement for short selling an S&P 500 ATM straddle is equal to 100% of the selling proceeds plus 20% of the current S&P 500 level, as in Johnson (2017).

The inclusion of margin requirements typically results in a reduction in the magnitude of returns and Sharpe ratios. Despite this fact, we consistently find that our timing portfolios, compared to constant-weight portfolios, improve the Sharpe ratio from 0.51 to a maximum of 0.62. Additionally, we observe an increase in excess return and Carhart (1997) 4-factor alpha (as well as alpha's t-statistic from 1.2 to a maximum of 1.8), as well as a drop in negative skewness, kurtosis, and maximum drawdowns.

What is the economic interpretation of our results? We show that our volatility-managed portfolios work under a similar logic as that in Moreira and Muir (2017). In their study, the volatility-managed portfolios yield improvements relative to market buy-and-hold, primarily driven by the observed tendency for the one-month equity premium to decline following periods of increased volatility during economic downturns. Since high risk is not compensated for by higher risk premia, reducing the weight on the market index in such times helps mitigate further losses or expedite the recovery of values from economic downturns. As a result, the drawdowns experienced during recessions become smaller. Our analysis replaces equity premium with VRP, and S&P 500 with volatility assets.

Note that in leading asset pricing models, everything is compensation for risk. The only way to increase returns is to take on more risk. It follows that timing portfolios will never generate an alpha. The fact that our timing portfolios generate an alpha implies that the short-horizon risk-risk premium relation is essentially negative in the data.

So far, our timing portfolios have been volatility-managed. This approach offers several advantages. First, it is easy to implement. Second, it is independent of specific econometric models and avoids potential model misspecification. Johnson (2017) and Cheng (2019), among others, also find that their respective timing strategies, based on ex-ante VRP measures, can improve performance relative to S&P 500 or constant-weight volatility portfolios. To differentiate our approach from existing literature, we consider a fourth strategy that uses a VAR model of (RV_t, VIX_t) to estimate ex-ante one-month VRP and then sets weight proportional to it:

$$f_t = VRP_t \equiv 1 - \frac{E_t[RV_{t+1}]}{VIX_t}. \tag{1.3}$$

We do find that such econometrics-based VRP measures typically help improve portfolio performance. However, it exhibits two disadvantages. First, it typically underperforms compared to timing portfolios based on volatility measures, especially the GARCH volatility forecasts, which consistently prove to be a robust timing factor relative to others. Second, it shows less robustness across different historical periods and volatility assets.

Last but not least, we examine which market conditions have contributed the most to the performance improvements in our timing portfolios. To this end, we partition our data into two regimes: a high-volatility regime and a low-volatility regime. We classify a month t as belonging to the high-volatility (low-volatility) regime if realized volatility (RV_t) exceeds (falls below) the 80th percentile of its unconditional distribution, approximately 20. Within each regime, we run various timing portfolio excess returns onto contemporanous constant-weight portfolio excess returns.

Our exercise is similar to Moreira and Muir (2017) which decompose portfolio performance into expansions and recessions, and find that volatility-managed portfolios reduce exposure to MKT more greatly in recessions. Our choice of low versus high-volatility regimes is motivated by Yang (2022) which shows that the negative volatility-VRP relation puzzle is predominantly observed in high-volatility periods, not necessarily recessions.[1]

We find that various timing portfolios produce positive alpha and help reduce beta/exposure relative to benchmark portfolios mostly during high-volatility regimes, not in low-volatility regimes. We further provide detailed regime-conditional predictability evidence in the Appendix. We find that, aligning with the view in Yang (2022), only large "structural" volatility spikes predict negative realized VRPs in the next month, while small volatility shocks in stable periods essentially predict positive VRPs.

Our findings thus help differentiate theories on the puzzling negative risk-return relations in the volatility asset market. Cheng (2019) suggests the puzzle is possibly driven by falling hedging demand induced by microstructure frictions or government's implicit guarantees when volatility turns high. Lochstoer and Muir (2022) explain the puzzle with extrapolative volatility expectations. Yang (2022) attributes the puzzle to investors' slow and consecutive learning about occasional large underlying structural breaks in volatility. An underlying break (such as those caused by Gulf War, Dot-com Bubble Burst, Global Financial Crisis, and European Debt Crisis) occurs in month t and then lasts for several months. Due to limited rationality, investors do respond sharply, but not sharply enough. Investors' volatility expectation and pricing of volatility assets consecutively rise. It thus appears that month t+1 realized VRP is negative (i.e., month t+1 volatility asset returns are positive). Since our portfolio already reduces positions when observing the volatility spike in month t, it outperforms constant-weight portfolios. In normal times without structural breaks, there is no such puzzling negative risk-return relation and therefore volatility-managed portfolios do not work. Our findings indeed support Yang (2022)'s

[1] It is also known that stock market volatility has its own dynamics that is not entirely driven by business cycles.

channel.

However, we note that the above analysis is conducted in an ex-post context. In an ex-post study of portfolio performance, we find that the enhancement is driven by periods of persistent large volatility spikes, which however does not mean there must exist an easy way to leverage this ex-post pattern to further enhance ex-ante timing strategy performance.

The fundamental challenge is that, ex-ante, it is difficult to predict the arrival of large volatility spikes.[2] Indeed, the unpredictability of volatility spikes is precisely the reason why selling volatility on average earns positive premiums. If volatility spikes were predictable, then there would be much less covariation between investors' SDF and volatility. Then volatility assets wouldn't have been thus excessively expensive in stable periods. In short, our timing portfolios exploit the fact that "although bad states (crises) are unpredictable, investors' slow response to them is."

Literature. Our research is related to three literatures. First, it is related to the factor timing literature (Moreira and Muir (2017), Cederburg, ODoherty, Wang, and Yan (2020), Barroso and Detzel (2021), Haddad, Kozak, and Santosh (2020)). A difference is that we time VRPs rather than premiums such as equity and value premiums, etc. Another important difference is that Moreira and Muir (2017)'s volatility managed portfolio works when the timing factor is realized volatility, which negatively predicts next-month returns. Recent work by Martin (2017) shows that implied volatility instead tends to positively predict next-month returns. In contrast, the volatility-VRP relation is negative regardless of how to measure volatility, and thus our timing portfolios work more or less equally

[2]Volatility spikes, once occurring, immediately result in large negative realized VRPs. During low-volatility periods, where volatility and VRP exhibit small movement, these large negative realized VRPs stand out as outliers that can swiftly attenuate away the positive relations between volatility and next-month realized VRPs (that should have been seen in stable periods). In fact, this is the main reason why we did not observe an increasing pattern of VRPs with volatility levels across the first several "low volatility" portfolios when following Moreira and Muir (2017) to sort portfolios based on the current month's volatility levels.

well whether the timing factor is realized, implied, or conditional volatility.

Other papers that consider VRP timing strategies include, for instance, Dörries, Korn, and Power (2024), Egloff, Leippold, and Wu (2010), Johnson (2017) and Cheng (2019). Our research differs in several aspects. First, we are the first to consider all three types of variance assets. Second, we are the first to study (model-free) volatility-managed, rather than VRP-managed, portfolios. Third, we study portfolio performance over different subsamples. Our paper is also related to Aragon, Chen, and Shi (2022), which show that hedge funds have superior ability to time realized volatility (VRPs) across several asset classes.

Second, our research relates to a literature that studies intertemporal risk-return relations in volatility asset markets (Bekaert and Hoerova (2014), Cheng (2019), Cheng (2020), Aït-Sahalia, Karaman, and Mancini (2020), Lochstoer and Muir (2022), Yang (2022)). This literature has recently emphasized a puzzling negative volatility-VRP relation, inconsistent with leading asset pricing theories. Our timing strategies precisely exploit this puzzle, and our ex-post regime-conditional performance analysis helps differentiate competing theories on the puzzle.

Third, our research is related to the large volatility risk premium literature. Among others, Dew-Becker, Giglio, Le, and Rodriguez (2017) study variance swaps; Eraker and Wu (2017) and Cheng (2019) study VIX futures; Carr and Madan (1998), Coval and Shumway (2001), Broadie, Chernov, and Johannes (2007), Carr and Wu (2009), Christoffersen, Jacobs, and Mimouni (2010), Andersen, Fusari, and Todorov (2015), Eraker (2021), and Eraker and Yang (2022) study S&P 500 straddles and options; Johnson (2017) and Lochstoer and Muir (2022) study all three assets. Ang, Hodrick, Xing, and Zhang (2006) and Bansal, Kiku, Shaliastovich, and Yaron (2014) estimate VRP in the cross-section of stock returns. Our constant-weight portfolio by construction has the same Sharpe ratio as the original volatility asset returns. We find numbers consistent with those reported in the papers that originally study each particular asset, lending credibility to our results.

2 Timing Strategy

2.1 Equity Premium Timing

To time equity premium, Moreira and Muir (2017) use a strategy consisting of two assets: the market index and a risk-free asset. Each month t, the weight in the market is

$$w_t^{mkt} = \frac{c}{RV_t}, \tag{2.1}$$

where c is a scaling positive constant whose value is set such that the portfolio has the same unconditional return variance as the market index. RV_t is market return realized variance. Note that portfolio excess return (relative to risk-free rate) is equal to

$$\begin{aligned} r_{t+1} - r_t^f &= w_t^{mkt} r_{t+1}^{mkt} + (1 - w_t^{mkt}) r_t^f - r_t^f \\ &= w_t^{mkt} (r_{t+1}^{mkt} - r_t^f) \\ &= \frac{c}{RV_t} (r_{t+1}^{mkt} - r_t^f). \end{aligned} \tag{2.2}$$

Thus, portfolio Sharpe ratio $E(r_{t+1} - r_t^f)/std(r_{t+1} - r_t^f)$ is independent of the scaling constant c. A researcher cannot manipulate the Sharpe ratio. The authors provide evidence that this strategy generates a significant alpha and a higher Sharpe ratio compared to the market index. The rationale behind the strategy is that during periods of increased realized volatility, particularly during economic downturns, monthly equity premiums tend to decrease. Consequently, reducing the weight assigned to the market index enhances the strategy's performance.

2.2 Variance Risk Premium Timing

2.2.1 Motivating Evidence

Recent literature, such as Cheng (2019) and Lochstoer and Muir (2022), has emphasized that an increase in market volatility typically also leads to a decrease in the conditional one-month Volatility Risk Premium (VRP). Cheng (2019) specifically refers to this negative relation as a "low-premium response puzzle," as established asset pricing theories with time-varying volatility, volatility-of-volatility, disaster risk, and jumps (Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), Wachter (2013), Dew-Becker, Giglio, Le, and Rodriguez (2017)) all predict a positive relation.

In Table 1, we provide additional confirmation of the negative relationship between volatility and VRP using OLS predictive regressions. To ensure robustness, we use the longest available data sample and consider three different measures of market volatility: realized volatility (RV), implied volatility (VIX), and a GARCH(1,1) conditional one-month volatility forecast. As shown, consistent with the literature, when volatility rises, the next-month realized returns on a variance swap tend to be more positive, that is, realized VRP falls.

[Table 1 about here.]

Table 2 presents out-of-sample R-squared values obtained from OLS predictive regressions, following Welch and Goyal (2008). As seen, all three volatility measures exhibit a positive and significant OOS R-squared, indicating their ability to predict next-month realized variance asset returns in a meaningful manner.

[Table 2 about here.]

Figures 1, 2, and 3 sort the time-series of variance swap returns ($\frac{RV_{t+1}}{VIX_t^2} - 1$) into five portfolios respectively based on previous month's realized volatility (RV_t), implied volatility

(VIX_t), and GARCH(1,1)'s one-month volatility forecast ($GARCH(1,1)_t$). These exercises are similar to Moreira and Muir (2017) Figure 1. As seen, the message is consistent across different volatility measures. First, average variance swap returns tend to weaken as we move from the "lowest volatility" portfolio to "highest volatility" portfolio, confirming an overall negative volatility-VRP relationship. Second, standard deviation tends to rise. Third, as a result, Sharpe ratio tends to fall in size. Lastly, the probability of a recession monotonically increases.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

The above evidence all suggests that when volatility increases, VRP tends to fall. Equivalently speaking, volatility assets tend to be priced "too cheap" and thus exhibit more positive returns going forward. If follows that there might exist a potential opportunity for timing VRP by reducing negative exposure on volatility assets at such times. In this paper, we consider three variance assets. In our benchmark analysis, we use a one-month variance swap contract. This allows us to consider the longest data sample 1990-2023. Later on, we extend our analysis to include VIX futures and S&P 500 straddles as alternative variance assets. Note that the specific trading rule for each variance asset is different and can impact portfolio returns. We start by defining our timing strategies.

2.2.2 Timing Portfolio Definitions

Similar to Moreira and Muir (2017), our strategy consists of two assets: a variance asset and a risk-free asset. In all three cases (variance swap, VIX futures, or straddle), we determine the weight in the variance asset, denoted as w_t^i, at the end of each month t. Specifically, we set it as

$$w_t^i = -cf_t, \tag{2.3}$$

where c is a positive scaling constant chosen in such a way that the portfolio has the same unconditional return variance as the stock market index. The variable f_t represents a timing factor that ideally demonstrates a positive relationship with conditional $VRP_t = 1 - \frac{E_t[RV_{t+1}]}{VIX_t^2}$, which signifies the relative expensiveness of variance assets. Thus, we sell more (less) variance assets in month t if VRP_t is higher (lower).

Consider first the case of variance swaps. When initiating a long position in a one-month variance swap contract in month t, we are obligated to pay a fixed leg in exchange for realized variance RV_{t+1} in month $t+1$. Carr and Madan (1998) show that the fixed leg can be approximated by VIX_t^2. Let W_t denote our initial capital at the end of month t. Our portfolio return realized in month t+1, in excess of the risk-free rate, is equal to

$$\begin{aligned} r_{t+1} - r_t^f &= \frac{W_t w_t^{vs} \frac{RV_{t+1} - VIX_t^2}{VIX_t^2} + W_t(1 + r_t^f)}{W_t} - 1 - r_t^f \\ &= w_t^{vs} \Big(\frac{RV_{t+1}}{VIX_t^2} - 1 \Big) \\ &= w_t^{vs} r_{t+1}^{vs} \\ &= -cf_t r_{t+1}^{vs}. \end{aligned} \tag{2.4}$$

The numerator in the first line represents our month t+1 gross payoff. The first term represents the payoff from our positions in variance swaps, with a notional value equal to a fraction w_t^{vs} of our capital W_t. A negative (positive) value of w_t^{vs} implies that we have short (long) positions. The second term means that our entire capital W_t can be allocated to the risk-free rate since variance swap positions do not require any upfront payment to initiate. The third equality follows from a definition of monthly variance swap return in terms of notional values:

$$r_{t+1}^{vs} \equiv \frac{RV_{t+1}}{VIX_t^2} - 1. \tag{2.5}$$

Consider then the case of VIX futures. In futures trading, the margin requirements tend to be relatively small, so for the purpose of this analysis, we will ignore the margin requirements. In this case, our portfolio return realized in month t+1, in excess of the risk-free rate, is equal to

$$\begin{aligned}
r_{t+1} - r_t^f &= \frac{W_t w_t^{vix} \frac{P_{t+1}^{vix}}{P_t^{vix}} + W_t(1 - w_t^{vix})(1 + r_t^f)}{W_t} - 1 - r_t^f \\
&= w_t^{vix}(1 + r_{t+1}^{vix}) + (1 - w_t^{vix})(1 + r_t^f) - 1 - r_t^f \\
&= w_t^{vix}(r_{t+1}^{vix} - r_t^f) \\
&= -cf_t(r_{t+1}^{vix} - r_t^f).
\end{aligned} \tag{2.6}$$

The numerator in the first line still represents our month t+1 gross payoff. The first and second terms respectively indicate that we divide the total wealth W_t into VIX futures and risk-free assets according to the weight $(w_t^{vix}, 1 - w_t^{vix})$. Note that the last term in equation (2.6) is slightly larger than the last term of equation (2.4) given the same variance asset return. The difference reflects the fact that when we short sell VIX futures, our margin account immediately receives cash inflows that can be allocated to risk-free assets. Note that the second line follows from a definition of monthly VIX futures return:

$$r_{t+1}^{vix} \equiv \frac{P_{t+1}^{vix}}{P_t^{vix}} - 1, \tag{2.7}$$

where P_t^{vix} represents the value of a one-month constant-maturity VIX futures portfolio or ETF.

Consider then the case of straddles. In options trading, particularly options short selling, margin requirements are typically substantial and cannot be ignored. The margin requirement for options short selling is usually equal to 100% of the proceeds from short selling, plus a portion that is proportional to the level of the underlying asset price adjusted by the option's moneyness. In Johnson (2017), a margin requirement equal to the short selling proceeds plus 20% of the current S&P 500 level is assumed when short selling an at-the-money (ATM) SPX straddle.

In this case, our portfolio return realized in month t+1, in excess of the risk-free rate, is equal to (for negative weight w_t^{st})

$$\begin{aligned} r_{t+1} - r_t^f &= \frac{W_t w_t^{st} \frac{P_{t+1}^{st}}{20\% \times SPX_t} - W_t w_t^{st} \frac{P_t^{st}}{20\% \times SPX_t}(1 + r_t^f) + W_t(1 + r_t^f)}{W_t} - 1 - r_t^f \\ &= w_t^{st}\Big(- r_{t+1}^{st,short} - \frac{P_t^{st}}{20\% \times SPX_t} r_t^f \Big) \\ &= -cf_t\Big(- r_{t+1}^{st,short} - \frac{P_t^{st}}{20\% \times SPX_t} r_t^f \Big), \end{aligned} \tag{2.8}$$

which has a similar form as the last terms of equations (2.4) and (2.6), given the same variance asset return, apart from the risk-free rate term. The numerator in the first line still presents our month t+1 gross payoff. The first term indicates that we short sell straddles with a total margin requirement that is equal to a fraction w_t^{st} of our capital W_t. When w_t^{st} is negative, the first term is negative, representing the payment to close short positions. The second term is positive, representing the additional interest earned on short selling proceeds. The third term represents interest earned on our capital W_t, which entirely can be allocated to the risk-free rate, noting that even the capital maintained as margin earns interest.

The key difference between equation (2.8) and equations (2.4) and (2.6) is the redefinition of short returns. In the second line, we've followed Johnson (2017) to define the monthly short straddle return as

$$r^{st,short} \equiv \frac{P_t^{st} - P_{t+1}^{st}}{20\% \times SPX_t}, \tag{2.9}$$

which explicitly incorporates margin requirements into the calculation and represents a return from a short-seller's perspective. Here, P_t^{st} represents the value of a one-month constant-maturity S&P 500 ATM straddle portfolio.

3 Portfolio Performance: Variance Swaps

3.1 Longest OOS Periods: Jan 1992 - Nov 2023

We now examine the empirical performance of the above defined portfolios. We start with variance swaps as the variance asset, which allows us to have a relatively long data sample back to 1990. Table 3 provides a summary of the various f_t we consider and their respective performance.

It is important to define a benchmark portfolio. In our benchmark portfolio, we maintain a constant weight on variance swaps each month, $f_t = const$. Dew-Becker, Giglio, Le, and Rodriguez (2017) show that one-month variance swaps exhibit significantly higher return volatility compared to S&P 500. In our sample, the return volatility is approximately 66% per annum. Consequently, in order for the constant-weight portfolio to match the return variance of S&P 500, the weight assigned to the variance swap should be relatively small, close to -0.066 in our sample.

Consistent with Dew-Becker, Giglio, Le, and Rodriguez (2017), we find that the mean return on variance swaps in our sample is -31% monthly. With an exposure of -0.066, the resulting mean portfolio return is roughly $-0.066 \times -31\% = 2\%$ monthly, or 24% annualized. As seen in Table 3, the benchmark constant-weight portfolio already delivers a substantial mean excess return (24% annually) and Sharpe ratio (1.54 annually). Although the portfolio returns exhibit larger negative skewness, kurtosis, and maximum drawdowns compared to the S&P 500, these statistics remain at reasonable and acceptable levels. In the long run, the constant-weight portfolio significantly outperforms S&P 500 (see Figure 5). Since in recent decades, S&P 500 outperforms Fama and French (2015) 5 factors and momentum, it follows that the constant-weight portfolio also outperforms

Fama-French 5 factors and momentum (not reported).

We also regress portfolio returns onto Carhart (1997) 4 factors. The MKT beta is 0.57, which is less than 1. This implies that the strategy enhances returns while simultaneously reducing risk exposure to the stock market. Further, portfolio returns are also correlated with SMB and MOM.

To further enhance portfolio performance, we consider four different f_t. The first three strategies are not directly based on VRP, but rather based on different volatility measures. These measures include realized volatility, implied volatility, and a GARCH one-month volatility forecast. The analysis in the last section suggets that our exposure on variance swaps should decrease in volatility. Thus, we respectively set

$$\begin{aligned} f_t &= 1/RV_t, \\ f_t &= 1/VIX_t, \\ f_t &= 1/GARCH(1,1)_t. \end{aligned} \tag{3.1}$$

As shown, all of the three timing portfolios exhibit improved performance compared to the constant-weight portfolio. Specifically, the Sharpe ratio can be enhanced from 1.54 to a maximum of 1.76. The negative skewness, kurtosis, and maximum drawdowns all fall. The improved performance can be naturally attributed to two possibilities.

The first possibility is purely mechanical. When there is higher (expected) volatility, the expected payoff of variance swaps increases. By reducing negative exposure to variance swaps during such periods, performance is improved. However, this mechanical reason alone is obviously insufficient for two key reasons. First, the results hold even when using VIX as the timing factor, which is directly linked to the price, rather than the expected payoff, of variance swaps. If the mechanical reason were the main driver, we would expect to observe opposite results for VIX-timed portfolios, but this is not the case. Second, leading asset pricing models (such as Bollerslev, Tauchen, and

Zhou (2009), Drechsler and Yaron (2011), Wachter (2013), Dew-Becker, Giglio, Le, and Rodriguez (2017)) all suggest that when volatility rises, the VRP should increase, with variance asset prices rising more than expected payoffs. To boost performance, we would therefore need to increase our negative exposure to variance swaps, but instead, we observe the opposite effect.

The second reason is that, as shown in Cheng (2019), among others, when volatility increases, VRP puzzlingly decreases: variance swap prices increase less than expected payoffs. By reducing negative exposure to variance swaps during these periods, performance is enhanced. Cheng (2019) shows that the negative volatility-VRP relation holds no matter which volatility measure to use. This is precisely consistent with what we find: the timing portfolios improve performance more or less equally no matter which volatility measure to use. We therefore believe that the second reason is the primary driver of improved performance. As seen from Table 3, various timing strategies further help reduce MKT beta, suggesting an even lower exposure to systematic market risk.

[Table 3 about here.]

Johnson (2017) and Cheng (2019) develop VRP timing portfolios based on ex-ante VRP measures. As a comparison, we also introduce a strategy that directly incorporates a VRP measure as the timing factor. To estimate VRP, in each month, we estimate a VAR model with two variables (RV_t, VIX_t) and five lags, utilizing all available historical realized and implied volatility data. The inclusion of implied volatility (VIX_t) as a model input is motivated by our belief that it contains information about the expected future realized volatility ($E_t[RV_{t+1}]$). We use volatility measures instead of variance measures as model inputs, as we believe VAR-type models better capture the dynamics of volatility, which can be more gradual compared to the sharp movements of variance. We find that this VRP-based timing strategy also significantly improves performance. As a comparison, we also report the performance of stock market buy-and-hold and equity premium timing strategies of Moreira and Muir (2017).

Figure 4 plots each month's portfolio weight on variance swaps for each strategy, w_t^{vs}. Mechanically, our timing strategies enhance performance due to a positive correlation between the month t weight w_t^{vs} and the subsequent month's realized variance swap return $\frac{RV_{t+1}}{VIX_t} - 1$. By comparing the middle and bottom panels of the figure, we can observe a certain degree of positive comovement between these two. Intuitively, when volatility spikes in month t, we reduce our exposure to variance swaps (increase w_t^{vs}) immediately in month t. Variance swap return realized in month t is already positive and large due to the volatility spike. But since variance swap is typically underpriced in month t, when next month t+1 arrives, payoff is typically higher than the price, resulting in another large positive return, confirming the profitability of our strategy.

[Figure 4 about here.]

This predictive correlation is further directly confirmed by Table 4. Additionally, Table 5 presents the contemporaneous correlation between the weights adopted in different strategies. As shown, there is a notable level of similarity among them. The strategy based on VRP exhibits a relatively lower correlation with the strategies based on volatility measures.

[Table 4 about here.]

[Table 5 about here.]

Figure 5 displays the cumulative portfolio value for each strategy, with all portfolios starting with an initial value of $1 in January 1992. The ending value of each portfolio in November 2023 is also provided. As we can see, the cumulative effect over this 30-year period is huge, with the GARCH-timed portfolio, for instance, generating an almost three-times ending value than the constant-weight portfolio.

[Figure 5 about here.]

Additionally, Figure 5 illustrates the drawdowns for each strategy. Moreira and Muir (2017) demonstrate significant improvements relative to market buy-and-hold, primarily due to the tendency of the equity premium to decline after a volatility spike occurs during recessions. Their strategy sharply reduces the weight on the market index in such times, helping to mitigate further losses or expedite the recovery of values from economic downturns. As a result, drawdowns become smaller in recessions. Our logic is similar. After a volatility spike in recessions, VRP tends to decrease. Consequently, we significantly diminish the negative weight on variance swaps. This serves to prevent further losses or expedite the recovery of values from economic downturns. As a result, drawdowns also become smaller in recessions. These patterns are evident from Figure 5 when examining events such as the 2008 Global Financial Crisis.

3.1.1 Where is the improved performance exactly generated?

To better understand where the improved performance is exactly generated, we partition our data into two regimes: a high-volatility regime and a low-volatility regime. We classify a month t as belonging to the high-volatility (low-volatility) regime if realized volatility (RV_t) exceeds (falls below) the 80th percentile of its unconditional distribution, approximately 20. Table 6 performs regression analyses conditional on low-volatility ($RV_t < 20$) and high-volatility ($RV_t > 20$) regimes. Conditional on each regime, we regress timing portfolio excess returns onto contemporanous constant-weight portfolio excess returns:

$$r_t^{vs,f_{t-1}} - r_{t-1}^f = \alpha + \beta(r_t^{vs,cw} - r_{t-1}^f) + \varepsilon_t, \tag{3.2}$$

for $f_t = \{1/RV_t, 1/IV_t, 1/GARCH(1,1)_t\}$. We can see two patterns. First, the timing portfolio's alpha relative to constant-weight portfolio is mostly due to high-volatility regimes. Second, the timing portfolio's exposure to constant-weight portfolio is much lower in high-volatility regimes. These two observations both suggest that our VRP timing portfolios perform better in high-volatility times than in low-volatility times. Our exercise is similar to Moreira and Muir (2017) which decompose portfolio performance into expansions and recessions, and find that volatility-managed portfolios reduce exposure to MKT more greatly in recessions. Our choice of low versus high-volatility regimes is motivated by Yang (2022) which shows that the negative volatility-VRP relation puzzle is predominantly observed in high-volatility periods, not necessarily recessions.

[Table 6 about here.]

We further provide detailed regime-conditional predictability evidence in the Appendix. We find that, aligning with the view in Yang (2022), only large "structural" volatility spikes predict negative realized VRPs in the next month, while small volatility shocks in stable periods essentially predict positive VRPs.

Our findings thus help differentiate theories on the puzzling negative risk-return relations in the volatility asset market. Cheng (2019) suggests the puzzle is possibly driven by changes in hedging demand induced by microstructure frictions or government's implicit guarantees when volatility turns high. Lochstoer and Muir (2022) explain the puzzle with extrapolative volatility expectations. Yang (2022) attributes the puzzle to investors' slow and consecutive learning about underlying large structural breaks in volatility. An underlying break (such as those caused by Gulf War, Dot-com Bubble Burst, Global Financial Crisis, and European Debt Crisis) occurs in month t and then lasts for several months. Investors do respond sharply, but not sharply enough. Investors' volatility expectation and pricing of volatility assets consecutively rise. It thus appears that month t+1 realized VRP is negative (i.e., month t+1 volatility asset returns are more positive). Our findings seem to support the channel of Yang (2022).

Finally, we note that the above analysis in this section is conducted in an ex-post context. In an ex-post examination of portfolio performance, we find that the enhancement is driven by periods of large volatility spikes. While one might be tempted to think that a regime-dependent timing strategy can be developed, ex-post findings do not automatically suggest a straightforward method to enhance ex-ante timing strategies, due to several fundamental challenges. We explain these challenges in the Appendix. Consequently, in practice, we prefer to adopt simple timing strategies.

3.2 Shorter OOS Periods: Aug 2006 - Nov 2023

It appears from Figure 5 that the outperformance of different timing strategies compared to the constant-weight portfolio is primarily driven by the first half of our sample period. We conduct a robustness check by considering only the second half of our sample as out-of-sample test periods. Table 7 provides a summary of the performance for each strategy, while Figures 6 and 7 illustrate the portfolio weights and cumulative portfolio values for each strategy. In this analysis, all portfolios start with an initial value of \$1 in August 2006, with the ending value of each portfolio in November 2023 also displayed.

[Table 7 about here.]

Remarkably, we find that timing strategies based on VIX, GARCH volatility forecast and VRP estimated using the VAR model continue to significantly enhance performance compared to the constant-weight portfolio. From Figure 7, we can clearly see that during both the 2008 Crisis and the COVID-19 Crisis, our timing portfolio helped contain losses and accelerate value recovery relative to a fixed-weight portfolio.

[Figure 6 about here.]

[Figure 7 about here.]

Lastly, it is worth noting that the GARCH volatility forecast represents a particularly robust and powerful timing factor for the VRP. Its superior ability to predict the VRP is particularly strong during recent 15 to 10 years. This remarkable performance prompts us to consider the underlying reasons behind its efficacy. One possibility could be the changing dynamics of market volatility driven by breaks in macroeconomic factors, market structure, or trading patterns. GARCH is better able to capture structural breaks, which has rendered the relationship between volatility and VRP more favorable to the GARCH-based forecasting approach.

3.3 Robustness: Using Actual Price Data

Up until now, we have utilized VIX-squared as an approximation for the price of a variance swap contract. However, recent literature, such as the work by Martin (2017), raises concerns about the accuracy of this approximation, particularly when volatility exhibits jump-like behaviors. To address this issue, we incorporate actual OTC quoted variance swap price data provided by Dew-Becker, Giglio, Le, and Rodriguez (2017). This limits our analysis to a relatively shorter period, from 1996 to 2013.

First, we observe a high correlation of 98.7% between the realized return on a one-month variance swap, when based on (i) VIX-squared and (ii) the actual price data. The difference between the two is quite negligible. Second, when examining the out-of-sample test results in this scenario, as presented in Table 8 and Figures 8 and 9, we continue to find that various timing strategies demonstrate improvements compared to the constant-weight portfolio. For instance, Sharpe ratios can be enhanced from 1.26 to a maximum of 1.66. All of the other statistics also improve.

[Table 8 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

3.4 Transaction Costs

Up to this point, we have not factored in transaction costs in our analysis. It is important to note that our portfolios are constructed by holding a one-month variance swap contract to maturity each month. Given the low trading frequency, transaction costs associated with frequent rebalancing are unlikely to have a substantial impact on our portfolio performance.

Furthermore, even if transaction costs were substantial, they would likely have a similar impact on both a constant-weight strategy and, say, a VIX-timed strategy. The key distinction between the two strategies lies in the weighting scheme, with the former maintaining constant weights while the latter adjusts the weights based on VIX each month. But because the average weights are similar across the two strategies (for instance, Table 3 shows that average weights for all the strategies, including the constant-weight strategy, are close to -7%), whether the transaction costs are fixed or proportional (such as bid-ask spread), they would likely affect the two portfolios similarly. To see this, assume that, for instance, the bid-ask spread implies that the shorting return on the original one-month variance swaps is reduced by 1% per month, from 31% to 30%. Then, the return on the constant-weight and VIX-timed portfolios will both reduce by $1\% \times 0.07 = 0.07\%$ per month or $0.07\% \times 12 = 0.84\%$ per annum. The transaction cost, therefore, does not affect the relative performance between these two portfolios.

Note a significant difference between the volatility asset timing in the current paper and the stock market index timing in Moreira and Muir (2017). In Moreira and Muir (2017), the stock market index is an asset that can be held for a long term. Thus, the more frequently or drastically one adjusts the weight on the market index in the volatility-timed portfolios, the worse the volatility-timed portfolio will perform relative to market buy-and-hold. In contrast, volatility assets all have limited maturities.[3] We need to rebalance

[3]Indeed, any non-dividend asset with a stationary price process cannot have unlimited maturity. Oth-

whether our portfolio has constant or time-varying weights. Thus, transaction costs are unlikely to affect the relative performance between the two portfolios. This logic applies to variance swaps, as well as VIX futures and S&P 500 straddles that we consider below.

Considering these facts, we can reasonably expect that transaction costs, whether significant or not, are unlikely to sharply dampen the relative performance of our timing strategies relative to the constant-weight benchmark. This relative performance is this paper's focus. In the next, we consider exchange-traded volatility assets and explicitly take another form of transaction costs, the margin requirements, into account.

4 Portfolio Performance: VIX Futures and S&P 500 Straddles

We now consider using two exchange-traded assets, VIX futures and S&P 500 straddles, as the volatility asset. An important transaction cost in exchange trading is the margin requirement, which limits investors' ability to take positions. For VIX futures, we do not consider margin requirements for two reasons. First, margin requirements for futures trading are typically small. Second, there exist ETFs such as SVIX which provides investors a fixed negative exposure to constant-maturity VIX futures at minimum fees. For straddles, we explicitly consider margin requirements.

4.1 VIX Futures

Unlike our previous exercise with one-month variance swap contracts, VIX futures and S&P 500 straddles do not have regular monthly maturities. To address this, the literature typically constructs portfolios on a daily basis using a front-month contract and a back-month contract, appropriately weighted to achieve an average maturity of one month. We first obtain daily returns provided by Johnson (2017) for the constant-maturity VIX

erwise, investors can immediately create an arbitrage strategy by buying low (slightly below the unconditional average) and selling high (slightly above the unconditional average) using limit orders.

futures/S&P 500 ATM straddle positions with a target maturity of one month, denoted as $r_{m,m+1/21}$. We then roll over these strategies each month to convert the daily returns into monthly returns $r_{m,m+1}$:

$$1 + r_{m,m+1} = \prod_{j=1}^{21} \left(1 + r_{m+\frac{j-1}{21},m+\frac{j}{21}}\right). \tag{4.1}$$

Figure 10 illustrates the time series of monthly returns on the three volatility investments we consider. Table 9 displays the correlation between them. While these returns exhibit strong positive correlations, they are not perfectly correlated. Notably, the average negative returns are more pronounced for variance swaps relative to VIX futures and straddles.

[Table 9 about here.]

[Figure 10 about here.]

To assess the most recent periods, particularly the Covid Crisis periods, we supplement the VIX futures and straddle returns obtained from Johnson (2017) with data purchased from CBOE and OptionMetrics. This allows us to extend our sample to the most recent time.

Table 10 presents a summary of our results for VIX futures. As shown, the constant-weight portfolio generates a Sharpe ratio of 0.61, which by construction is equal to the (negative) Sharpe ratio of the original VIX futures portfolios. Our findings therefore align closely with those in Eraker and Wu (2017). In their study, they report an annualized return of -40% and an annualized standard deviation of $\sqrt{252} \times 4\% = 64\%$ for one-month constant-maturity VIX futures portfolios, implying a Sharpe ratio of -0.63. Our timing portfolios, on the other hand, can enhance the Sharpe ratio to as high as 0.78. All the other statistics also improve. The most notable improvement is in the Carhart (1997) 4-factor alpha, from 3.1% (statistically insignificant) to 6.3% (statistically significant).

[Table 10 about here.]

Plus, as shown in Table 11, we continue to find that the timing portfolios generate positive alpha and reduce exposure relative to constant-weight portfolios mostly in high-volatility regimes. In low-volatility regimes, the timing portfolios even increase rather than reduce exposure, evidence again consistent with "no low-premium response puzzle" in low-volatility periods. In sum, ex-post, our timing strategies simply work significantly better in high-volatility periods.

[Table 11 about here.]

Figures 11 and 12 plot the portfolio weights and cumulative portfolio values, respectively, for VIX futures-based portfolios. In this analysis, all portfolios start with an initial value of $1 in April 2004, with the ending value of each portfolio in November 2023 also displayed. Over this nearly 20-year period, the GARCH-timed portfolio, for instance, generates nearly twice the value of the constant-weight portfolio. From Figure 12, we can again clearly see that during both the 2008 Crisis and the COVID-19 Crisis, our timing portfolio helped contain losses and accelerate value recovery relative to a fixed-weight portfolio. This explains the smaller drawdowns.

[Figure 11 about here.]

[Figure 12 about here.]

Cheng (2019) implements a "Cash or Short" timing strategy, where all portfolio capital is allocated to a short VIX futures contract (subject to margin constraints) each day if the estimated ex-ante VRP (Q minus P) is positive, and to a cash account otherwise. The portfolio is then scaled to having a similar unconditional return variance as S&P 500. He uses a test period from 2004 to 2015, and reports a Sharpe ratio of 0.87, an excess return of 16.6% (both annualized), and a maximum daily drawdown of -26.4%.

We replicate various timing strategies during the same period. Although our strategy involves monthly rebalancing instead of daily rebalancing given that there are VIX futures ETFs directly available to investors, we find that the results are highly comparable. For the VAR strategy, we observe a Sharpe ratio of 0.98, an excess return of 15% (both annualized), and a maximum monthly drawdown of -25.4%.

Interestingly, as shown in Figure 12, the VAR-VRP timing strategy shows poorer performance during the more recent 2016-2023 period. In contrast, the simpler timing strategy based on volatility measures, such as GARCH forecasts, performs markedly better in recent periods. This observation suggests that the simplicity of our volatility-based timing strategy may confer advantages in terms of effectiveness and robustness.

4.2 S&P 500 ATM Straddles

Table 12 presents a summary of our findings based on S&P 500 straddles. The constant-weight portfolio generates a Sharpe ratio of 0.51. Note that this Sharpe ratio may appear relatively small because it incorporates the influence of margin constraints. Our findings align with previous studies by Carr and Madan (1998), Coval and Shumway (2001), Eraker (2021), and Eraker and Yang (2022), which all suggest that a reasonable range for the Sharpe ratio of S&P 500 near-ATM straddles is between -0.4 and -1.

[Table 12 about here.]

As shown in the table, our timing portfolios have the ability to improve the Sharpe ratio from 0.51 to a maximum of 0.62. Although the improvement may be relatively smaller compared to the VIX futures case, it highlights the robustness of our timing portfolios, even after accounting for margin constraints.

Plus, Table 13 again shows that the timing portfolios generate positive alpha and reduce exposure relative to constant-weight portfolios mostly in high-volatility regimes. In low-volatility regimes, the timing portfolios even increase rather than reduce exposure, evidence consistent with "no low-premium response puzzle" in low-volatility periods.

[Table 13 about here.]

In Table 12, we report two average portfolio weights. First, the weight w_t defined in equation (2.8), which measures the fraction of month-t total capital that is deposited as margin. Because the margin requirement is usually higher than the market value of the straddles we can trade, w_t does not measure our portfolio's real exposure to percent price movement in straddles. Second, the modified weight $\frac{w_t P_t^{st}}{20\% \times SPX_t}$ measures the true exposure. As seen, the true exposure is much smaller, on average from -0.12 to -0.14. Figure 13 plots the time series of the two portfolio weights for various strategies.

[Figure 13 about here.]

Figure 14 plots cumulative portfolio values for various strategies. In this analysis, all portfolios start with an initial value of $1 in Jan 1996, with the ending value of each portfolio in Dec 2022 also displayed. Over this 27-year period, the VIX-timed portfolio, for instance, generates nearly twice the value of the constant-weight portfolio. From Figure 14, we can again clearly see that during both the 2008 Crisis and the COVID-19 Crisis, our timing portfolio helped contain losses and accelerate value recovery relative to a fixed-weight portfolio, explaining the smaller drawdowns.

[Figure 14 about here.]

Johnson (2017) considers a strategy where each day, he buys (sells subject to margin requirements) one-month S&P 500 ATM straddles if "slope" (the second principal component) of VIX term structure falls within the lowest (highest) 20% of historical realized "slopes." Presumably, the effectiveness of this strategy may be influenced by the initial

OOS time period, which impacts the historical distribution of realized "slopes." Differently, our approach relies on simple volatility measures and exhibits robustness across various subsamples.

5 Conclusion

We develop VRP timing strategies that are analogous to equity premium timing strategies based on realized volatility, as in Moreira and Muir (2017). Our strategies involve trading two assets: a variance asset and a risk-free asset. To begin, we examine a benchmark portfolio which sells a fixed weight of variance asset each month. While this simple strategy already delivers remarkable long-term returns, we show that the portfolio's performance can be significantly enhanced by incorporating various timing factors, including several volatility measures and an ex-ante VRP measure. We find the simple volatility-managed strategies are particularly effective and robust. Our findings remain robust in both older and recent times and across three variance assets: variance swaps, VIX futures, and S&P 500 straddles. Our findings are unlikely affected by bid-ask spreads and hold after accounting for margin requirements.

Our portfolios improve performance by reducing negative exposure to variance assets once observing an increase in volatility. Essentially, we exploit the puzzle of the negative volatility-VRP relationship, which has been highlighted in several previous studies such as Cheng (2019). Notably, we find that, ex-post, various timing portfolios generate positive alpha and reduce beta/exposure to constant-weight variance asset portfolio returns almost only during high-volatility regimes, not in low-volatility regimes. Our such findings are consistent with Yang (2022) which estimates a regime-switch model and finds that the puzzling negative volatility-VRP relation is mostly driven by volatility-spikes.

However, we also note that the performance analysis is conducted in an ex-post context. Ex-post findings do not automatically suggest a straightforward method to enhance ex-ante timing strategies, mostly because volatility spikes are hard to predict. We leave the exploration of regime-conditional timing strategies for future research.

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Appendix A Regime-Conditional Predictive Regressions

We present further evidence regarding regime-conditional predictability. Figure 15 illustrates the coefficients and t-statistics from the unconditional OLS predictive regressions using the whole sample. We draw upon three variance asset returns (variance swaps, VIX futures, and straddles) and two volatility measures: RV and IV. As observed, the negative or weak relationship between month-t volatility and month-t+1 realized VRP is mainly driven by the occurrence of large negative realized VRP in month-t+1, which is typically associated with large volatility spikes in month t. In other words, the puzzling negative predictive relation is driven by a bunch of "outliers" located in the lower right corner of the graph.

[Figure 15 about here.]

In Figure 16, we plot the predictive regression coefficients and t-statistics conditional on the absence of such large spikes in month t as well as month t+1. Notably, once we isolate away the influence of these large spikes, all the predictive relationships exhibit statistically positive signs. We conclude that, in long-lasting stable periods, small volatility shocks positively predict realized VRP in the next month.

[Figure 16 about here.]

The above evidence of course corroborates our own findings that volatility-managed variance asset portfolios exhibit much stronger performance during high-volatility regimes (see Tables 6, 11 and 13). The enhanced performance of our timing portfolios relative to the original variance assets is essentially driven by a small bunch of "outling" periods. This is the case for all three variance assets.

Finally, we note that the above analysis is conducted in an ex-post context. In an ex-post examination of portfolio performance, we find that the enhancement is driven by periods of large persistent spikes. While one might be tempted to think that a regime-dependent timing strategy can be developed, ex-post findings do not automatically suggest a straightforward method to enhance ex-ante timing strategies.

One fundamental challenge is that it is difficult to predict random occurrence of a large volatility spike, which, once occurring, immediately results in large negative realized VRPs. During low-volatility periods, where volatility and risk premia exhibit small movement, these substantial negative realized VRPs stand out as outliers that can swiftly attenuate away the positive relations between volatility and next-month realized VRPs (that should have been seen in stable periods). In fact, this is the main reason why we did not observe an increasing pattern of VRPs with volatility levels across the first several "low-volatility" portfolios when following Moreira and Muir (2017) to sort portfolios based on the current month's volatility levels (see Figures 1, 2, and 3). The positive relations between VRPs and volatility levels are attenuated away due to the occurrence of random rare large volatility spikes in the subsequent month, which the portfolio sorting method does not account for.

Thus, we conclude that although we observe that ex-post portfolio performance is predominantly driven by persistent large volatility spikes, this does not necessarily imply that there is an easy way to leverage this pattern to enhance ex-ante timing strategy performance. Thus, in practice, we prefer to adopt simpler timing strategies.

Table 1. Motivating evidence: in-sample predictive regressions

The table displays in-sample OLS predictive regression coefficients and R-squared. LHS is next-month realized return on a one-month variance swap (annualized in percentage). RHS is a market volatility measure: realized volatility, implied volatility, and GARCH(1,1) one-month volatility forecast (annualized in percentage). Sample period is monthly Jan 1990 - Nov 2023. t-stat is based on Newey-West standard errors with 3 lags.

$\frac{RV_{t+1}}{VIX_t^2} - 1 = const + \beta X_t + \varepsilon_{t+1}$			
$X_t =$	RV_t	IV_t	$GARCH(1,1)_t$
β	9.82	9.79	16.63
t-stat	(2.00)	(1.42)	(2.47)
R^2	1.4%	0.86%	2.21%

Table 2. Motivating evidence: OOS R-squared.

The table displays OOS OLS predictive regression R-squared. We keep the longest OOS periods, Mar 1990 - Nov 2023. We follow Welch and Goyal (2008) to compute OOS R-squared as

$$R^2_{OOS} = 1 - \frac{\sum_{t=0}^{T-1}(r^{vs}_{t+1} - \hat{\mu}_t)^2}{\sum_{t=0}^{T-1}(r^{vs}_{t+1} - \bar{r}_t)^2}, \tag{A.1}$$

where $\hat{\mu}_t$ is the filtered value of the expected variance swap return using data only up until month t to estimate OLS parameters $\hat{\alpha}, \hat{\beta}$: $r^{vs}_{t+1} = \hat{\alpha} + \hat{\beta} X_t + \varepsilon_{t+1}$. The denominator $\bar{r}^{vs}_t$ is the historical mean of variance swap returns up until month t.

$X_t =$	RV_t	IV_t	$GARCH(1,1)_t$
	3.26%	2.63%	4.61%

Table 3. Out-of-sample strategy performance: longest periods.

In-sample: Jan 1990 - Dec 1991. Out-of-sample: Jan 1992 - Nov 2023. All variables are annualized. RV_t is realized volatility, VIX_t is implied volatility, $GARCH(1,1)_t$ is GARCH(1,1)'s forecast of next-month stock market return volatility. Each month-end t, GARCH(1,1) is fitted to historical daily stock market return since 1986. Results are robust to using other starting time, such as 1990, 1970 etc. $1 - \frac{E_t[RV_{t+1}]}{VIX_t}$ is conditional volatility risk premium in returns. To obtain $E_t[RV_{t+1}]$, each month t, a VAR of (RV_t, VIX_t) with 5 lags is fitted to historical monthly data since 1990. Results are robust to different lags. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

	VRP timing					EP timing	
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$	buy-hold	$\frac{1}{RV_t}$
excess return	0.239	0.248	0.253	0.273	0.268	0.075	0.090
	(7.40)	(7.57)	(7.90)	(8.44)	(8.08)	(2.75)	(3.21)
std	0.154	0.154	0.154	0.154	0.154	0.154	0.154
S.R.	1.54	1.61	1.63	1.76	1.74	0.48	0.58
skewness	-4.40	-4.13	-4.06	-3.74	-2.81	-0.76	-0.83
kurtosis	28.94	34.36	28.19	24.08	18.48	4.55	16.68
max. drawdown	-0.35	-0.42	-0.34	-0.32	-0.31	-0.19	-0.35
average weight w_t	-0.07	-0.06	-0.07	-0.07	-0.07	1	1.11
	Fama-French 3 + Mom						
α	0.184	0.202	0.206	0.222	0.219		0.033
	(5.93)	(6.45)	(6.75)	(7.53)	(7.17)		(1.37)
β_{MKT}	0.57	0.46	0.48	0.51	0.49		0.67
	(5.81)	(6.27)	(6.08)	(6.95)	(7.18)		(8.67)
β_{HML}	0.13	0.11	0.07	0.10	0.05		0.04
	(1.31)	(1.60)	(1.07)	(1.32)	(0.63)		(0.70)
β_{SMB}	0.16	0.09	0.10	0.12	0.14		-0.04
	(2.71)	(1.47)	(1.89)	(2.10)	(2.24)		(-0.61)
β_{MOM}	0.12	0.14	0.13	0.15	0.17		0.14
	(3.43)	(3.40)	(3.33)	(3.86)	(4.81)		(3.70)
R^2	0.33	0.21	0.23	0.26	0.25		0.40

Table 4. Predictive correlation.

The table displays the correlation between portfolio weight and next-month variance swap realized return. Test period is Jan 1992 - Nov 2023.

	w_t^{var}	w_t^{garch}	w_t^{iv}	w_t^{rv}
$\frac{RV_{t+1}}{VIX_t^2} - 1$	0.12	0.13	0.07	0.08

Table 5. Correlation between weights.

The table displays the correlation between portfolio weights for different strategies. Test period is Jan 1992 - Nov 2023.

	w_t^{var}	w_t^{garch}	w_t^{iv}	w_t^{rv}
w_t^{var}	1.00	0.39	0.18	0.24
w_t^{garch}	0.39	1.00	0.78	0.84
w_t^{iv}	0.18	0.78	1.00	0.82
w_t^{rv}	0.24	0.84	0.82	1.00

Table 6. Conditional regressions: longest periods.

Test period is Jan 1992 - Nov 2023. All variables are annualized. We define month t as low-volatility (high-volatility) regime if $RV_t < 20$ ($RV_t > 20$). Conditional on the two regimes respectively, we run f_t-timing portfolio excess returns onto $f_t = const$ portfolio excess returns. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

	VRP timing				
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$
	Low-volatility regime				
α		0.03	0.01	0.03	0.04
		(0.59)	(0.31)	(1.49)	(1.55)
$\beta_{r^{vs,cw}_{t+1}}$		1.01	1.07	1.08	0.96
		(7.60)	(12.99)	(20.42)	(11.87)
R^2		0.81	0.92	0.96	0.81
	High-volatility regime				
α		0.01	0.02	0.03	0.07
		(0.43)	(1.86)	(1.95)	(1.57)
$\beta_{r^{vs,cw}_{t+1}}$		0.43	0.51	0.54	0.65
		(7.75)	(14.68)	(9.99)	(5.59)
R^2		0.93	0.94	0.92	0.68

Table 7. Out-of-sample strategy performance: recent periods.

In-sample: Jan 1990 - July 2006. Out-of-sample: Aug 2006 - Nov 2023. All variables are annualized. RV_t is realized volatility, VIX_t is implied volatility, $GARCH(1,1)_t$ is GARCH(1,1)'s forecast of next-month stock market return volatility. Each month-end t, GARCH(1,1) is fitted to historical daily stock market return since 1986. Results are robust to using other starting time, such as 1990, 1970 etc. $1 - \frac{E_t[RV_{t+1}]}{VIX_t}$ is conditional volatility risk premium in returns. To obtain $E_t[RV_{t+1}]$, each month t, a VAR of (RV_t, VIX_t) with 5 lags is fitted to historical monthly data since 1990. Results are robust to different lags. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

	VRP timing					EP timing	
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$	buy-hold	$\frac{1}{RV_t}$
excess return	0.144	0.145	0.154	0.168	0.155	0.085	0.083
	(3.17)	(3.35)	(3.57)	(3.82)	(3.43)	(2.15)	(2.03)
std	0.163	0.163	0.163	0.163	0.163	0.163	0.163
S.R.	0.87	0.87	0.94	1.02	0.94	0.52	0.50
skewness	-3.90	-4.21	-3.75	-3.58	-3.40	-0.73	-2.09
kurtosis	21.05	28.98	21.44	19.47	20.10	4.41	24.19
max. drawdown	-0.30	-0.38	-0.29	-0.29	-0.30	-0.19	-0.36
average weight w_t	-0.06	-0.05	-0.06	-0.06	-0.05	1	1.08
			Fama-French 3 + Mom				
α	0.090	0.099	0.104	0.118	0.106		0.029
	(2.23)	(2.41)	(2.57)	(3.00)	(2.58)		(0.74)
β_{MKT}	0.64	0.52	0.56	0.57	0.55		0.63
	(5.00)	(4.97)	(5.12)	(5.51)	(5.33)		(5.48)
β_{HML}	-0.01	-0.05	-0.08	-0.05	-0.04		-0.03
	(-0.04)	(-0.37)	(-0.71)	(-0.42)	(-0.28)		(-0.32)
β_{SMB}	0.08	0.07	0.01	0.06	0.08		0.01
	(0.83)	(0.53)	(0.13)	(0.56)	(0.66)		(0.08)
β_{MOM}	0.10	0.13	0.12	0.13	0.17		0.13
	(1.92)	(2.00)	(2.00)	(2.22)	(3.04)		(2.00)
R^2	0.38	0.24	0.28	0.30	0.27		0.35

Table 8. Out-of-sample strategy performance: using actual price data

Data is based on monthly one-month variance swap prices provided in Dew-Becker, Giglio, Le, and Rodriguez (2017). In-sample: Jan 1990 - Dec 1995. Out-of-sample: Jan 1996 - Sep 2013. All variables are annualized. RV_t is realized volatility, VIX_t is implied volatility, $GARCH(1,1)_t$ is GARCH(1,1)'s forecast of next-month stock market return volatility. Each month-end t, GARCH(1,1) is fitted to historical daily stock market return since 1986. Results are robust to using other starting time, such as 1990, 1970 etc. $1 - \frac{E_t[RV_{t+1}]}{VIX_t}$ is conditional volatility risk premium in returns. To obtain $E_t[RV_{t+1}]$, each month t, a VAR of (RV_t, VIX_t) with 5 lags is fitted to historical monthly data since 1990. Results are robust to different lags. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

	VRP timing					EP timing	
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$	buy-hold	$\frac{1}{RV_t}$
excess return	0.210	0.263	0.232	0.246	0.278	0.052	0.082
	(4.24)	(5.36)	(4.81)	(5.07)	(5.53)	(1.33)	(2.07)
std	0.165	0.165	0.165	0.165	0.165	0.165	0.165
S.R.	1.26	1.58	1.39	1.48	1.66	0.32	0.50
skewness	-5.43	-3.42	-5.12	-4.78	-2.81	-0.87	-0.23
kurtosis	46.94	25.12	46.64	41.38	19.89	4.33	6.51
max. drawdown	-0.45	-0.37	-0.45	-0.43	-0.35	-0.19	-0.21
average weight w_t	-0.07	-0.08	-0.07	-0.08	-0.08	1	1.18
				Fama-French 3 + Mom			
α	0.170	0.220	0.192	0.205	0.237		0.038
	(4.15)	(5.73)	(4.98)	(5.39)	(6.17)		(1.21)
β_{MKT}	0.54	0.52	0.50	0.52	0.53		0.76
	(4.06)	(5.73)	(4.51)	(5.18)	(6.65)		(7.87)
β_{HML}	-0.02	0.05	-0.00	0.00	-0.08		-0.01
	(-0.16)	(0.45)	(-0.01)	(0.03)	(-0.73)		(-0.10)
β_{SMB}	0.12	0.08	0.11	0.09	0.10		-0.05
	(1.73)	(1.04)	(1.52)	(1.16)	(1.17)		(-0.67)
β_{MOM}	0.13	0.17	0.15	0.16	0.19		0.13
	(3.66)	(4.50)	(4.26)	(4.57)	(5.14)		(3.47)
R^2	0.30	0.27	0.26	0.27	0.30		0.53

Table 9. Correlation between constant-maturity variance asset returns.

The table displays the correlation between three constant-maturity variance asset returns. Data is monthly Apr 2004 - June 2019.

	r_t^{vs}	r_t^{vix}	$-r_t^{st,short}$
r_t^{vs}	1.00	0.69	0.87
r_t^{vix}	0.69	1.00	0.74
$-r_t^{st,short}$	0.87	0.74	1.00

Table 10. Out-of-sample strategy performance: VIX futures.

Data is based on one-month constant-maturity VIX futures returns provided in Johnson (2017), augmented with the most recent data obtained from CBOE. We roll-over daily returns into monthly returns. In-sample: Jan 1990 - Mar 2004. Out-of-sample: Apr 2004 - Nov 2023. All variables are annualized. RV_t is realized volatility. VIX_t is implied volatility. $GARCH(1,1)_t$ is GARCH(1,1)'s forecast of next-month stock market return volatility. Each month-end t, GARCH(1,1) is fitted to historical daily stock market return since 1986. Results are robust to using other starting time, such as 1990, 1970 etc. $1-\frac{E_t[RV_{t+1}]}{VIX_t}$ is conditional volatility risk premium in returns. To obtain $E_t[RV_{t+1}]$, each month t, a VAR of (RV_t, VIX_t) with 5 lags is fitted to historical monthly data since 1990. Results are robust to different lags. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

	VRP timing					EP timing	
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1-\frac{E_t[RV_{t+1}]}{VIX_t}$	buy-hold	$\frac{1}{RV_t}$
excess return	0.095	0.106	0.112	0.123	0.107	0.080	0.078
	(2.52)	(2.80)	(2.96)	(3.20)	(2.74)	(2.27)	(2.18)
std	0.156	0.156	0.156	0.156	0.156	0.156	0.156
S.R.	0.61	0.68	0.72	0.78	0.68	0.51	0.50
skewness	-2.86	-1.96	-2.03	-1.59	-1.67	-0.73	-2.00
kurtosis	16.49	9.65	9.91	6.76	9.98	4.69	23.81
max. drawdown	-0.28	-0.25	-0.26	-0.18	-0.25	-0.19	-0.36
average weight w_t	-0.22	-0.25	-0.26	-0.26	-0.23	1	1.09
			Fama-French 3 + Mom				
α	0.031	0.052	0.052	0.063	0.051		0.025
	(1.12)	(1.57)	(1.66)	(2.04)	(1.44)		(0.74)
β_{MKT}	0.78	0.65	0.72	0.72	0.66		0.63
	(8.59)	(7.75)	(8.86)	(9.31)	(7.53)		(5.72)
β_{HML}	0.11	0.04	-0.01	0.05	0.03		-0.06
	(0.94)	(0.38)	(-0.12)	(0.49)	(0.20)		(-0.60)
β_{SMB}	0.03	0.05	0.03	0.03	-0.07		0.07
	(0.94)	(0.45)	(0.38)	(0.28)	(-0.58)		(0.55)
β_{MOM}	0.07	0.11	0.11	0.11	0.14		0.14
	(1.51)	(1.92)	(2.16)	(2.10)	(2.56)		(2.44)
R^2	0.61	0.40	0.49	0.49	0.35		0.37

Table 11. Conditional regressions: VIX futures.

Test period is Apr 2004 - Nov 2023. All variables are annualized. We define month t as low-volatility (high-volatility) regime if $RV_t < 20$ ($RV_t > 20$). Conditional on the two regimes respectively, we run f_t-timing portfolio excess returns onto $f_t = const$ portfolio excess returns. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

		VRP timing			
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$
		Low-volatility regime			
α		-0.01	-0.00	0.00	-0.01
		(-0.47)	(-0.26)	(0.30)	(-0.64)
$\beta_{r_{t+1}^{vix,cw}}$		1.20	1.20	1.22	1.09
		(17.85)	(16.11)	(38.75)	(13.11)
R^2		0.88	0.93	0.96	0.80
		High-volatility regime			
α		0.02	0.03	0.04	0.08
		(1.00)	(1.87)	(2.13)	(1.41)
$\beta_{r_{t+1}^{vix,cw}}$		0.43	0.55	0.52	0.49
		(5.19)	(24.02)	(7.29)	(3.55)
R^2		0.89	0.96	0.91	0.55

Table 12. Out-of-sample strategy performance: S&P 500 ATM straddles.

Data is based on (short) one-month constant-maturity ATM S&P 500 straddle returns provided in Johnson (2017), augmented with the most recent data obtained from OptionMetrics. We roll-over daily returns into monthly returns. In-sample: Jan 1990 - Dec 1995. Out-of-sample: Jan 1996 - Dec 2022. All variables are annualized. RV_t is realized volatility. VIX_t is implied volatility. $GARCH(1,1)_t$ is GARCH(1,1)'s forecast of next-month stock market return volatility. Each month-end t, GARCH(1,1) is fitted to historical daily stock market return since 1986. Results are robust to using other starting time, such as 1990, 1970 etc. $1 - \frac{E_t[RV_{t+1}]}{VIX_t}$ is conditional volatility risk premium in returns. To obtain $E_t[RV_{t+1}]$, each month t, a VAR of (RV_t, VIX_t) with 5 lags is fitted to historical monthly data since 1990. Results are robust to different lags. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

	VRP timing					EP timing	
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$	buy-hold	$\frac{1}{RV_t}$
excess return	0.092	0.101	0.112	0.107	0.101	0.067	0.069
	(2.25)	(2.48)	(2.75)	(2.64)	(2.58)	(2.13)	(2.17)
std	0.162	0.162	0.162	0.162	0.162	0.162	0.162
S.R.	0.51	0.55	0.62	0.58	0.55	0.41	0.43
skewness	-4.07	-2.72	-3.01	-3.36	-2.90	-0.77	-1.77
kurtosis	28.61	13.96	17.32	21.70	21.37	4.27	23.04
max. drawdown	-0.39	-0.28	-0.34	-0.35	-0.38	-0.19	-0.39
average weight w_t	-0.60	-0.78	-0.77	-0.73	-0.62	1	1.06
average weight $\frac{w_t P_t^{st}}{20\% \times SPX_t}$	-0.12	-0.13	-0.14	-0.13	-0.12		
	Fama-French 3 + Mom						
α	0.049	0.057	0.067	0.064	0.061		0.020
	(1.20)	(1.57)	(1.79)	(1.73)	(1.74)		(0.70)
β_{MKT}	0.35	0.32	0.34	0.31	0.27		0.66
	(2.48)	(3.57)	(3.06)	(3.29)	(3.24)		(7.84)
β_{HML}	0.11	0.12	0.07	0.09	0.06		0.02
	(0.90)	(1.17)	(0.80)	(0.96)	(0.60)		(0.28)
β_{SMB}	0.29	0.26	0.27	0.27	0.29		-0.01
	(3.68)	(3.53)	(3.57)	(3.42)	(2.41)		(-0.18)
β_{MOM}	0.04	0.07	0.08	0.07	0.09		0.12
	(0.92)	(1.51)	(1.75)	(1.61)	(2.11)		(2.92)
R^2	0.19	0.15	0.16	0.14	0.12		0.40

Table 13. Conditional regressions: S&P 500 ATM straddles.

Test period is Jan 1996 - Dec 2022. All variables are annualized. We define month t as low-volatility (high-volatility) regime if $RV_t < 20$ ($RV_t > 20$). Conditional on the two regimes respectively, we run f_t-timing portfolio excess returns onto $f_t = const$ portfolio excess returns. t-stat (in parenthesis) is based on Newey-West s.e. with 3 lags.

		VRP timing			
f_t	const	$\frac{1}{RV_t}$	$\frac{1}{VIX_t}$	$\frac{1}{GARCH(1,1)_t}$	$1 - \frac{E_t[RV_{t+1}]}{VIX_t}$
		Low-volatility regime			
α		0.01	0.01	-0.00	-0.00
		(0.31)	(1.27)	(-0.01)	(-0.06)
$\beta_{r_{t+1}^{st,cw}}$		1.21	1.19	1.25	1.13
		(12.88)	(25.67)	(19.46)	(8.16)
R^2		0.89	0.94	0.97	0.84
		High-volatility regime			
α		0.02	0.03	0.04	0.06
		(0.78)	(1.86)	(1.71)	(1.43)
$\beta_{r_{t+1}^{st,cw}}$		0.51	0.66	0.55	0.50
		(5.12)	(16.35)	(5.87)	(3.17)
R^2		0.87	0.95	0.87	0.56

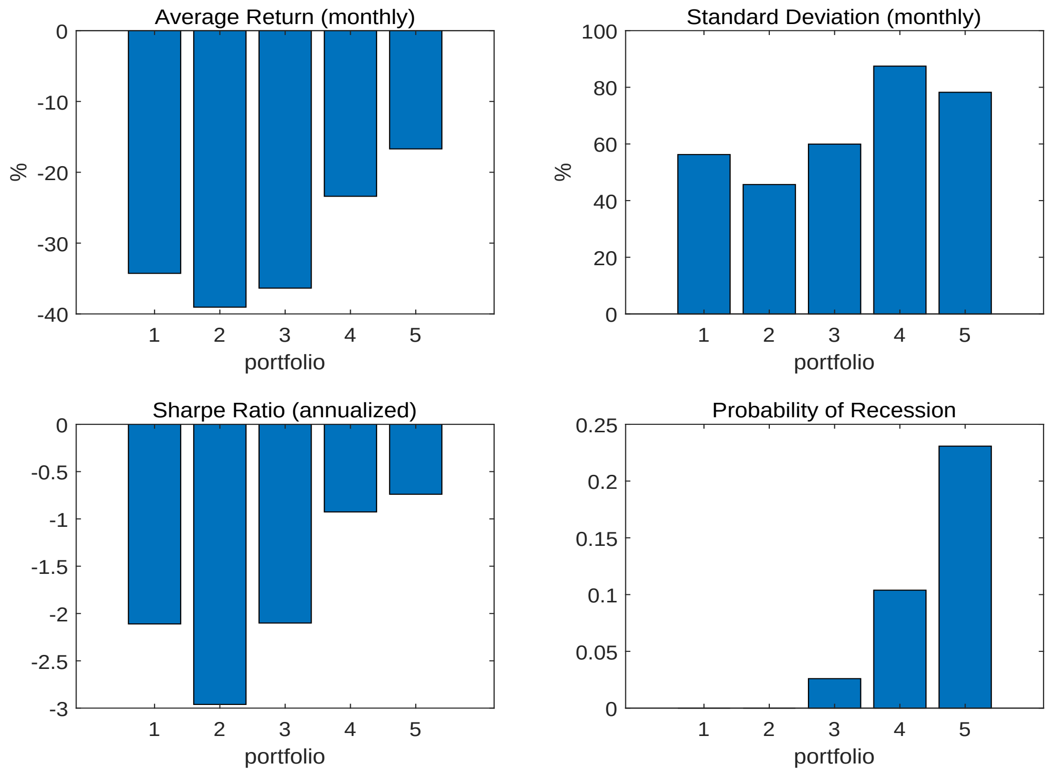


Figure 1. Sorts on previous month's realized volatility. Sample is monthly Jan 1990 - Nov 2023. We use the monthly time-series of realized volatility to sort the following month's variance swap returns into five portfolios. Portfolio "1" ("5") looks at the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We show (monthly) average returns, (monthly) standard deviation of returns, (annualized) Sharpe ratio of returns, and probability of NBER recessions by computing the average of an NBER recession dummy.

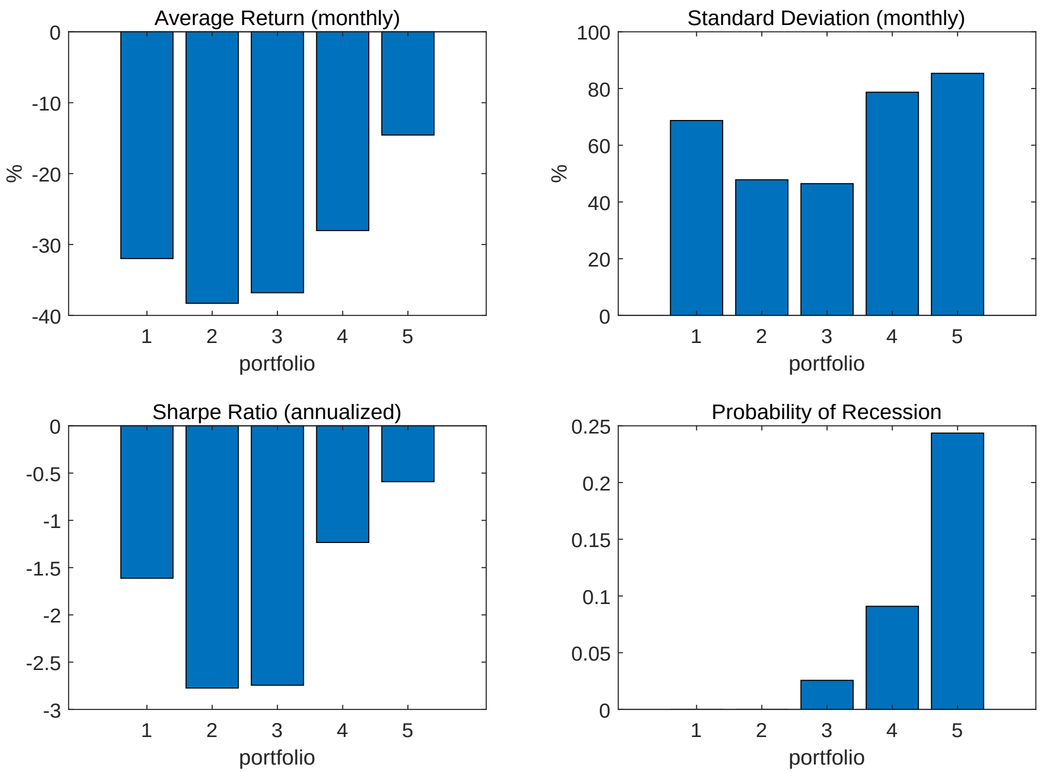


Figure 2. Sorts on previous month's implied volatility. Sample is monthly Jan 1990 - Nov 2023. We use the monthly time-series of implied volatility to sort the following month's variance swap returns into five portfolios. Portfolio "1" ("5") looks at the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We show (monthly) average returns, (monthly) standard deviation of returns, (annualized) Sharpe ratio of returns, and probability of NBER recessions by computing the average of an NBER recession dummy.

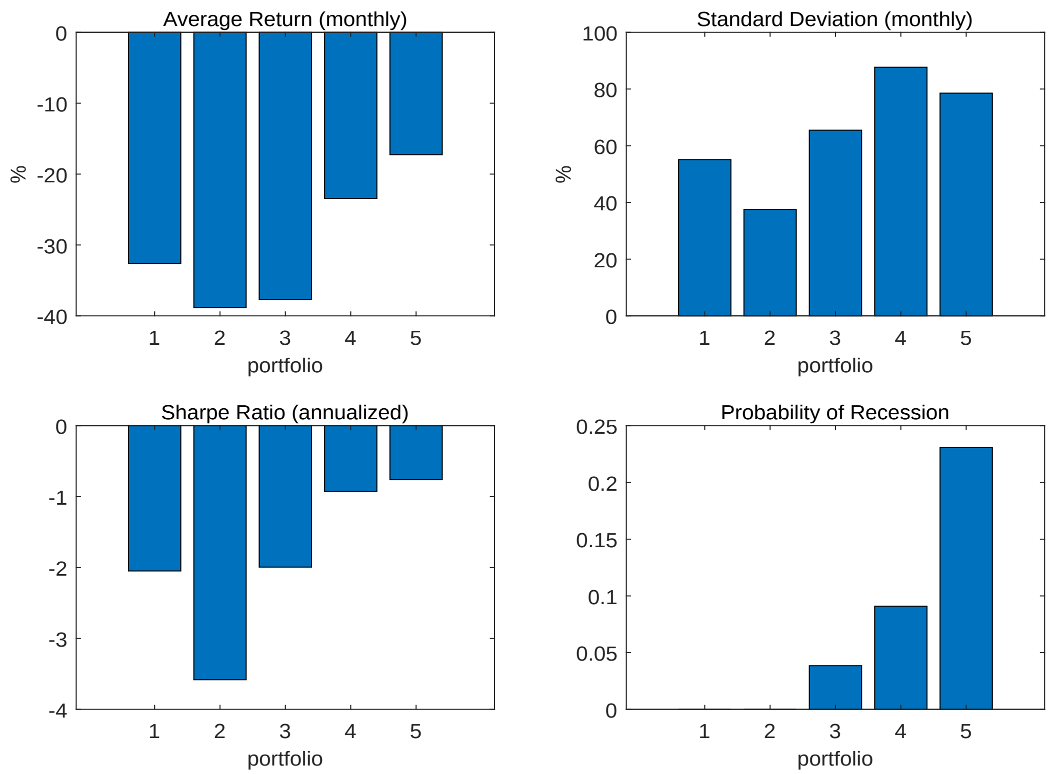


Figure 3. Sorts on previous month's GARCH 1-month volatility forecast. Sample is monthly Jan 1990 - Nov 2023. We use the monthly time-series of GARCH(1,1)'s 1-month volatility forecast to sort the following month's variance swap returns into five portfolios. Portfolio "1" ("5") looks at the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We show (monthly) average returns, (monthly) standard deviation of returns, (annualized) Sharpe ratio of returns, and probability of NBER recessions by computing the average of an NBER recession dummy.

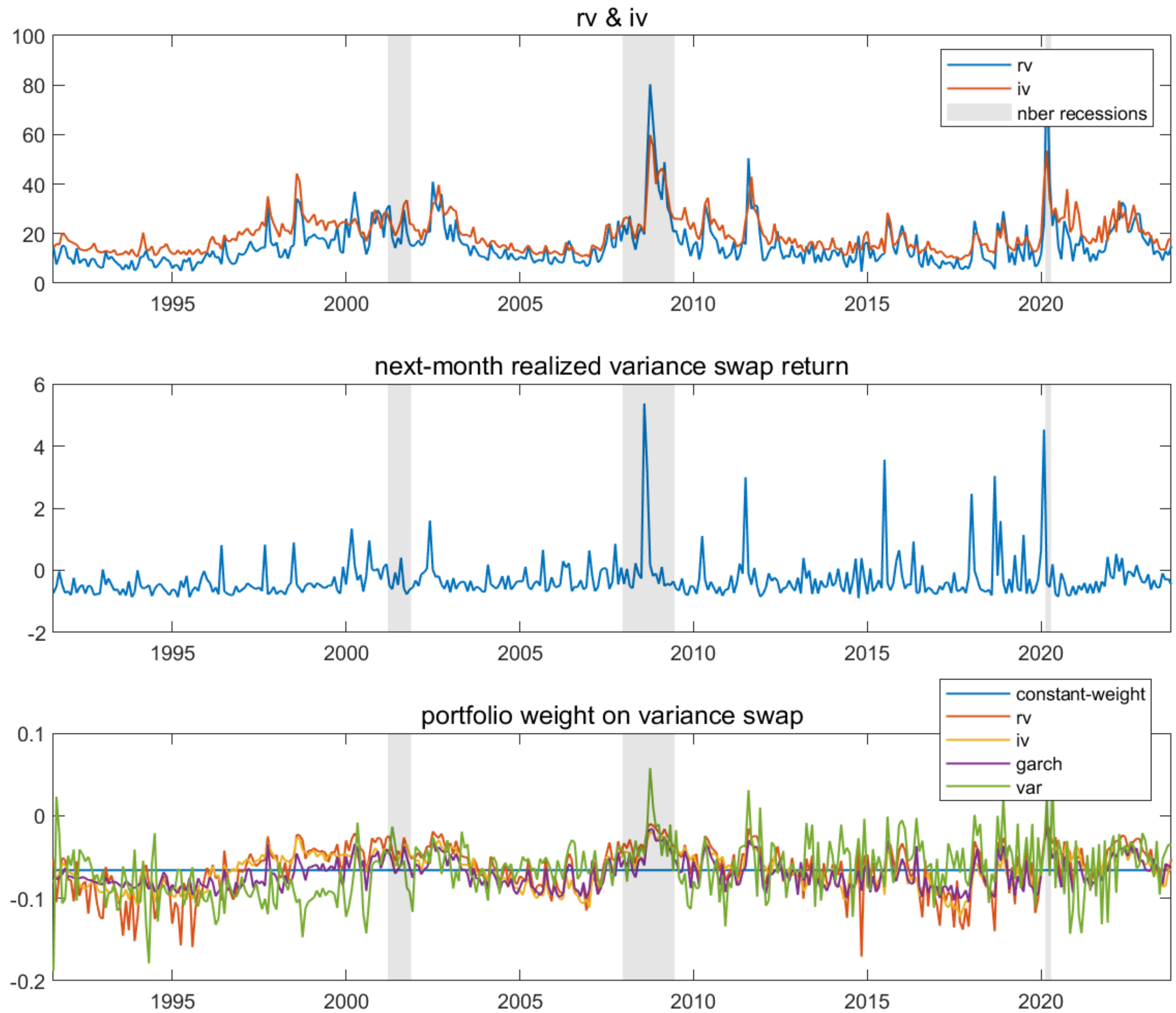


Figure 4. Portfolio weight on variance swaps: longest periods. Test period is Jan 1992 - Nov 2023. The top panel plots implied and realized volatility. The middle panel plots next-month realized return on a one-month variance swap. The bottom panel plots portfolio weight on the variance swap for each strategy.

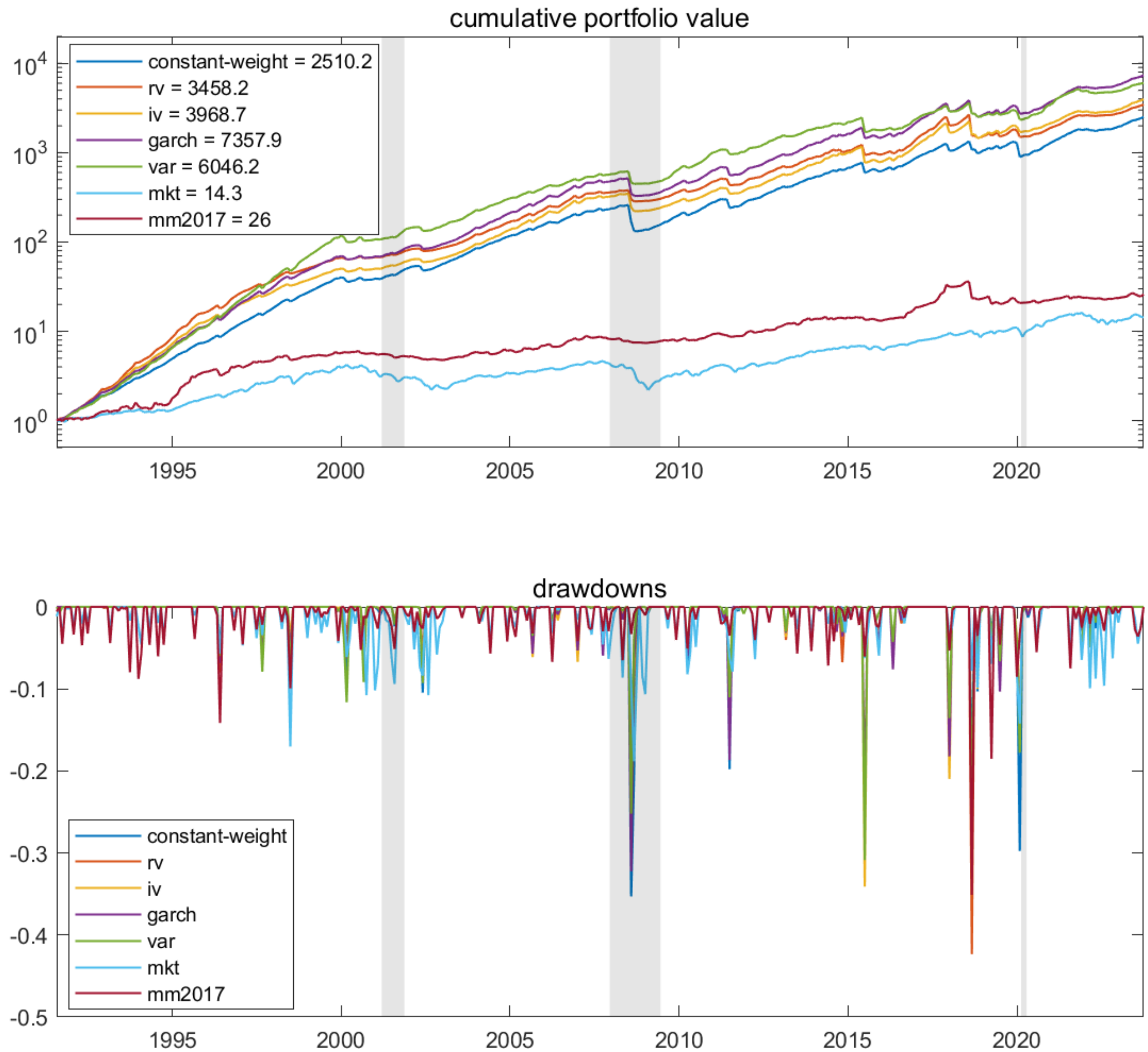


Figure 5. Cumulative portfolio values: longest periods. Starting value is $1 on Jan 1992 for each strategy. Test period is Jan 1992 - Nov 2023. The top panel plots cumulative portfolio value for each strategy. Ending value for each strategy is attached. The bottom panel plots drawdowns for each strategy.

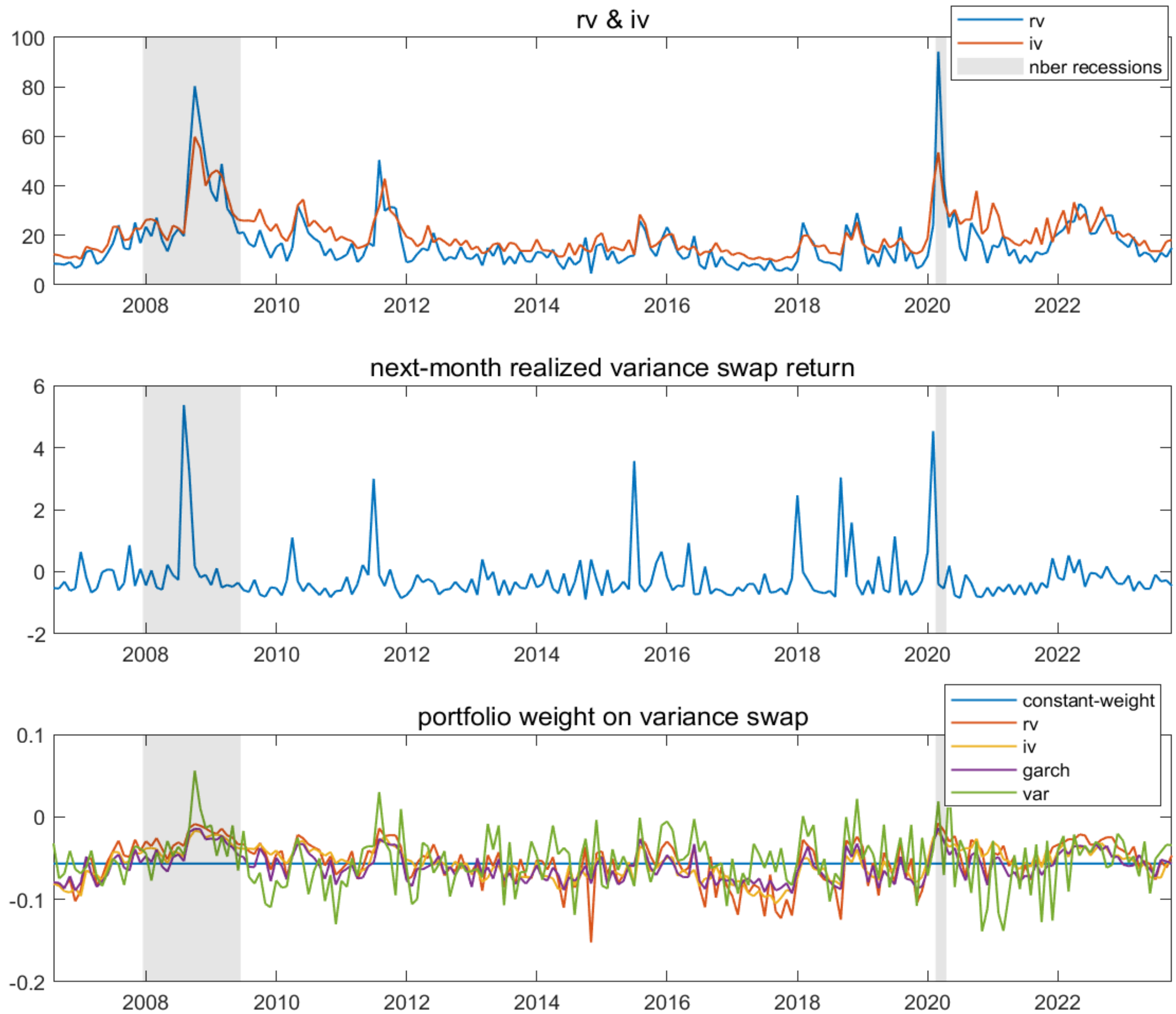


Figure 6. Portfolio weight on variance swaps: recent periods. Test period is Aug 2006 - Nov 2023. The top panel plots implied and realized volatility. The middle panel plots next-month realized return on a one-month variance swap. The bottom panel plots portfolio weight on the variance swap for each strategy.

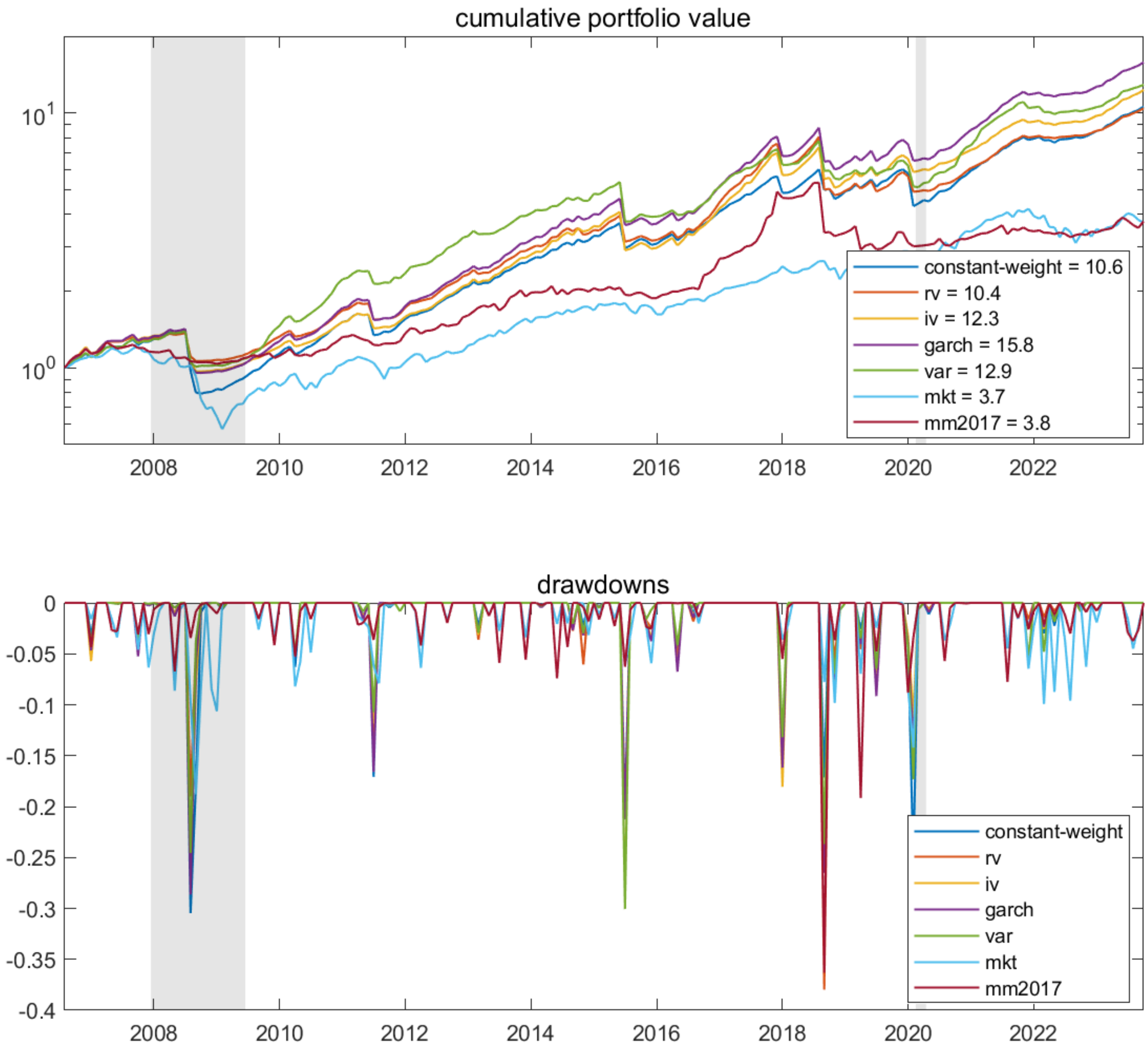


Figure 7. Cumulative portfolio values: recent periods. Starting value is $1 on Aug 2006 for each strategy. Test period is Aug 2006 - Nov 2023. The top panel plots cumulative portfolio value for each strategy. Ending value for each strategy is attached. The bottom panel plots drawdowns for each strategy.

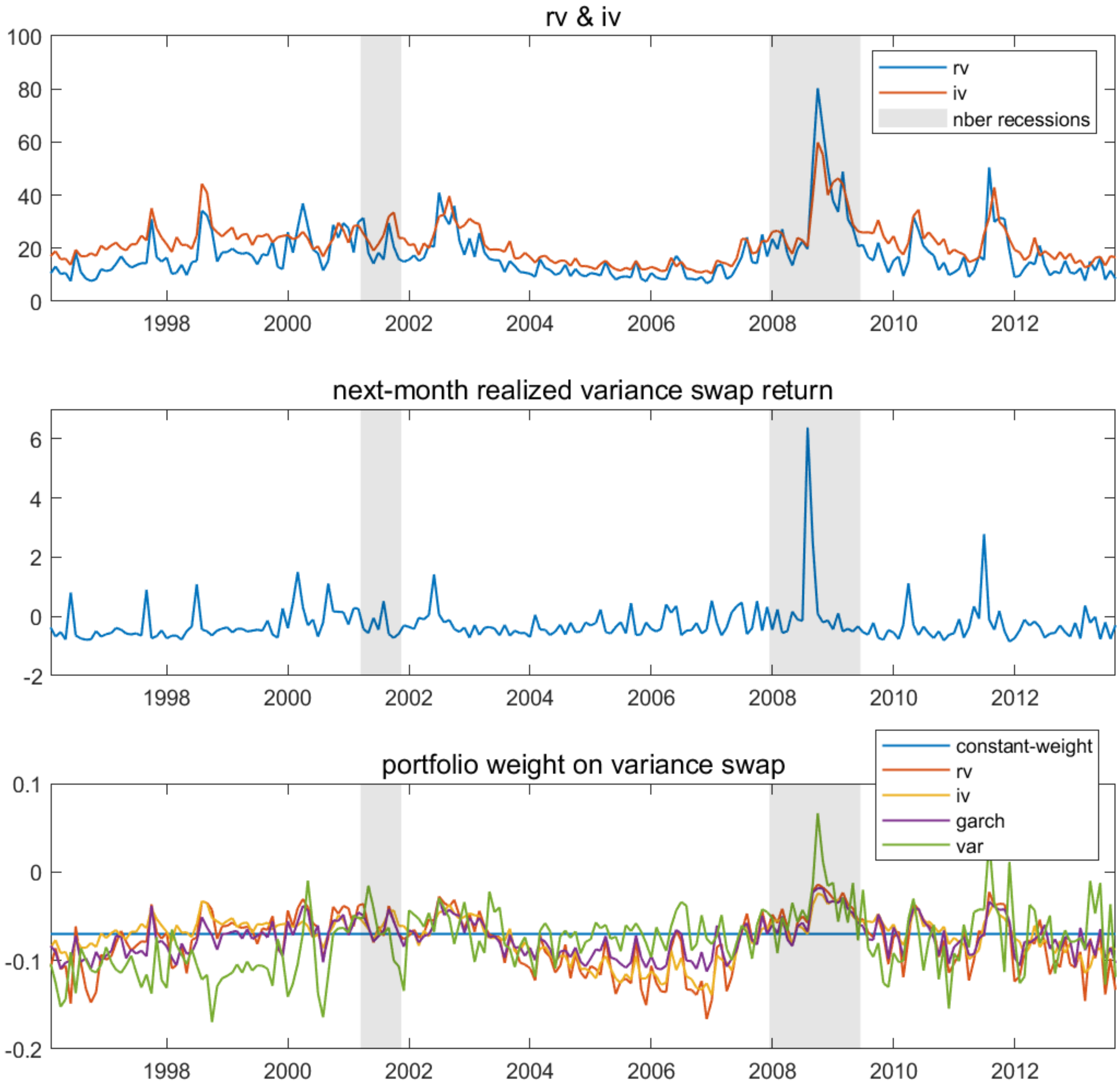


Figure 8. Portfolio weight on variance swaps: using actual price data. Test period is Jan 1996 - Sep 2013. The top panel plots implied and realized volatility. The middle panel plots next-month realized return on a one-month variance swap using data provided in Dew-Becker, Giglio, Le, and Rodriguez (2017). The bottom panel plots portfolio weight on the variance swap for each strategy.

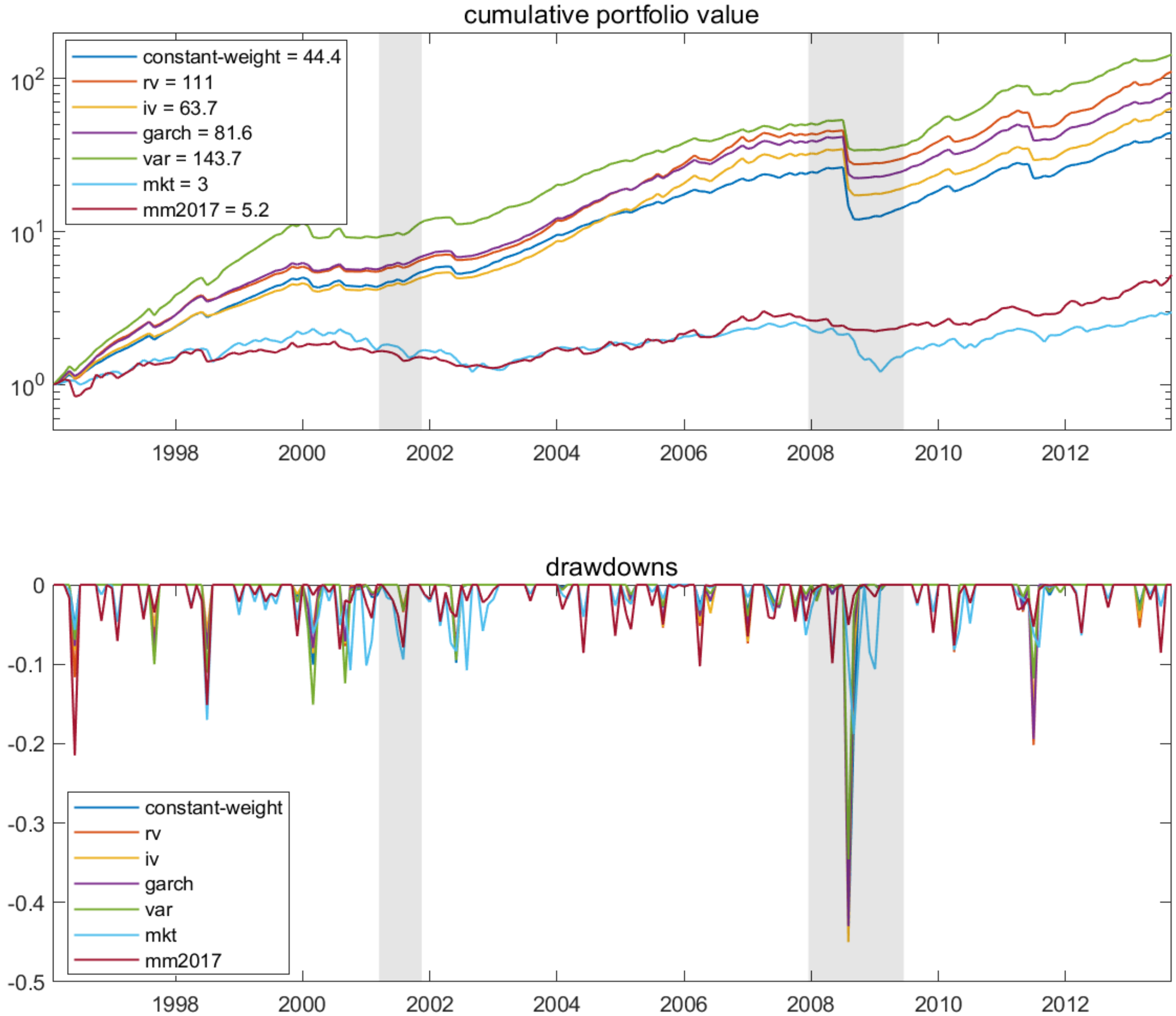


Figure 9. Cumulative portfolio values: using actual price data. Data is based on variance swap price data provided in Dew-Becker, Giglio, Le, and Rodriguez (2017). Starting value is $1 on Jan 1996 for each strategy. Test period is Jan 1996 - Sep 2013. The top panel plots cumulative portfolio value for each strategy. Ending value for each strategy is attached. The bottom panel plots drawdowns for each strategy.

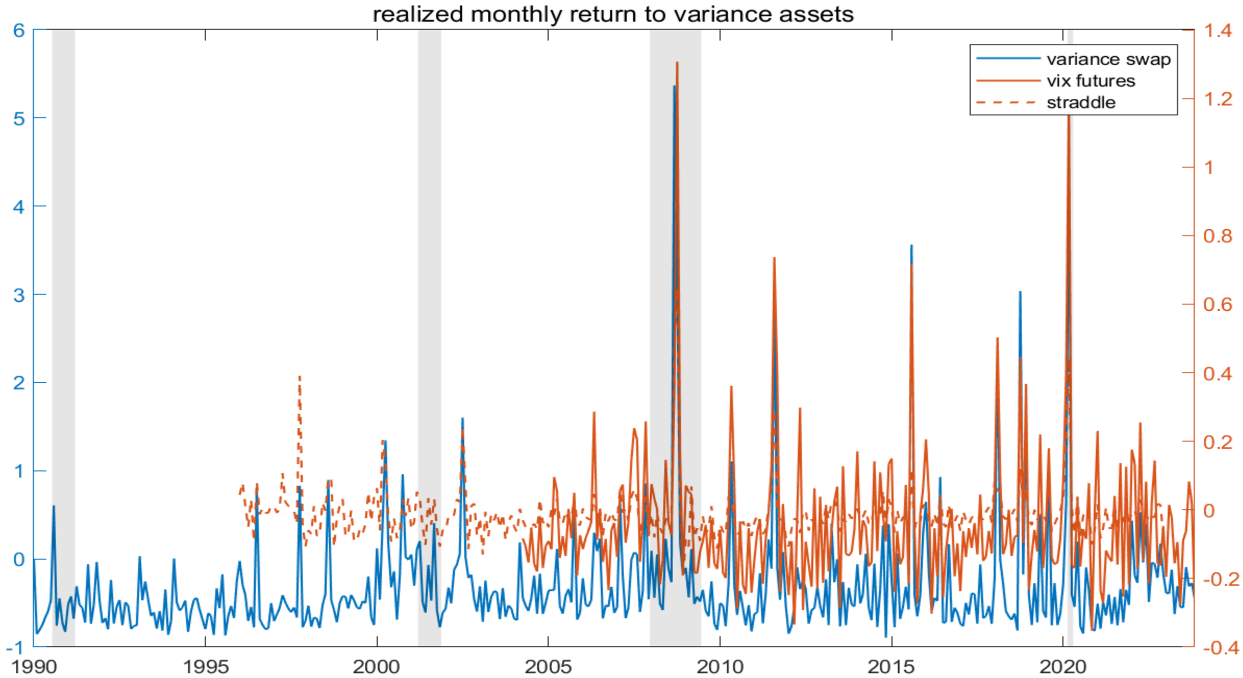


Figure 10. A comparison of three variance asset returns. We first get daily returns for one-month constant-maturity VIX-futures/S&P 500 ATM straddle positions provided by Johnson (2017). Within each month, we implement a roll-over strategy to generate the total return for that specific month. We then plot the obtained monthly return. For straddles, we use (negative) short returns provided by Johnson (2017), which assumes a margin requirement equal to 20% of current S&P 500 level, since our timing porfolios prescribe short selling variance assets. We augment the VIX futures and straddle returns from Johnson (2017) with the most recent time data obtained from CBOE and OptionMetrics.

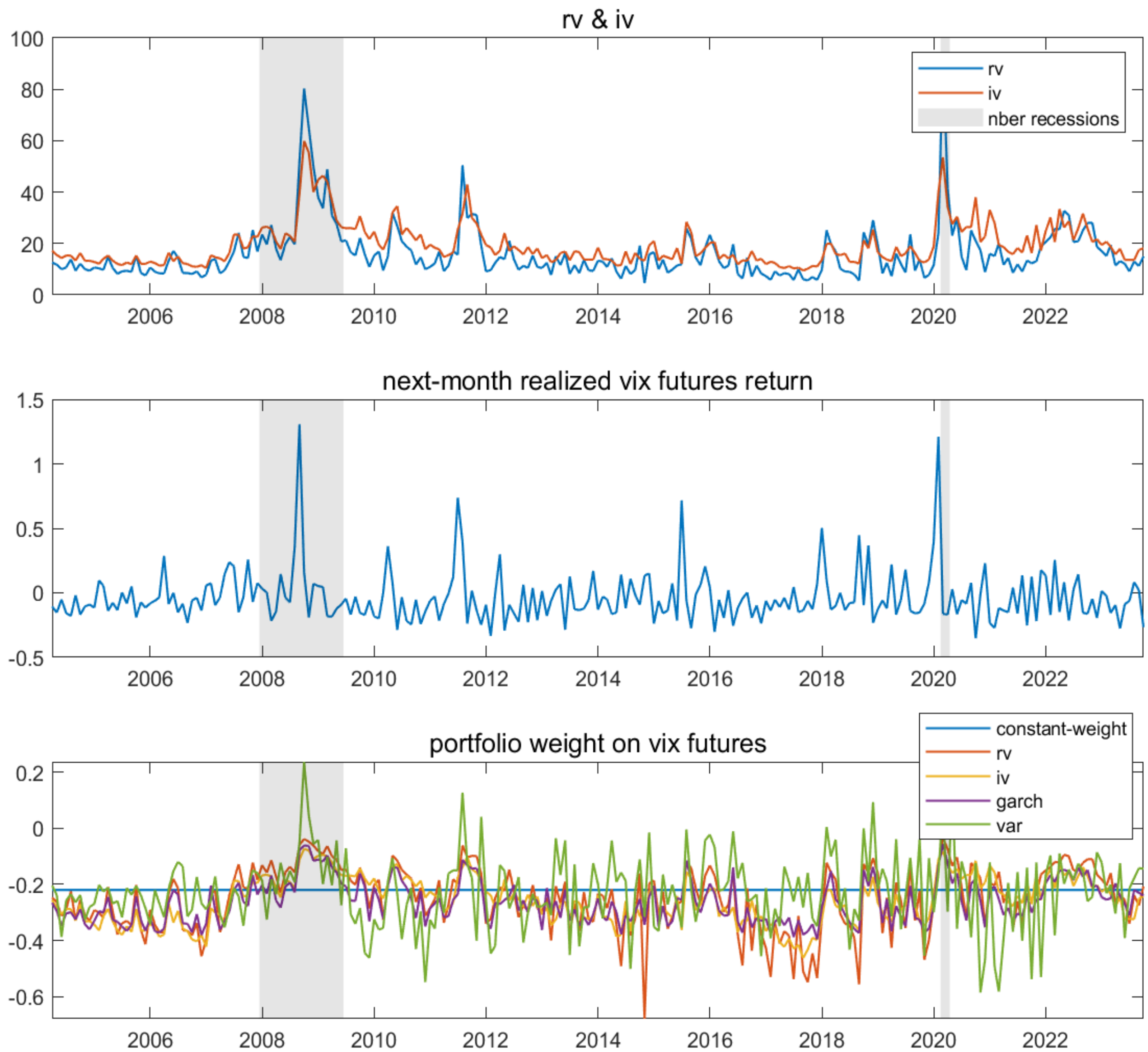


Figure 11. Portfolio weight on VIX futures. We first get daily returns for a one-month constant-maturity VIX futures position provided by Johnson (2017). Within each month, we implement a roll-over strategy to generate the total return for that specific month. We augment the data from Johnson (2017) with the most recent time data obtained from CBOE. Test period is Apr 2004 - Nov 2023. The top panel plots implied and realized volatility. The middle panel plots next-month realized return on a constant-maturity one-month VIX futures. The bottom panel plots portfolio weight on VIX futures for each strategy.

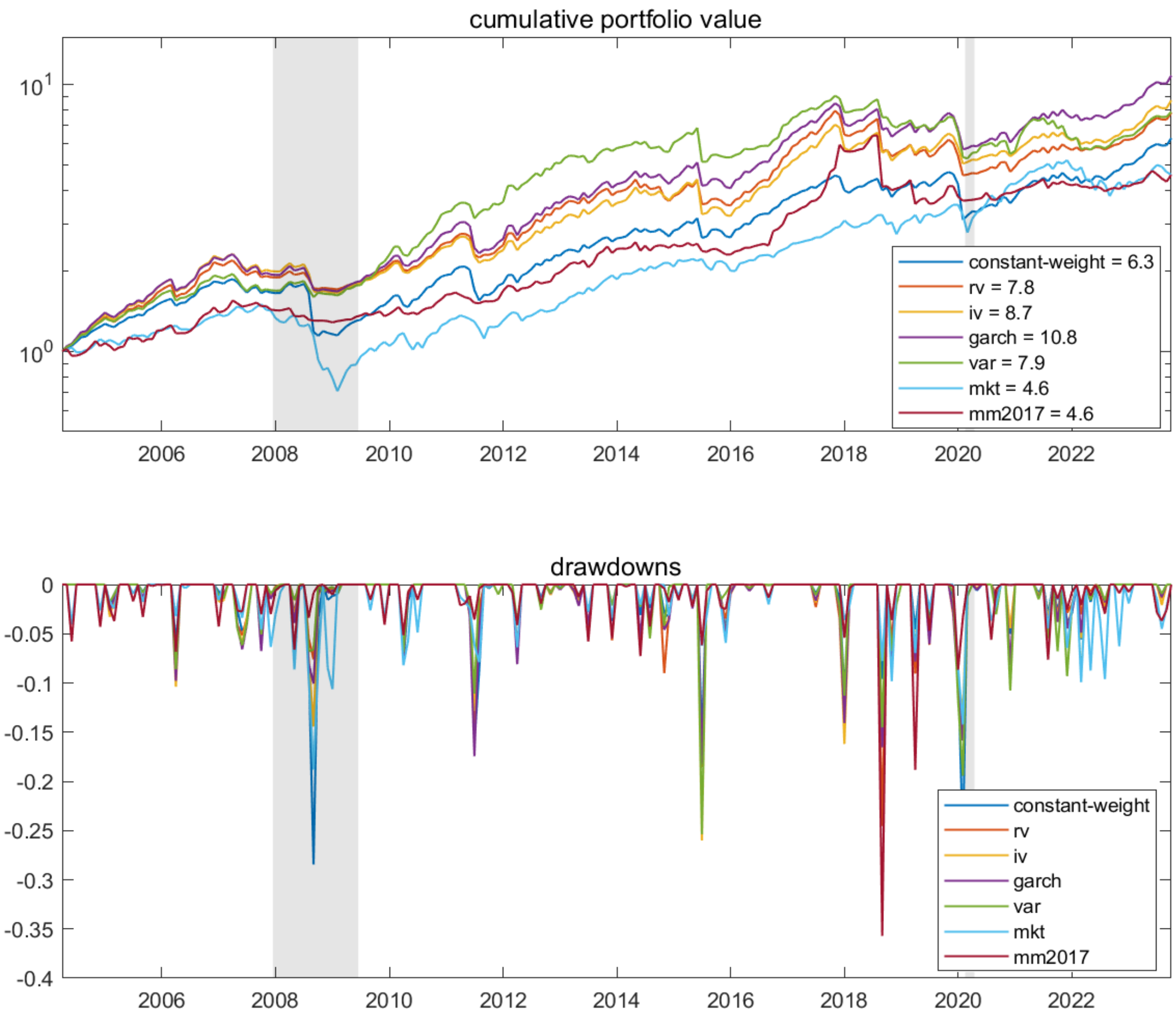


Figure 12. Cumulative portfolio values: VIX futures. Starting value is $1 on Apr 2004 for each strategy. Test period is Apr 2004 - Nov 2023. The top panel plots cumulative portfolio value for each strategy. Ending value for each portfolio is attached. The bottom panel plots drawdowns for each strategy.

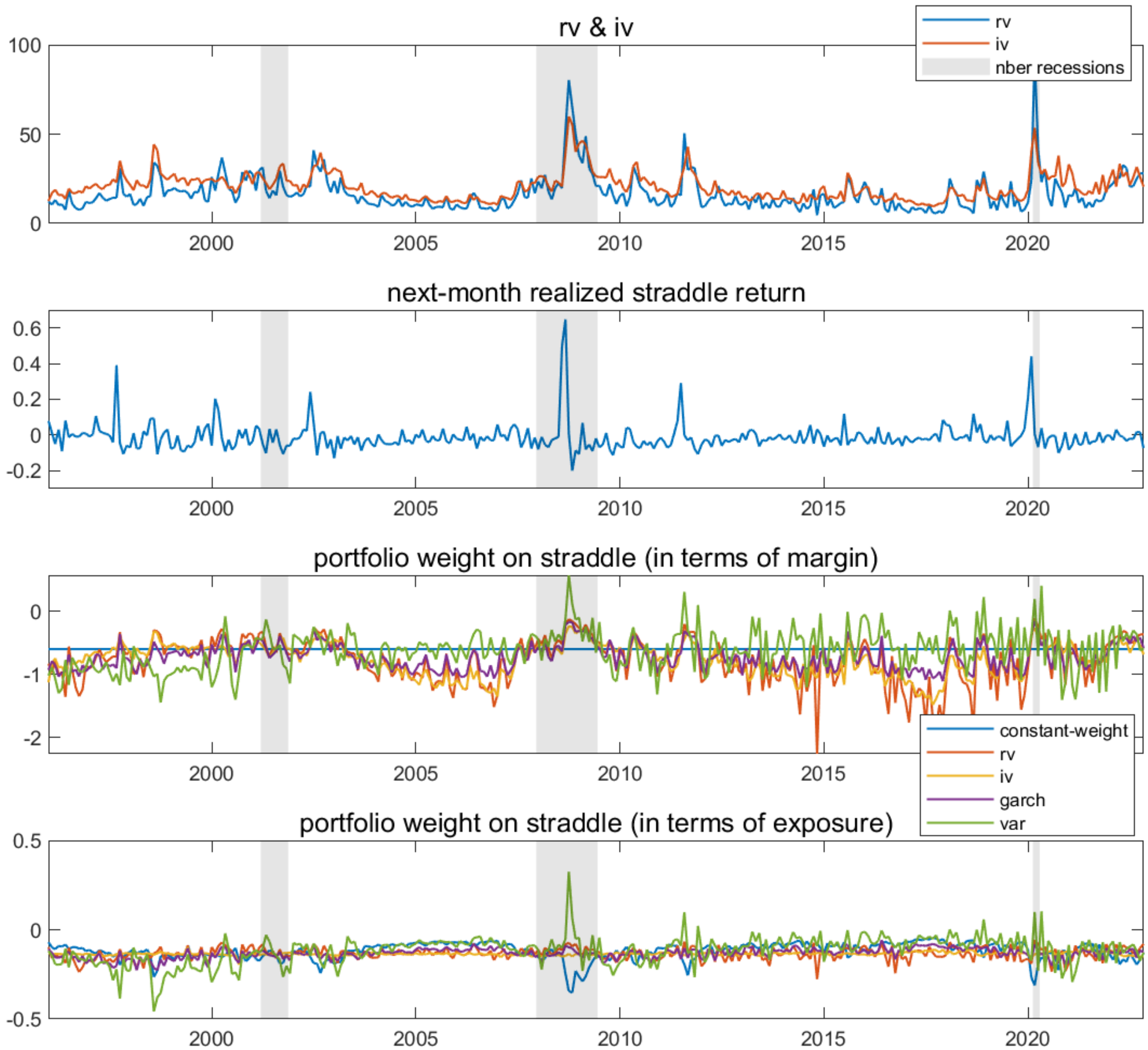
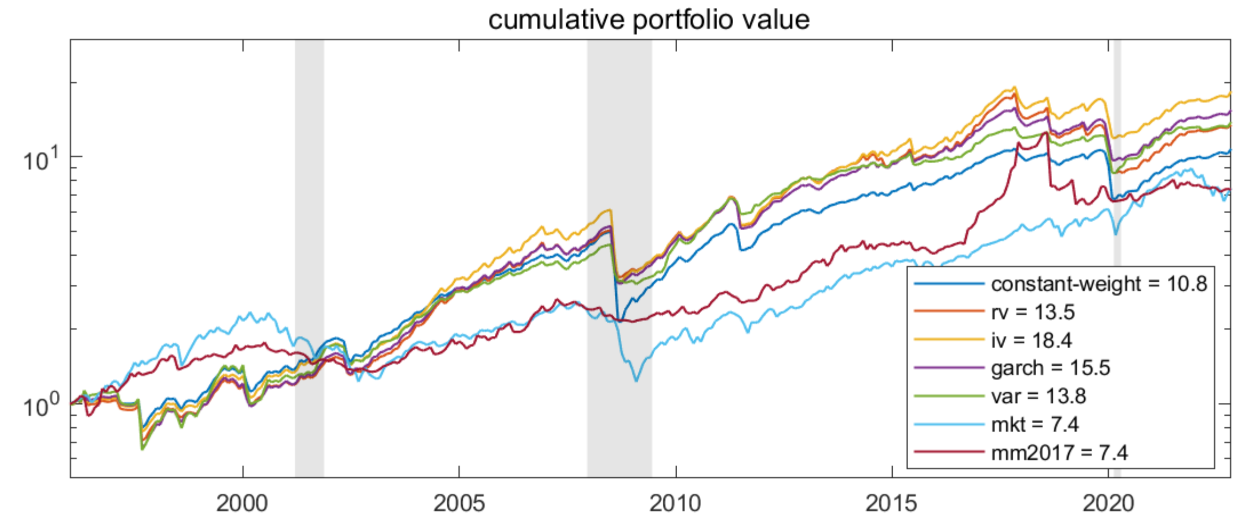


Figure 13. Portfolio weight on S&P 500 ATM straddles. We first get daily returns for a one-month constant-maturity S&P 500 ATM straddle (short) position provided by Johnson (2017). Within each month, we implement a roll-over strategy to generate the total return for that specific month. We use short returns as provided by Johnson (2017), which assumes a margin requirement equal to 20% of current S&P 500 level. We augment the data from Johnson (2017) with the most recent time data obtained from OptionMetrics. Test period is Jan 1996 - Dec 2022. The top panel plots implied and realized volatility. The second panel plots next-month realized return on a constant-maturity one-month S&P 500 ATM straddle. The third and fourth panel respectively plots portfolio weight on the straddle for each strategy in terms of deposited margin and straddles' market values.



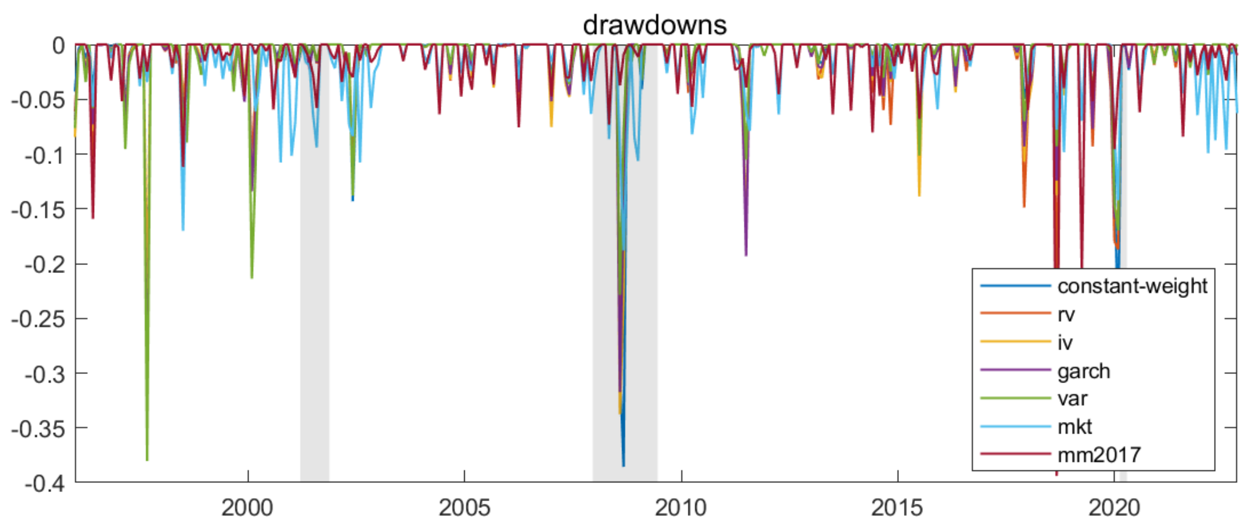


Figure 14. Cumulative portfolio values: S&P 500 ATM straddles. Starting value is \$1 on Jan 1996 for each strategy. Test period is Jan 1996 - Dec 2022. The top panel plots cumulative portfolio value for each strategy. Ending value for each portfolio is attached. The bottom panel plots drawdowns for each strategy.

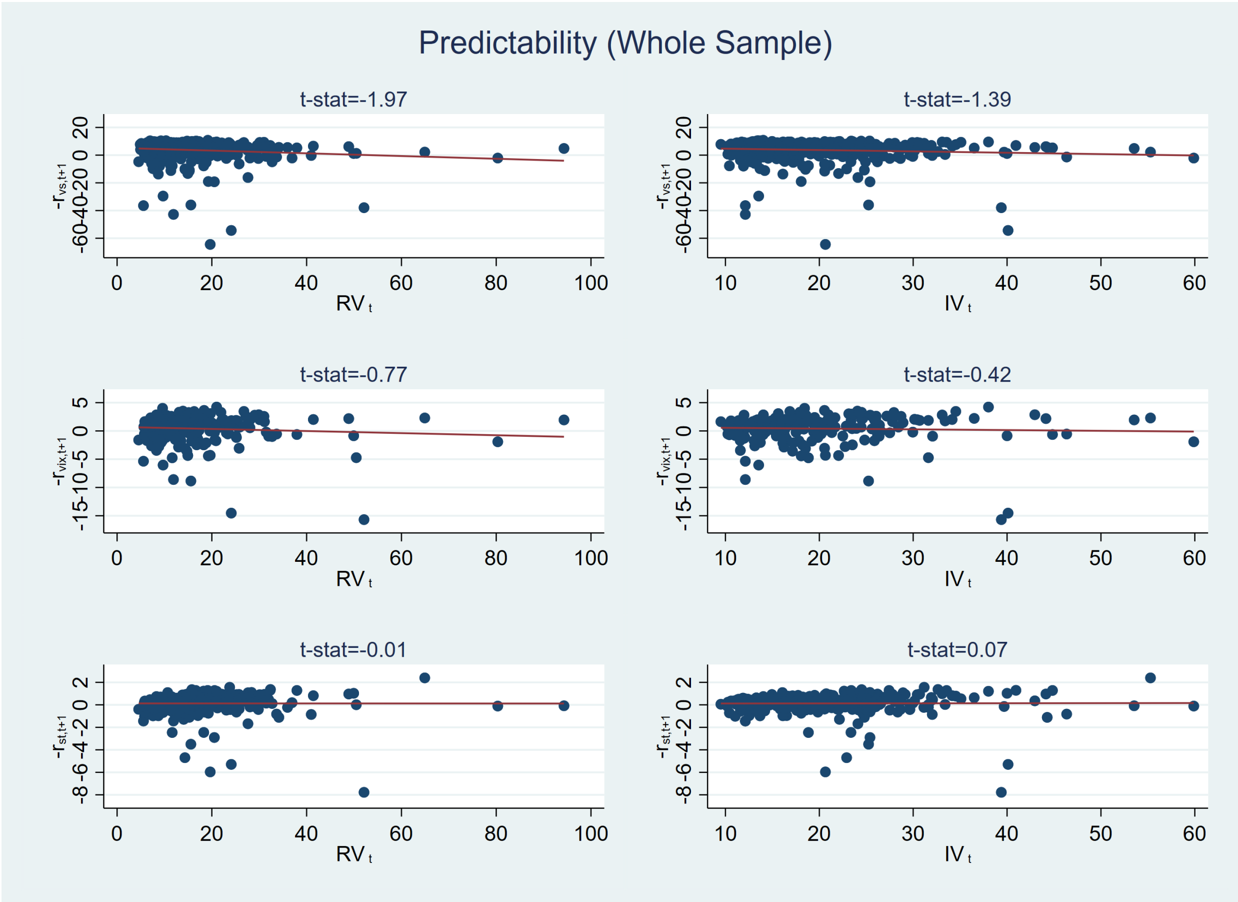


Figure 15. Predictive regressions: unconditional. We run predictive regressions of next-month (negative) variance swap returns, (negative) constant-maturity VIX futures portfolio returns, and (negative) constant-maturity straddle portfolio returns onto this month's realized volatility (RV) and implied volatility (VIX). We use full-sample. t-stat is heteroscedasticity robust. Data is as in the main text.

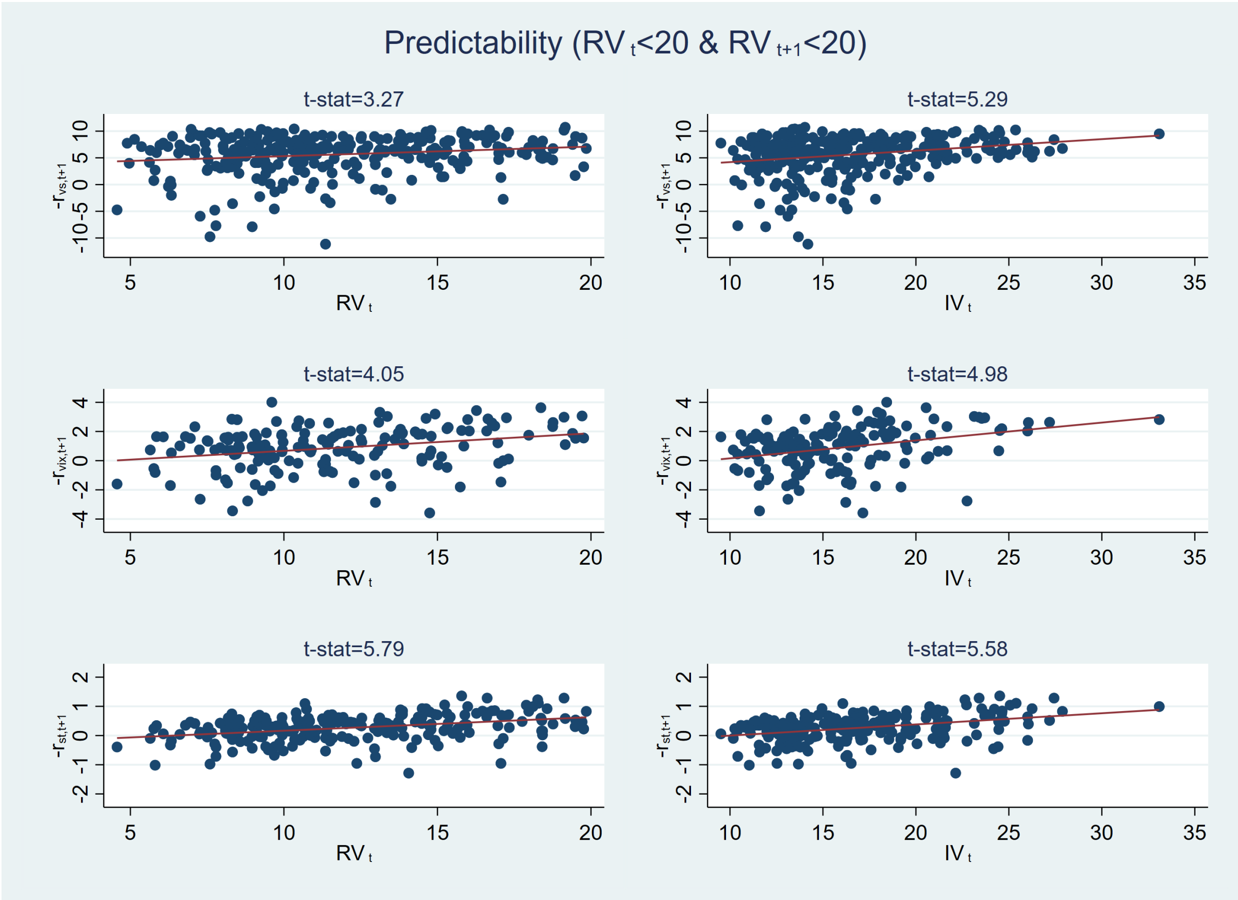


Figure 16. Predictive regressions: conditional on low-volatility regime throughout. We run predictive regressions of next-month (negative) variance swap returns, (negative) constant-maturity VIX futures portfolio returns, and (negative) constant-maturity straddle portfolio returns onto this month's realized volatility (RV) and implied volatility (VIX). We condition on both this and next month's RV<20. t-stat is heteroscedasticity robust. Data is as in the main text.