Construction of Generic Credit Curves for Valuation and Risk Management of Illiquid Credit Exposure

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Abstract — Trading books of most financial institutions contain significant exposure to illiquid defaultable instruments that are subject to mark-to-market accounting. Due to unavailability of regular market quotes of credit spreads for such instruments, one must make use of proxies for valuation and manage risks. Credit curve proxies are typically arbitrary, resulting in inaccurate risk management and high valuation uncertainties. In addition, a typical bias toward over-conservativeness results in higher regulatory capital calculation. We propose an approach for the construction of generic credit curves that establishes consistency with the default risk of the wider credit market, implicitly solving the various problems with arbitrary proxies. The approach consists in adopting the universe of credit rated issuers with visible CDS spread market quotes to form the underlying portfolio of an imaginary synthetic Collateralized Debt Obligation (CDO), in which credit ratings are associated with specific tranches or subordinations. AAA-rated spreads constitute the portfolio loss risk of a super senior tranche while CCC-rated spreads are aligned with equity (first-loss) tranche losses, with all other ratings forming the mezzanine subordinations. Vasicek's portfolio loss distribution is used to imply the loss distribution of the portfolio, allowing for the construction of generic credit spread for each desired maturity, credit subordination, currency, sector and rating.

Keywords: Credit spreads, Credit Default Swaps, Collateralized Debt Obligation, Vasicek portfolio loss distribution

I Introduction

The valuation and risk management aspects of exposure to illiquid credit instruments in trading books of financial institutions played a central role during and in the subsequent years from the Global Financial Crisis of 2008/09. Those positions were subject to mark-to-market accounting, and one problem originated from the required mark down due to the absence of market implied credit curves as a result of Level 3 classification under FASB 157[1]. Moreover, regulatory capital requirements calculated from Value at Risk (VaR) were heavily underestimated due to the lack of volatility information of credit curves from such illiquid positions, ultimately leading to the liquidity crisis that ensued as a result of the perceived vulnerability of financial institutions. Although now somewhat tamed, the problem with valuation and risk management of illiquid credit instruments in trading books remains high on the agenda of financial institutions and regulators. One way around the issue is to use appropriate generic credit curves.

Generic credit curves are basically proxies that aggregate information from visible market credit curves used to infer the credit curves for the illiquid instruments. They represent a generalization of credit risk for different combinations of denomination currency, credit subordination (senior unsecured, subordinated), sector, credit rating and maturity. For instance, an illiquid Canadian dollar, senior unsecured, BBB corporate bond exposure requires one to construct a generic credit curve with exactly those attributes for subsequent valuation and risk management activities.

Most commercial and in-house development of models and methodologies for construction of credit curves adopt a specific group of representative issuers for credit rating, sector, currency and some-

[1] FASB 157 is a statement issued in 2006 by the Financial Accounting Standards Board (FASB) that requires all publicly-traded firms in the United States to classify their assets into three categories, Level 1, Level 2, or Level 3, according to the level of uncertainty with which fair values can be determined. Based on this, regulators have prescribed the write down of Level 3 assets, which are the most illiquid and therefore the highest fair valuation uncertainty. More details on asset valuation classification can be found in `http://www.fasb.org/cs/BlobServer?blobkey=id&blobwhere=1175823288587&blobheader=application/pdf&blobcol=urldata&blobtable=MungoBlobs`.

times geography, applying some form of averaging directly to credit spreads as a model assumption. One such example is described in [6], where generic curves are formed via mean spreads at each maturity, median spreads at each maturity, and what the authors call "best fit" method which minimizes the sum of squared differences between the individual spreads and the estimated generic curve spreads. One criticism is that such approaches fail to capture the implicit market dependence structure between different cohorts (defined by specific ratings), geographies and sectors. And how to handle the cases for which a statistically representative set of issuers does not exist

In the subsequent sections we discuss the construction of generic credit curves that establishes consistency with the default risk of the wider credit market. Where marking-to-*market* is not possible, these curves may be used as inputs in order to mark-to-*model* illiquid credit exposure. More importantly, they may be used to calculate VaR and other risk measures (expected shortfall, stress testing, etc.) by providing live information on market volatility otherwise unavailable.

II Construction of generic credit curves

II.1. Preliminary considerations and assumptions

One example where generic credit curves apply are those non-US issuers representing large global corporations with dealings across the global financial system and debt issuance in both USD and local currency. Although USD issuances are usually more liquid with visible market quotes, local currency restrictions and the limited liquidity of local markets make valuation and risk management impossible tasks for most local currency exposure without generic credit curves. And then there are issuers for which even USD-denominated CDS quotes are very limited. The goal is to be able to generate such generic credit curves that are consistent with market expectations of credit defaults subdivided by credit subordination, sector, rating and maturity as illustrated by Figure II.1. The level of granularity can even go one step further depending on availability of quoted CDS spreads on specific regions and countries.

Another important point is that we use only CDS spreads to construct generic credit curves, thus alienating the various other credit-sensitive instruments (e.g. bonds, credit linked notes, etc) from which default probabilities may also be implied. The main reason is that we are constructing credit

Debt Tier	Sector	Rating	Tenor
Senior Unsecured	Basic Materials	AAA	1Y
Subordinated	Consumer Goods	AA	3Y
	Consumer Services	A	5Y
	Enegery	BBB	10Y
	Financial	BB	
	Government	B	
	Healthcare	CCC	
	Industrials		
	Technology		
	Telecom		
	Utilities		

Figure II.1: Generic credit curves depth

curves, and CDS spreads represent the direct market expectation on credit defaults while other instruments embed other risk factors such as interest rates and funding. For instance, when using bonds prices, one must take into account the interest rate and funding cost (i.e., term liquidity premium) portions of bond prices in order to bootstrap credit spreads. While it is straightforward to capture the interest rates portion from yield curves, working out the funding cost requires knowledge of asset swap market quotes, which are often not visible even for the most liquid bond issuances. In addition, bonds and other credit instruments suffer from lack of standardization, non-transparent price discovery (OTC transactions) and lack of coverage for the full term structure of a credit curve. Finally, some of these other credit instruments embed redemption or call features, which are hard to value and thus interfere with the accurate computation of credit spreads.

II.2. The synthetic CDO approach

Most typical approaches for the creation of generic credit curves adopt some form of averaging as assumption to consolidate credit spreads directly. As an example, the process would start by aggregating a representative sample of issuers with quoted and liquid CDS spreads for senior debt (subordination) in industrials (sector), and that are rated BBB. Then, an arithmetic mean would be applied to those CDS spreads across all maturities to construct the Senior-Industrials-BBB generic credit curve. The inconsistency of this approach becomes obvious when credit spreads on all maturities are not available for all issuers of the adopted representative group. One way around would be to limit the process to only issuers with a complete credit curve, but this would exclude valuable market information embedded in the credit spreads of those issuers removed. More importantly, there are sector-ratings for which no CDS spreads quotes exist at all for which the construction of credit curves under such approaches is not doable.

Another problem with similar approaches is that it performs calculation on the CDS spreads directly, thus limiting the construction of credit curves to the currencies in which there market CDS spreads. But the cases in which generic credit curves are most needed are precisely those currencies with limited market liquidity in the CDS market. Finally, an important aspect of credit markets is the interdependence of credit event expectations among different sectors and ratings in the wider market. This is completely missed by marginal computation of credit spread averages.

We propose to solve these problems by first making the universe of quoted CDS spreads into the portfolio of an imaginary synthetic Collateralized Debt Obligation (CDO), where credit ratings are associated with specific tranches or CDO subordinations. In this construct, AAA-rated and CCC-rated CDS spreads respectively imply the credit loss expectations of super senior and equity (first-loss) tranches, while all other ratings constitute the credit loss expectations of the mezzanine CDO subordinations. Then, as illustrated in Figure II.2 for a certain maturity in the credit spread term-structure, one needs only to know the loss probability distribution of the synthetic credit portfolio in order to infer credit spreads for each rating acting as tranches of the CDO. Calibrating loss distributions for maturities in the credit spread term-structure allows us to construct complete generic credit curves for each rating, sector and credit subordination.

Our approach eliminates inconsistency as each maturity's loss distribution allows us to infer all the points in the credit curve, whether or not a number of issuers in the portfolio contributed a complete credit term structure. Because the approach works on the survival probability space, credit curves resulting from the term structure of survival probabilities may be generated in virtually any currency, as it will be subsequently discussed. And finally, the interdependence among credit spreads of different ratings is captured through the measures of dependence embedded in portfolio loss distributions.

As shown in Figure II.2, we have a total of $n_k = 7$ tranches or ratings (AAA, AA, A, BBB, BB, B and CCC) and a total of $n_j = 4$ maturities (1-year, 3-year, 5-year and 10-year). We define attachment/detachment points ku_j for each tranche and for each maturity T_j, where $k = 1, 2, \dots n_k$ and $j = 1, 2, \dots n_j$.. The variable u_j is simply a dummy parameter that is calibrated alongside the loss distribution of the portfolio given that the subordinations of the imaginary CDO are unknown.

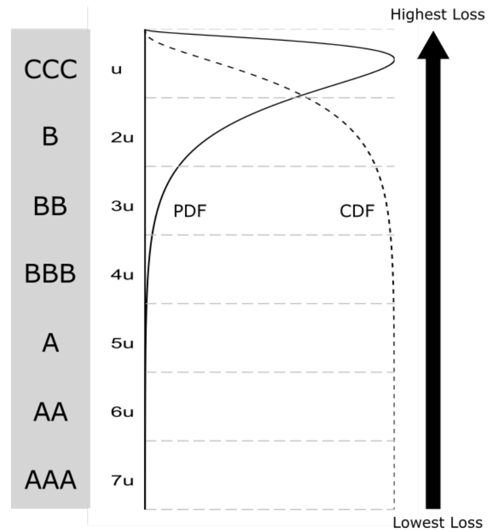

Figure II.2: Synthetic CDO concept

II.3. Vasicek portfolio loss distribution

The next step is to model the loss probability distribution of the underlying credit portfolio of our synthetic CDO. The Gaussian copula model emerged during the late 1990's as the market standard for pricing portfolio credit derivatives. It was popularized by [9] and [10], becoming the prevalent approach used by practitioners and academics throughout the past decade. This model was not new, but in fact a simplified form of earlier one-factor models proposed by Vasicek in [12] and [3] and almost identical to the original CreditMetrics model introduced by Gupton in [11]. Its main breakthrough was the analogy established between techniques used in credit risk management of loan portfolios and portfolios of credit securities in derivatives such as CDOs.

Our approach goes back to the foundations of the Gaussian copula model proposed earlier by Vasicek, which provides a closed-form solution for the loss probability distribution given a large number of issuers in the underlying credit portfolio. Vasicek's model is founded on the principles outlined by the Merton model of a firm's capital structure[8], which establishes that credit default happen when the value of a firm's assets falls below the contractual value of its debt obligations within a certain horizon. In Merton's model, the debtor's assets dynamics is assumed to follow the Geometric Brownian Motion (GBM)

$$\frac{dA_i}{A_i} = \mu_i dt + \sigma_i dW_i$$

where, for each debtor i in a credit portfolio, A_i

represents the debtor's assets, μ_i and σ_i are respectively the mean and standard deviation of changes in the debtor's assets, and dW_i is a Brownian Motion. This gives the value of the assets at a horizon T as

$$\log A_i(T) = \log A_i(0) + (\mu_i - \frac{1}{2}\sigma_i^2)T + \sigma_i\sqrt{T}z_i,$$

where z_i is a standard normal variable. In this setup, the probability of a debtor to default on its obligations by a horizon T is the probability of its assets A_i falling below its nominal debt obligations B_i, given by

$$\begin{aligned} p_i &= \mathrm{P}\left[A_i(T) < B_i\right] \\ &= \mathrm{P}\left[z_i < \frac{\log\frac{B_i}{A_i(0)} - (\mu_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\right] \\ &= \Phi\left[\frac{\log\frac{B_i}{A_i(0)} - (\mu_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\right] \\ &= \Phi(z_i). \end{aligned}$$

where Φ is the cumulative standard normal distribution function.

Now consider a portfolio with n such debt obligations in equal nominal amounts, and let the probability of default of any debtor be $p_i = p$. Also, allow for the random portion z_i of the assets of any two debtors to be correlated with correlation coefficient ρ. In this case, z_i can be expressed by

$$z_i = \Phi^{-1}(p) = y\sqrt{\rho} + \varepsilon_i\sqrt{1-\rho},$$

where y and $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_i, \ldots \varepsilon_n$ are mutually independent and identically distributed standard normal variables. This is the well established one-factor modeling of portfolios where y is interpreted as a common portfolio factor that emerges from a property of equicorrelated marginal normal distributions. In economic terms, the factor y accounts for the systemic risk (e.g., wider economy) while the term ε_i quantifies the specific risk of each firm i. From this we can derive the loss probability of obligor i conditional on the systemic factor y as

$$\begin{aligned} \mathrm{P}\left[\varepsilon_i < \frac{\Phi^{-1}(p) - y\sqrt{\rho}}{\sqrt{1-\rho}} \middle| y = Y\right] &= \\ \mathrm{P}\left[d_i = 1 \middle| y = Y\right] & \\ \Rightarrow p(Y) = \Phi\left[\frac{\Phi^{-1}(p) - Y\sqrt{\rho}}{\sqrt{1-\rho}}\right], & \end{aligned} \quad \text{(II.1)}$$

with $d_i = \mathbf{1}_{(A_i(T)<B_i)}$ as the unit step function to define a default indicator, i.e., $d_i = 1$ in case of a default and zero otherwise. The total relative loss of the credit portfolio is then given by

$$d = \sum_{i=1}^{n} w_i d_i,$$

$$\sum_{i=1}^{n} w_i = 1,$$

where the w_i's represent the weights of the portfolio constituents. Noting that $d \in [0,1]$, we have

$$\begin{aligned} \mathrm{E}\left[d \middle| Y\right] =& \sum_{i=1}^{n} w_i E\left[d_i \middle| Y\right] \\ =& \sum_{i=1}^{n} w_i \mathrm{P}\left[d_i = 1 \middle| Y\right] \\ =& p(Y)\sum_{i=1}^{n} w_i = p(Y), \end{aligned}$$

and

$$\begin{aligned} \mathrm{VAR}\left[d \middle| Y\right] =& \sum_{i=1}^{n} w_i^2 \mathrm{VAR}\left[d_i \middle| Y\right] \\ =& p(Y)\left[1 - p(Y)\right]\sum_{i=1}^{n} w_i^2. \end{aligned}$$

For very large portfolios, i.e., $n \to \infty$, we have

$$\lim_{n\to\infty}\sum_{i=1}^{n} w_i^2 \Rightarrow \lim_{n\to\infty}\mathrm{VAR}\left[d \middle| Y\right] = 0,$$

Thus, the cumulative probability function (CDF) of losses up to a proportion $x \in [0,1]$ of the total portfolio is given by

$$\begin{aligned} F(x) = \mathrm{P}\left[d \le x\right] \underset{n\to\infty}{\longrightarrow} & \mathrm{P}\left[\mathrm{E}\left[d \middle| Y\right] \le x\right] = \\ & \mathrm{P}\left[p(Y) \le x\right] = \\ & \mathrm{P}\left[Y \ge p^{-1}(x)\right] \Rightarrow \end{aligned}$$

$$F(x) = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right], \quad \text{(II.2)}$$

with the probability density function (PDF) from [3] derived as

$$f(x) = \sqrt{\frac{1-\rho}{\rho}} \exp\left[-\frac{1}{2\rho}\Big(\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)\Big)^2 + \frac{1}{2}\left(\Phi^{-1}(x)\right)^2\right]. \tag{II.3}$$

II.4. Bootstrapping default probabilities from CDS spreads

In order to calibrate generic curves via the Vasicek portfolio loss distributions in II.4II.2, we need to avail of default probabilities implied from market expectations. These default probabilities have a term structure and can be bootstrapped from market quotes of the CDS spreads. A CDS is a swap of cash for credit event protection where the protection buyer agrees to pay a regular fee (*spread*) to the protection seller whom, in turn, is liable to pay the contractual value (*notional*) to the protection buyer in case of a credit event[2] that happens before the contractual expiration (*maturity*) of the agreement. Note that the CDS acts as an insurance to the protection buyer against credit events (defaults), and even the fee paid is often called *premium*.

Financially, the CDS effectively has two legs, a premium and a protection leg. Mathematically, the premium leg of a CDS with notional 1 and maturity T_j can be written as

$$\begin{aligned} &\text{Premium Leg PV} = \\ &S_j \sum_{n=1}^{N} \{(t_n - t_{n-1}) \\ &\times [(1-\alpha)B_n - \alpha B_{n-1}]\,[(1-\alpha)q_n - \alpha q_{n-1}]\}\,, \end{aligned}$$

where

- t_n represents the premium payment dates in years from inception date $t_0 = 0$ and with maturity date $t_N = T_j$,
- S_j is the spread or premium agreed to be paid until credit event or maturity T_j is reached,
- B_n is the discount factor computed from the risk-free rate on date t_n,
- q_n is the risk-neutral survival probability of the underlying credit by date t_n, and
- $\alpha \in [0,1]$ is the parameter that takes into account the accrued premium depending on the assumption of when default happens in the premium payment cycle.

Note that the payment frequency of a CDS is determined by the difference $t_n - t_{n-1}$, mostly quarterly for CDSs on developed markets currencies and semi-annually for emerging markets. Also, because this is a discrete representation of the CDS premium leg, a common assumption is that default happens in the middle of the premium payment period, or $\alpha = 0.5$.

Similarly, the protection leg of the same CDS with notional 1 and and maturity T_j can be written as

$$\text{Protection Leg PV} = (1-R)\sum_{n=1}^{m\cdot N} B_n\,[q_n - q_{n-1}]\,,$$

where t_n, B_n and q_n are defined as previously,

- m is a annual frequency in we assume credit events may happen, and
- R is the recovery rate.

We choose $m = 12$ (monthly discretization) for computational convenience, as [1] shows that the difference between daily discretization ($m = 365$) and monthly discretization ($m = 12$) produces an absolute error which is well within bid-ask spreads of market quotes. Also, we adopt a typical practitioner's assumption on the recovery rate, or 40% for senior unsecured debt and 20% for subordinated debt as supported by studies such as [4].

CDS spreads are quoted at par, i.e., the premium to be paid by protection buyers that would equalize the cash flows of the premium leg with the ones from the protection leg. This implies that

$$\begin{aligned} &S_j \sum_{n=1}^{N} \{(t_n - t_{n-1})\,[(1-\alpha)B_n - \alpha B_{n-1}] \\ &\times [(1-\alpha)q_n - \alpha q_{n-1}]\} \\ &- (1-R)\sum_{n=1}^{m\cdot N} B_n\,[q_n - q_{n-1}] = 0 \end{aligned} \tag{II.4}$$

The only unknown that needs to be calculated from II.4 is the probability of survival term structure $q_1, q_2, \ldots, q_n, \ldots, q_N$. To that end, we adopt the market practice presented in [**?**] which uses a reduced-form (or intensity-based) model as the solution to the probability of survival term structure, or

[2] Since CDS contracts are standardized under the International Swaps and Derivatives Association (ISDA), the various definitions of credit events are available from the 2014 ISDA Credit Derivatives Definitions (`http://www2.isda.org/asset-classes/credit-derivatives/2014-isda-credit-derivatives-definitions/`).

$$q_n = e^{-h(t_n)t_n} \tag{II.5}$$

Here h is a hazard rate stepwise function, basically a constant from one maturity date T_{j-1} to another T_j in the term structure. Starting with $q_0 = 1$, II.4 is applied recursively through all maturities $T_1, T_2, \ldots, T_j, \ldots, T_{n_j}$ in order to bootstrap $h(T_j)$ for each maturity. This results in the term structure of survival probabilities that can then be calculated with II.5.

This process applies for CDS quotes of one single issuer. In order to imply the representative term structure for a certain rating k and maturity T_j with a number of issuers n_i, we calculate the respective survival probability as the geometric mean

$$q_{kj} = \left(q_{kj}^{(1)} q_{kj}^{(2)} q_{kj}^{(3)} \ldots q_{kj}^{(i)} \ldots q_{kj}^{(n_i)}\right)^{\frac{1}{n_i}}.$$

Finally, the default probabilities can be calculated from survival probabilities as simply $1 - q_{kj}$.

II.5. Calibration of generic credit curves

We now show the application of II.2 to generate credit spreads for different ratings and through the term-structure of survival probabilities as in II.5. Consider the tranche k of the CDO defined by its detachment point ku_j and attachment point $(k-1)u_j$ for maturity T_j. The cumulative expected loss of this tranche is defined by

$$\mathrm{E}_k\left[\mathrm{Loss}_j\right] = \mathrm{E}\left[\left((x - ku_j)^- - (k-1)u_j\right)^+\right],$$

where x is a random variable representing the losses in the portfolio, $(A)^- = \min(A, 0)$ and $(A)^+ = \max(A, 0)$. We can also express this expectation through II.2:

$$\mathrm{E}_k\left[\mathrm{Loss}_j\right] = u_j \left\{ k\Phi\left[\frac{\sqrt{1-\rho_j}\Phi^{-1}(ku_j) - \Phi^{-1}(p_j)}{\sqrt{\rho_j}}\right] -(k-1)\Phi\left[\frac{\sqrt{1-\rho_j}\Phi^{-1}((k-1)u_j) - \Phi^{-1}(p_j)}{\sqrt{\rho_j}}\right]\right\}.$$

On the other hand, the same expectation can be expressed through the survival probabilities bootstrapped from the CDS quotes of each respective rating and maturity. We have

$$\mathrm{E}_k\left[\mathrm{Loss}_j\right] = u_j(1 - q_{kj}),$$

where q_{kj} is defined by II.5 for maturity T_j and rating k. Combining these two results leads to the error function

$$\varepsilon_k(u_j, \rho_j, p_j) = u_j \left\{ k\Phi\left[\frac{\sqrt{1-\rho_j}\Phi^{-1}(ku_j) - \Phi^{-1}(p_j)}{\sqrt{\rho_j}}\right] - (k-1)\Phi\left[\frac{\sqrt{1-\rho_j}\Phi^{-1}((k-1)u_j) - \Phi^{-1}(p_j)}{\sqrt{\rho_j}}\right] - (1 - q_{kj})\right\}$$

Calibrating the loss distribution of our synthetic CDO means finding the values of the three unknowns (u_j, ρ_j, p_j) that minimize the sum of the squares of this error function, i.e.,

$$\min \sum_{k=1}^{n_k} \mathbf{1}_k \epsilon_k^2(u_j, \rho_j, p_j),$$

where

$$\mathbf{1}_k = \begin{cases} 1 & \text{if } q_{kj} \text{ is available,} \\ 0 & \text{otherwise.} \end{cases}$$

Since we have three parameters to be calibrated, we require at least three equations for the error functions ϵ_k, i.e., at least three groups of issuer of a certain rating from which we were able to bootstrap survival probabilities such that $\sum_{k=1}^{n_k} \mathbf{1}_k \geq 3$. However, approximations may be used where this does not apply. For instance, if $\sum_{k=1}^{n_k} \mathbf{1}_k = 2$, we may fix the parameter u_j and then calibrate (ρ_j, p_j). Or in case $\sum_{k=1}^{n_k} \mathbf{1}_k = 1$, we may fix (u_j, ρ_j) and find p_j.

II.6. Generic credit curves in different currencies

One fundamental premise of our approach lies on the fact that, in an ideal world, each issuer's default dynamics should be *unique* and *independent* of the currency in which debt is denominated. The uniqueness argument suggests that each issuer possesses a unique probability of survival term structure from which debt and CDS spreads can be derived in any currency. This is not solid mathematically, given that survival probabilities are implied from currency denominations under different risk-neutral measures. However, this not a significant error in practice and an important assumption we adopted.

On independence between the credit default and the foreign exchange (FX) dynamics, while this is reasonable during the “risk-on” portion of economic cycles, we acknowledge that default-FX independence is less realistic during “risk-off” periods. Perhaps this is the reason why “over 85% of the emerging market debt outstanding (in U.S. dollar terms) is denominated in developed market currencies” [7]. USD-denominated debt from emerging markets issuers is more attractive to market participants, and therefore more liquid, because it acts as a natural hedge against default-FX dependence. But we argue that this problem can be mitigated by hedging the FX exposure in the FX markets, as most of the the more sophisticated market participants do.

Thus, ignoring the mathematical rigor of measure theory, we assume the flexibility of constructing credit spread term structures in any currency through II.4, provided the term structure of the survival probabilities is given. For instance, if we are able to calibrate a survival probability term structure for USD-Senior Unsecured-Technology-BBB, we may now construct the complete generic credit curve for CHF-Senior Unsecured-Technology-BBB through II.4, where the only difference is that B_n is the CHF discount factor. The same goes to bootstrapping the survival probabilities through CDS quotes of different currency denominations. We have in fact tested this assumption by comparing probabilities of survival for single issuers with CDS quotes in different currencies. Although not exactly the same as our assumption would determine, the differences found in the probabilities of survival for different maturities were negligible.

III Results

From the background discussed previously, the steps for the construction of generic credit curves are listed below.

1. Define the target generic credit curves by currency-subordination-sector.

2. Select a representative set of issuers with CDS quotes for the respective subordination-sector on all available ratings.

3. Bootstrap survival probability term structures from all available ratings according to process in Section II.4.

4. Calibrate the Vasicek loss probability distribution according to discussion in Sections II.5 and II.6.

5. Generate credit spreads according to the Vasicek loss probability distribution for all ratings of the target currency-subordination-sector.

6. Perform the previous step for all maturities in order to generate a complete term structure for all target currency-subordination-sector-rating.

	AAA	AA	A	BBB	BB	B	CCC	Total
AUD	2	16	59	94	27	6	2	206
BRL				1				1
CAD		18	108	179	39	10	5	359
CHF	3	14	21	28	14	2	3	85
CNH		1						1
CNY			1	1				2
COP				1				1
CZK				1				1
DKK							1	1
EUR	35	220	919	1130	397	202	70	2973
GBP	1	45	164	141	52	18	8	429
HKD		5	5					10
HUF					1			1
IDR				1				1
ILS			1		1			2
JPY	14	95	320	326	65	24	6	850
KRW	1	4	4	2				11
MXN		1		2				3
NOK		6	11	32	21	7	2	79
PEN				1				1
PLN			1					1
RON				1				1
RUB				7	4	1		12
SEK		1	33	71	80	69	14	268
SGD	2	8	7	20	1	2		40
TRY				2				2
USD	46	274	1155	1474	552	312	86	3899
ZAR				6				6
Total	104	708	2809	3521	1254	653	197	9246

	BAS	GOOD	SVCS	ENRG	FIN	GOVT	HEAL	INDU	TECH	TELC	UTIL	Total
AUD	16	34	34	13	60	7	2	16	2	17	14	215
BRL						1						1
CAD	34	52	77	19	77	2	12	42	9	25	32	381
CHF	11	8	5	3	31	10		10	2	4	2	86
CNH						1						1
CNY					2							2
COP						1						1
CZK					1							1
DKK					1							1
EUR	204	339	378	179	1127	132	118	348	91	169	224	3309
GBP	24	43	57	9	209	7	5	44	4	31	33	466
HKD				1	6	4		1				12
HUF						1						1
IDR						1						1
ILS						1					1	2
JPY	87	171	122	51	291	60	37	147	47	36	66	1115
KRW				2	2	4				2	1	11
MXN				1	1	1						3
NOK	9	11	13	4	13	2		11	3	10	4	80
PEN						1						1
PLN						1						1
RON						1						1
RUB	2			2	7	2				1		14
SEK	25	42	54	11	47	1	10	47	13	24	16	290
SGD	3	2	9	2	12	7		1	1	5	1	43
TRY				1		1						2
USD	279	461	507	275	1294	280	192	479	170	236	313	4486
ZAR	1	1			2	1					1	6
Total	695	1164	1256	573	3183	530	376	1146	342	560	708	10533

Figure III.1: Number of quotes by rating / sector as of 23 March 2015

The CDS data used in this exercise come from the market data provider Markit, more specifically its CDS composite report. Not surprisingly, availability of CDS spread market quotes is heavy concentrated in USD, EUR and JPY as well as in the financials sector, as showed in Figure III.1. Certain currencies offer less than 20 quotes (CNY, HKD, IDR, KRW and NZD), and others none at all (e.g., MYR). Even currencies with more abundant coverage lack market quotes for certain ratings, such as AAA in CAD. Overall, the availability of market quotes concentrate on the mezzanine subordinations of our synthetic CDO (AA, A, BBB, BB, B) while significant hard to value positions may be found in the books of financial institutions for CCC and especially AAA ratings. These are the typical cases where the synthetic CDO approach works well in deriving missing market information.

One example is JPY Technology, shown in Figure III.2 with missing market information for AAA, B, and CCC ratings in all maturities[3].Figure III.2 also shows the newly generated survival probabilities on the missing ratings for all maturities. As these are cumulative probabilities, it is immediately obvious on all maturities that the ascending nature of the numbers is maintained from CCC to AAA, and that the first loss tranche (CCC) reflects its significantly higher risk profile as compared with the upper subordinations. Another observation from Figure III.2 is the fact that the survival probabilities decrease from 1Y to 10Y maturities, reflecting the fact that default expectations increase for longer exposures to non-distressed credit. Finally, Figure III.2 also shows the complete credit spread matrix with generic credit curves in basis points (bps) for all ratings.

From another angle, Figure III.3 shows these results in the form of the Vasicek PDF and CDF. Note that the probability mass of the PDF from 1Y to 10Y maturities spreads out more evenly as we move to higher maturities. This is exactly as expected given that default becomes more likely the lower the tranche subordination is, with survival expectation moving upwards in the subordination scale. Overall, the results were in line with expectation of the loss dynamics of credit portfolios.

	1y	3y	5y	10y
AAA				
AA	99.81	98.75	96.81	92.06
A	99.69	98.38	95.61	88.75
BBB	99.38	97.93	92.83	78.65
BB	97.42	91.78	83.86	67.52
B				
CCC				

	1y	3y	5y	10y
AAA	99.99	99.95	99.25	96.71
AA	99.81	98.75	96.81	92.06
A	99.69	98.38	95.61	88.75
BBB	99.38	97.93	92.83	78.65
BB	97.42	91.78	83.86	67.52
B	91.50	79.67	71.86	51.39
CCC	71.58	51.34	49.25	29.25

	1y	3y	5y	10y
AAA	0.19	1.94	9.37	19.10
AA	13.23	25.90	38.25	47.34
A	20.01	34.05	52.93	68.00
BBB	35.43	46.04	88.71	134.16
BB	124.92	168.99	203.69	223.06
B	593.34	437.43	394.55	390.95
CCC	2960.96	1468.03	1030.70	866.41

Figure III.2: Survival probabilities / Implied CDS spreads for JPY Technology as of 23 March 2015

[3]We here show only the most liquid standard maturities of 1Y, 3Y, 5Y and 10Y. Actual credit curves tend to be more granular, including points for maturities 2Y, 4Y, 7Y and even beyond 10Y

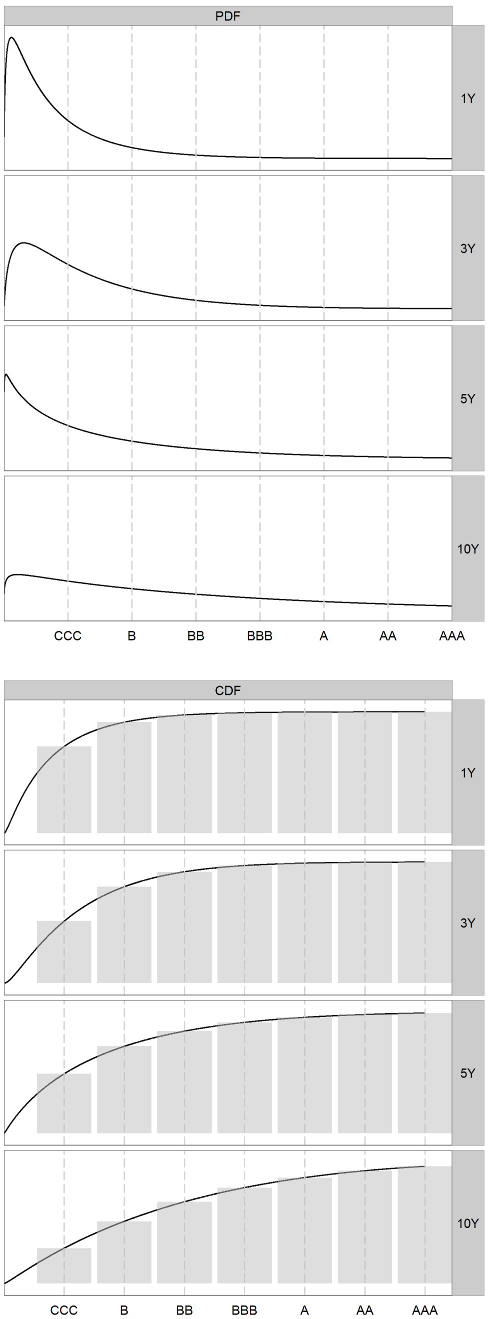

Figure III.3: PDF / CDF for JPY Technology as of 23 March 2015

IV Conclusion

In this article we discussed the importance of having generic credit curves consistent with the wider credit markets in order to value and risk manage illiquid exposure to credit instruments. Our proposed approach to construct such curves involved modelling the universe of rated credit issuers with CDS market quotes as a synthetic CDO, in which the ratings represented the different tranches of this CDO. We showed how Vasicek loss portfolio distributions can be calibrated for each maturity of a credit curve, allowing for the generation of survival probabilities and CDS spreads for ratings without available market quotes with satisfactory results.

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