1. $$Pr(\alpha_1, \dots \alpha_n \mid \beta) = \frac{Pr(\alpha_1, \dots, \alpha_n, \beta)}{Pr(\beta)} = \frac{Pr(\alpha_1, \dots, \alpha_n, \beta)}{Pr(\alpha_2, \dots, \alpha_n, \beta)} \cdot \frac{Pr(\alpha_2, \dots, \alpha_n, \beta)}{Pr(\beta)}$$ (chain rule)

$$= Pr(\alpha_1 \mid \alpha_2, \dots, \alpha_n, \beta) \cdot \frac{Pr(\alpha_2, \dots, \alpha_n, \beta)}{Pr(\alpha_3, \dots, \alpha_n, \beta)} \cdot \frac{P(\alpha_3, \dots, \alpha_n, \beta)}{Pr(\beta)}$$

$$= Pr(\alpha_1 \mid \alpha_2, \dots, \alpha_n, \beta) \cdot Pr(\alpha_2 \mid \alpha_3, \dots, \alpha_n, \beta) \cdot \frac{Pr(\alpha_3, \dots, \alpha_n, \beta)}{Pr(\alpha_4, \dots, \alpha_n, \beta)} \cdots \frac{Pr(\alpha_n, \beta)}{Pr(\beta)}$$

$$= Pr(\alpha_1 \mid \alpha_2, \dots, \alpha_n, \beta)\, Pr(\alpha_2 \mid \alpha_3, \dots, \alpha_n, \beta) \dots Pr(\alpha_n \mid \beta)$$

2. $P(\text{oil}) = 0.5$ $\quad P(\text{gas}) = 0.2$ $\quad P(\text{nothing}) = 0.3$ $\quad P(\text{test} \mid \text{oil}) = 0.9$ $\quad P(\text{test} \mid \text{gas}) = 0.3$

$P(\text{test} \mid \text{nothing}) = 0.1$

$$P(\text{test}) = P(\text{test} \mid \text{oil}) \cdot P(\text{oil}) + P(\text{test} \mid \text{gas}) \cdot P(\text{gas}) + P(\text{test} \mid \text{nothing}) \cdot P(\text{nothing})$$

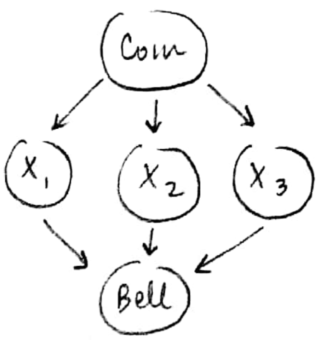
$$= (0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3)$$

$$= 0.54$$

$$P(\text{oil} \mid \text{test}) = \frac{P(\text{test} \mid \text{oil}) \cdot P(\text{oil})}{P(\text{test})} = \frac{0.9 \cdot 0.5}{0.54}$$

$$= 0.8333$$

3.




| Coin | $\Theta_{coin}$ |
|---|---|
| a | 1/3 |
| b | 1/3 |
| c | 1/3 |

| Coin | $X_1$ | $\Theta_{X_1 \mid Coin}$ |
|---|---|---|
| a | h | 0.2 |
| a | t | 0.8 |
| b | h | 0.4 |
| b | t | 0.6 |
| c | h | 0.8 |
| c | t | 0.2 |

| Coin | $X_2$ | $\Theta_{X_2 \mid Coin}$ |
|---|---|---|
| a | h | 0.2 |
| a | t | 0.8 |
| b | h | 0.4 |
| b | t | 0.6 |
| c | h | 0.8 |
| c | t | 0.2 |

| Coin | $X_3$ | $\Theta_{X_3 \mid Coin}$ |
|---|---|---|
| a | h | 0.2 |
| a | t | 0.8 |
| b | h | 0.4 |
| b | t | 0.6 |
| c | h | 0.8 |
| c | t | 0.2 |

| $X_1$ | $X_2$ | $X_3$ | Bell | $\Theta_{Bell \mid X_1 X_2 X_3}$ |
|---|---|---|---|---|
| h | h | h | on | 1 |
| h | h | t | off | 0 |
| h | t | h | off | 0 |
| h | t | t | off | 0 |
| t | h | h | off | 0 |
| t | h | t | off | 0 |
| t | t | h | off | 0 |
| t | t | t | on | 1 |

4. a) general formula: $I(V, \text{Parents}(V), \text{Non-Descendants}(V))$

$I(A, \emptyset, BE)$ ; $I(B, \emptyset, AC)$ ; $I(C, A, BDE)$ ; $I(D, AB, CE)$ ; $I(E, B, ACDFG)$ ;

$I(F, CD, ABE)$ ; $I(G, F, ABCDEH)$ ; $I(H, EF, ABCDG)$

b) d-separated(A, F, E) is false ; the path ADBE is unblocked. D is open because it is convergent and F is a descendant of D. B is open because it is divergent and $B \notin \{F\}$.

d-separated(G, B, E) is true ; all paths from G to E must pass through B or H. B is sequential and given so it is closed. H is convergent and not given so it is closed.

d-separated(AB, CDE, GH) is true ; all paths from A to G, A to H, B to G, and B to H are blocked by at least one node from $\{C, D, E\}$.

c) $Pr(a,b,c,d,e,f,g,h) = Pr(g|f) \cdot Pr(h|f,e) \cdot Pr(f|c,d) \cdot Pr(e|b) \cdot Pr(c|a) \cdot Pr(d|a,b) \cdot Pr(a) \cdot Pr(b)$

d) $Pr(A=1, B=1) = Pr(A=1) \cdot Pr(B=1) = 0.2 \cdot 0.7 = 0.14$

$$Pr(E=0|A=0) = \frac{Pr(E=0, A=0)}{P(A=0)} = \frac{Pr(E=0) \cdot Pr(A=0)}{Pr(A=0)} = Pr(E=0) = 0.66$$

$$Pr(E=0) = Pr(E=0|B=0) \cdot Pr(B=0) + Pr(E=0|B=1) \cdot Pr(B=1)$$
$$= (0.1)(0.3) + (0.9)(0.7) = 0.66$$

5. a)

| | A | B | A ⇒ B |
|---|---|---|---|
| $w_0$ | T | T | T |
| $w_1$ | T | F | F |
| $w_2$ | F | T | T |
| $w_3$ | F | F | T |

$\therefore \{w_0, w_2, w_3\}$ are the models of $\alpha$

b) $Pr(\alpha) = Pr(w_0) + Pr(w_2) + Pr(w_3)$
$= 0.3 + 0.1 + 0.4 = 0.8$

c) $P(A=T, B=T | \alpha) = 0.3/0.8 = 0.375$

$P(A=T, B=F | \alpha) = 0$

$P(A=F, B=T | \alpha) = 0.1/0.8 = 0.125$

$P(A=F, B=F | \alpha) = 0.4/0.8 = 0.5$

d) $A \Rightarrow \neg B \rightarrow \neg A \vee \neg B$ ; satisfied by worlds 1, 2, 3

$$Pr(A \Rightarrow \neg B | \alpha) = \frac{Pr(A \Rightarrow \neg B \wedge \alpha)}{Pr(\alpha)} = \frac{0.1 + 0.4}{0.8} = 0.625$$