ALGORITHMS, FALL 2018, HOMEWORK 9

Due Thursday, November 8 at 11:59pm.
Worth 2% of the final grade.

1. Suppose that you have two strings, S_1, S_2, each consisting of characters from some small alphabet. The lengths of the strings are k and n respectively. You can assume that $n \geq k$ but not that k is necessarily some small value.

   If we alternate reading characters from one string and then the other, we create a "blend" of the two strings. For example if
   $S_1 = DYNOGRASOAMA$
   and
   $S_2 = AMICPRMMINGISZING$
   we can create this blend:
   DYNAMICPROGRAMMINGISSOAMAZING

   Note that the length of a blend is $n+k$, and there can be many blends.

   Now for the problem:
   Given three strings S_1, S_2 and C, determine if C is a blend of S_1 and S_2. If S_1 and S_2 have size k and n respectively, the time complexity should be $O(kn)$.

   You should address the following in your answer:
   Formulate the problem recursively, including an informal justification for its correctness. (How can you categorize subproblems that will be helpful to solve first?) How many distinct subproblems are there? Mention what you consider to be a base case. What is the size of the dynamic programming table that you will use, how do you fill it in, and where are the base cases and final answer located?

2. You must support the following two operations on a set of numbers: (1) insertion, and (2) "large-delete" which is to delete the largest half of the values in the set. In other words if there are k elements you delete the elements with the $\lceil k/2 \rceil$ largest values. For example, large-delete$\{5, 3, 7, 9, 1, 8\} = \{3, 1, 5\}$.
   Both operations should take constant time, amortized. Assume that you start with an empty data structure.
   You can use whatever data structure you like for the set. A small amount of the total credit for this problem will be given for making a non-complicated choice.

   (a) Describe your structure and how you will handle the operations algorithmically. Explain what the worst-case time complexity is for one operation, and for n operations.

   (a) Analyze with the accounting method.

   (b) Analyze with the potential method.