CECS 229: HW 1 (Divisibility)

Spring 2021

Remember, we will not be collecting or grading homework. The homework is optional but highly recommended. Quiz questions will be similar to the homework questions but not identical. Solutions to these problems are posted on BeachBoard.

1. True or false? If it is false, provide a counterexample and if true, prove why. Note: $a, b, c \in \mathbb{Z}$

   a. If $a \neq 0$, then $a \mid a$ ⟹ use def'n $a = a \cdot k$

   Show $k \in \mathbb{Z}$

   $k = 1$ ✓

   Because $a = a \cdot k$ and $k \in \mathbb{Z}$, then $a \mid a$ ✓

   $\boxed{\text{True}}$

   b. If $bc \mid a$ then $c \mid a$ (assume $b, c \neq 0$)

   Given $a = bc \cdot k$, $k \in \mathbb{Z}$

   $a = c \cdot bk$, $k \in \mathbb{Z}$

   $b, k \in \mathbb{Z}$ ∴ $b \cdot k \in \mathbb{Z}$

   $a = c \cdot m$, $m = bk \in \mathbb{Z}$

   ∴ by def'n, $c \mid a$ ✓

   $\boxed{\text{True}}$

   c. If $a \mid b$, then $a \mid b^{10}$ (assume $a \neq 0$)

   Given $b = a \cdot k$, $k \in \mathbb{Z}$

   $b^{10} = a \cdot m$: Show $m \in \mathbb{Z}$

   Subst. $b = ak$ into $b^{10} = am$

   $(ak)^{10} = am$

   $\frac{a^{10} k^{10}}{a} = \frac{am}{a}$

   $m = a^9 k^{10}$

   $a, k \in \mathbb{Z}$ ∴ $a^9 k^{10} \in \mathbb{Z}$

   so $m \in \mathbb{Z}$

   thus $a \mid b^{10}$

   $\boxed{\text{True}}$

2. 19 divides which of the following numbers?
   a. 342 True
   b. 771 False
   c. 1273 True
   d. 1735 False

3. Let $a \in \mathbb{Z}$. Find an integer, $b \neq 1$, so that $b \mid \underbrace{(a^2 + 3a + 2)}_{(a+1)(a+2)}$ is ALWAYS a true statement.

Two cases: ① a is even ② a is odd

① a is even

$(a+1)(a+2)$

even + odd = odd; even + even = even

odd × even = even

② a is odd

$(a+1)(a+2)$

odd + odd = even; odd + even = odd

even × odd = even

a^2+3a+2 is ALWAYS even.

$\boxed{b=2}$