Economic Complexity Theory

César A. Hidalgo
Director, Center for Collective Learning (CCL)
IAST, Toulouse School of Economics
CIAS, Corvinus University of Budapest

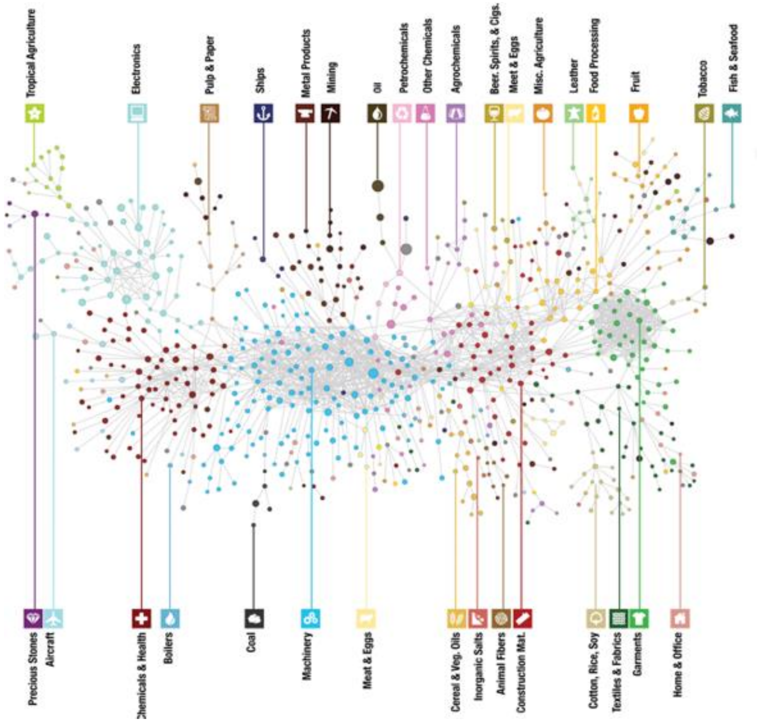
Thomas Thwaites

The world works not because a few people know a lot, but because many people know a little.

Economic complexity helps us understand how multiple factors, such as knowledge come together, without having to define them.

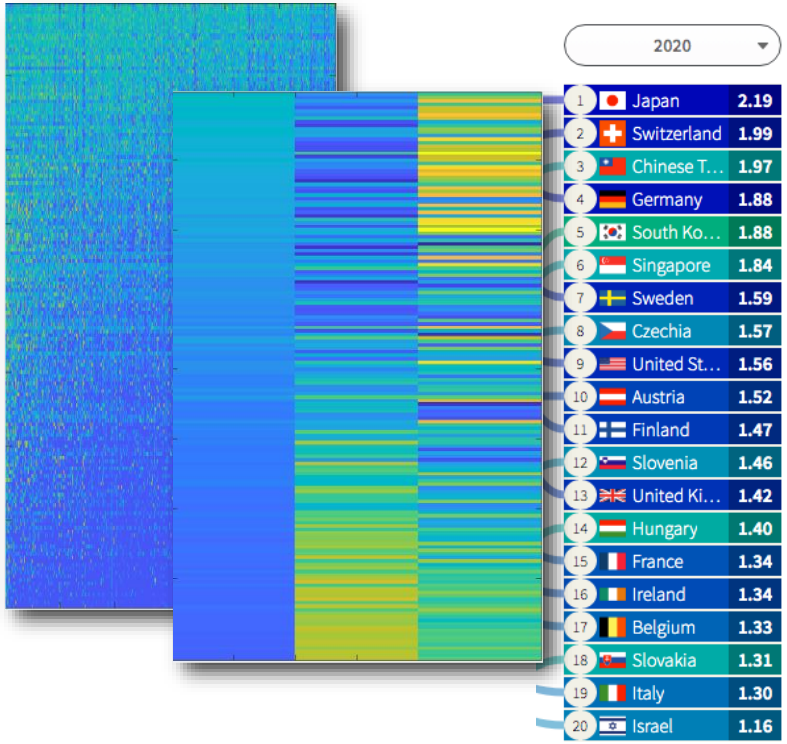
Two Key Concepts

Relatedness



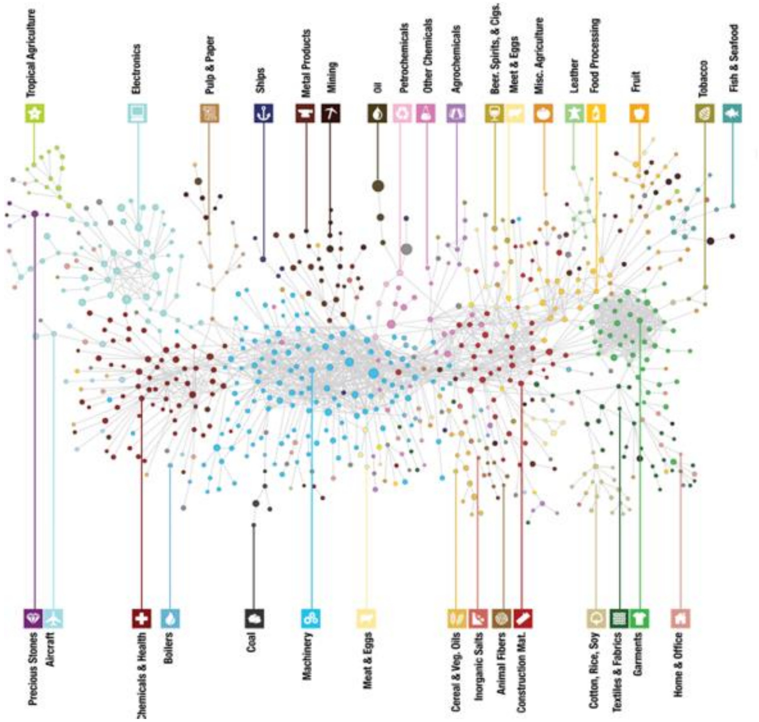
Hidalgo et al. Science (2007)

Complexity Indexes



Hidalgo & Hausmann. PNAS (2009)

Relatedness

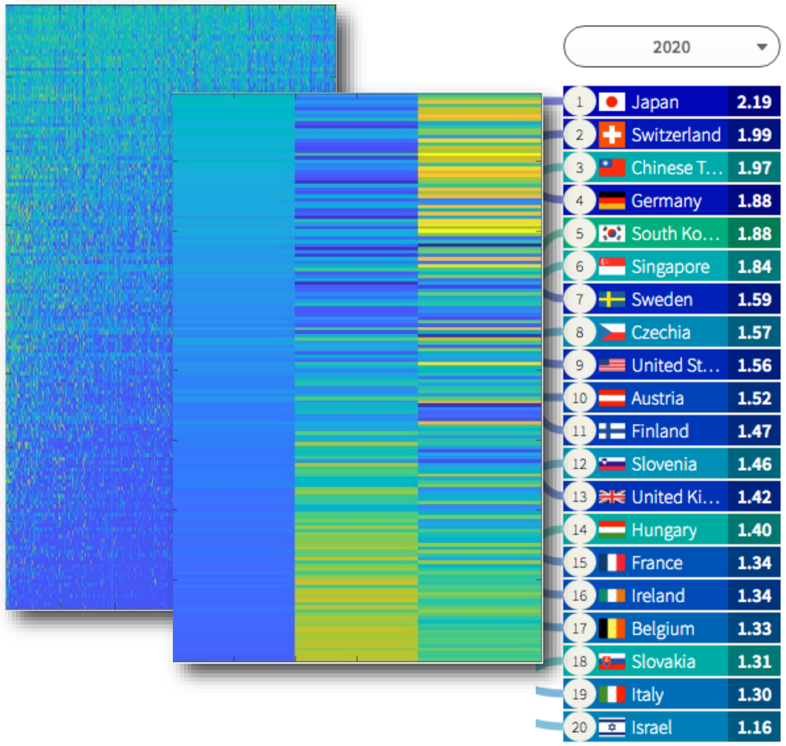


Hidalgo et al. Science (2007)

It is the idea of measuring the affinity or potential of an economy (country, region, city) in an activity, in a way that is specific to the economy and the activity.

Used extensively on work on economic geography, development economics, and innovation studies (e.g. Hidalgo et al. 2007, Neffke et al. 2011, 2013, Kogler et al 2015, Guevara et al. 2016, Boschma et al. 2013, Cicerone et al. 2020, Barbieri et al. 2020, Juhasz et al. 2020, Innocenti et al. 2019, Poncet and Waldemar 2015, etc.)

Complexity Indexes
(e.g. ECI)



Hidalgo & Hausmann. PNAS (2009)

Estimates the economic potential of the combined presence of multiple capabilities explaining variations in future economic growth (Balsalobre, 2019; Chávez et al., 2017; Domini, 2022; Hausmann et al., 2014; Hidalgo and Hausmann, 2009; Koch, 2021; Ourens, 2012; Poncet and de Waldemar, 2013; Stojkoski et al., 2023b, 2016; Vallim and Monasterio, 2023; Weber et al., 2021), inequality (Bandeira Morais et al., 2018; Ben Saâd and Assoumou-Ella, 2019; Chu and Hoang, 2020; Hartmann et al., 2017; Le Caous and Huarng, 2020; Lee and Vu, 2019; Sbardella et al., 2017), and emissions (Can and Gozgor, 2017; Doğan et al., 2021; Lapatinas et al., 2019; Mealy and Teytelboym, 2020; Neagu, 2019; Romero and Gramkow, 2021), among other outcomes.

Who cares about economic complexity?


Economic Complexity is the First Mission of Malaysia's New Industrial Master Plan
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Home > Mission > Mission 1: Advance economic complexity
Mission 1: Advance economic complexity
Integrate in global
Value Chain
High Value
R&D
Design
Assembly
Logistics
SMEs
SMEs
SMEs
SMEs
Integrate in other
Machinery and Equipment (M&E)
Advanced inspection solution
Implantable devices
Minimally invasive surgical tool
The Research, Development, Commercialisation, and Innovation (RDCI) cycle plays a pivotal role in enhancing economic complexity and cultivating high-skilled talent, facilitating the introduction of innovative products and services that drive job creation and economic expansion. Central to this approach is

Economic Complexity in the recent Draghi Report to the EU

The future of European Competitiveness. “Draghi Report” 2024

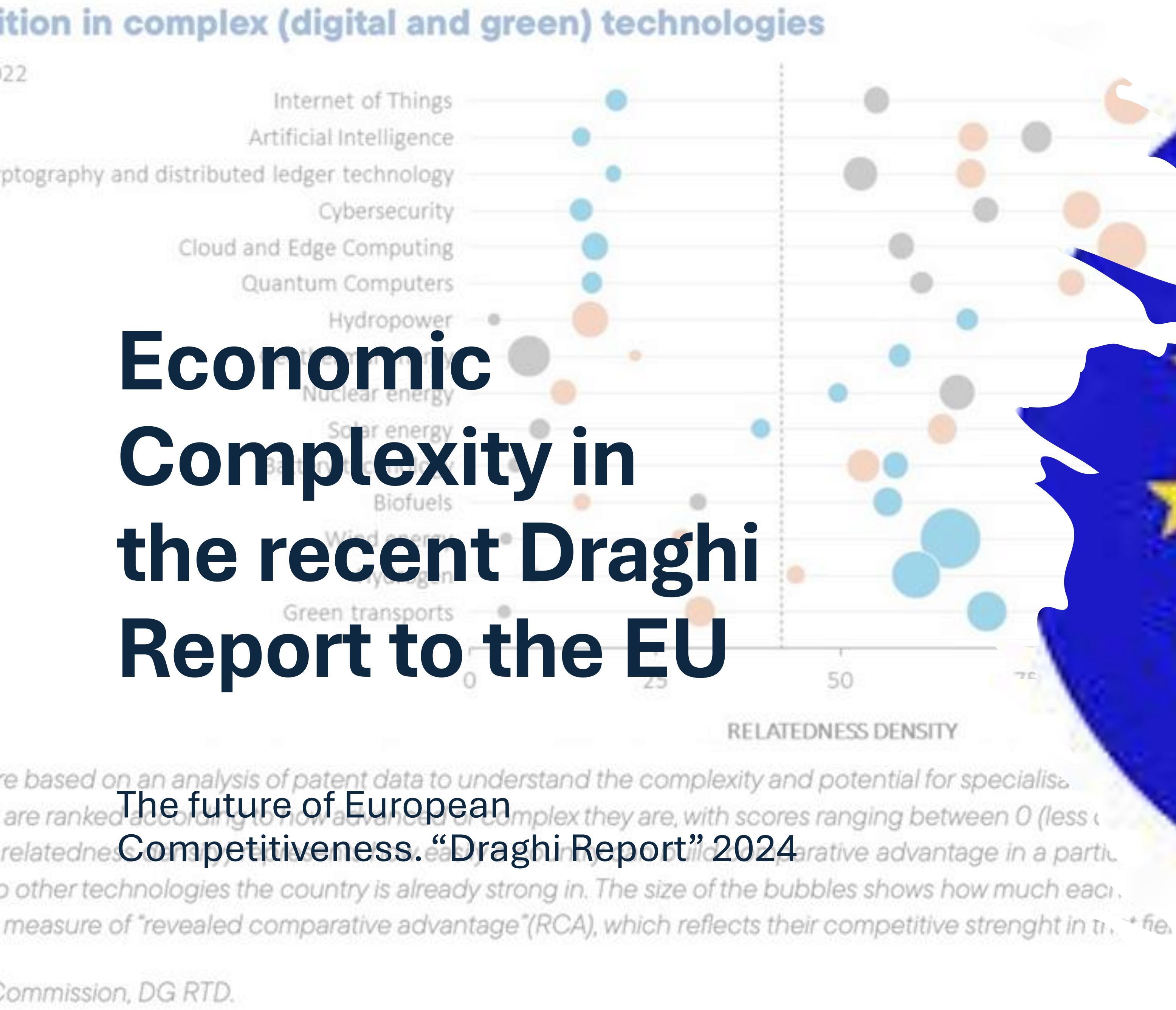
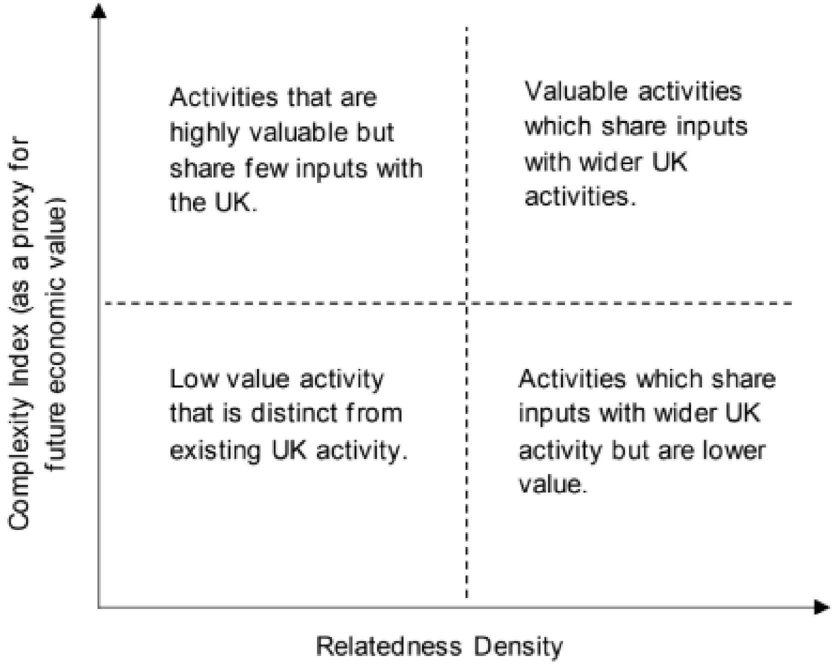


Fig 1. – Smart Specialisation Quadrant Plot



Government Office for Science

Technology and Science Insights (TSI)

TSI(24)039

Version: 20th December 2024

Where is it possible to develop UK domestic productive capability in Emerging and Disruptive Technologies?

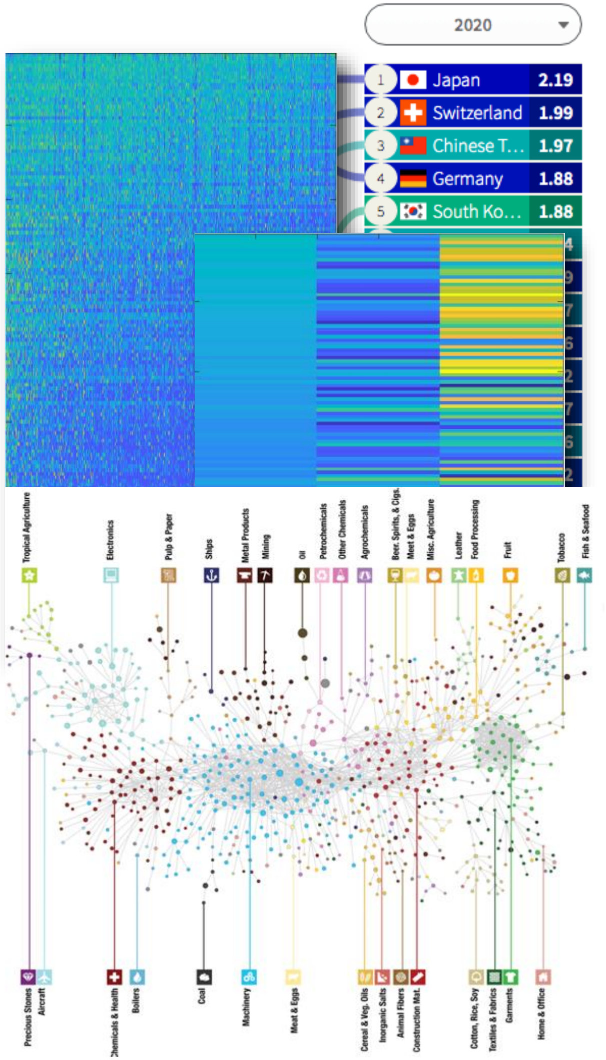
This is experimental work. There is good academic evidence that economic complexity is a significant predictor of long-term economic growth. This has been used to inform industrial policy by many national governments. This is the first time, to our knowledge, that this approach been used within HMG to compare a range of emerging technologies. This analysis has been conducted at pace to inform the Industrial Strategy and has not been subject to expert review.

Used extensively in Kazakhstan

ECI mentioned in several presidential speeches (2019, 2020, 2023, 2024) and industrial policy laws (2021, 2022).

Part of the 2029 national development plan (and of the previous 2025 development plan)

Yet…



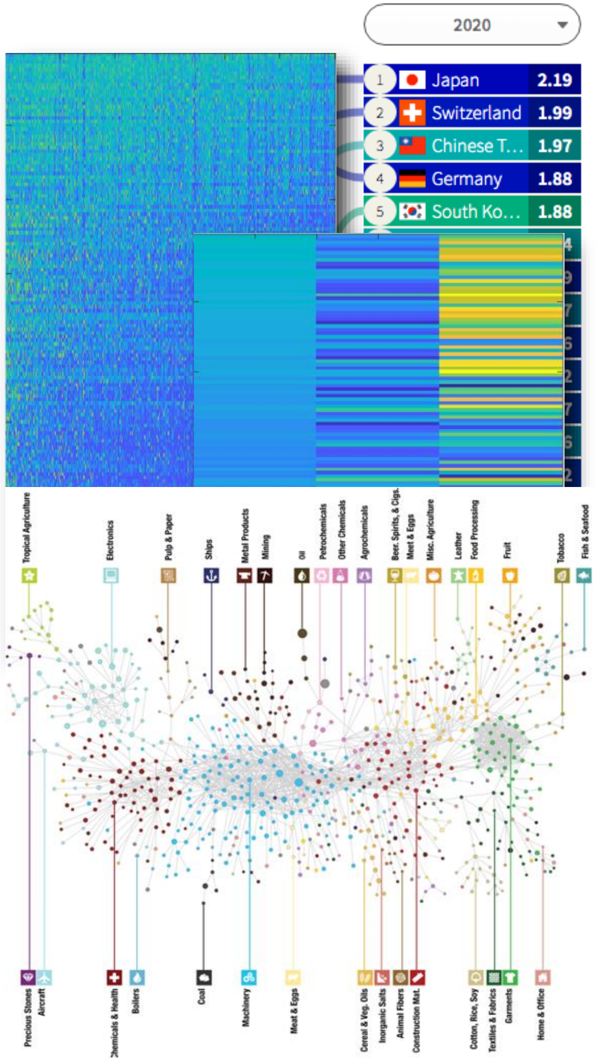
Despite their popularity as empirical methods the theoretical front still lags…

For a long-time there has been a puzzle in economic complexity involving a missing connection between empirical methods and a theoretical model. (E.g. What does ECI mean? What does the product space shape tells us about the distribution of capabilities in the world?)

The lack of that connection has led to criticisms and confusion

ECI is an ad-hoc measure
Is not a measure of “complexity”
ECI is a measure of diversity or adjusted diversity
ECI is not the right measure of economic complexity

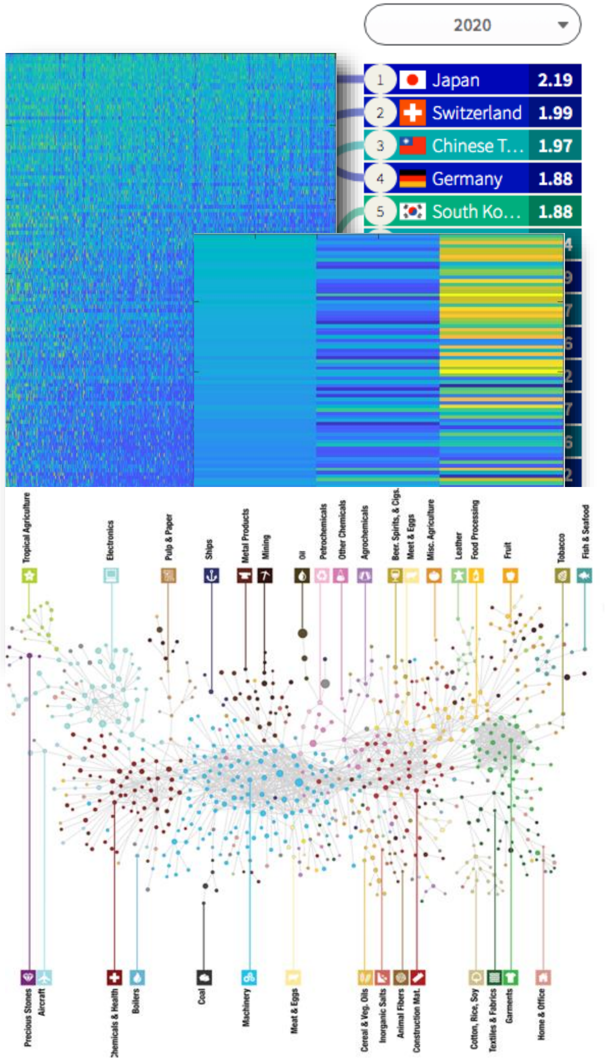
In this presentation…



We calculate the eigenvectors for a model of capabilities (a generalized version of Shockley's innovation or Kramer's O-Ring model) and show analytically that—in the one capability model—ECI separates economies between those that are above average and below average in terms of complexity.

We extend this result numerically to models involving multiple capabilities, heterogenous capabilities, noise, and other production functions and show that in a multi-capability heterogenous model ECI recovers perfectly the average probability that an economy is endowed with a capability.

In this presentation…



We show that differences in the structures observed in networks of related activities (the product space, the research space, etc.) can also be explained in the light of this model.

We expand this into a dynamic model that we can use to explore different growth mechanisms and explain the onset of growth and divergence.

Let…

Economies be endowed with capabilities or factors that are required by the activities they produce.

r_{cb} is the probability economy c has capability b

q_{cb} is the probability activity p requires capability b

For example, products are like words and countries are endowed with letter tiles in a proverbial game of scrabble.

E.g. building a software-as-a-service company requires web designers, devops engineers, backend engineers, front-end engineers, salespeople, fast internet connection, etc.

Hidalgo & Stojkoski (2025)

And let’s start simple…

In a model with **one** capability, output is given by:

$$Y_{cp} = A(1 - q_p(\underbrace{1 - r_c}))$$

Probability activity *p* requires the capability

Probability economy c lacks the capability

Output is one minus the probability that a “country” doesn’t have a capability that the “product” requires.

(and it is also the ReLU function for q=1)

Hidalgo & Stojkoski (2025)

With N_b capabilities

Output

Prob activity p needs capability b

$$Y_{cp} = A \prod_{b=1}^{N_b} (1 - q_{p,b}(1 - r_{c,b}))$$

Prob economy c doesn't have capability b

When $q=1$ for all b and $r_{cb}= r_c$ this reduces to the task models proposed by Micheal Kremer (O-Ring) (1993) and William Shockley (1957)

Hidalgo & Stojkoski (2025)

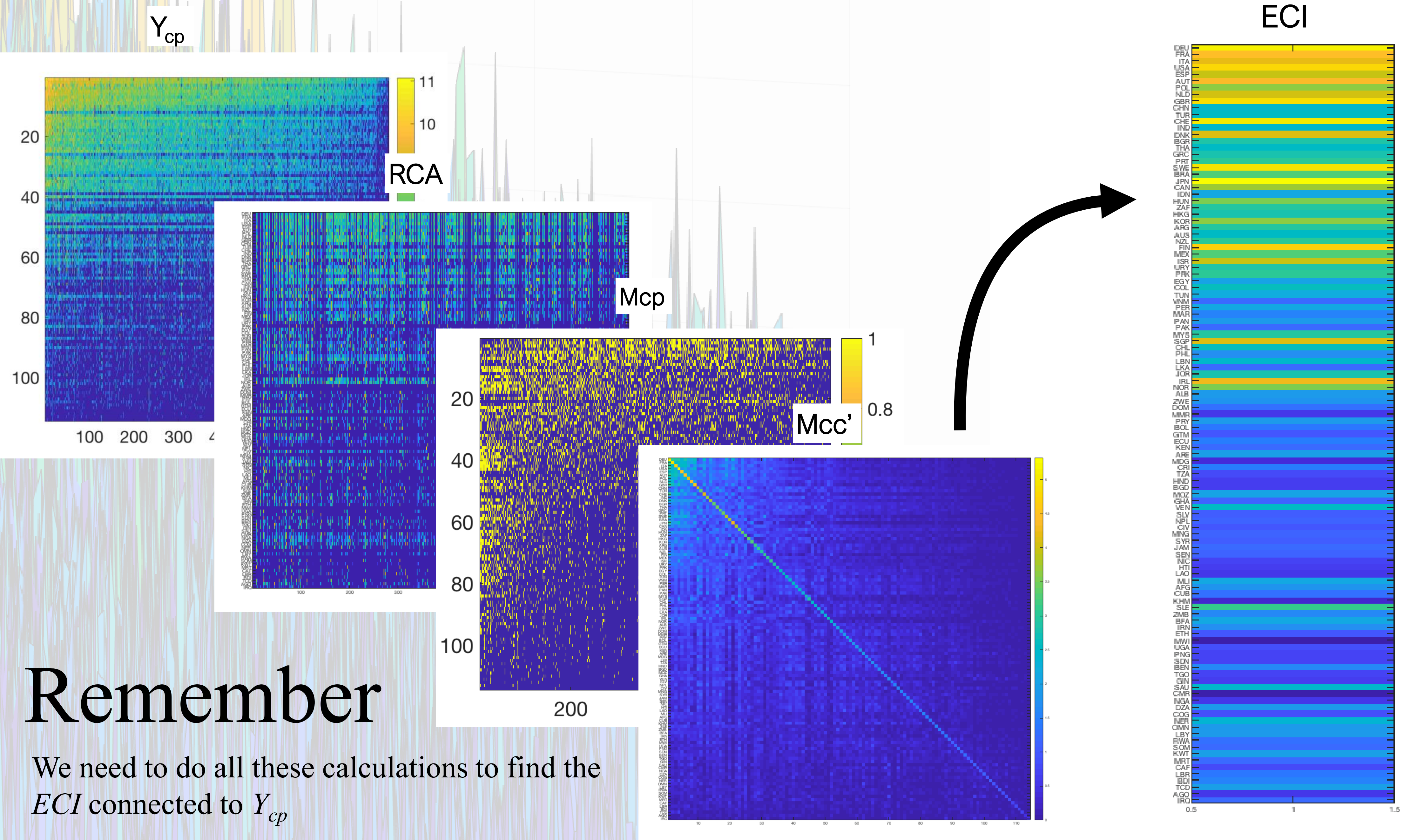
So.. in the single capability model

Output is given by the matrix:

$$Y_{cp} = A \begin{bmatrix} 1 - q_1(1 - r_1) & 1 - q_2(1 - r_1) & \dots \\ 1 - q_1(1 - r_2) & \dots & \dots \\ \dots & \dots & 1 - q_N(1 - r_N) \end{bmatrix}$$

But to estimate the economic complexity index or ECI of an output matrix we need to perform several manipulations

Hidalgo & Stojkoski (2025)



Remember

We need to do all these calculations to find the *ECI* connected to Y_{cp}

We start going from Y_{cp} to R_{cp}; the Revealed Comparative Advantage matrix

$$R_{cp} = \frac{Y_{cp} Y}{Y_c Y_p}$$

Going forward, it will be useful to notice that the sum operator applied to Y_{cp} transforms variables into averages. For example,

$$Y_p = \sum_p (1 - q_p(1 - r_c))$$

$$Y_p = N_p - (1 - r_c) \sum_p q_p$$

$$Y_p = N_p(1 - (1 - r_c)\langle q \rangle)$$

So R_{cp} is:

$$R_{cp} = \frac{(1 - q_p(1 - r_c))(1 - \langle q \rangle (1 - \langle r \rangle))}{(1 - q_p(1 - \langle r \rangle))(1 - \langle q \rangle (1 - r_c))}.$$

Now, setting $R_{cp} \geq 1$ and working through the algebra we obtain the condition for $M_{cp}=1$

$$(r_c - \langle r \rangle)(q_p - \langle q \rangle) \geq 0$$

That is, countries with above (below) average r specialize in products with above (below) average q

For example…

$$M_{cp} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Now we need to derive $M_{cc'}$

We start from the standard economic complexity formulation

$$ECI_c = \frac{1}{M_c} \sum_p M_{cp} PCI_p$$

$$PCI_p = \frac{1}{M_p} \sum_c M_{cp} ECI_c$$

And replace the second equation on the first one to obtain:

$$ECI_c = \sum_{c'} M_{cc'} ECI_{c'}$$

$$M_{cc'} = \frac{1}{M_c} \sum_p \frac{M_{cp} M_{c'p}}{M_p}$$

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First eigenvector of $M_{cc'}$

$$M_{cc'}\mathbf{1} = \sum_{c'} \frac{1}{M_c} \sum_p \frac{M_{cp} M_{c'p}}{M_p}$$

$$M_{cc'}\mathbf{1} = \frac{1}{M_c} \sum_p \frac{M_{cp} M_p}{M_p}$$

$$M_{cc'}\mathbf{1} = \frac{1}{M_c} \sum_p M_{cp} = \mathbf{1} \longrightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

This is well known, since Mcc' is a stochastic matrix (meaning it always has a leading eigenvector of *1s*)

For the second eigenvector there are three key cases

- The number of economies and activities is even (e.g. 100 economies 200 activities)

- The number of economies is odd, and the number activities is even (e.g. 101 economies 200 activities)

- The number of economies and of activities is odd (e.g. 101 economies 201 activities)

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In the first case

(even,even)

$$M_{cc'} = \frac{1}{M_c} \sum_p \frac{M_{cp} M_{c'p}}{M_p}$$

$$M_{cc'} = \frac{1}{M_p} \quad \text{if} \quad r_c \,\&\, r_{c'} \geq \langle r \rangle \quad \& \quad r_c \,\&\, r_{c'} < \langle r \rangle$$

$$M_{cc'} = 0 \quad \text{otherwise}$$

$$M_{cp} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \longrightarrow \quad M_{cc'} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

Hidalgo & Stojkoski (2025)

Second eigenvector of $M_{cc'}$

$$M_{cc'} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

$M_{cc'}$ is symmetric, so eigenvectors are orthogonal..
then second eigenvector is

$$e_c^2 = ECI = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$e_c^2 = ECI_c = 1 \qquad \text{if} \quad r_c \geq \langle r \rangle$$
$$e_c^2 = ECI_c = -1 \qquad \text{if} \quad r_c < \langle r \rangle$$

ECI tells us which economies are above or below average in their capability endowment

In the second case

$$M_{cp} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \longrightarrow M_{cc'} = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/6 & 1/6 & 1/3 & 1/6 & 1/6 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$M_{cc'} = \frac{1}{M_p} \quad \text{if} \quad c = c'$$

$$M_{cc'} = \frac{1}{M_p} \quad \text{if} \quad r_c \,\&\, r_{c'} > \langle r \rangle \quad \& \quad r_c \,\&\, r_{c'} < \langle r \rangle$$

$$M_{cc'} = \frac{1}{M_c} \sum_p \frac{M_{cp} M_{c'p}}{M_p} \quad \text{if} \quad r_c \,\&\, r_{c'} = \langle r \rangle \quad \& \quad c \neq c'$$

$$M_{cc'} = 0 \quad \text{otherwise}$$

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Second eigenvector of $M_{cc'}$

$$M_{cc'} = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/6 & 1/6 & 1/3 & 1/6 & 1/6 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \end{bmatrix} \qquad e_c^2 = ECI_c = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}$$

$$\begin{aligned} e_c^2 &= ECI_c = 1 && \text{if} \quad r_c > \langle r \rangle \\ e_c^2 &= ECI_c = -1 && \text{if} \quad r_c < \langle r \rangle \\ e_c^2 &= ECI_c = 0 && \text{if} \quad r_c = \langle r \rangle \end{aligned}$$

ECI is not “fooled” by diversity!

In the third case

$$M_{cp} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \rightarrow M_{cc'} = \begin{bmatrix} 3/10 & 3/10 & 3/10 & 1/20 & 1/20 \\ 3/10 & 3/10 & 3/10 & 1/20 & 1/20 \\ 6/35 & 6/35 & 11/35 & 6/35 & 6/35 \\ 1/20 & 1/20 & 3/10 & 3/10 & 3/10 \\ 1/20 & 1/20 & 3/10 & 3/10 & 3/10 \end{bmatrix}$$

$$M_{cc'} = \frac{1}{M_c}(1 + \frac{1}{N_p}) \quad \text{if} \quad r_c \,\&\, r_{c'} > \langle r \rangle \quad \text{or} \quad r_c \,\&\, r_{c'} < \langle r \rangle$$

$$M_{cc'} = \frac{1}{M_c}(\frac{1}{N_p}) \quad \text{if} \quad r_c > \langle r \rangle \quad \& \quad r_{c'} < \langle r \rangle \text{ and vice versa}$$

$$M_{cc'} = \frac{1}{N_c}(\frac{1}{N_p} + \frac{N_c - 1}{M_p}) \quad \text{if} \quad c = c' \quad \& \quad r_c = \langle r \rangle$$

$$M_{cc'} = \frac{1}{N_c}(1 + \frac{1}{N_p}) \quad \text{if} \quad c \neq c' \quad \& \quad \text{if} \quad r_c = \langle r \rangle$$

$$M_{cc'} = \frac{1}{M_c}(1 + \frac{1}{N_p}) \quad \text{if} \quad r_{c'} = \langle r \rangle$$

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Second eigenvector of $M_{cc'}$

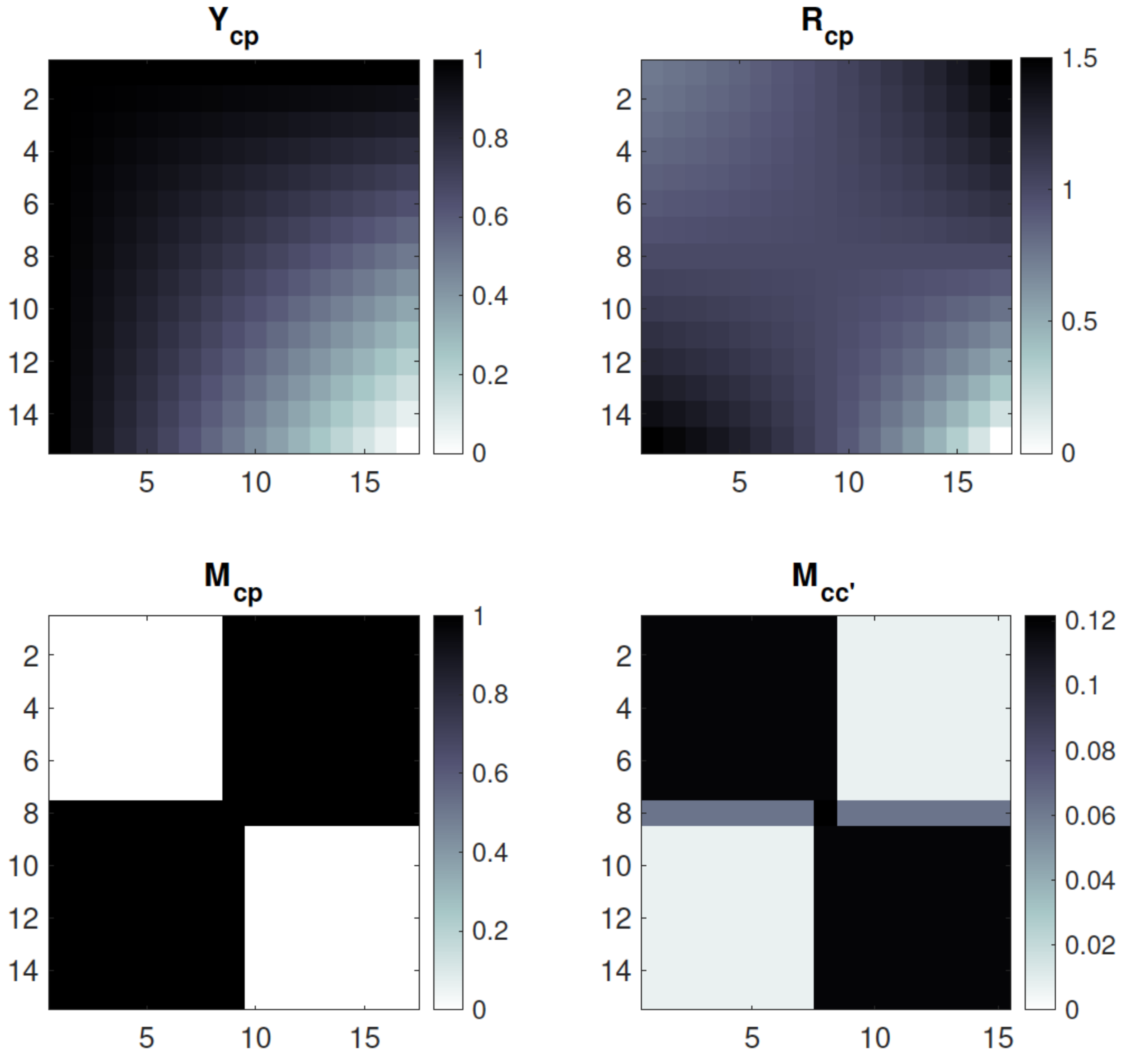
$$M_{cc'} = \begin{bmatrix} 3/10 & 3/10 & 3/10 & 1/20 & 1/20 \\ 3/10 & 3/10 & 3/10 & 1/20 & 1/20 \\ 6/35 & 6/35 & 11/35 & 6/35 & 6/35 \\ 1/20 & 1/20 & 3/10 & 3/10 & 3/10 \\ 1/20 & 1/20 & 3/10 & 3/10 & 3/10 \end{bmatrix}$$

$$e_c^2 = ECI_c = \begin{bmatrix} a \\ a \\ 0 \\ -a \\ -a \end{bmatrix}$$

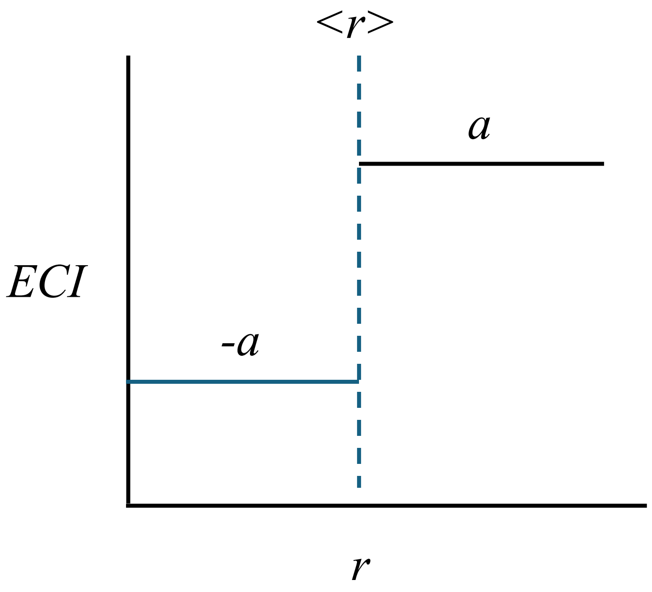
$$\begin{aligned} e_c^2 &= ECI_c = a && \text{if} \quad r_c > \langle r \rangle \\ e_c^2 &= ECI_c = -a && \text{if} \quad r_c < \langle r \rangle \\ e_c^2 &= ECI_c = 0 && \text{if} \quad r_c = \langle r \rangle \end{aligned}$$

Again, ECI is not “fooled” by diversity!

The single capability model



$$e_c^2 = ECI_c = a \quad \text{if} \quad r_c > \langle r \rangle$$
$$e_c^2 = ECI_c = -a \quad \text{if} \quad r_c < \langle r \rangle$$
$$e_c^2 = ECI_c = 0 \quad \text{if} \quad r_c = \langle r \rangle$$



Hidalgo & Stojkoski (2025)

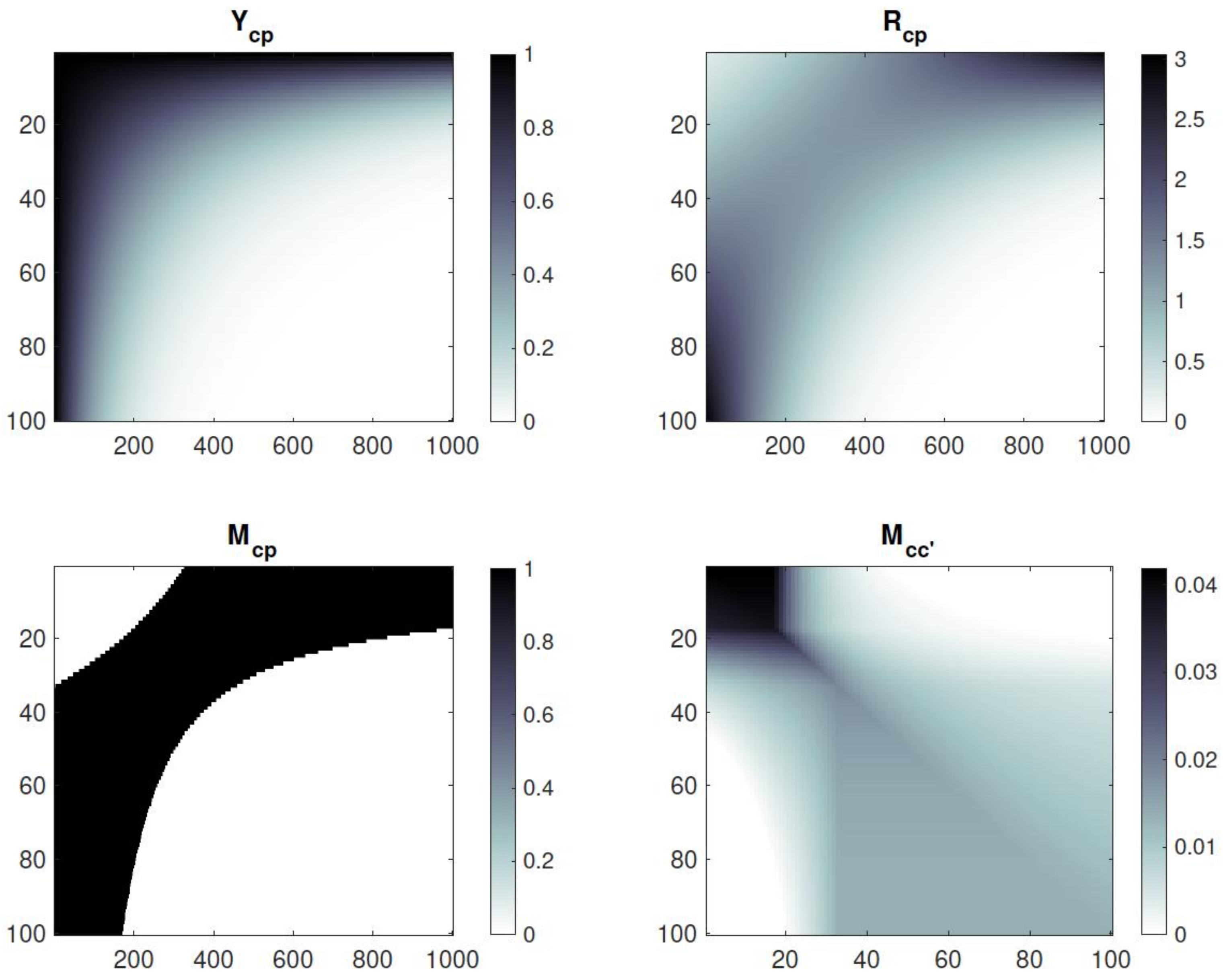
Let's go back to the Multi-Capability Model…

$$Y_{cp} = A \prod_{b=1}^{N_b} (1 - q_{p,b}(1 - r_{c,b}))$$

And consider first constant probabilities across capabilities…

$$Y_{cp} = (1 - q_p(1 - r_c))^{N_b}$$

Hidalgo & Stojkoski (2025)



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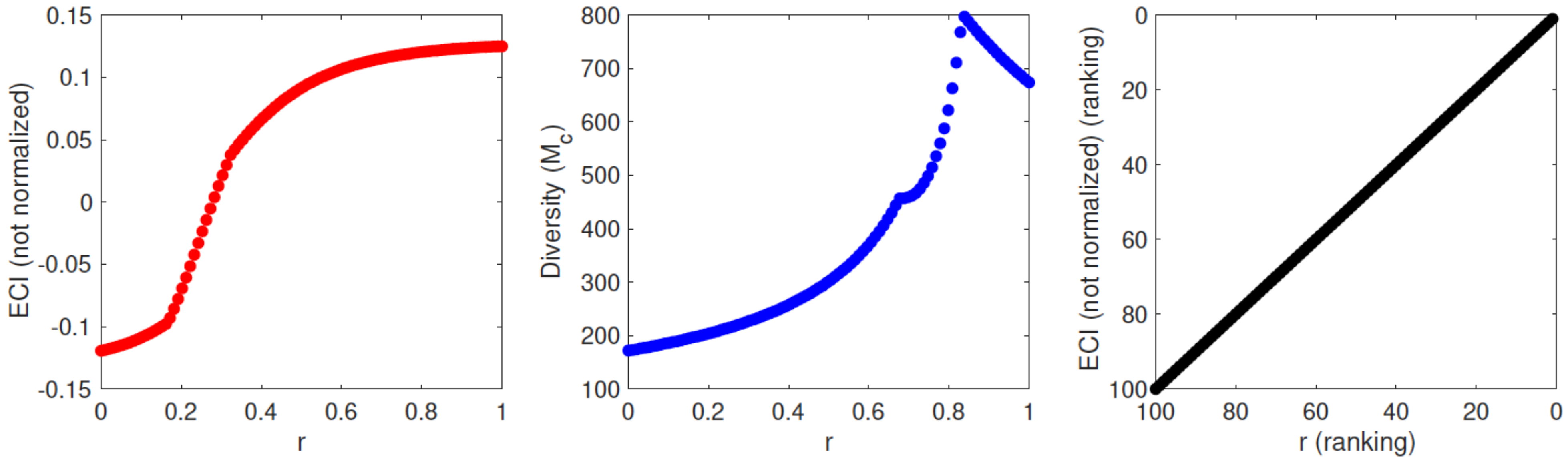
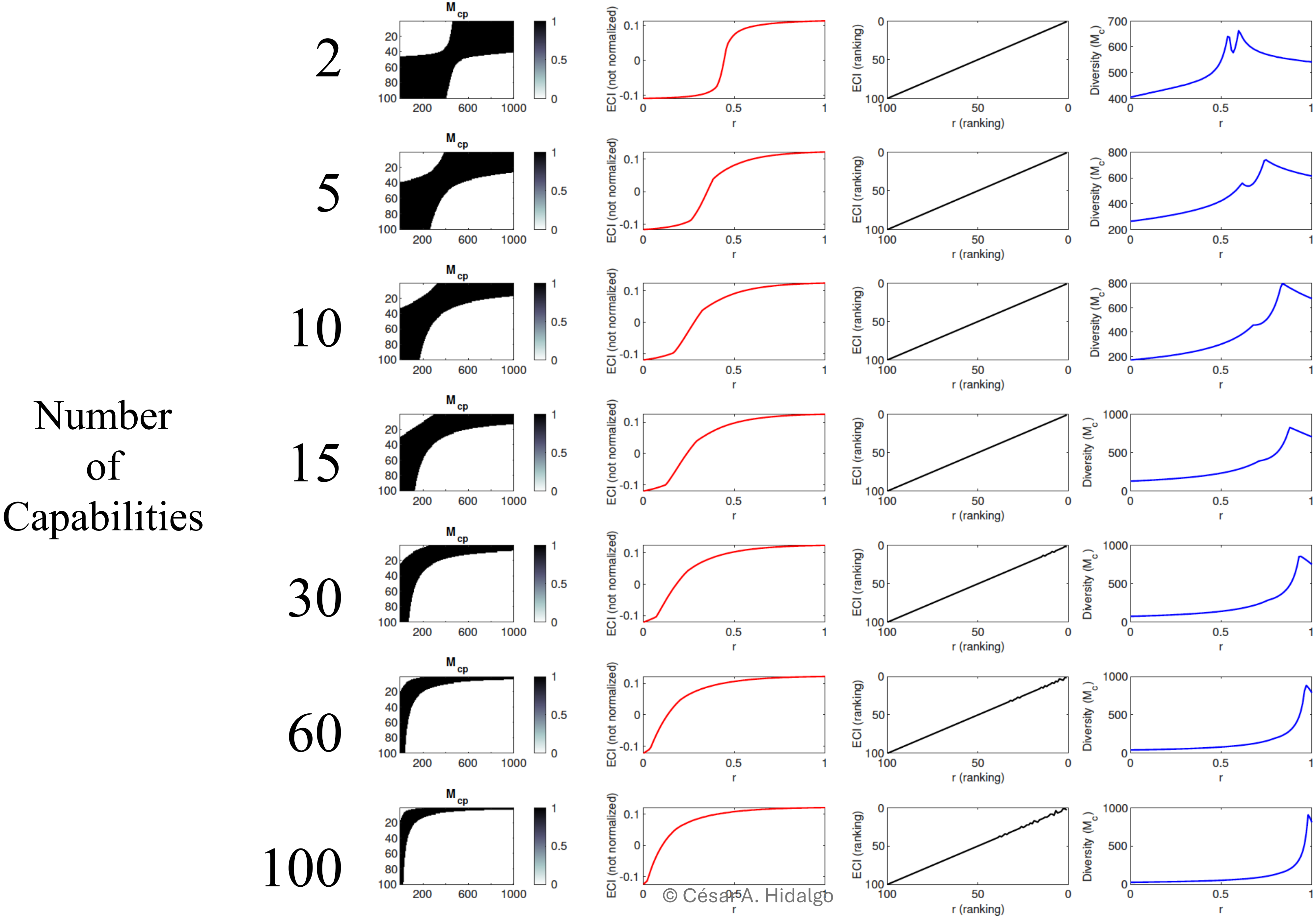


Figure 5: Comparison between the key parameters representing economies in the model (r), the second eigenvector of the $M_{cc'}$ matrix (ECI), and the diversity (M_c) of economies in the model. This example clearly illustrates that ECI is able to recover r well, especially when we compare the ranking of both variables. Diversity is not an ideal estimator of r as it peaks for economies with r lower than the maximum.

Hidalgo & Stojkoski (2025)



Hidalgo & Stojkoski (2025)

Let's go back to the full multicapability model

$$Y_{cp} = A \prod_{b=1}^{N_b} (1 - q_{p,b}(1 - r_{c,b}))$$

And consider capabilities as a mix of random and non-random capabilities (to make them heterogeneous and idiosyncratic)

$$r_{c,b} = \alpha r_c + (1 - \alpha)\,\mathrm{random}(0, 1),$$

$$q_{p,b} = \alpha q_p + (1 - \alpha)\,\mathrm{random}(0, 1).$$

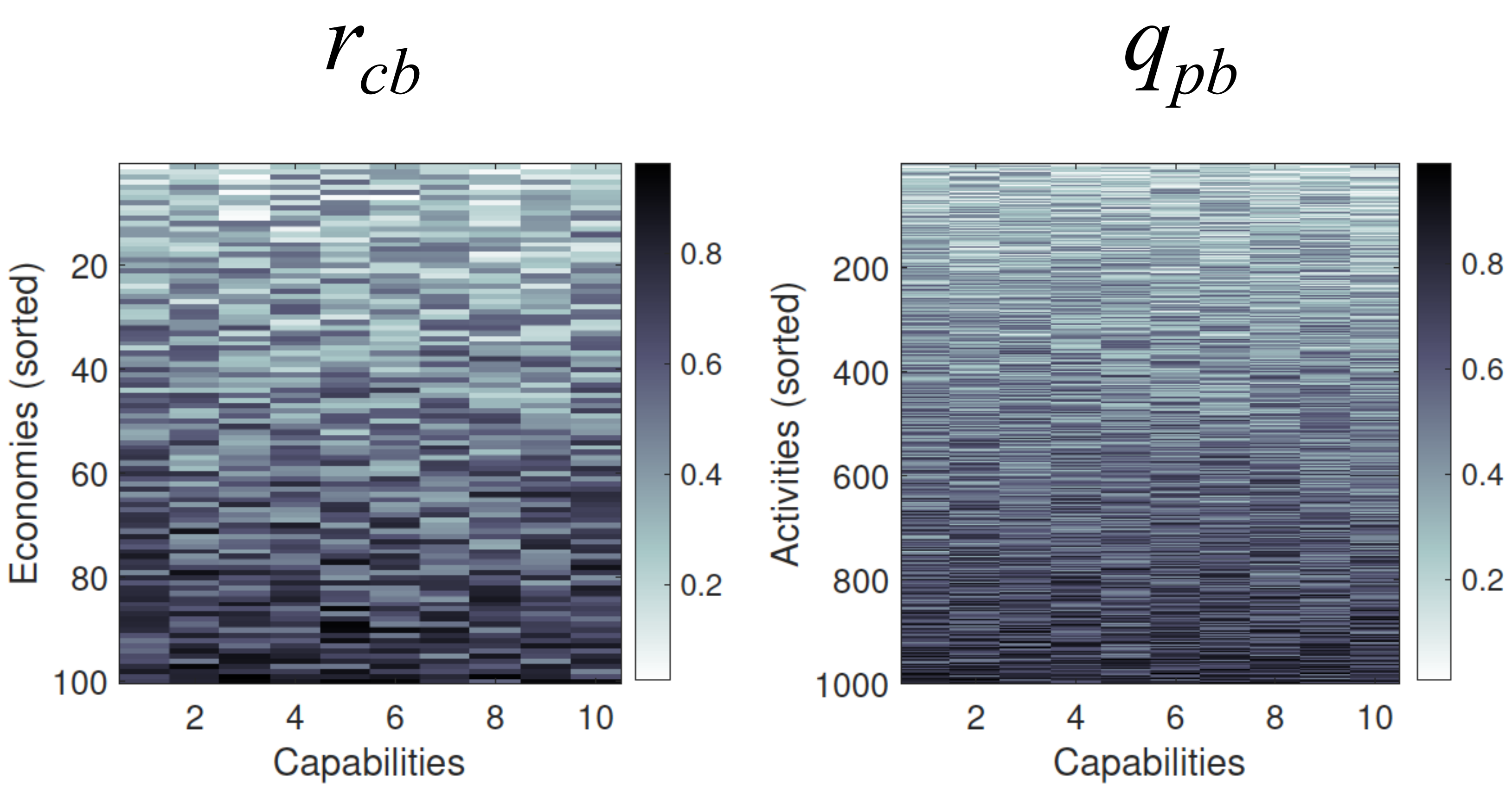
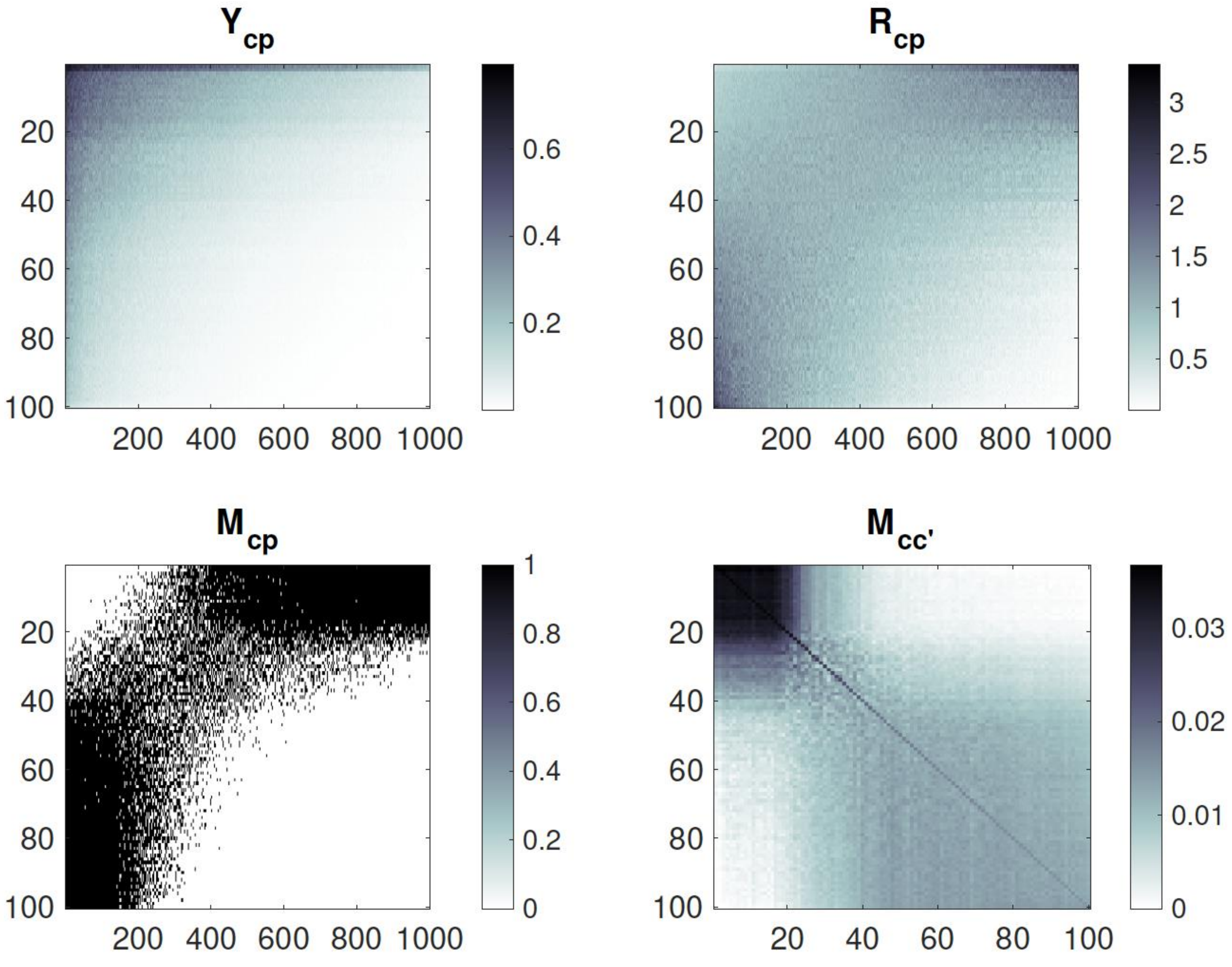
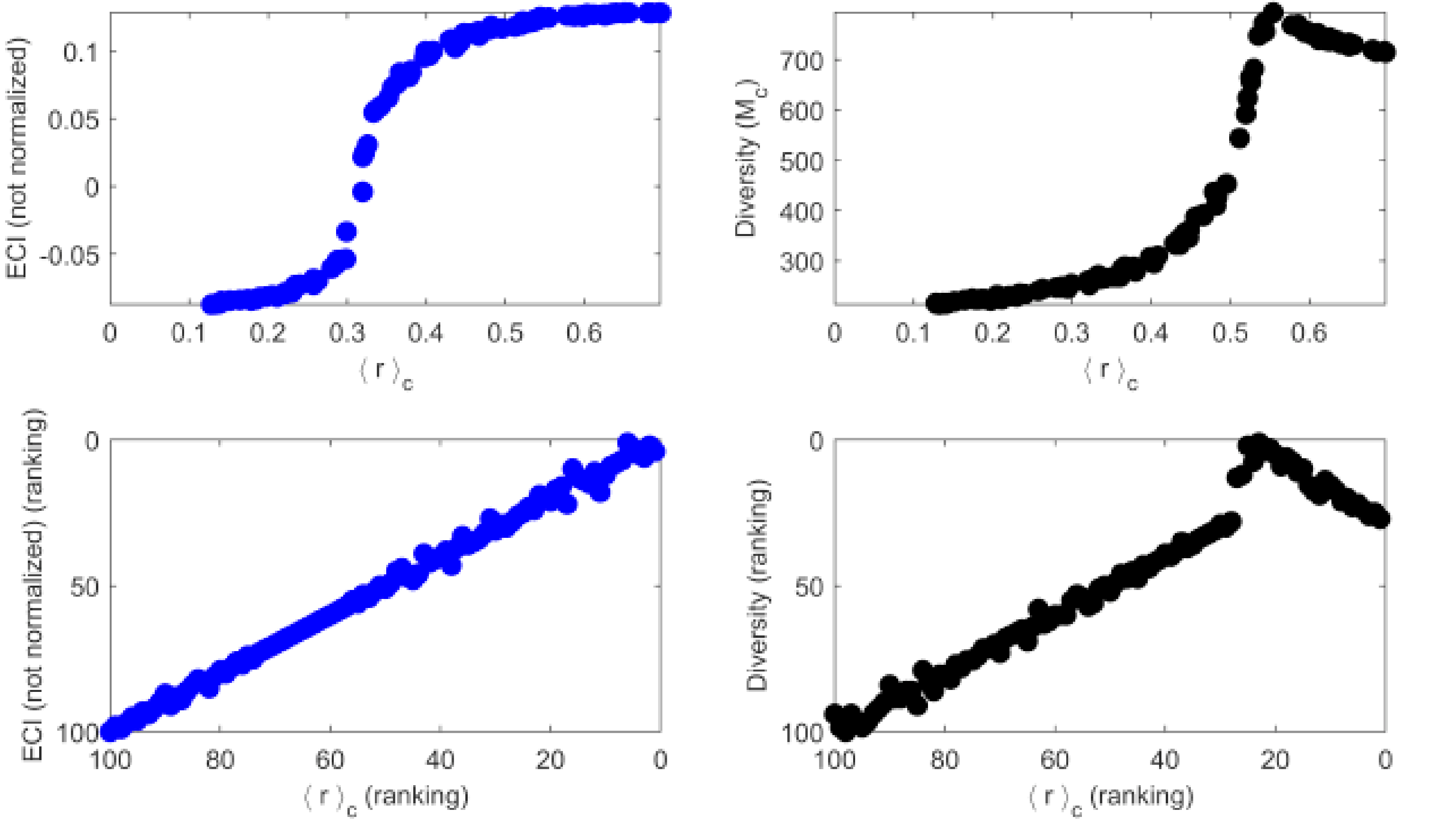


Figure 9: Parametrization of $r_{c,b}$ and $q_{p,b}$ for ten capabilities in a model where the probability that an economy is endowed with a capability, or that an activity requires it, is 1/2 of a linearly spaced baseline in the $[0, 1]$ interval and 1/2 random.

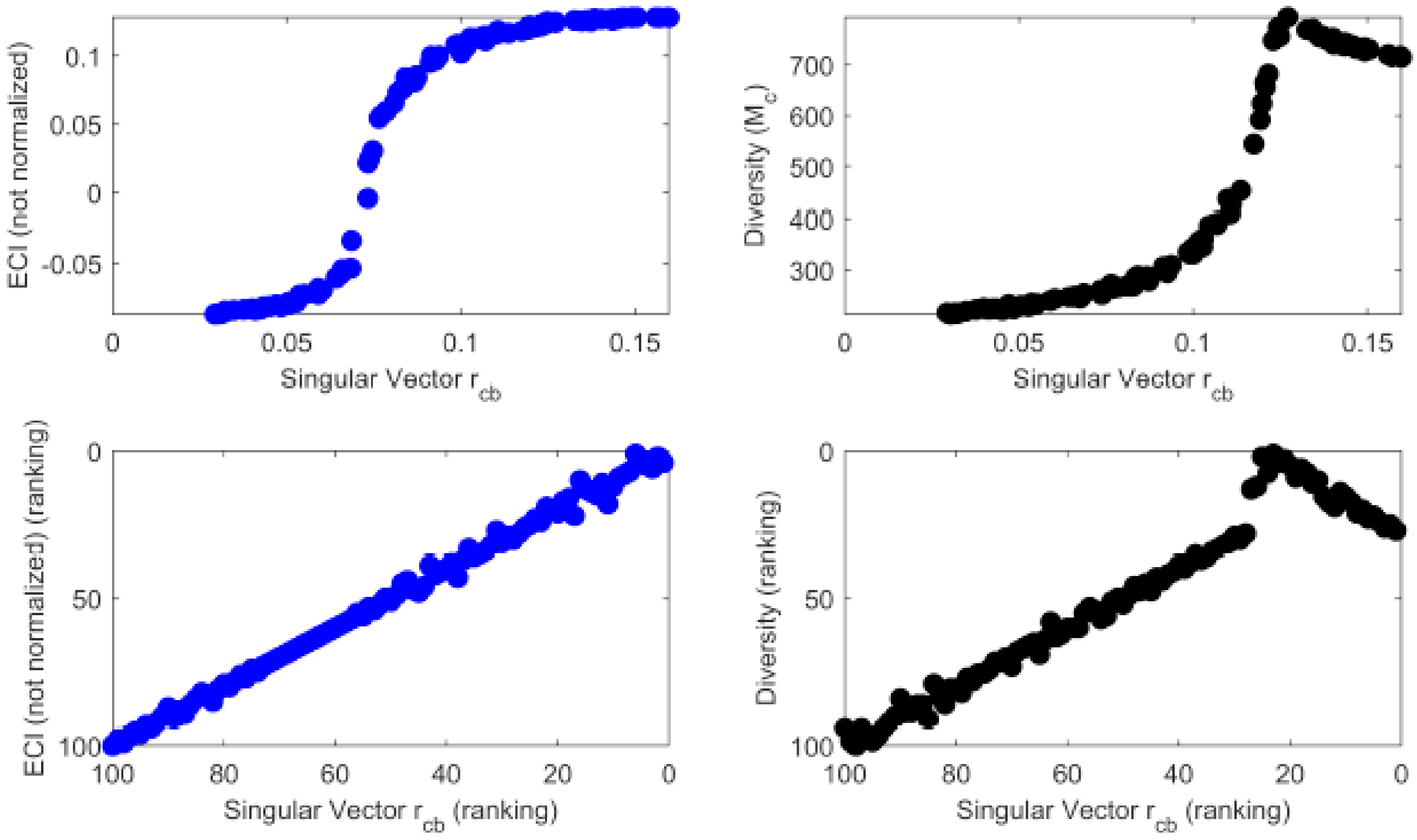


Y_{cp}
R_{cp}
M_{cp}
$M_{cc'}$
20
40
60
80
100
200
400
600
800
1000
0.6
0.4
0.2
3
2.5
2
1.5
1
0.5
0.8
0
0.03
0.02
0.01

ECI recovers average capability endowment



ECI recovers leading eigenvector of capability endowment



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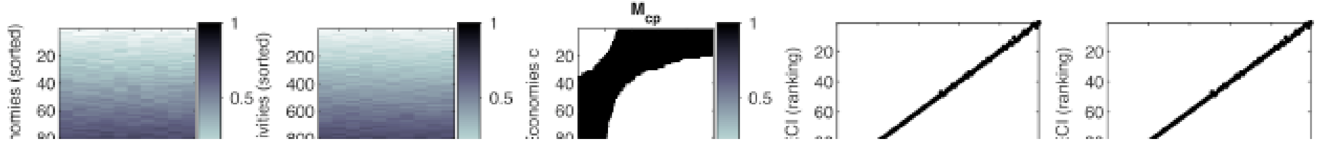
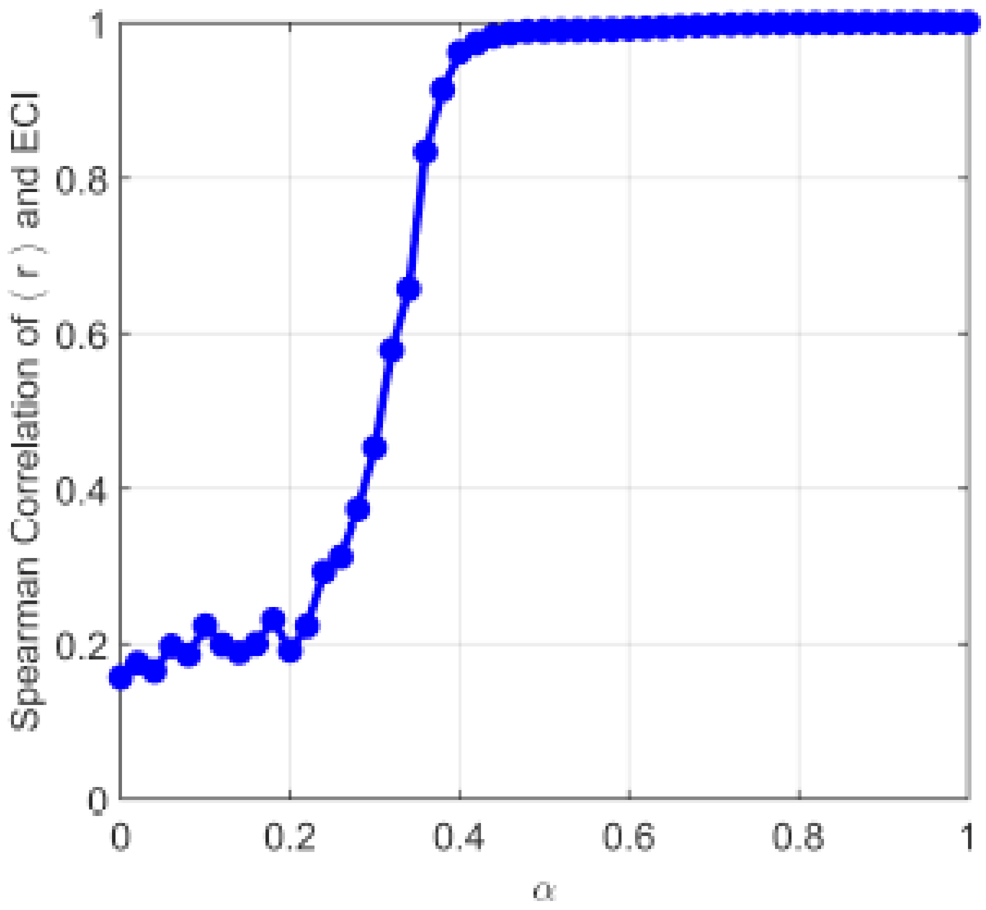
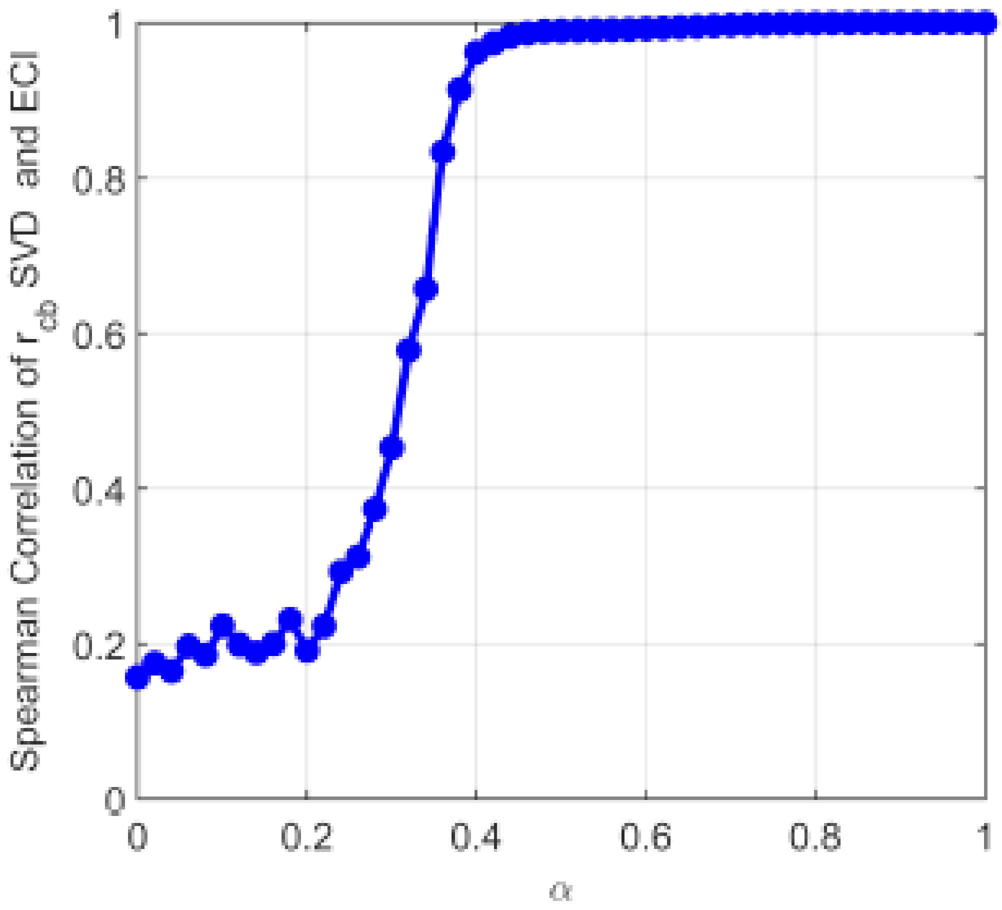
Mostly

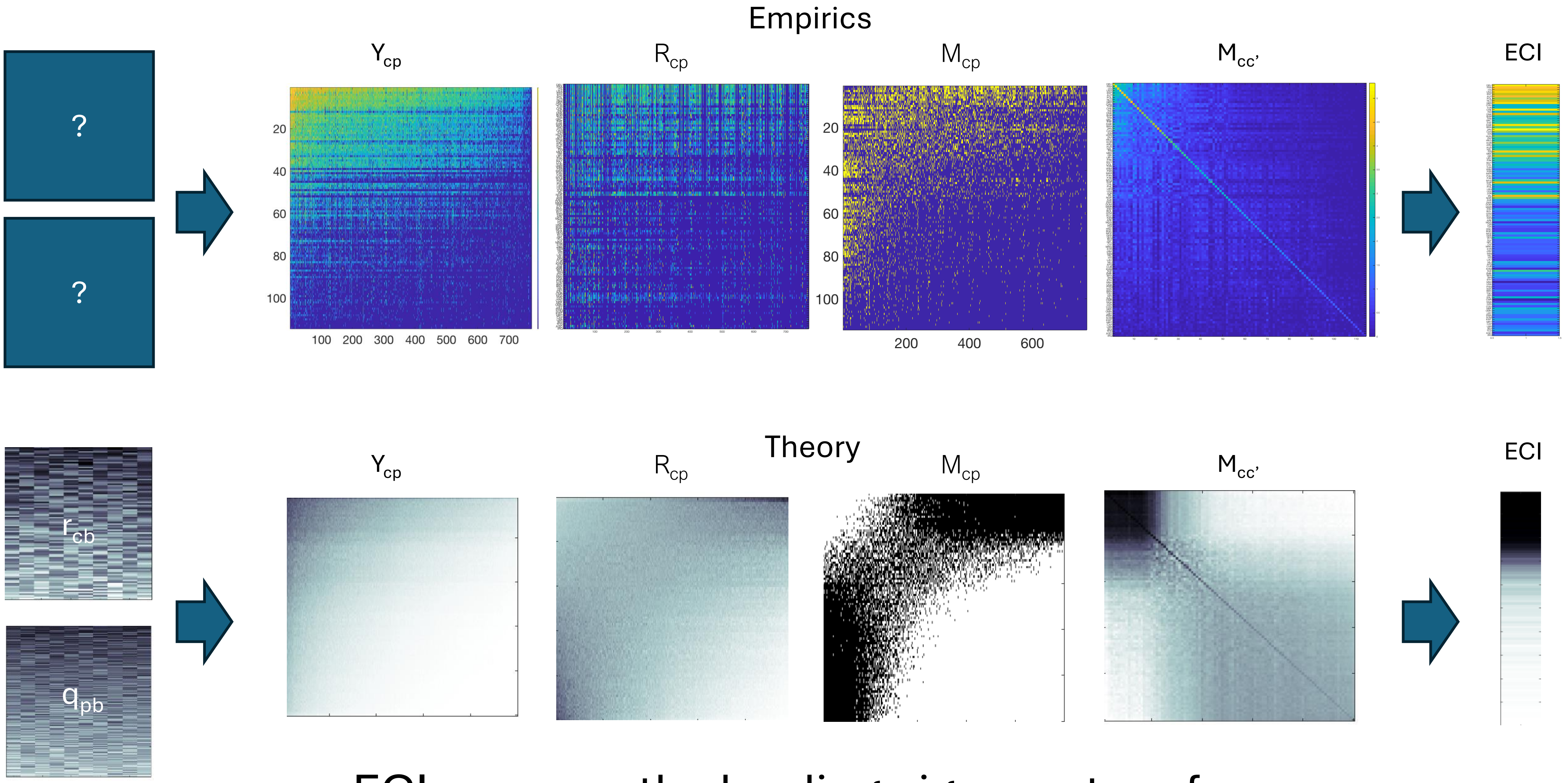






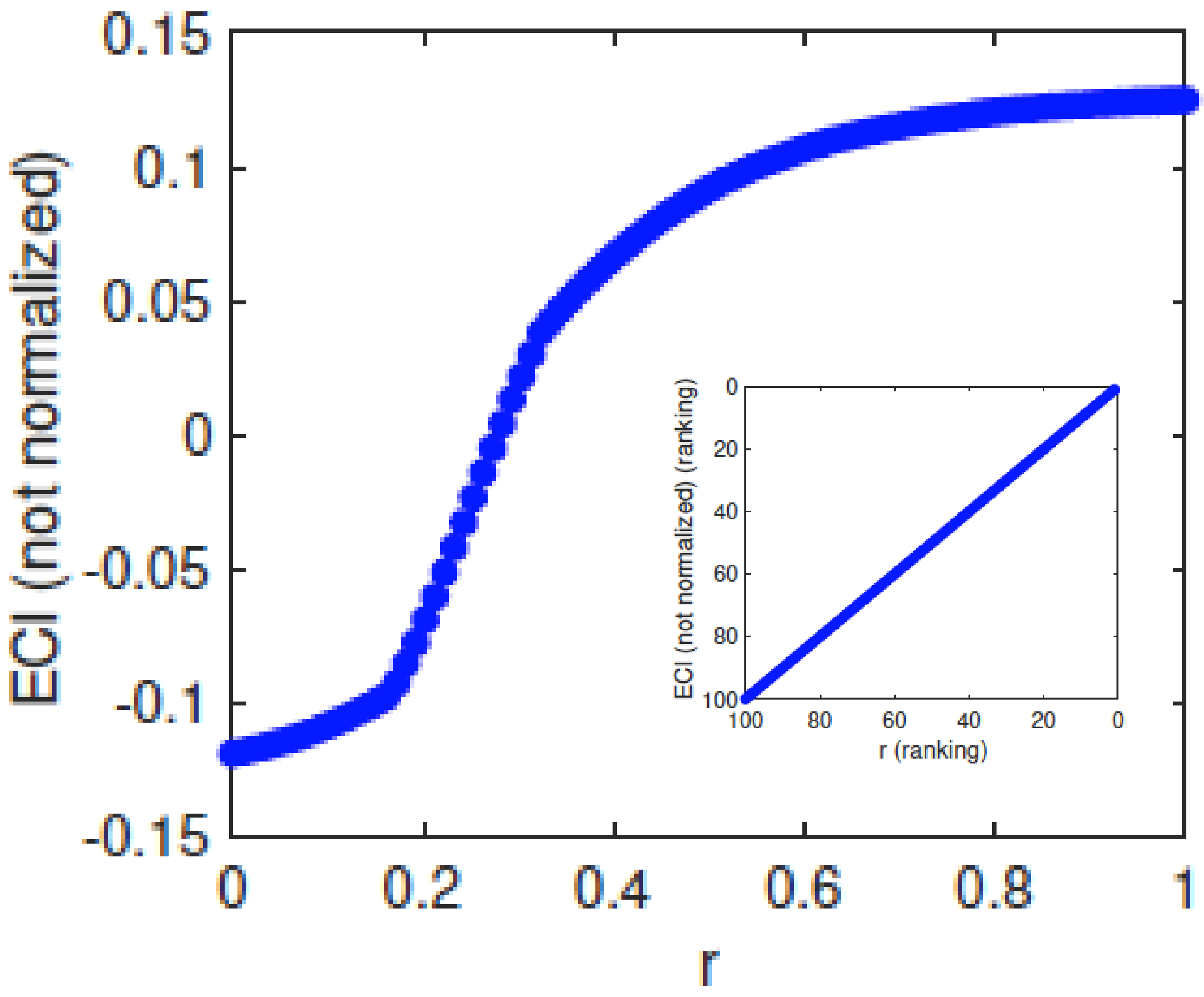
Capabilities

What are we saying?



ECI recovers the leading eigenvector of r_{cb}

For the Multi-Capability Model:



ECI is a monotonic function of the probability an economy is endowed with a capability, and it is perfectly rank correlated with that probability.

It is a spline telling us if an economy belongs to the high or the low capability cluster. This property holds even for substantial levels of noise in capability endowments (e.g. 50% random)

Hidalgo & Stojkoski (2025)

But there is more…

Consider other production functions:

$$Y_{cp} = A(K_c/K_p)^\gamma$$

In this case:

$$R_{cp} = \frac{(K_c/K_p)^\gamma \sum_c K_c^\gamma \sum_p (1/K_p)^\gamma}{(K_c/K_p)^\gamma \sum_c K_c^\gamma \sum_p (1/K_p)^\gamma} = 1.$$

Hidalgo & Stojkoski (2025)

And how about:

$$Y_{cp} = B + f_c g_p$$

In this case:

$$R_{cp} = \frac{(B + f_c g_p) \sum_{c,p} (B + f_c g_p)}{\sum_c (B + f_c g_p) \sum_p (B + f_c g_p)}$$

Leads to the condition:

$$(f_c - \langle f \rangle)(g_p - \langle g \rangle) \geq 0$$

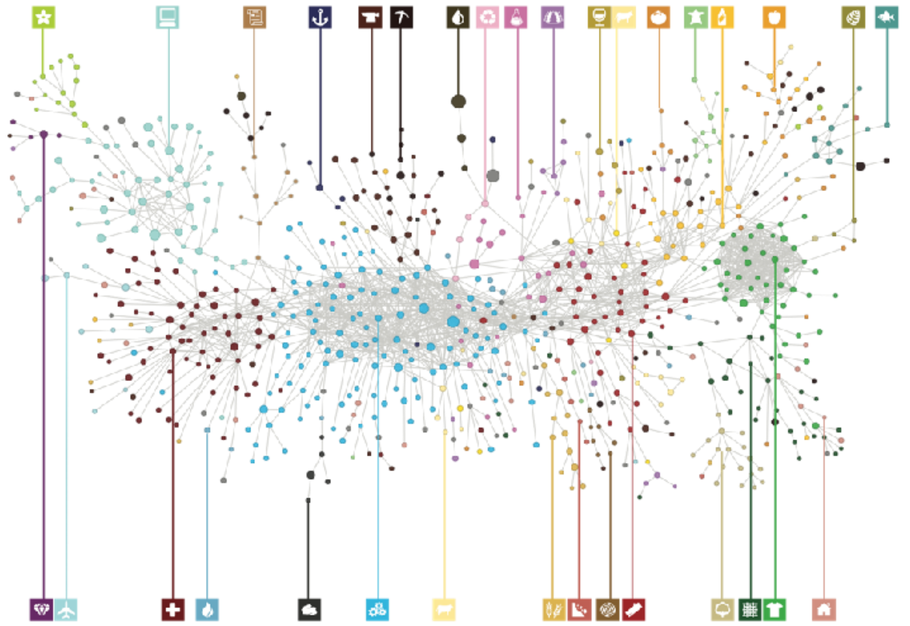
Which is the condition of the one capability model!

Hidalgo & Stojkoski (2025)

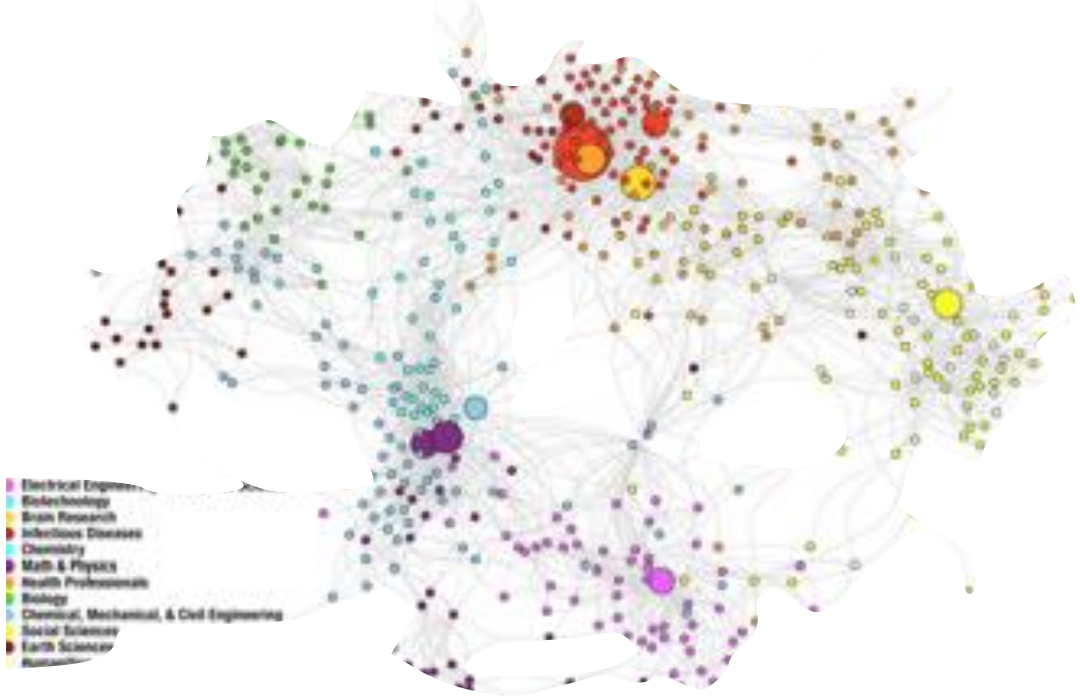
What about relatedness?

Two key observations:

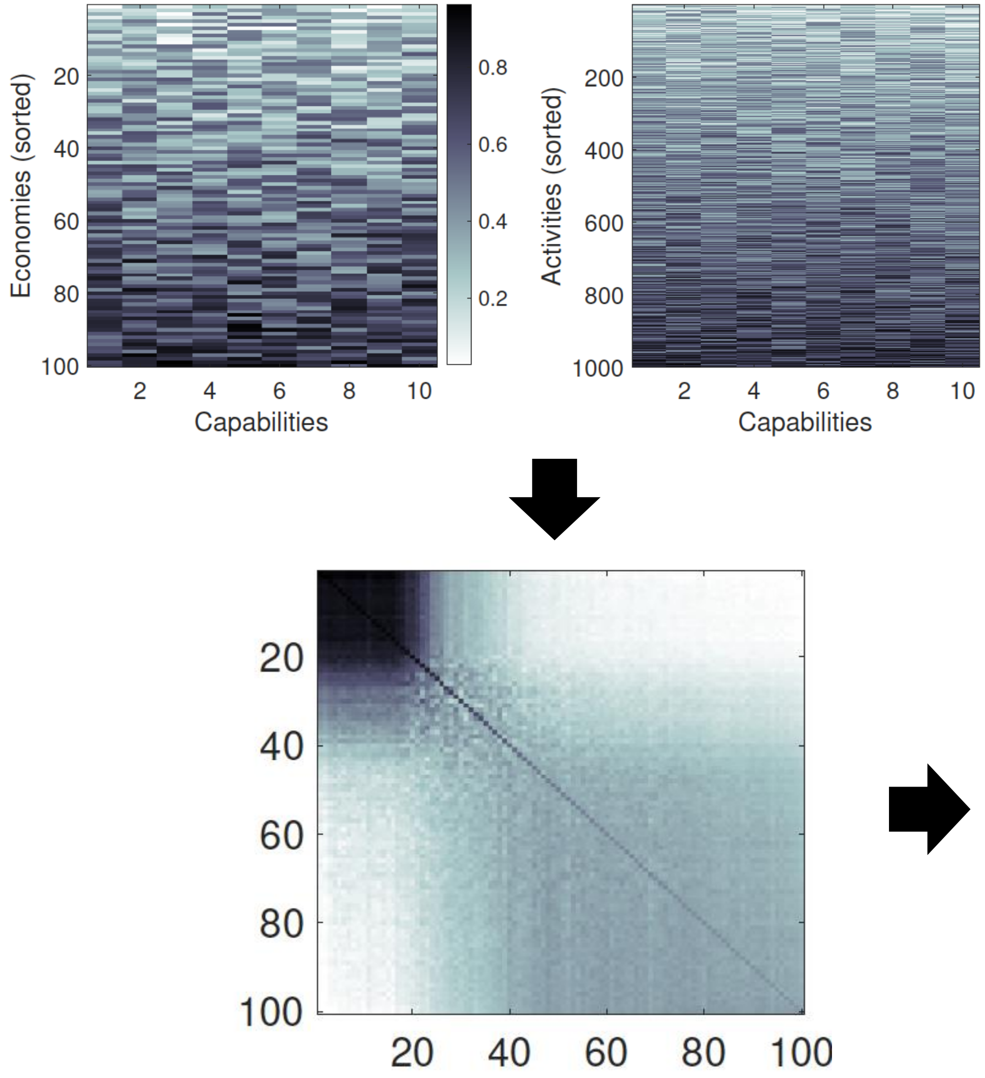
The core of the product space is populated by high complexity activities (Hidalgo et al. 2007)

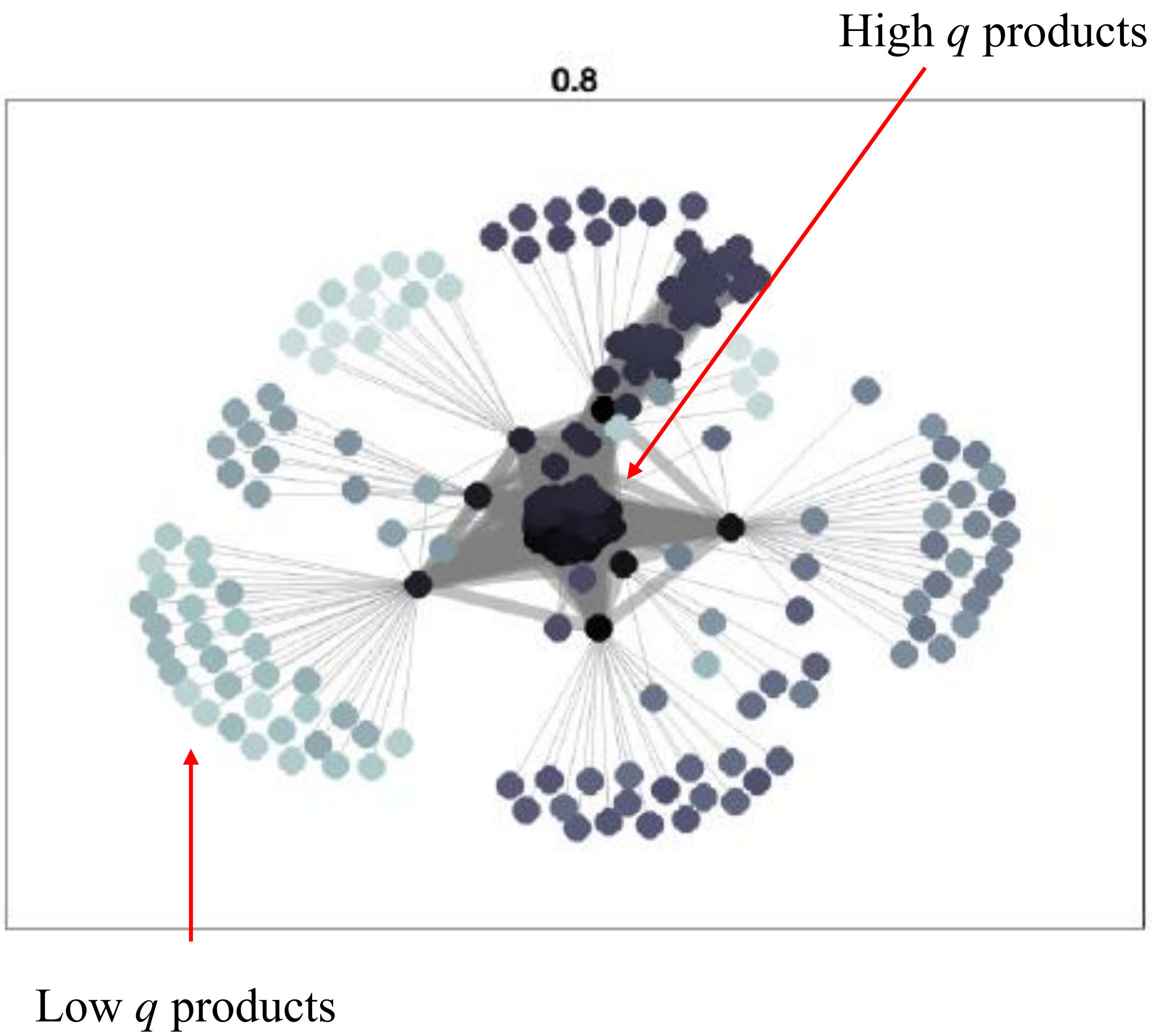


The research space (papers) or tech space (patents) is shaped like a ring, when estimated using citations among fields or patterns of co-publication (Kogler et al. 2015, Guevara et al. 2016, Borner 2012)



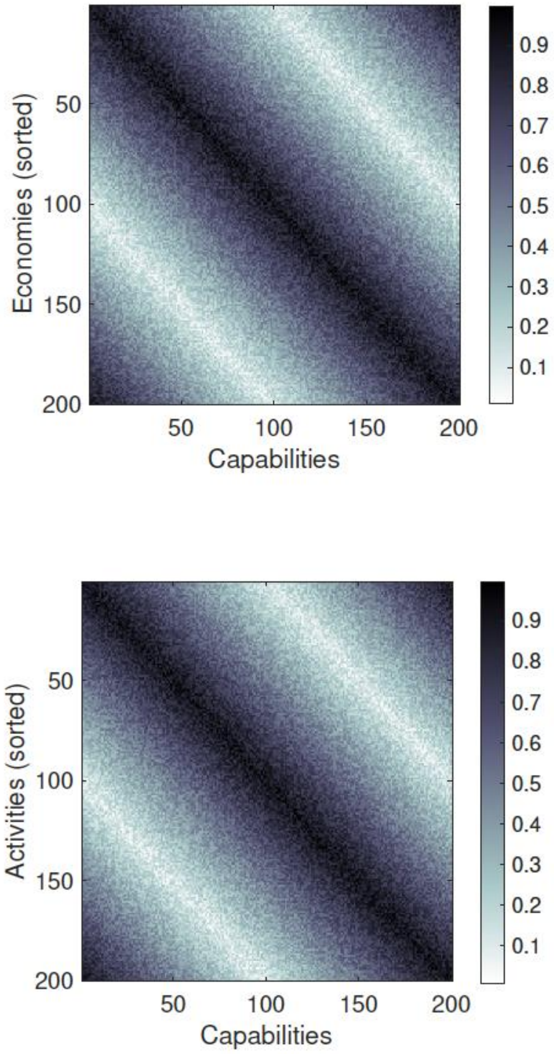
Correlated capability endowments imply a product space with a core composed of high complexity activities

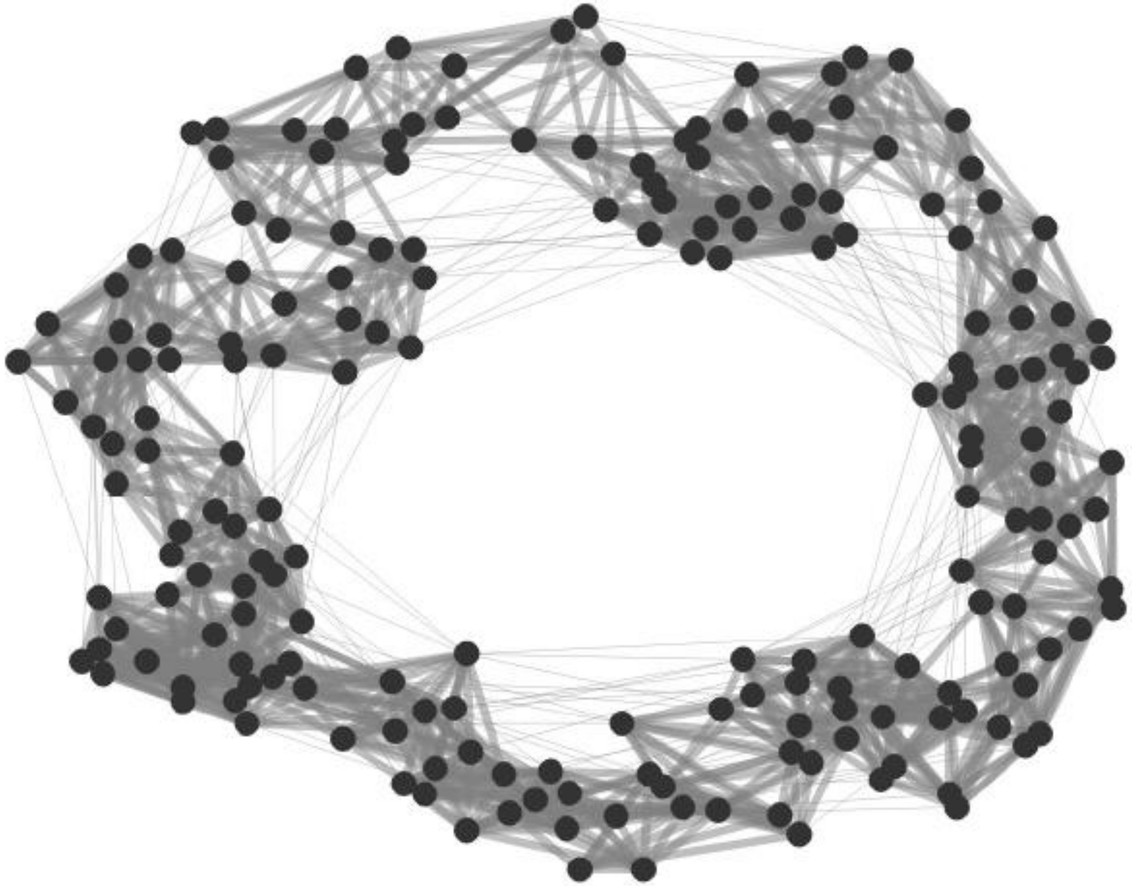




Hidalgo & Stojkoski (2025)

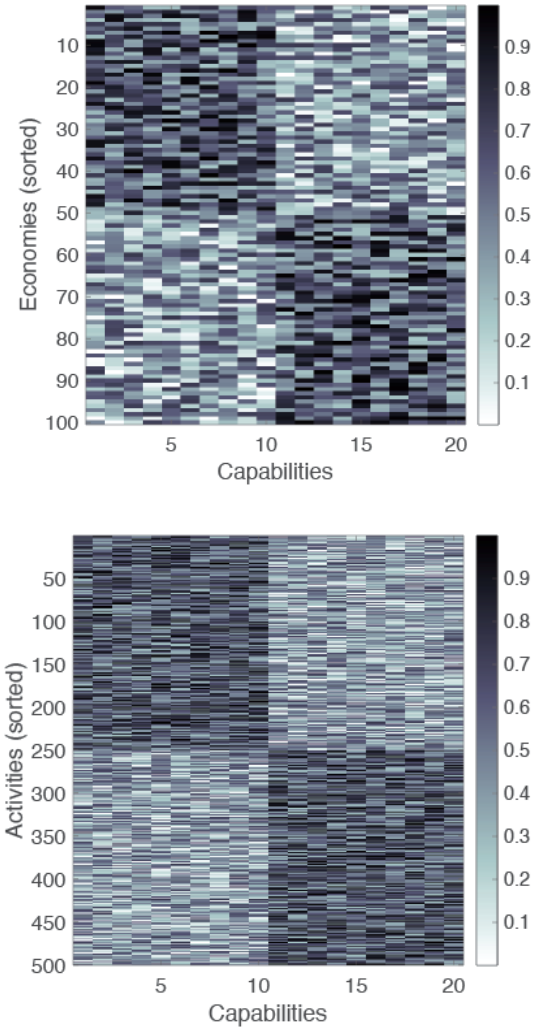
Symmetric circulant Toeplitz matrices lead to a circular research space

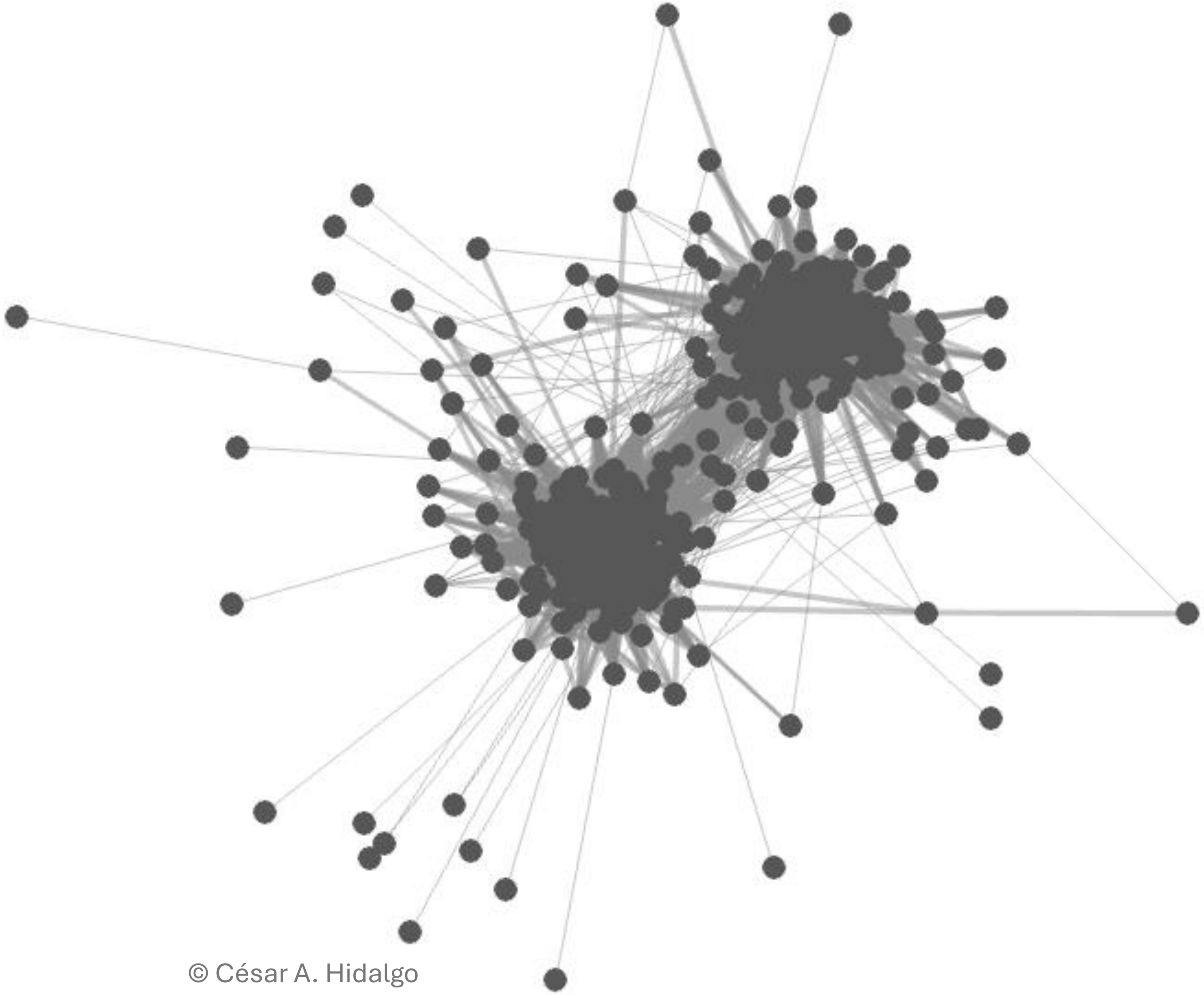




Hidalgo & Stojkoski (2025)

Dumbbell Network
(Block diagonal with substantial noise)



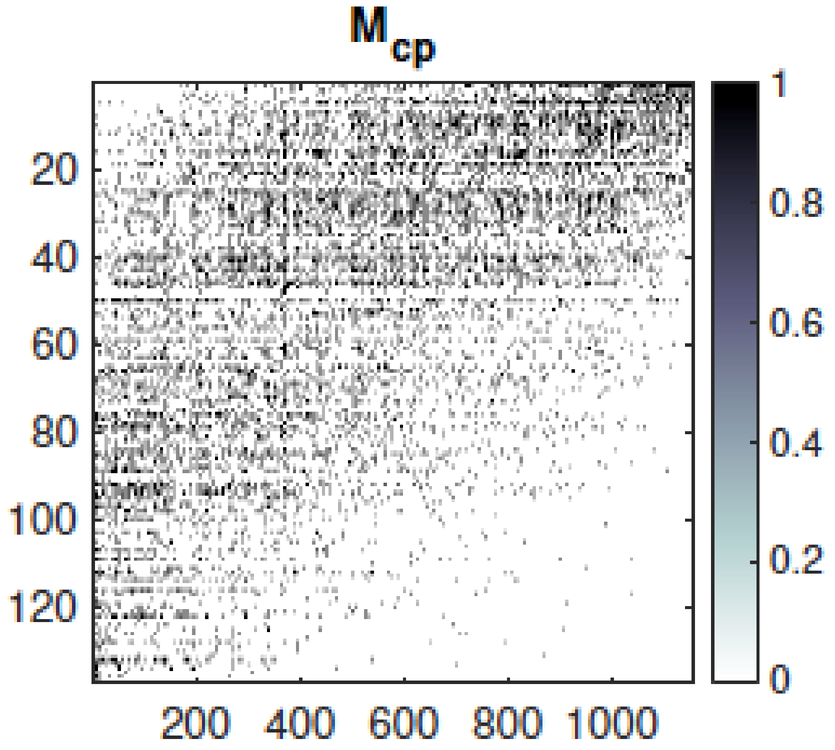


Why does this work?

Unlike previous work (my fault), assuming this is a model of the binary specialization matrix M_{cp}, here we assume it is a model of the output matrix Y_{cp}.

This makes an enormous difference, since M_{cp} is a matrix that has a clear signature of specialization (empty upper left triangle).

Jumping directly to M_{cp} with the model skips the normalization steps needed for this to work (ECI is strongly connected to RCA, since the condition comes from there)

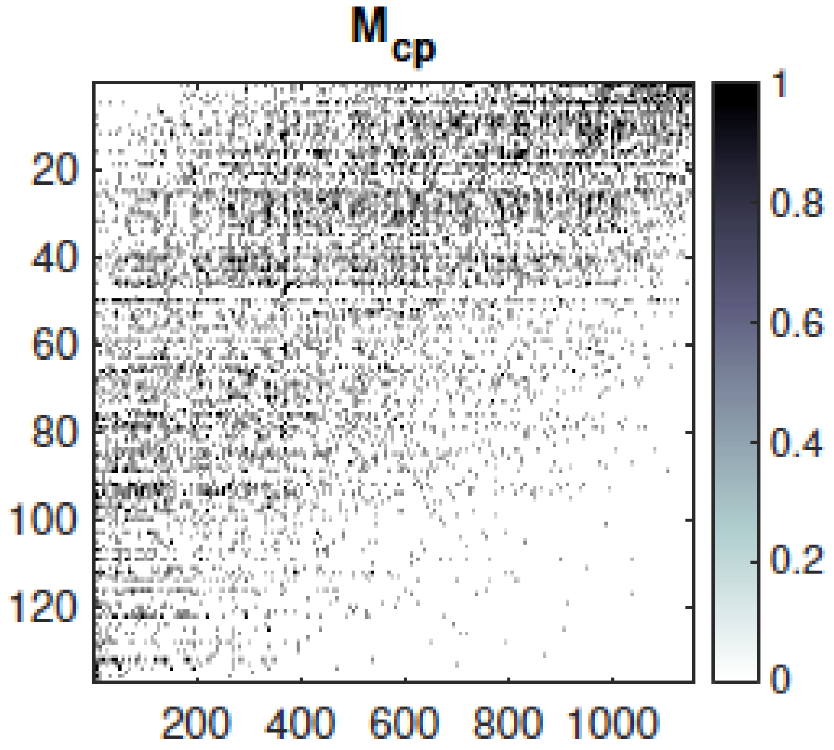


Empirical M_{cp} for Trade Data

Why does this work?

Using a capability model is useful, but not essential. What matters is to have a non-multiplicatively-separable production function $Y_{cp}=B+f_c g_p$ where output is not perfectly proportional to factors.

The condition comes from the interaction between the shifted term (B) and the factor terms.



Empirical M_{cp} for Trade Data

But what about the *dynamics*?

The Dynamical Theory of Economic Complexity

We start from the same function

$$Y_{cp} = \prod_b (1 - q_{pb}(1 - r_{cb}))$$

and differentiate

$$\frac{dY_{cp}}{dt} = \sum_b \frac{\partial Y_{cp}}{\partial r_{cb}} \frac{dr_{cb}}{dt} + \sum_b \frac{\partial Y_{cp}}{\partial q_{pb}} \frac{dq_{pb}}{dt}$$

It is useful to define a few things first

$$Y_{cp} = (1 - q_{pb}(1 - r_{cb})) \prod_{b' \neq b} (1 - q_{pb'}(1 - r_{cb'}))$$

Complementarity of other capabilities (contribution to the output of activities requiring capability *b*)

$$E_{cpb} = \prod_{b' \neq b} (1 - q_{pb'}(1 - r_{cb'}))$$

With this

$$\frac{dY_{cp}}{dt} = \sum_b E_{cpb} q_{pb} \frac{dr_{cb}}{dt} + \sum_b E_{cpb} (r_{cb} - 1) \frac{dq_{pb}}{dt}$$

Let’s stop for a second… two channels for growth

$$\frac{dY_{cp}}{dt} = \sum_b E_{cpb} q_{pb} \frac{dr_{cb}}{dt} + \sum_b E_{cpb}(r_{cb} - 1)\frac{dq_{pb}}{dt}$$

Channel 1, capability accumulation. Positive if dr_{cb}/dt is positive. Economies grow output by accumulating capabilities.

Channel 2, technological change. It is positive if dq_{pb}/dt is negative. (E.g. producing films become easier with the rise of smartphones and cheap video editing software (output increase because technology makes production less capability intensive)).

Now… to focus on the simplest case of capability accumulation, we will assume…

Capability requirements (q's) are constant and only one capability change at a time.

$$\frac{dY_{cp}}{dt} = \sum_b E_{cpb} q_{pb} \frac{dr_{cb}}{dt} + \sum_b E_{cpb} (r_{cb} - 1) \frac{dq_{pb}}{dt}$$

Then..

$$\frac{dY_{cp}}{dt} = E_{cpb} q_{pb} \frac{dr_{cb}}{dt}$$

So, we need a model for dr_{cb}/dt…

Dynamical model of capability accumulation

Temporal change of capability *b* in economy *c*

$$\frac{dr_{cb}}{dt} = (1 - r_{cb}) \sum_p i_{cp} \left(\frac{q_{pb}}{q_p} + \sum_{p'} \phi_{pp'} \frac{q_{p'b}}{q_p} \right)$$

Saturation/depreciation term
(capabilities are bounded at 1)

Investment of economy
c in activity *p*

Intensity of
use of capability *b*
in activity *p*

Intensity of
use of capability *b*
in related activities *p'*

Investment assumption

We assume each economy invests a fraction of the output generated in an activity in that activity (e.g. car manufacturers invest in car manufacturing)

$$i_{cp} = \gamma Y_{cp}$$

Then

$$\frac{dr_{cb}}{dt} = \gamma(1 - r_{cb}) \sum_p Q_{pb} \prod_b (1 - q_{pb}(1 - r_{cb}))$$

Where Q is the direct and indirect intensity of use of the capability

$$Q_{pb} = \frac{q_{pb}}{q_p} + \sum_{p'} \phi_{pp'} \frac{q_{p'b}}{q_p}$$

Now, let's solve the single capability version, and then move to the multicapability model

Single capability dynamical model

It is convenient to rearrange the equation into the form

$$\frac{dr_c}{dt} = \gamma(1 - r_c)\left(\sum_p (1 - q_p)Q_p + r_c \sum_p q_p Q_p\right)$$

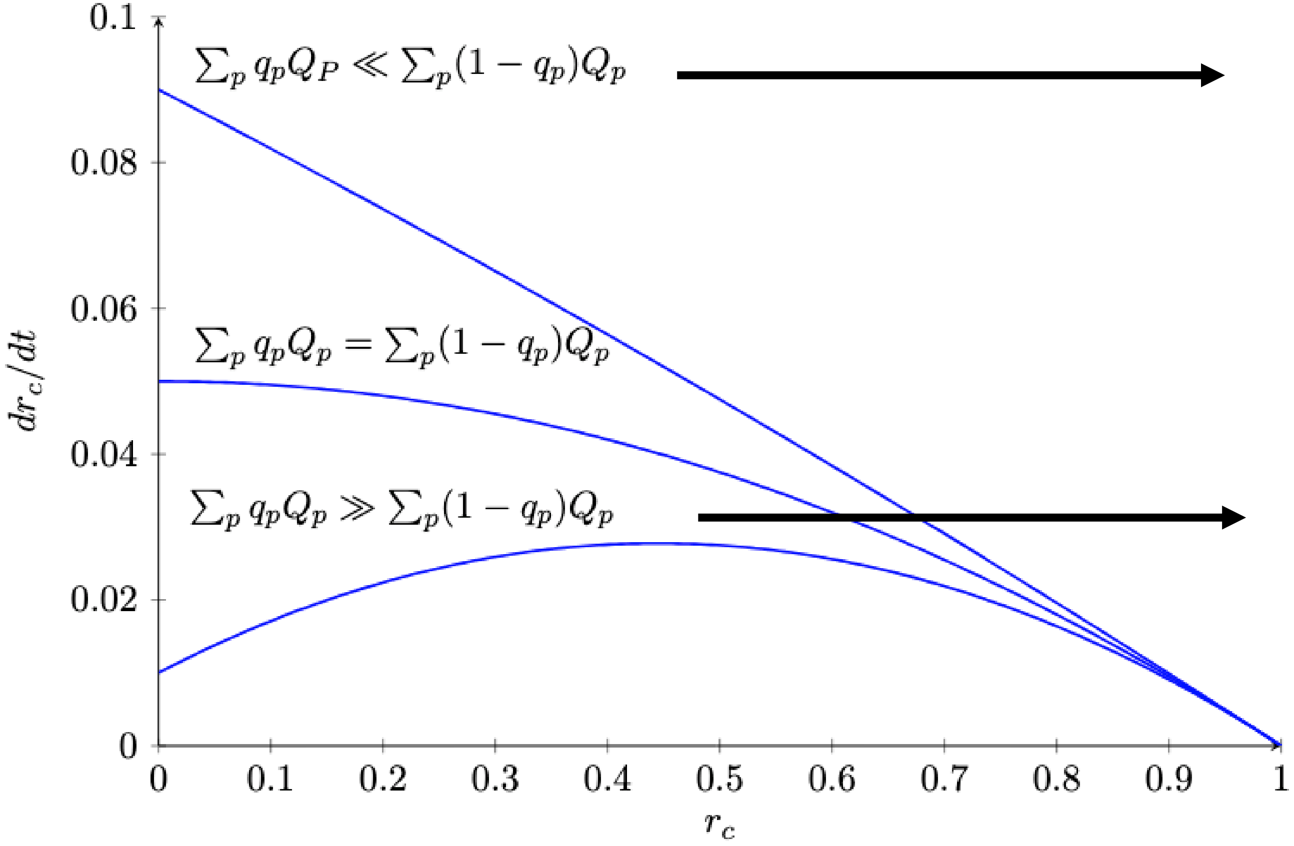
Intensity of use of capability
In activities that do not require it

Intensity of use of capability
in activities that require it

Availability of the capability

Single capability model

$$\frac{dr_c}{dt} = \gamma(1 - r_c)\Big(\sum_p (1 - q_p)Q_p + r_c \sum_p q_p Q_p\Big)$$



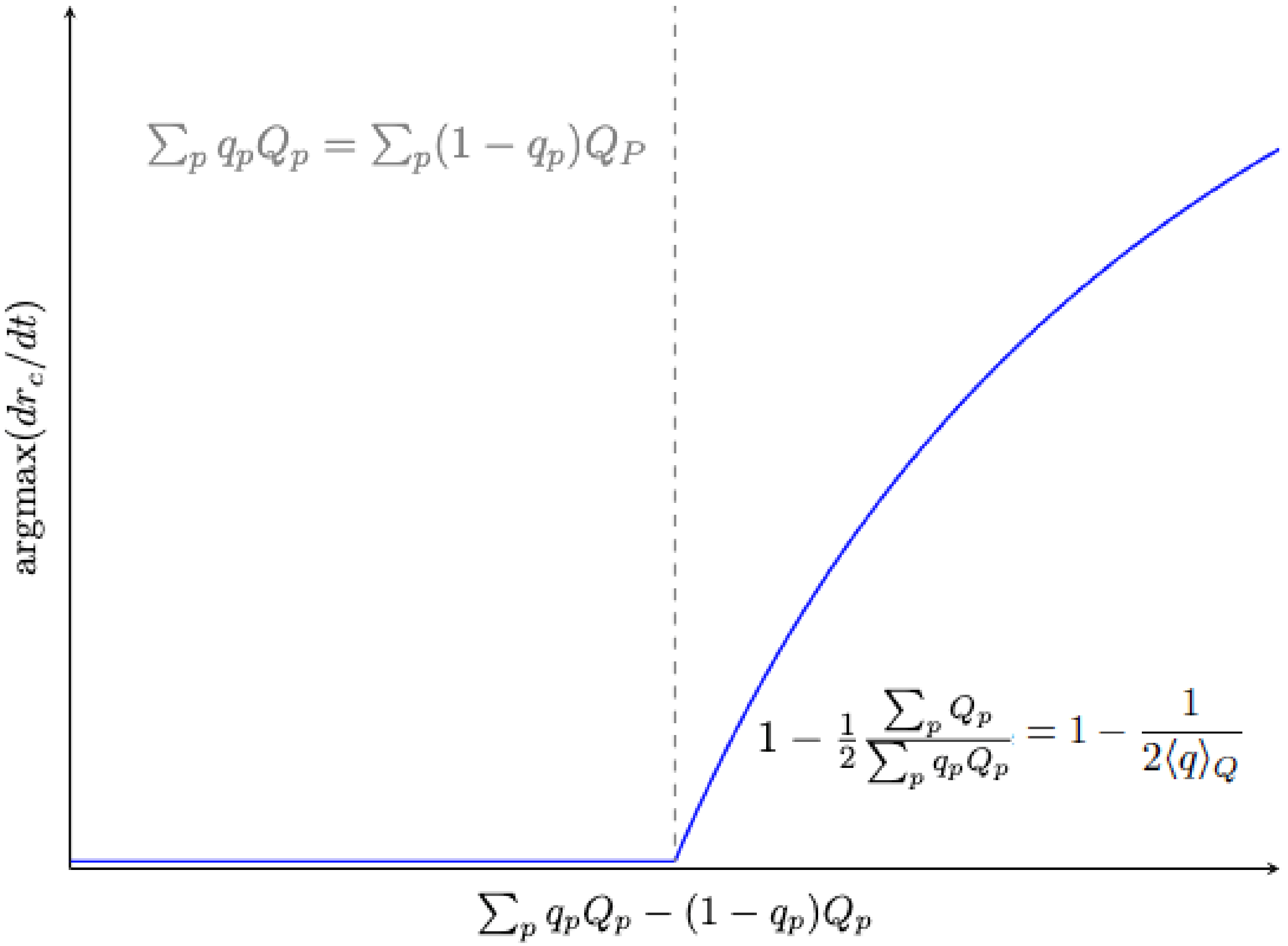
Two regimes

Convergence:
When capabilities are not much required, economies with lower capability endowments accumulate capabilities faster (e.g. Solow world)

Divergence:
When activities are intense in the capability, economies with low capability endowments accumulate capabilities slower

Single capability model

$$\frac{dr_c}{dt} = \gamma(1 - r_c)\left(\sum_p (1 - q_p)Q_p + r_c \sum_p q_p Q_p\right)$$



Location of maximum growth rate experiences a phase transition when activities become more intense in their need of capabilities

Single capability model

$$\frac{dr_c}{dt} = \gamma(1-r_c)\left(\sum_p (1-q_p)\underline{Q_p} + r_c \sum_p q_p \underline{Q_p}\right)$$

Q = spillovers

$$\text{argmax}(dr_c/dt) \to r_c = 0 \quad \text{if} \quad \sum_p q_p Q_p < \sum_p (1-q_p) Q_p$$

$$\text{argmax}(dr_c/dt) \to r_c = 1 - \frac{1}{2}\frac{\sum_p Q_p}{\sum_p q_p Q_p} \quad \text{if} \quad \sum_p q_p Q_p > \sum_p (1-q_p) Q_p$$

Multi capability model

$$\frac{dr_{cb}}{dt} = \gamma(1-r_{cb})\left(\sum (1-q_{pb})\underline{Q_{pb}E_{cpb}} + r_{cb} \sum q_{pb}\underline{Q_{pb}E_{cpb}}\right)$$

Q = spillovers
E = complementarity

$$\text{argmax}\left(\frac{dr_{cb}}{dt}\right) \to r_{cb} = 0 \quad \text{if} \quad \sum_p (1-q_{pb})Q_{pb}E_{cpb} > \sum_p q_{pb}Q_{pb}E_{cpb}$$

$$\text{argmax}\left(\frac{dr_{cb}}{dt}\right) \to r_{cb} = 1 - \frac{1}{2\langle q \rangle_{QE}} \quad \text{if} \quad \sum_p (1-q_{pb})Q_{pb}E_{cpb} < \sum_p q_{pb}Q_{pb}E_{cpb}$$

Kinematics

$$\frac{dr_{cb}}{dt} = \gamma(1 - r_{cb})\left(\sum_p (1 - q_{pb})Q_{pb}E_{cpb} + r_{cb}\sum_p q_{pb}Q_{pb}E_{cpb}\right)$$

Can be arranged into the form

$$\frac{dx}{dt} = A(1 - x)(B + Cx)$$

Known as a Ricatti equation (Jacoppo Ricatti 1676-1754), which we can solve through partial integration

$$\frac{dx}{(1 - x)(B + CX)} = Adt$$

Multi capability model

The analytical solution is

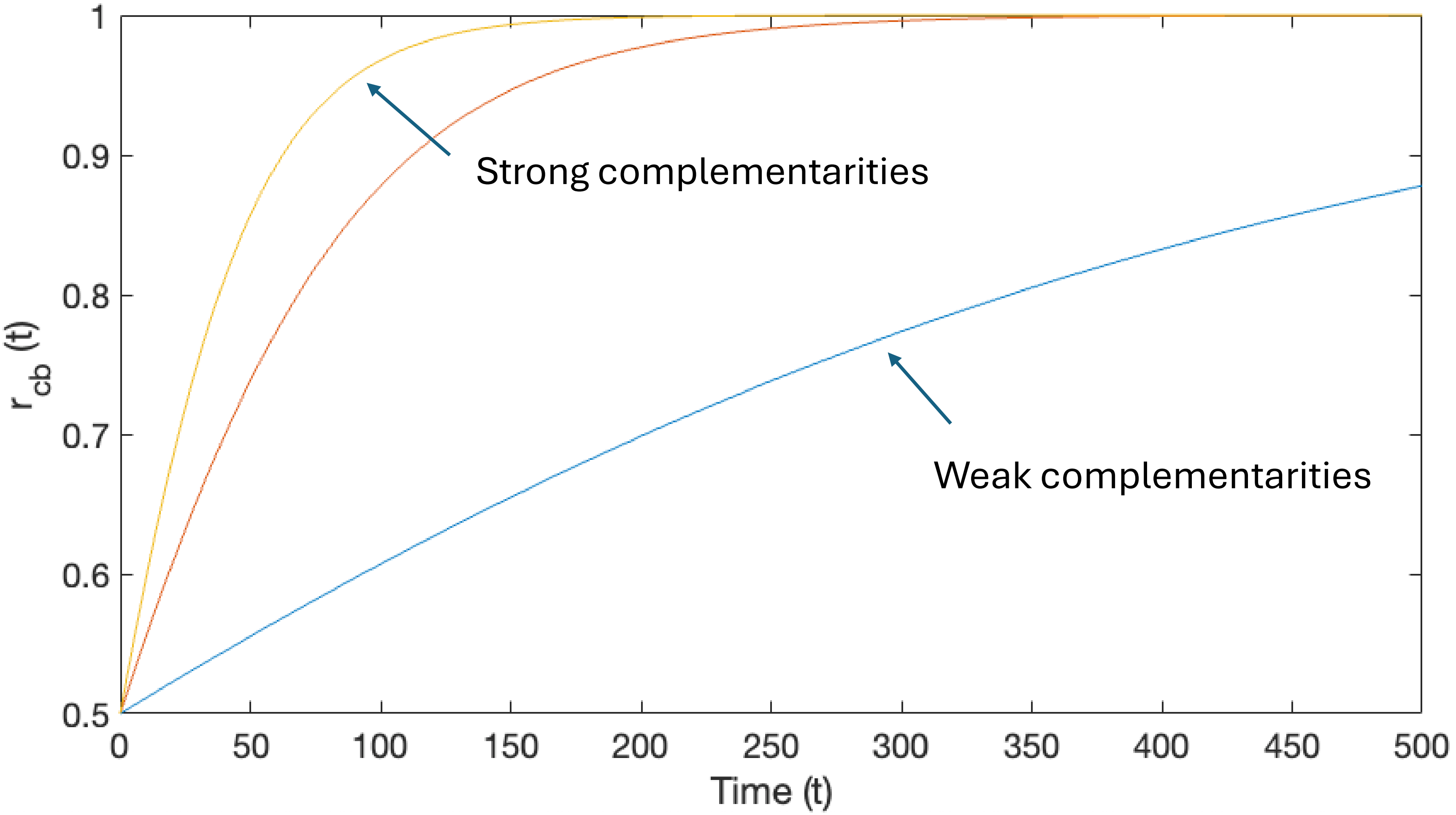
$$r_{cb}(t) = \frac{\rho^0(\langle q\rangle_{QE} - 1) + (1 - \rho^0\langle q\rangle_{QE})e^{\gamma t \sum_p Q_p E_{cpb}}}{\rho^0\langle q\rangle_{QE} + (1 - \rho^0\langle q\rangle_{QE})e^{\gamma t \sum_p Q_p E_{cpb}}}$$

replacing $1 - r_c^0$ with ρ^0

Looks "scary," but it is just a ratio between two terms of the form *A+Bexp(CT)*.
In the case of no spillovers ($\phi_{pp'}$=0) we can simplify this to:

$$r_{cb}(t) = \frac{\rho^0(q-1) + (1-\rho^0 q)e^t}{\rho^0 q + (1-\rho^0 q)e^t}$$

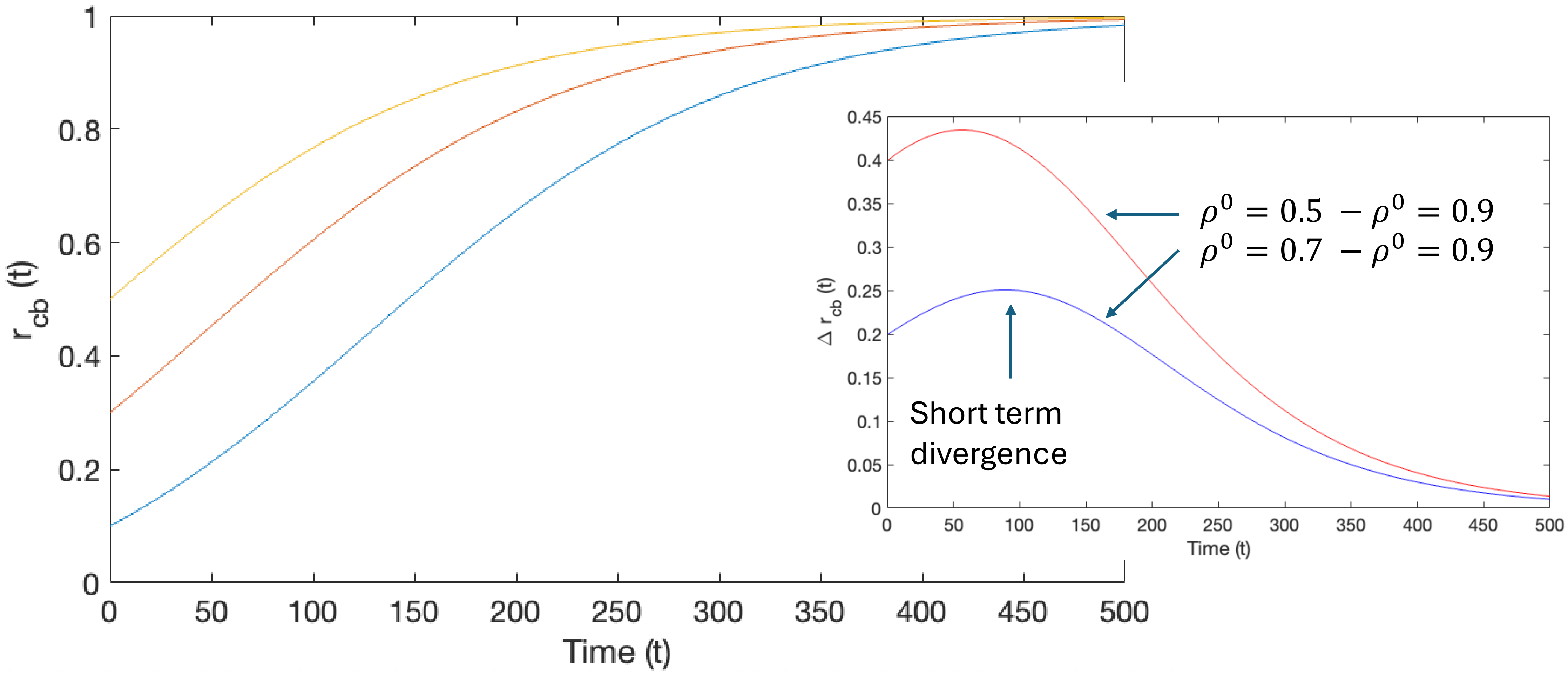
Kinematics



$$r_{cb}(t) = \frac{\rho^0(q-1) + (1-\rho^0 q)e^t}{\rho^0 q + (1-\rho^0 q)e^t}$$

rho=0.5, gamma=0.05, q=0.7, E=0.1,0.5,0.9

Kinematics



$$r_{cb}(t) = \frac{\rho^0(q-1) + (1-\rho^0 q)e^t}{\rho^0 q + (1-\rho^0 q)e^t}$$

rho=0.9.0.7,0.5, gamma=0.05, q=0.9, E=0.25

Discussion

From empirics to theory:
We have provided a mechanistic foundation for the Economic Complexity Index (ECI), showing it classifies economies monotonically based on their capability endowment.

Explaining structure:
The same framework helps us explain the shape of relatedness networks—like the product space or research space—allowing us to ”read” coarse properties of capability distributions from network visualizations.

Dynamics of growth:
By modeling capability accumulation, we can show divergence emerges when activities become more capability intense and can study how complementarities and spillover affect the speed of capability accumulation.

Discussion

Why this matters:

Provides a principled way to link “micro”-level capabilities with macro-level outcomes.

Bridges development theory, innovation studies, and policy.

Offers testable predictions about when and how economies diverge.

Takeaway:

Economic complexity is not just an empirical toolkit—it is becoming a general theory of how knowledge and capabilities drive growth, inequality, and structural transformation.