

$$b = af$$

$$c = \sqrt{a^2 + b^2}$$

$$\begin{aligned}
t(f) &= \frac{c-a}{v_{Lisa}} - \frac{b}{v_{Bus}} \\
&= \frac{\sqrt{a^2 + b^2} - a}{v_{Lisa}} - \frac{af}{v_{Bus}} \\
&= \frac{\sqrt{a^2(1+f^2)} - a}{v_{Lisa}} - \frac{af}{v_{Bus}} \\
&= a \left(\frac{\sqrt{1+f^2} - 1}{v_{Lisa}} - \frac{f}{v_{Bus}} \right)
\end{aligned}$$

$$\begin{aligned}
\frac{dt(f)}{df} &= \frac{da \left(\frac{\sqrt{1+f^2}-1}{v_{Lisa}} - \frac{f}{v_{Bus}} \right)}{df} \\
&= a \left(\frac{d \frac{\sqrt{1+f^2}-1}{v_{Lisa}}}{df} - \frac{d \frac{f}{v_{Bus}}}{df} \right) \\
&= a \left(\frac{\frac{d\sqrt{1+f^2}}{df}}{v_{Lisa}} - \frac{\frac{df}{df}}{v_{Bus}} \right) \\
&= a \left(\frac{\frac{1}{2\sqrt{1+f^2}} \cdot \frac{d1+f^2}{df}}{v_{Lisa}} - \frac{1}{v_{Bus}} \right) \\
&= a \left(\frac{f}{v_{Lisa} \sqrt{1+f^2}} - \frac{1}{v_{Bus}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{dt(f)}{df} = 0 \\
\iff & a \left(\frac{f}{v_{Lisa} \sqrt{1+f^2}} - \frac{1}{v_{Bus}} \right) = 0 \\
\iff & a \frac{f}{v_{Lisa} \sqrt{1+f^2}} = a \frac{1}{v_{Bus}} \\
\iff & \frac{f}{\sqrt{1+f^2}} = \frac{v_{Lisa}}{v_{Bus}} \\
\iff & \left(\frac{f}{\sqrt{1+f^2}} \right)^2 = \left(\frac{v_{Lisa}}{v_{Bus}} \right)^2 \\
\iff & \frac{f^2}{1+f^2} = \frac{v_{Lisa}^2}{v_{Bus}^2} \\
\iff & \frac{1+f^2}{f^2} = \frac{v_{Bus}^2}{v_{Lisa}^2} \\
\iff & \frac{1}{f^2} = \frac{v_{Bus}^2 - v_{Lisa}^2}{v_{Lisa}^2} \\
\iff & f^2 = \frac{v_{Lisa}^2}{v_{Bus}^2 - v_{Lisa}^2} \\
\iff & f = \sqrt{\frac{v_{Lisa}^2}{v_{Bus}^2 - v_{Lisa}^2}} \\
\iff & f = \frac{v_{Lisa}}{\sqrt{v_{Bus}^2 - v_{Lisa}^2}}
\end{aligned}$$