高等数学微积分推导

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微积分公式推导

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1　高等数学微积分公式推导

1.1　常见函数的不定积分

1. $\int ln|x|dx = ln|x|(x-1)+C$ 推导：

$$\begin{aligned} \int ln|x|dx &= xln|x| - \int \frac{1}{x}dx \\ &= ln|x|(x-1)+C \end{aligned} \tag{1}$$

2. $\int xln|x|dx =$ 推导：

$$\begin{aligned}\int xln|x|dx &= \frac{1}{2}\int ln|x|dx^2 \\ &= \frac{1}{2}(x^2ln|x| - \int x^2 \cdot \frac{1}{x}dx \\ &= \frac{1}{2}(x^2ln|x| - \frac{1}{2}x) + C\end{aligned} \tag{2}$$

1.2 含有 $ax+b$ 的积分

1. $\int \frac{dx}{ax+b} = \frac{1}{a}ln|ax+b| + C$
   推导：

$$\begin{aligned}\int \frac{dx}{ax+b} &= \frac{1}{a}\frac{d(ax+b)}{ax+b} \\ &= \frac{1}{a}ln|ax+b| + C\end{aligned} \tag{3}$$

2. $\int (ax+b)^\mu dx = \frac{1}{a(u+1)}(ax+b)^{\mu+1}$ 推导：

$$\int (ax+b)^\mu dx = \frac{1}{a}\int (ax+b)^\mu dax+b \tag{4}$$

$$= \frac{1}{a(u+1)}(ax+b)^{\mu+1} \quad (\mu \neq -1) \tag{5}$$

3. $\int \frac{x}{ax+b}dx = \frac{1}{a^2}(ax+b-bln|ax+b|) + C$ 推导：

$$\int \frac{x}{ax+b}dx = \frac{1}{a^2}\int \frac{ax}{ax+b}dax \tag{6}$$

$$= \frac{1}{a^2}\int \frac{ax+b-b}{ax+b}dax \tag{7}$$

$$= \frac{1}{a^2}\int 1d(ax+b) - \frac{b}{a^2}\int \frac{1}{ax+b}d(ax+b) \tag{8}$$

$$= \frac{1}{a^2}(ax+b) - \frac{b}{a^2}ln(ax+b) + C \tag{9}$$

$$= \frac{1}{a^2}(ax+b-bln|ax+b|) + C \tag{10}$$

4. $\int \frac{x^2}{ax+b}dx = \frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2 ln|ax+b|\right] + C$ 推导:

$$\begin{aligned}\int \frac{x^2}{ax+b}dx &= \frac{1}{a}\int \frac{ax^2}{ax+b}dx \\ &= \frac{1}{a}\int xdx - \frac{b}{a}\int \frac{x}{ax+b}dx \\ &= \frac{x^2}{2a} - \frac{b}{a^3}(ax+b-bln|ax+b|) + C\end{aligned} \tag{11}$$

5. $\int \frac{dx}{x(ax+b)} = -\frac{1}{b}ln|\frac{ax+b}{x}| + C$ 推导:

$$\begin{aligned}\int \frac{dx}{x(ax+b)} &= \frac{1}{b}(\frac{1}{x} - \frac{a}{ax+b})dx \\ &= \frac{ln|x|}{b} - \frac{ln|ax+b|}{b} = -\frac{1}{b}ln|\frac{ax+b}{x}| + C\end{aligned} \tag{12}$$

6. $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}ln|\frac{ax+b}{x}| + C$ 推导:

$$\begin{aligned}\int \frac{dx}{x^2(ax+b)} &= \int \frac{\frac{-a}{b^2}x + \frac{1}{b}}{x^2} + \frac{a^2}{b^2}\frac{1}{ax+b}dx \\ &= -\frac{1}{bx} + \frac{a}{b^2}ln(|\frac{ax+b}{x}|) + C\end{aligned} \tag{13}$$

7. $\int \frac{x}{(ax+b)^2}dx = \frac{1}{a^2}(ln|ax+b| + \frac{b}{ax+b}) + C$ 推导:

$$\begin{aligned}\int \frac{x}{(ax+b)^2}dx &= \frac{1}{a^2}\int \frac{ax+b-b}{(ax+b)^2}d(ax+b) \\ &= \frac{1}{a^2}\left(ln|ax+b| + \frac{b}{ax+b}\right)\end{aligned} \tag{14}$$

8. $\int \frac{x^2}{(ax+b)^2}dx = \frac{1}{a^3}\left(ax+b-2bln|ax+b| - \frac{b^2}{ax+b}\right) + C$ 推导:

$$\begin{aligned}\int \frac{x^2}{(ax+b)^2}dx &= \frac{1}{a^3}\left(\int 1 - 2ab\frac{x}{(ax+b)^2} - \frac{b^2}{(ax+b)^2}\right)d(ax+b) \\ &= \frac{1}{a^3}\left(ax+b - \frac{2b}{a}(ln|ax+b|)\cdot a - \frac{2b}{a}(\frac{b}{ax+b})\cdot a\right) - \\ &\int \frac{b^2}{ax+b}d(ax+b) \\ &= \frac{1}{a^3}\left(ax+b-2bln|ax+b| - \frac{b^2}{ax+b}\right) + C\end{aligned} \tag{15}$$

9. $\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2} ln|\frac{ax+b}{x}| + C$ 推导:

$$
\begin{aligned}
\int \frac{dx}{x(ax+b)^2} &= -\frac{1}{a}\int \frac{1}{x} d\frac{1}{ax+b} \\
&= -\frac{1}{a}\left(\frac{1}{(ax+b)x} + \int \frac{1}{x^2(ax+b)}\right) \\
&= -\frac{1}{a}\left(\frac{1}{(ax+b)x} - \frac{1}{bx} + \frac{a}{b^2} ln|\frac{ax+b}{x}|\right) \\
&= \frac{1}{b(ax+b)} - \frac{1}{b^2} ln|\frac{ax+b}{x}| + C
\end{aligned}
\tag{16}
$$

1.3 含有 $\sqrt{ax+b}$

1. $\int \sqrt{ax+b}dx = \frac{2}{3a}\sqrt{(ax+b)^3} + C$ 推导:

$$
\begin{aligned}
\int \sqrt{ax+b}dx &= \frac{1}{a}\int \frac{(ax+b)^{1/2}}{d}(ax+b) \\
&= \frac{2}{3}\sqrt{(ax+b)^3}
\end{aligned}
\tag{17}
$$

2. $\int x\sqrt{ax+b}dx = \frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3} + C$ 推导:

$$
\begin{aligned}
\int x\sqrt{ax+b}dx &= \frac{1}{a^2}\int (ax+b-b)\sqrt{ax+b}dax \\
&= \frac{1}{a^2}(ax+b)^{\frac{3}{2}} - \frac{2b}{3}(ax+b)^{\frac{3}{2}}dax \\
&= \frac{1}{a^2}\left[\frac{2}{5}(ax+b)^{\frac{5}{2}} - \frac{2b}{3}(ax+b)^{\frac{3}{2}}\right] + C \\
&= \frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3} + C
\end{aligned}
\tag{18}
$$

3. $\int x^2\sqrt{ax+b}dx = \frac{2}{105a^3}(15a^2x^2 - 12abx + 8b^2)\sqrt{(ax+b)^3} + C$ 推导:

$$\begin{aligned}
\int x^2\sqrt{ax+b}dx &= \frac{1}{a^3}\int (ax+b)^2\sqrt{ax+b} - 2abx\sqrt{ax+b} - b^2\sqrt{ax+b}dax \\
&= \frac{1}{a^3}\left(\frac{2}{7}(ax+b)^{\frac{7}{2}} - 2ab\frac{2}{15a}(3ax-2b)\sqrt{(ax+b)^3} - \frac{2b^2}{3}(ax+b)^{\frac{3}{2}}\right) + \\
&= \frac{2\sqrt{(ax+b)^3}}{105a^3}\left(15(ax+b)^2 - 14b(3ax-2b) - 35b^2\right) + C \\
&= \frac{2\sqrt{(15a^2x^2 - 12abx + 8b^2)}}{105a^3}(ax+b)^3 + C
\end{aligned} \tag{19}$$

4. $\int \frac{x}{\sqrt{ax+b}}dx = \frac{2}{3a^2}(ax-2b)\sqrt{ax+b} + C$ 推导:

$$\begin{aligned}
\int \frac{x}{\sqrt{ax+b}}dx &= \int \frac{1}{a^2}\frac{ax+b-b}{\sqrt{ax+b}}dax \\
&= \frac{1}{a^2}\left(\int \sqrt{ax+b} - (ax+b)^{-\frac{1}{2}})dax + b\right) \\
&= \frac{1}{a^2}\left(\frac{2}{3}(ax+b)^{1/2}(ax+b) - 2b\right) + C \\
&= \frac{2}{3a^2}\left(\frac{2}{3}(ax+b)^{1/2}(ax+b) - 3b\right) + C \\
&= \frac{2}{3a^2}(ax-2b)\sqrt{ax+b} + C
\end{aligned} \tag{20}$$

5. $\int \frac{x^2}{\sqrt{ax+b}}dx = \frac{2}{15a^3}(3a^2x^2 - 4abx + 8b^2)\sqrt{ax+b} + C$ 推导:

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax+b}}dx &= \frac{1}{a^3}(\frac{(ax+b)^2 - 2abx - b^2}{\sqrt{ax+b}})dax \\
&= \frac{1}{a^3}\left(\frac{2}{5}(ax+b)^{\frac{5}{2}}\right) - 2ab\frac{2}{3a}(ax-2b)\sqrt{ax+b} - 2b^2\sqrt{ax+b} + C \\
&= \frac{2}{15a^3}\left(3(ax)^2 + 6abx + 3b^2 - 10abx + 20b^2 - 15b^2\right)\sqrt{ax+b} + C \\
&= \frac{2}{15a^3}\left(3(ax)^2 - 4abx + 8b^2\right)\sqrt{ax+b} + C
\end{aligned} \tag{21}$$

6.

$$\int \frac{dx}{x\sqrt{ax+b}}dx = \begin{cases} \frac{1}{\sqrt{b}}ln|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}| + C & (b>0) \\ \frac{2}{\sqrt{-b}}arctan\sqrt{\frac{ax+b}{-b}} + C & (b<0) \end{cases} \tag{22}$$

7. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}$ 推导:

$$\int \frac{dx}{x^2\sqrt{ax+b}} = \int \frac{dx}{x\sqrt{ax+b}}\frac{1}{x} \tag{23}$$

8. $\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$ 推导:

$$\begin{aligned} \int \frac{\sqrt{ax+b}}{x}dx &= \int \frac{\sqrt{ax+b}}{\sqrt{x}}d2\sqrt{x} \\ &= 2\sqrt{ax+b} + \int b\sqrt{\frac{x}{ax+b}} \cdot \frac{1}{x^2} \cdot \sqrt{x}dx \\ &= 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}dx \end{aligned} \tag{24}$$

9. $\int \frac{\sqrt{ax+b}}{x^2}dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}$ 推导:

$$\begin{aligned} \int \frac{\sqrt{ax+b}}{x^2}dx &= -\int \sqrt{ax+b}d\frac{1}{x} \\ &= \frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}dx \end{aligned} \tag{25}$$

1.4 含有 $x^2 \pm a^2$

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}arctan\frac{x}{a} + C$ 推导:

$$\begin{aligned} \int \frac{dx}{x^2+a^2} &= \int \frac{1}{a}\frac{d\frac{x}{a}}{\frac{x}{a}^2 + 1^2} \\ &= \frac{1}{a}arctan(\frac{x}{a}) + C \end{aligned} \tag{26}$$

2. $\int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2}\int \frac{dx}{(x^2+a^2)^{n-1}}$

3. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} ln|\frac{x-a}{x+a}| + C$

$$\begin{aligned}\int \frac{dx}{x^2-a^2} &= \frac{1}{2a}\int \frac{1}{x-a} - \frac{1}{x+a} dx \\ &= \frac{1}{2a} ln|\frac{x-a}{x+a}|\end{aligned} \tag{27}$$

1.5 含有 $ax^2 + b(a > 0)$

1. $\int \frac{dx}{ax^2+b} = \begin{cases} \frac{1}{\sqrt{ab}} arctan\sqrt{\frac{a}{b}}x + C & (b > 0) \\ \frac{1}{2\sqrt{-ab}} ln|\frac{\sqrt{a}x-\sqrt{-b}}{\sqrt{a}x+\sqrt{-b}}| + C \end{cases}$

推导:

$$\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{a}} \int \frac{1}{x^2+\frac{b}{a}} dx \tag{28}$$

$$= \begin{cases} \frac{1}{a}\int \frac{1}{x^2+\left(\sqrt{\frac{b}{a}}\right)^2} dx = \frac{1}{a}\left(\sqrt{\frac{a}{b}}\right) arctan\sqrt{\frac{a}{b}}x + C = \frac{1}{\sqrt{ab}} arctan\sqrt{\frac{a}{b}}x + C \quad (b \\ \frac{1}{a}\int \frac{1}{x^2-\left(\sqrt{\frac{-b}{a}}\right)^2} dx = \int \frac{1}{2a} \cdot \sqrt{\frac{a}{-b}} ln|\frac{x-\sqrt{\frac{b}{a}}}{x+\sqrt{\frac{b}{a}}}| dx = \frac{1}{2\sqrt{-ab}} ln|\frac{\sqrt{a}x-\sqrt{-b}}{\sqrt{a}x+\sqrt{-b}}| + C \end{cases} \tag{29}$$

2. $\int \frac{x}{ax^2+b} dx = \frac{1}{2a} ln|ax^2 + b| + C$ 推导:

$$\int \frac{x}{ax^2+b} dx = \frac{1}{2a}\int \frac{1}{ax^2+b} dax^2 + b = \frac{1}{2a} ln|ax^2 + b| + C \tag{30}$$

3. $\int \frac{x^2}{ax^2+b} dx = \frac{x}{a} - \frac{b}{a}\int \frac{ax}{ax^2+b}$
4. $\int \frac{dx}{x(ax^2+b)} = \frac{1}{2b} \ln \frac{x^2}{|ax^2+b|} + C$
5. $\int \frac{dx}{x^2(ax^2+b)} = -\frac{1}{bx} - \frac{a}{b}\int \frac{dx}{ax^2+b}$
6. $\int \frac{dx}{x^3(ax^2+b)} = \frac{a}{2b^2} ln\frac{|ax^2+b|}{x^2} - \frac{1}{2bx^2} + C$
7. $\int \frac{dx}{(ax^2+b)^2} = \frac{x}{2b(ax^2+b)} + \frac{1}{b}\int \frac{dx}{ax^2+b}$

1.6 含有 $ax^2+bx+c(a>0)$ 的积分

- $\int \frac{dx}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} arctan \frac{2ax+b}{\sqrt{4ac-b^2}+C} & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2-4ac}} ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + C & (b > 4ac) \end{cases}$
- $\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{dx}{ax^2+bx+c}$

1.6.1 含有 $\sqrt{x^2+a^2}$

- $\int \frac{dx}{\sqrt{x^2+a^2}} = arsh\frac{x}{a} + C_1 = ln(x+\sqrt{x^2+a^2}) + C$
- $\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$
- $\int \frac{x}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2} + C$
- $\int \frac{x}{\sqrt{(x^2+a^2)^3}} dx = -\frac{1}{\sqrt{x^2+a^2}} + C$
- $\int \frac{x^2}{\sqrt{x^2+a^2}} dx = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2} ln\left(x+\sqrt{x^2+a^2}\right) + C$
- $\int \frac{x^2}{\sqrt{\left(x^2+a^2\right)^3}} = -\frac{x}{\sqrt{x^2+a^2}} + ln\left(x+\sqrt{x^2+a^2}\right) + C$
- $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\frac{\sqrt{x^2+a^2}-a}{|x|} + C$
- $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{x^2+a^2}{a^2x} + C$
- $\int \sqrt{x^2+a^2} dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C$
- $\int \sqrt{\left(x^2+a^2\right)^3} dx = \frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2} + \frac{3}{8}a^4 ln(x+\sqrt{x^2+a^2}) + C$
- $\int x\sqrt{x^2+a^2} dx = \frac{1}{3}\sqrt{\left(x^2+a^2\right)^3} + C$
- $\int x^2\sqrt{x^2+a^2} dx = \frac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2} - \frac{a^4}{8}\ln(x+\sqrt{x^2+a^2}) + C$
- $\int \frac{\sqrt{x^2+a^2}}{x} dx = \sqrt{x^2+a^2} + aln\frac{\sqrt{x^2+a^2}-a}{|x|} + C$
- $\int \frac{\sqrt{x^2+a^2}}{x^2} dx = -\frac{\sqrt{x^2+a^2}}{x} + ln(x+\sqrt{x^2+a^2}) + C$

1.7 含有 $\sqrt{x^2-a^2}(a>0)$ 的积分

- $\int \frac{dx}{\sqrt{x^2-a^2}} = \frac{x}{|x|} arch\frac{|x|}{a} + C_1 = ln|x+\sqrt{x^2-a^2}| + C$
- $\int \frac{dx}{\sqrt{\left(x^2-a^2\right)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$

- $\int \frac{x}{x^2-a^2}dx = \sqrt{x^2-a^2} + C$
- $\int \frac{x}{\sqrt{(x^2-a^2)}}dx = -\frac{1}{\sqrt{x^2-a^2}} + C$
- $\int \frac{x^2}{\sqrt{x^2-a^2}}dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}ln|x + \sqrt{x^2-a^2}| + C$
- $\int \frac{x^2}{\sqrt{(x^2-a^2)}}x = -\frac{x}{\sqrt{x^2-a^2}} + ln|x + \sqrt{x^2-a^2}| + C$
- $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}arccos\frac{a}{|x|} + C$
- $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{(x^2-a^2)}{a^2x} + C$
- $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a}{2}ln|x + \sqrt{x^2-a^2}| + C$
- $\int \sqrt{\left(x^2-a^2\right)^3} = \frac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2} + \frac{3}{8}a^4 \ln|x + \sqrt{x^2-a^2}| + C$
- $\int x\sqrt{x^2-a^2}dx = \frac{1}{3}\sqrt{\left(x^2-a^2\right)^3} + C$
- $\int x^2\sqrt{x^2-a^2}dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln|x + \sqrt{x^2-a^2}| + C$
- $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - aarccos\frac{a}{|x|} + C$
- $\int \frac{\sqrt{x^2-a^2}}{x^2}dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln|x + \sqrt{x^2-a^2}| + C$

1.8 含有 $\sqrt{a^2-x^2}(a>0)$ 的积分

- $\int \frac{dx}{\sqrt{a^2-x^2}} = arcsin\frac{x}{a} + C$
- $\int csc^2xdx = -cotx + C$
- $\int \sec x \tan xdx = \sec x + C$
- $\int \csc x \cot xdx == -\csc x + c$
- $\int \sin^2 x = \frac{x}{2} - \frac{1}{4}\sin 2x + C$
- $\int \cos^2 dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$
- $\int \sin^n xdx = -\frac{1}{n}\sin^{n-1}\cos x + \frac{n-1}{n}\int \sin^{n-1} xdx$
- $\int \cos^n xdx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} dx$
- $\int \frac{dx}{\sin^n x} = -\frac{1}{n-1}\cdot\frac{\cos x}{sin^{n-1}} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} x}dx$
- $\int \frac{dx}{cos^n x} = \frac{1}{n-1}\cdot\frac{\sin x}{\cos^{n-1}(x)} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x}$
- $\int \cos^m x\sin^x dx = \frac{1}{m+n}cos^{m-1}xsin^{n+1}x + \frac{m-1}{m+n}\int \cos^{m-2} x\sin^n xdx = -\frac{1}{m+n}cos^{m+1}xsin^{n}$
  $\frac{m-1}{m+n}\int cos^m xsin^{n-2}xdx$

- $\int \sin ax \cos bx dx = -\frac{1}{2(a+b)} \cos(a+b)x - \frac{1}{2(a-b)} cos(a-b)x + C$
- $\int \sin ax \sin bx dx = -\frac{1}{2(a+b)} \sin(a+b)x + \frac{1}{2(a-b)} \sin(a-b)x + C$
- $\int \cos ax \cos bx dx = \frac{1}{2(a+b)} \sin(a+b)x + \frac{1}{2(a-b)} \sin(a-b)x + C$
- $\int \frac{dx}{a+b\sin x} = \frac{2}{\sqrt{a^2-b^2}} arctan \frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}} + C(a^2 > b^2)$
- $\int \frac{dx}{a+b\sin x} = \frac{1}{\sqrt{b^2-a^2}} ln \left|\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}\right| + C(a^2 < b^2)$
- $\int \frac{dx}{a+b\cos x} = \frac{2}{a+b}\sqrt{\frac{a+b}{a-b}} archtan\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) + C(a^2 > b^2)$
- $\int \frac{dx}{a+b\cos x} = \frac{1}{a+b}\sqrt{\frac{a+b}{a-b}} \ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{a+b}{a-b}}}{\tan\frac{x}{2}-\sqrt{\frac{a+b}{a-b}}} + C(a^2 < b^2)\right|$
- $\int \frac{dx}{a^2 cos^2 x + b^2 cos^2 x} = \frac{1}{ab} \arctan\left(\frac{b}{a}\tan x\right) + C$
- $\int \frac{dx}{a^2 \cos^x - b^2 \sin^2 x} = \frac{1}{2ab} \ln\left|\frac{b\tan x + a}{b\tan x - a}\right| + C$
- $\int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$
- $\int x^2 \sin ax dx = -\frac{1}{a} x^2 \cos ax + \frac{2}{a^2} x \sin ax + \frac{2}{a^3} \cos ax + C$
- $\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{1}{a} + C$
- $\int x^2 \cos ax dx = \frac{1}{a} x^2 \sin ax + \frac{2}{a^2} x \cos ax - \frac{2}{a^3} \sin ax + C$