

1. (a)  $U(n)$  is an unbalanced sort.  $B(N)$  is a balanced sort.

Worst case scenario for a balanced sort would be  $B(n) = U(\frac{n}{c}) + U(n - \frac{n}{c}) + dn$ .

Worst case scenario for an unbalanced sort is  $U(n) = B(n - u) + udn$ .

If we substitute, we are left with the expression

$$B(n) = B(\frac{n}{c} - u) + ud\frac{n}{c} + B(\frac{cn-n}{c} - u) + ud\frac{cn-n}{c} + dn$$

we can bring together all of the constant terms to

$$B(n) = B(\frac{n}{c} - u) + B(\frac{cn-n}{c} - u) + (\frac{n}{c} + \frac{cn-n}{c} + 1)ud$$

Consolidating the constants gets us

$$B(n) = B(\frac{n}{c} - u) + B(\frac{cn-n}{c} - u) + dun + du$$

$$B(n) \leq 2B(\frac{n}{c}) + dun + du$$

The runtime of this is  $O(n \log n)$

- (b) If  $du = n$  at any point of this Quicksort, quicksort is actually root dominated, and the runtime becomes  $O(n^2)$ .