Measuring Planet Mass, Radius, and Density for system GJ 436 b

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1. Introduction

Astronomers have discovered a vast number of exoplanets by measuring either the mass (M) or the radius (R) of the object. Nevertheless, these measurements were not compared to understand the existing relationship before. In 2016, Chen & Kipping built a model that established the connection between the two measurements (MR) and different object types. In this project we will measure M and R for the planet GJ 436 b—a Neptune-like planet orbiting an M-type star—and compare the MR relationship to the Chen & Kipping model. Moreover, we calculate the planet density (ρ) based on the M and R measurements. Lastly, we analyze the similarities between our results and that of exoplanets of similar mass, radius, and density. Measuring these values (M, R, ρ) can help us to further understand the properties of GJ 436 b.

2. Methods

The following methodology was applied in an effort to calculate the mass, radius, and the resulting density of the planet GJ 436 b based on radial velocity and transit photometry data. For information on the treatment of propagation of errors in our calculations, see the Appendix.

2.1 Planet mass based on radial velocity data

The radial velocity (RV) detection method records light emitted from stars to calculate the "wobble" which a star experiences from the gravitational effects of the star-planet system using the Doppler effect. The signal that we detect is the RV amplitude, K (m/s), which is proportional to the mass of the planet. We use the following equation

$$K \sim 28.4 \left(\frac{a}{AU}\right)^{-\frac{1}{2}} \left(\frac{m \sin(i)}{M_J}\right) \left(\frac{M_*}{M_\odot}\right)^{-\frac{1}{2}} (1-e^2)^{-\frac{1}{2}}$$

to solve for the mass of GJ 436 b by rearranging it for the term m. Here, 28.4 is a constant coefficient in m/s, a is the semi-major axis of the planet (0.0291 AU), i gives the inclination (85.8 degrees), M_J is the mass of Jupiter (1.898×10^{27} kg), M_* is the star's mass (0.452 $M_\odot$), and e is the eccentricity (0.1912). For our K value, we use a best fit model from the RV data in

Maness et al. (2007) and take half the distance between the crest and trough of one period. To find m, the planetary mass, we can rearrange the above equation to

$$m = \frac{KM_J}{28.4\sin(i)}\left(\frac{a}{AU}\right)^{\frac{1}{2}}(0.452)^{\frac{1}{2}}(1-e^2)^{\frac{1}{2}}$$

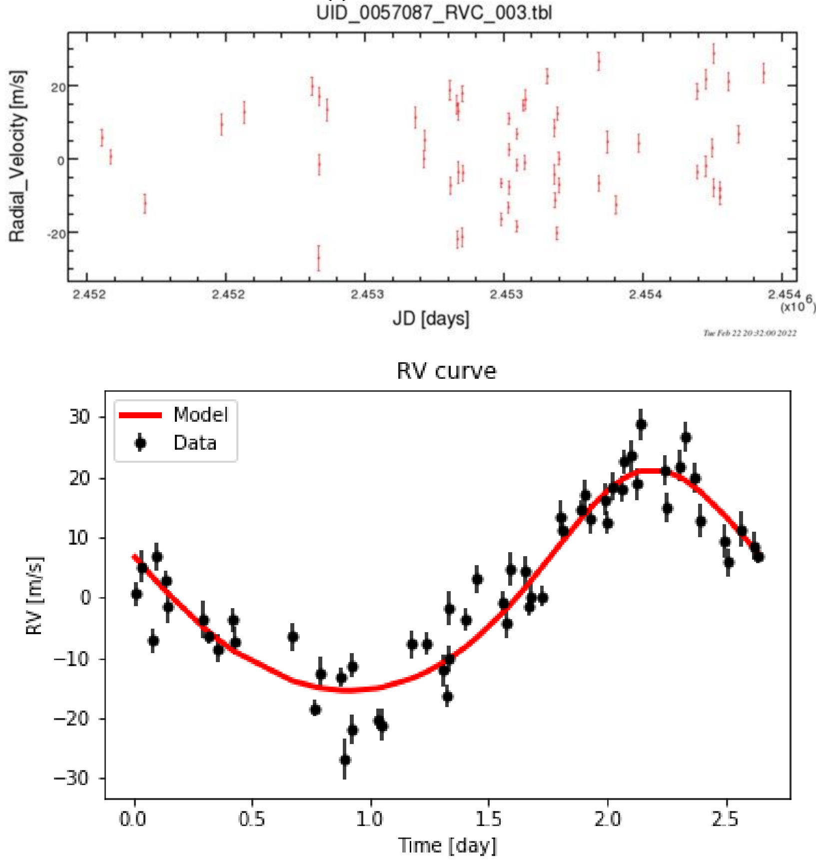

Figure 1: (top) Raw Radial Velocity data from Maness et al. (2007). (bottom) Data plotted with best fit curve showing the measured RV amplitude as a function of time for one orbit, about 2.6 days.

2.2 Planet radius based on transit data

The transit photometry method involves surveying the light from stars to analyze when part of the light is blocked by an orbiting planet passing between its parent star and Earth. The fraction of light blocked during a transit is directly related to the ratio of the planet radius to the radius of its parent star. We use the following equation

$$f = \left(\frac{R_p}{R_*}\right)^2$$

to solve for the radius of GJ 436 b by rearranging it for the term R_p. Here, f is the dimensionless ratio of the starlight blocked by the planet and R_* is the radius of the host star ($0.464R_\odot$). We use a best-fit curve of the transit data from Deming et al. (2007) to approximate the transit depth for *f*. To find R_p, the planetary radius, we can rearrange the equation to

$$R_p = R_* \sqrt{f}$$

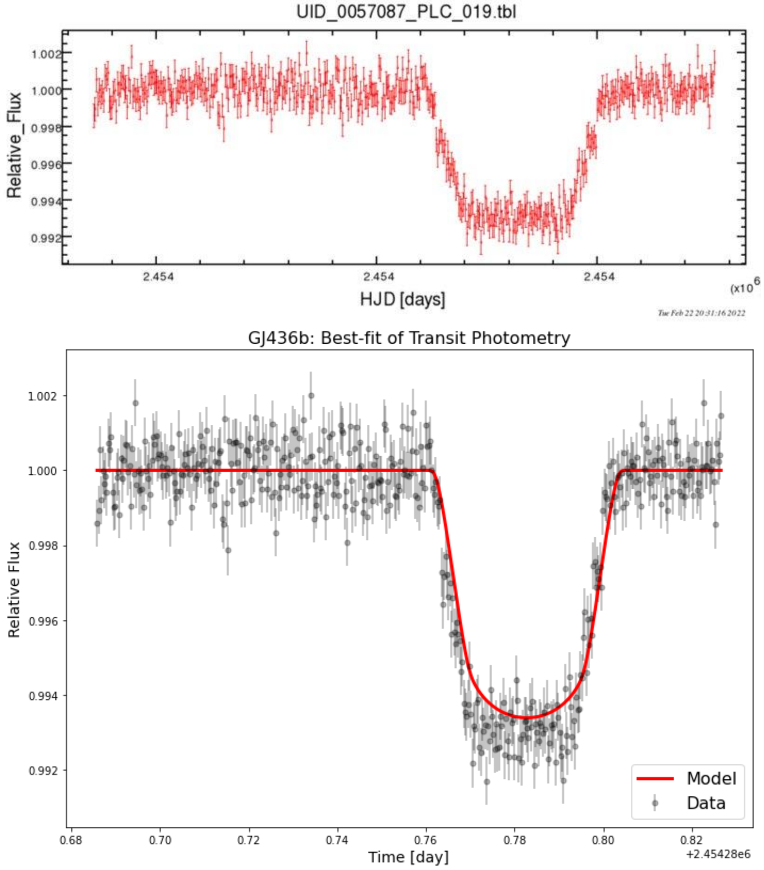

Figure 2: (top) Raw photometric light curve data from Deming et al. (2007). (bottom) Data plotted with best fit curve showing the fraction of starlight blocked as a function of time during transit.

2.3 Planet density based on the mass and radius measurements

The density, ρ, of any object can be found by dividing its mass by its volume. We have already obtained the mass of the planet in Section 2.1, so all that remains is to use the radius found in Section 2.2 to calculate the volume. We assume spherical symmetry, so the volume can be calculated by

$$V = \frac{4}{3}\pi r^3$$

Therefore, the density is given by

$$\rho = \frac{3m}{4\pi\left(R_p\right)^3}$$

3. Results

From the equations above, we calculate a mass of 23 ± $3M_{\oplus}$ ((1.37 ± 0.18)× 10^{26} kg) and a radius of 3.52 ± $0.19R_{\oplus}$ ((2.24 ± 0.12)× 10^4 km) for GJ 436 b. For the mass calculation, our value is roughly 4% larger than the actual mass of the planet, and our radius is about 15% less than the actual radius. Our mass calculation is consistent with the accepted value of this planet, although our error is a rather large percentage of the best estimate, however our radius lies outside of our calculated range. Because we overestimated the mass and underestimated the radius of the planet, the calculated density is significantly larger than the accepted value. After performing the corresponding unit conversions, the resulting density is found to be ρ = 2.9 ± 0.2 g/cm^3 which is about 1.6 times larger than the actual density of GJ 436 b (ρ = 1.78 g/cm^3). The density of Earth is ρ = 5.51 g/cm^3

In their 2016 paper, Chen & Kipping assert a power law which approximates the relationship between the mass and the radius of an object in three different planetary size bins. For planets similar to the size of GJ 436 b, the relationship is $R \sim M^{0.59}$ (relative to the size of Earth). Using our calculated mass, we should expect a radius of about R = $3.75R_{\oplus}$. Our results are inconsistent with this prediction, however, so is the actual radius of the planet which is about 10% larger than this prediction.

To confirm the validity of our results, we found exoplanets with similar properties to GJ 436 b from NASA's Exoplanet Archive. The selected planet population includes a total of 32 exoplanets with masses between 15-$30M_{\oplus}$ and radii between 2-$5R_{\oplus}$.

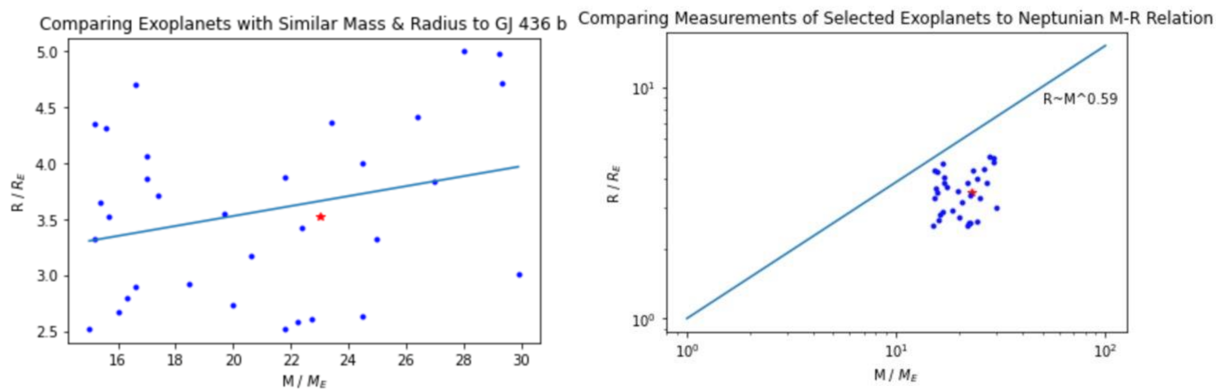

Figure 3: MR plot for 32 exoplanets with similar properties to GJ 436 b. (left) Simple plot with best fit line. (right) Chen & Kipping MR relationship with selected planet population. Note: plot on the right uses a logarithmic scale.

While there is no distinctive trend between planet mass and radius in this population, the left plot does show a small, positive correlation between mass and radius as we expect. The plot on the right compares our selected population to the predicted radii from the Chen & Kipping model, yet all these planets fall below the predicted values. This could be due to a systematic error in our calculations. However, our calculations for the mass and radius of GJ 436 b appear to be in good agreement with exoplanets exhibiting similar properties.

4. Conclusion

Measuring the mass, radius, and the density of a planet can help us to further understand its properties. The mass of a planet can confirm if the exoplanet can develop an atmosphere and retain it through evolutionary processes. By measuring the radius, we can determine how close a planet is to its parent start. If the planet is too close or too far, it is unlikely that the chemistry seen on Earth will occur, which alludes to a planet's habitability. Lastly, the measured density offers insight into the planet's composition such as gas, ice, or terrestrial worlds. In this project, we measured the following values for system GJ 436 b: M = 23 ± 3$M_{\oplus}$, R = 3.52 ± 0.19$R_{\oplus}$, and density = 2.9 ± 0.2 g/cm^3. This overestimation in mass and underestimation in radius resulted in a greatly overestimated density, yet we still expect GJ 436 b to be a gaseous planet with a small, rocky core. Although our values differ from those previously calculated by NASA, this project provided us with an opportunity to perform the calculations ourselves. Future research will require a refinement of the calculation techniques used in this study, including multiple data sets and more careful propagation of errors, perhaps utilizing existing programs for handling such uncertainties.

Contributions

Alex did research and compared GJ 436 b to similar exoplanets & M-R relation. Ashley and Mariana wrote the project report. Missie calculated the radius. Yuanhao calculated the mass.

Appendix: Propagation of Uncertainties

The following statistical techniques were used to calculate the errors of our final measurements.

Rule for Uncertainty in Products/Quotients

If $q = \frac{x \times \ldots \times z}{u \times \ldots \times w}$ then, $\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \cdots + \left(\frac{\delta z}{|z|}\right)^2 + \left(\frac{\delta u}{|u|}\right)^2 + \cdots + \left(\frac{\delta w}{|w|}\right)^2}$

Rule for Uncertainty in a Power

If $q = x^n$ then, $\frac{\delta q}{|q|} = |n|\frac{\delta x}{|x|}$

While this technique is not preferred or recommended, we treated most planetary parameters as known constants, which have no error. Therefore, the propagation of our uncertainties dealt only with the measured value in our data sets (K or f) and the property we solved for (M or R). For the mass calculation, the quantities for sin(i), a, $M_*/M_\odot$, and e were treated as known constants. To calculate the error in the mass measurement, we used

$$\frac{\delta m}{|m|} = \frac{\delta K}{|K|}$$

where the fractional uncertainty in the mass is exactly equal to the fractional uncertainty of the measure RV amplitude. For the radius calculation, only the stellar radius R_* was treated as a constant, so this error is likely much more accurate. To calculate the error, we used

$$\frac{\delta R}{|R|} = \frac{1}{2}\frac{\delta f}{|f|}$$

where the fractional uncertainty in the radius is equal to half the fractional uncertainty in the measured radiative flux according to the Rule for Uncertainty in a Power. The error in the density measurement must consider the error in both measurements. We can assume that these errors were independent, since different data sets were used to calculate each value. Therefore, their errors should be added in quadrature according to the Rule for Uncertainty in a Quotient, where

$$\frac{\delta \rho}{|\rho|} = \sqrt{\left(\frac{\delta m}{|m|}\right)^2 + \left(3 \cdot \frac{\delta R}{|R|}\right)^2}$$

Again, in the future propagation of errors should be treated more carefully instead of assuming constant coefficients, but this technique allows us to estimate the magnitude of the uncertainty in each calculation.

References

Chen, J., & Kipping, D. (2016). Probabilistic forecasting of the masses and radii of other worlds. *The Astrophysical Journal*, *834*(1), 17.

Deming, D., Harrington, J., Laughlin, G., Seager, S., Navarro, S. B., Bowman, W. C., & Horning, K. (2007). Spitzer transit and secondary eclipse photometry of GJ 436b. *The Astrophysical Journal*, *667*(2), L199.

Maness, H. L., Marcy, G. W., Ford, E. B., Hauschildt, P. H., Shreve, A. T., Basri, G. B., ... & Vogt, S. S. (2007). The M Dwarf GJ 436 and its Neptune-Mass Planet. *Publications of the Astronomical Society of the Pacific*, *119*(851), 90.

NAE. *Counts by Radius 2 < R ≤ 6 R_Earth*. Planetary systems. Retrieved February 23, 2022, from https://exoplanetarchive.ipac.caltech.edu/cgi-bin/TblView/nph-tblView?app=ExoTbls&config=PS&addcol=pl_rade&constraint=pl_rade+in+%282%2C6%5D

NASA. *Exoplanet-catalog – exoplanet exploration: Planets beyond our solar system*. NASA. Retrieved February 23, 2022, from https://exoplanets.nasa.gov/exoplanet-catalog/6534/gj-436-b/

Ucl-Exoplanets, A. (2021). *UCL-exoplanets/pylightcurve: A python package for modeling and analysing transit light-curves.* GitHub. Retrieved February 23, 2022, from https://github.com/ucl-exoplanets/pylightcurve