$$n = senk(norm(l - c) - norm(r - c))$$

$$= senk((l - c)/len(l - c) - (r - c)/len(r - c))$$

$$= senk((l - c)/\sqrt{((l_x - c_x)^2 + (l_y - c_y)^2)} - (r - c)/\sqrt{((r_x - c_x)^2 + (r_y - c_y)^2)})$$

$$n_x = \frac{l_y - c_y}{\sqrt{(l_x - c_x)^2 + (l_y - c_y)^2}} - \frac{r_y - c_y}{\sqrt{(r_x - c_x)^2 + (r_y - c_y)^2}}$$

$$n_x = \frac{\sqrt{(r_x - c_x)^2 + (r_y - c_y)^2} \cdot (l_y - c_y)}{\sqrt{(r_x - c_x)^2 + (r_y - c_y)^2} \cdot \sqrt{(l_x - c_x)^2 + (l_y - c_y)^2}} - \frac{\sqrt{(l_x - c_x)^2 + (l_y - c_y)^2} \cdot (r_y - c_y)}{\sqrt{(l_x - c_x)^2 + (l_y - c_y)^2} \cdot \sqrt{(r_x - c_x)^2 + (r_y - c_y)^2}}$$

$$n_x = \frac{\sqrt{(r_x - c_x)^2 + (r_y - c_y)^2} \cdot (l_y - c_y) - \sqrt{(l_x - c_x)^2 + (l_y - c_y)^2} \cdot (r_y - c_y)}{\sqrt{(l_x - c_x)^2 + (l_y - c_y)^2} \cdot \sqrt{(r_x - c_x)^2 + (r_y - c_y)^2}}$$

$$n_y = -\frac{l_x - c_x}{\sqrt{(l_x - c_x)^2 + (l_y - c_y)^2}} + \frac{r_x - c_x}{\sqrt{(r_x - c_x)^2 + (r_y - c_y)^2}}$$

$$b = af$$

$$c = \sqrt{a^2 + b^2}$$

$$
\begin{aligned}
t(f) &= \frac{c - a}{v_{Lisa}} - \frac{b}{v_{Bus}} \\
&= \frac{\sqrt{a^2 + b^2} - a}{v_{Lisa}} - \frac{af}{v_{Bus}} \\
&= \frac{\sqrt{a^2(1 + f^2)} - a}{v_{Lisa}} - \frac{af}{v_{Bus}} \\
&= a\left(\frac{\sqrt{1 + f^2} - 1}{v_{Lisa}} - \frac{f}{v_{Bus}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dt(f)}{d\theta} &= \frac{da\left(\frac{\sqrt{1+f^2}-1}{v_{Lisa}} - \frac{f}{v_{Bus}}\right)}{df} \\
&= a\left(\frac{d\frac{\sqrt{1+f^2}-1}{v_{Lisa}}}{df} - \frac{d\frac{f}{v_{Bus}}}{df}\right) \\
&= a\left(\frac{\frac{d\sqrt{1+f^2}}{df}}{v_{Lisa}} - \frac{\frac{df}{df}}{v_{Bus}}\right) \\
&= a\left(\frac{\frac{1}{2\sqrt{1+f^2}} \cdot \frac{d1+f^2}{df}}{v_{Lisa}} - \frac{1}{v_{Bus}}\right) \\
&= a\left(\frac{f}{v_{Lisa}\sqrt{1+f^2}} - \frac{1}{v_{Bus}}\right)
\end{aligned}
$$

$$\frac{dt(f)}{d\theta} = 0$$

$$\Longleftrightarrow \qquad a\left(\frac{f}{v_{Lisa}\sqrt{1+f^2}} - \frac{1}{v_{Bus}}\right) = 0$$

$$\Longleftrightarrow \qquad a\frac{f}{v_{Lisa}\sqrt{1+f^2}} = a\frac{1}{v_{Bus}}$$

$$\Longleftrightarrow \qquad \frac{f}{\sqrt{1+f^2}} = \frac{v_{Lisa}}{v_{Bus}}$$

$$\Longleftrightarrow \qquad \left(\frac{f}{\sqrt{1+f^2}}\right)^2 = \left(\frac{v_{Lisa}}{v_{Bus}}\right)^2$$

$$\Longleftrightarrow \qquad \frac{f^2}{1+f^2} = \frac{v_{Lisa}^2}{v_{Bus}^2}$$

$$\Longleftrightarrow \qquad \frac{1+f^2}{f^2} = \frac{v_{Bus}^2}{v_{Lisa}^2}$$

$$\Longleftrightarrow \qquad \frac{1}{f^2} = \frac{v_{Bus}^2 - v_{Lisa}^2}{v_{Lisa}^2}$$

$$\Longleftrightarrow \qquad f^2 = \frac{v_{Lisa}^2}{v_{Bus}^2 - v_{Lisa}^2}$$

$$\Longleftrightarrow \qquad f = \sqrt{\frac{v_{Lisa}^2}{v_{Bus}^2 - v_{Lisa}^2}}$$

$$\Longleftrightarrow \qquad f = \frac{v_{Lisa}}{\sqrt{v_{Bus}^2 - v_{Lisa}^2}}$$