$$
\begin{aligned}
t(\theta) &= v_{Lisa}\sqrt{x^2 + (x\tan\theta)^2} - v_{Bus}x\tan\theta \\
&= v_{Lisa}x\sqrt{1+\tan^2\theta} - v_{Bus}x\tan\theta \\
&= v_{Lisa}x\sqrt{\frac{1}{\cos\theta}} - v_{Bus}x\tan\theta \\
&= v_{Lisa}\frac{x}{\sqrt{\cos\theta}} - v_{Bus}x\tan\theta
\end{aligned}
$$

$$
\begin{aligned}
\frac{dt(\theta)}{d\theta} &= \frac{dv_{Lisa}\frac{x}{\sqrt{\cos\theta}} - v_{Bus}x\tan\theta}{d\theta} \\
&= v_{Lisa}\frac{\frac{dx}{d\theta}\sqrt{\cos\theta} - x\frac{d\sqrt{\cos\theta}}{d\theta}}{\sqrt{\cos\theta}^2} - v_{Bus}x\frac{d\tan\theta}{d\theta} \\
&= v_{Lisa}\frac{-x}{\sqrt{\cos\theta}^2}\cdot\frac{1}{2\sqrt{\cos\theta}}\cdot\frac{d\cos\theta}{d\theta} - \frac{v_{Bus}x}{\cos^2\theta} \\
&= v_{Lisa}\frac{x\sin\theta}{2\cos^{\frac{3}{2}}\theta} - v_{Bus}\frac{x}{\cos^2\theta}
\end{aligned}
$$

$$
\begin{aligned}
& & \frac{dt(\theta)}{d\theta} &= 0 \\
\iff & & v_{Lisa}\frac{x\sin\theta}{2\cos^{\frac{3}{2}}\theta} - v_{Bus}\frac{x}{\cos^2\theta} &= 0 \\
\iff & & v_{Lisa}\frac{x\sin\theta}{2\cos^{\frac{3}{2}}\theta} &= v_{Bus}\frac{x}{\cos^2\theta} \\
\iff & & v_{Lisa}\frac{\sin\theta}{\cos^{\frac{3}{2}}\theta} &= v_{Bus}\frac{2}{\cos^2\theta} \\
\iff & & \sin\theta\sqrt{\cos\theta} &= 2\frac{v_{Bus}}{v_{Lisa}} \\
\iff & & \frac{\sin\theta\cos\theta}{\sqrt{\cos\theta}} &= 2\frac{v_{Bus}}{v_{Lisa}} \\
\iff & & \frac{\sin 2\theta}{\sqrt{\cos\theta}} &= 4\frac{v_{Bus}}{v_{Lisa}}
\end{aligned}
$$

$$b = af$$
$$c = \sqrt{a^2 + b^2}$$

$$\begin{aligned}
t(f) &= \frac{c-a}{v_{Lisa}} - \frac{b}{v_{Bus}} \\
&= \frac{\sqrt{a^2+b^2}-a}{v_{Lisa}} - \frac{af}{v_{Bus}} \\
&= \frac{\sqrt{a^2(1+f^2)}-a}{v_{Lisa}} - \frac{af}{v_{Bus}} \\
&= a\left(\frac{\sqrt{1+f^2}-1}{v_{Lisa}} - \frac{f}{v_{Bus}}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{dt(f)}{d\theta} &= \frac{da\left(\frac{\sqrt{1+f^2}-1}{v_{Lisa}} - \frac{f}{v_{Bus}}\right)}{df} \\
&= a\left(\frac{d\frac{\sqrt{1+f^2}-1}{v_{Lisa}}}{df} - \frac{d\frac{f}{v_{Bus}}}{df}\right) \\
&= a\left(\frac{\frac{d\sqrt{1+f^2}}{df}}{v_{Lisa}} - \frac{\frac{df}{df}}{v_{Bus}}\right) \\
&= a\left(\frac{\frac{d\sqrt{1+f^2}}{df}}{v_{Lisa}} - \frac{1}{v_{Bus}}\right) \\
&= a\left(\frac{\frac{1}{2\sqrt{1+f^2}} \cdot \frac{d1+f^2}{df}}{v_{Lisa}} - \frac{1}{v_{Bus}}\right) \\
&= a\left(\frac{f}{v_{Lisa}\sqrt{1+f^2}} - \frac{1}{v_{Bus}}\right)
\end{aligned}$$

$$
\begin{aligned}
&& \frac{dt(f)}{d\theta} &= 0 \\
\iff && a\left(\frac{f}{v_{Lisa}\sqrt{1+f^2}} - \frac{1}{v_{Bus}}\right) &= 0 \\
\iff && a\frac{f}{v_{Lisa}\sqrt{1+f^2}} &= a\frac{1}{v_{Bus}} \\
\iff && \frac{f}{\sqrt{1+f^2}} &= \frac{v_{Lisa}}{v_{Bus}} \\
\iff && \left(\frac{f}{\sqrt{1+f^2}}\right)^2 &= \left(\frac{v_{Lisa}}{v_{Bus}}\right)^2 \\
\iff && \frac{f^2}{1+f^2} &= \frac{v_{Lisa}^2}{v_{Bus}^2} \\
\iff && \frac{1+f^2}{f^2} &= \frac{v_{Bus}^2}{v_{Lisa}^2} \\
\iff && \frac{1}{f^2} &= \frac{v_{Bus}^2 - v_{Lisa}^2}{v_{Lisa}^2} \\
\iff && f^2 &= \frac{v_{Lisa}^2}{v_{Bus}^2 - v_{Lisa}^2} \\
\iff && f &= \sqrt{\frac{v_{Lisa}^2}{v_{Bus}^2 - v_{Lisa}^2}} \\
\iff && f &= \frac{v_{Lisa}}{\sqrt{v_{Bus}^2 - v_{Lisa}^2}}
\end{aligned}
$$