1. $P \to \neg Q$, $Q \to \neg P$

P	Q	¬P	¬Q	P → ¬Q	Q → ¬P
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

equal ✓

$P \leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	¬P	¬Q	P ↔ ¬Q	(P∧¬Q)	(¬P∧Q)	((P∧¬Q)∨(¬P∧Q))
T	T	F	F	F	F	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	F	T	T
F	F	T	T	F	F	F	F

equal ✓

2. (Smoke → Fire) → (¬Smoke → ¬Fire) = Δ

Neither

Smoke	Fire	(Smoke → Fire)	¬Smoke	¬Fire	(¬Smoke → ¬Fire)	Δ
T	T	T	F	F	T	T
T	F	F	F	T	T	T
F	T	T	T	F	F	F
F	F	T	T	T	T	T

(Smoke → Fire) → ((Smoke ∨ Heat) → Fire) = Δ

Neither

Smoke	Fire	Heat	(Smoke → Fire)	(Smoke ∨ Heat)	(S∨H) → F	Δ
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

((Smoke ∧ Heat) → Fire) ↔ ((Smoke → Fire) ∨ (Heat → Fire)) = Δ   Valid

Smoke	Heat	Fire	(Smoke ∧ Heat)	(S∧H)→Fire	S→F	H→F	(S→F)∨(H→F)	Δ
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

3.

a)   Y = is mythical   I = is immortal   H = is horned   M = is mammal   G = is magical

$$Y \to I,\ \neg Y \to (\neg I \wedge M),\ (I \vee M) \to H,\ H \to G$$

b)

$Y \to I$	$\neg Y \to (\neg I \wedge M)$	$(I \vee M) \to H$	$H \to G$
$(\neg Y \vee I)$	$\neg(\neg Y) \vee (\neg I \wedge M)$	$\neg(I \vee M) \vee H$	$(\neg H \vee G)$
	$(Y \vee \neg I) \wedge (Y \vee M)$	$(\neg I \wedge \neg M) \vee H$	
		$(\neg I \vee H) \wedge (\neg M \vee H)$	

→ $(\neg Y \vee I) \wedge (Y \vee \neg I) \wedge (Y \vee M) \wedge (\neg I \vee H) \wedge (\neg M \vee H) \wedge (\neg H \vee G)$

c)

Mythical			Magical		Horned	
1. $\neg Y \vee I$	8. $\neg I$	1,7	7. $\neg G$		7. $\neg H$	
2. $Y \vee \neg I$	9. M	3,7	8. $\neg H$	6,7	8. $\neg M$	5,7
3. $Y \vee M$	10. H	5,9	9. $\neg M$	5,8	9. Y	3,8
4. $\neg I \vee H$	11. G	6,10	10. $\neg I$	4,8	10. I	1,9
5. $\neg M \vee H$	12. $\neg M \vee G$	5,6	11. $\neg Y$	1,10	11. H	4,7
6. $\neg H \vee G$	13. $\neg Y \vee H$	1,4	12. Y	3,12	12. False	7,11
7. $\neg Y$	14. $I \vee M$	1,3	13. False	11,12		

All of the variables could be assigned without a contradiction so we can't conclude mythical.

∴ the unicorn is magical

∴ the unicorn is horned

4. Figure 1

Deterministic? each side of the _or_ gates are mutually exclusive (unsat) ✓

Decomposable? each side of all the _and_ gates have different variables ✓

Smooth? the inner _or_ gates have different variables (one side has D and C while the other side only has C) so this is not smooth ✗

Figure 2

Decomposable? each side of all the _and_ gates have different variables ✓

Deterministic? the inner _or_ gates aren't mutually exclusive because $(\neg A \wedge B) \wedge (\neg A \wedge B)$ is sat ✗

Smooth? each side of all the _or_ gates have the same variables ✓

5. a)

A B	¬A ¬B	¬A ∧ B	¬B ∧ A	(¬A∧B)∨(¬B∧A)
T T	F F	F	F	F
T F	F T	F	T	T
F T	T F	T	F	T
F F	T T	F	F	F

$$0.07 + 0.27 = 0.34$$

$$w(A, \neg B) = w(A) \cdot w(\neg B)$$
$$= 0.1 \cdot 0.7$$
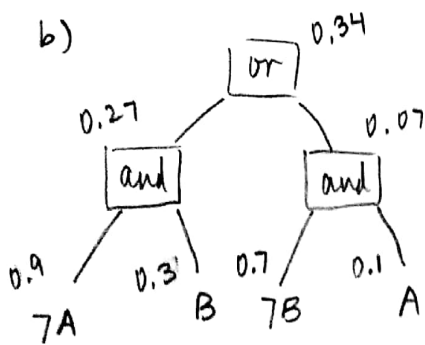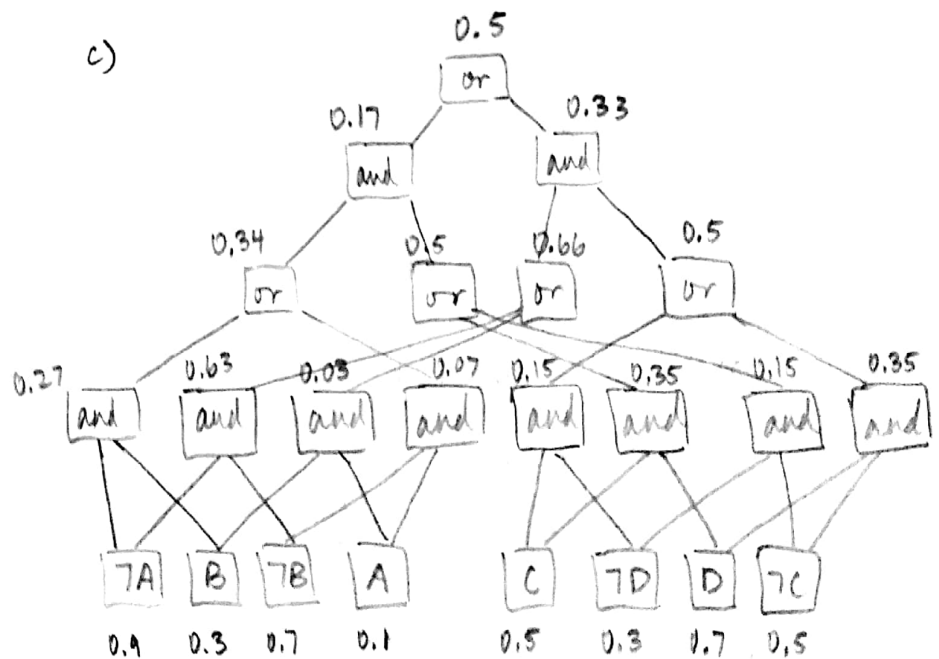$$= 0.07$$

$$w(\neg A, B) = w(\neg A) \cdot w(B)$$
$$= 0.9 \cdot 0.3$$
$$= 0.27$$

b)

The count on the root is the same as the Weighted Model count for the formula.

c)

The Weighted Model Count is 0.5