STANDARD COLLOCATION AND COEFFICIENT COMPARISON METHODS FOR THE SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS BY POWER SERIES POLYNOMIAL AS BASIS FUNCTION

BY

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Certification

This is to certify that this project work was carried out by **BASSEY, Blessing Itoro** with Matriculation number: **12/55EB077** in the Department of Mathematics, Faculty of physical science, University of Ilorin, Ilorin, Nigeria.

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Dedication

This research work is dedicated to the Almighty GOD, the Author and the Finisher of my faith, for giving me the strength and grace all through my stay in the school.

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Abstract

This project deals with the numerical approximation of $3rd$ and $4th$ order ordinary differential equation using the standard collocation and coefficient comparison methods. The power series is used as the basis function in the two cases considered. Numerical examples are given to illustrate the reliability and efficiency of the two methods. Both methods produced numerical solutions but that of standard collocation method gives a better approximation.

Contents

Chapter 1

GENERAL INTRODUCTION

1.1 INTRODUCTION

Mathematics (from Greek word (máthéma) means knowledge, study, learning) is the study of quantity, structure space and change.

Mathematics is an abstract science but which has applications in all branches of science and sphere of life, e,g Natural sciences, Engineering, Medicine and the Social sciences.

Different techniques and approaches are used to obtain the solution of the equations, Many researchers have worked on Ordinary Differential Equation using perturbed method, finite difference method, collocation method to mention a few. Differential Equations:Differential equations first came into existence with the invention of calculus by Newton and Leibniz. Isaac Newton listed three kinds of differential equations Also differencial Equations are equations or functional equations involving derivatives of the unknown functions Also, a differential equation is a mathematical equation that relates some function with its derivatives. In application the functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two. Because such relations are extremely common differential equations play a prominent role in many disciplines including engineering, physics, economics and biology. In mathematics, differential equations are studied from several different perspectives, mostly concerned with their solutions, the set of functions that satisfy the equation.

In this study, we solve linear differential equation by assuming an approximate solution of different degrees, using the Standard Collocation method and the variable Coefficient Comparison method, using Power series Polynomial as basis function.

1.2 DEFINITIONS OF RELEVANT TERMS.

1.2.1 Equation

An equation describes the relationship between the independent variable x and the dependent variable y.

$$y = 2x \tag{1.1}$$

an equality sign "=" is needed in every equation.

1.2.2 Differential Equation

An equation which involves dependent variable and its derivative with respect to the independent variable is called a differential equation. For example

$$\frac{d^2y}{dx^2} + y = 0 \tag{1.2}$$

1.2.3 Collocation

This is the evaluation of an equation at some equally spaced interior points.

1.2.4 Initial condition

Initial Solution is the condition that is specified at the initial/starting point of a given equation; More generally, an initial condition is specified at the same value of the independent variable example is

$$y(0) = A \text{ and } y'(0) = B$$

1.2.5 Absolute Error

Error is the difference between the exact and the approximate solution when evaluated at any given point in the interval under consideration.
Mathematically, it is represented as

$$\text{Absolute Error } = |Exact - Approximate|$$

1.2.6 Power Series

Power series is any series that can be written in the form,

$$\sum_{n=0}^{\infty} a_n (x - x_0)^n \tag{1.3}$$

where a_n are numbers, the a_n's are often called the coefficients of the series.

1.3 AIM AND OBJECTIVES OF THE STUDY.

The aim of this study is to solve $3rd$ and $4th$ order ordinary differential equation using the Standard Collocation Method and Variable Coefficient Comparison Method with Power Series as the basis function.

The objectives are:-

1. To discuss both methods on general higher order ordinary differential equation.

2. To investigate and compare the efficiency and accuracy of the proposed methods on the numerical examples considered.
3. To compare results of the two proposed methods in term of accuracy and error.

1.4 LIMITATION OF THE STUDY

This project is limited just the numerical approximation of linear differential equations only, because the power series method alone is difficult to apply to most non linear equations.

Chapter 2

METHODOLOGY.

2.1 INTRODUCTION

In this project we shall solve the $3rd$ and $4th$ order ordinary differential equation using two proposed numerical methods namely:- Standard Collocation Method and Coefficient Comparison Method

2.1.1 STANDARD COLLOCATION METHOD.

Over the years, several researchers considered the collocation method as ways of generating numerical solution to ordinary D.E. the Collocation method is dated as far back as 1956 in the research carried out by Lanczos(1956) and Brunne(1996).Lanczos introduced the standard collocation method with some selected points. Now, using the standard collocation method we assume an approximate solution of the form

$$y(x) = \sum_{n=0}^{N} a_n x^n \tag{2.1}$$

Where N is the degree of our approximant $a_n (n \geq 0)$ are constants to be determined. Hence, having an ODE in the form of (1.4)

$$p(x)y''' + q(x)y' + r(x)y = 0 \tag{2.2}$$

$$y(0) = 1$$

$$y'(0) = 1$$

Now, differentiating (2.1) 3-times, we obtain

$$\left.\begin{aligned} y' &= \textstyle\sum_{n=1}^{N} na_n\, x^{n-1} \\ y'' &= \textstyle\sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} \\ y''' &= \textstyle\sum_{n=3}^{N} n(n-1)(n-2)a_n\, x^{n-3}. \end{aligned}\right\} \tag{2.3}$$

Now, substituting (2.4) into (2.3), we have

$$p(x)\sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} + q(x)\sum_{n=1}^{N} na_n\, x^{n-1} + r(x)\sum_{n=0}^{N} a_n\, x^n = 0 \tag{2.4}$$

We then collocate (2.5) using

$$x_k = \frac{a+(b-a)}{N}$$

$$k = 1, 2, ...[N-(n-1)]$$

N is the degree of our approximant and n is the order of the D.E

Thus, equation (2.5) gives rise to $[N-(n-1)]$ system of algebraic linear equations in $[N-(n-1)]$ unknown constant. The extra equations are obtained using the boundary conditions given in (2.3). These equations are then solved simultaneously using MAPPLE software in order to obtain the unknown constants, which are then substituted back into (2.2) to get the *Approximate Solution*

2.1.2 COEFFICIENT COMPARISON METHOD

In order to solve equation (2.3) we also assume the approximate solution of power series as in (2.1)

$$y(x) = \sum_{n=0}^{N} a_n\, x^n$$

Also differentiating to get (2.4) and substituting back into (2.3), in this method, we collect like terms. this technique entails comparing the coefficient of independent variable x of different degrees on the right hand side and equating corresponding to the left hand side of the same residual equation.

$$\begin{pmatrix} C_{11} & C_{21} & C_{13} & \cdots & C_{NN} \\ C_{21} & C_{22} & C_{23} & \cdots & C_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ C_{N1} & C_{N2} & C_{N3} & \cdots & C_{NN} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_N \end{pmatrix}$$

Solving the matrix using MAPPLE software, yield, the values of a_0 to a_n. After which we now substitute back the values of a_0 to a_n to yield the *Approximate solution.*

Chapter 3

DEMONSTRATION OF STANDARD COLLOCATION METHOD

3.1 NUMERICAL EXAMPLE 1

Consider the ODE

$$y''' + 4y'' - 5y' = 0 \tag{3.1}$$

together with the initial conditions:

$$y(0) = 4,\ y'(0) = -7,\ y''(0) = 23.$$

The exact solution is given as

$$y(x) = 5 - 2e^{x} + e^{-5x} \tag{3.2}$$

Solution

The general form of a power series is

$$y(x) = \sum_{n=0}^{N} a_n\, x^n \tag{3.3}$$

but for this problem, we shall consider, from $n = 0$ to $n = 10$ therefore,

$$y(x) = \sum_{n=0}^{10} a_n\, x^n \tag{3.4}$$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$$y' = \sum_{n=1}^{N} na_n\, x^{n-1} \tag{3.5}$$

$$y'(x) = a_1+2a_2x+3a_3x^2+4a_4x^3+5a_5x^4+6a_6x^5+7a_7x^6+8a_8x^7+9a_9x^8+10a_{10}x^9$$

$$y'' = \sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} \tag{3.6}$$

$$y''(x) = 2a_2+6a_3x+12a_4x^2+20a_5x^3+30a_6x^4+42a_7x^5+56a_8x^6+72a_9x^7+90a_{10}x^8$$

$$y''' = \sum_{n=3}^{N} n(n-1)(n-2)a_n\, x^{n-3} \tag{3.7}$$

$$y'''(x) = 6a_3+24a_4x+60a_5x^2+120a_6x^3+210a_7x^4+336a_8x^5+504a_9x^6+720a_{10}x^7$$

Now, substituting the expansion of (3.5) to (3.7) into (3.1), and after simplification we obtain

$$\begin{aligned} -50a_{10}x^9 - 45a_9x^8 + 360a_10x^8 - 40a_8x^7 + 288a_9x^7 + 720a_10x^7 \\ -35a_7x^6 + 224a_8x^6 + 504a_9x^6 - 30a_6x^5 + 618a_7x^5 + 336a_x^5 - 25a_5x^4 \\ +120a_6x^4 + 210a_7x^4 - 20a_4x^3 + 80a_5x^3 + 120a_6x^3 - 15a_3x^2 + 48a_4x^2 \\ +60a_5x^2 - 10a_2x + 24a_3x + 24a_4x + -5a_1 + 8a_2 + 6a_3 = 0 \end{aligned} \tag{3.8}$$

Using the boundary conditions,

$$y(0) = 4; \Longrightarrow a_0 = 4$$

$$y'(0) = -7; \Longrightarrow a_1 = -7$$

$$y''(0) = 23; \Longrightarrow 2a_2 = 23 \Longrightarrow a_2 = 11.5$$

Now, collocating (3.8) using

$$x_k \;=\; \frac{a+(b-a)}{N}$$

$$k = 1, 2, ...[N-(n-1)]$$

When $k = 1, x_1 = \frac{1}{10} = 0.1$

$$\begin{aligned} -8.25a_3 + 2.860a_4 + 0.6775a_5 + 0.13170a_6 + 0.022645a_7 \\ +0.0035800a_8 + 0.0053235a_9 + 0.00007550a_{10} = -115.5 \end{aligned} \tag{3.9}$$

When $k = 2, x_2 = \frac{2}{10} = 0.2$

$$\begin{aligned} 10.20a_3 + 6.560a_4 + 3a_5 + 1.14240a_6 + 0.387520a_7 \\ +0.121344098a_8 + 0.03582720a_9 + 0.010112a_{10} = -104 \end{aligned} \tag{3.10}$$

When $k = 3, x_3 = \frac{3}{10} = 0.3$

$$\begin{aligned} 11.85a_3 + 10.9800a_4 + 7.3575a_5 + 4.13910a_6 + 2.037250a_7 \\ +0.971028a_8 + 0.4274415a_9 + 0.180099450a_{10} = -92.5 \end{aligned} \tag{3.11}$$

When $k = 4, x_4 = \frac{4}{10} = 0.4$

$$\begin{aligned} 13.20a_3 + 16a_4 + 14.080095a_5 + 10.44480a_6 + 6.952960a_7 \\ +4.2926080a_8 + 2.506752009a_9 + 1.402470400a_{10} = -81 \end{aligned} \tag{3.12}$$

When $k = 5, x_5 = \frac{5}{10} = 0.5$

$$\begin{aligned} 14.25a_3 + 21.5a_4 + 23.4375a_5 + 21.56250a_6 + 17.828125a_7 \\ +13.6875a_8 + 9.94921875a_9 + 6.93359375a_{10} = -69.5 \end{aligned} \tag{3.13}$$

When $k = 6, x_6 = \frac{6}{10} = 0.6$

$$\begin{aligned} 15a_3 + 27.360a_4 + 35.64a_5 + 39.13920a_6 + 38.46720a_7 \\ +35.4585600a_8 + 30.82095360a_9 + 25.69812480a_{10} = -58 \end{aligned} \tag{3.14}$$

When $k = 7, x_7 = \frac{7}{10} = 0.7$

$$\begin{aligned} 15.45a_3 + 33.460a_4 + 50.8375a_5 + 64.92990a_6 + 74.5390a_7 \\ +79.5307240a_8 + 80.41897395a_9 + 78.0369925a_{10} = -46.5 \end{aligned} \tag{3.15}$$

When $k = 8, x_8 = \frac{8}{10} = 0.8$

$$\begin{gathered}15.60a_3 + 39.680a_4 + 69.129a_5 + 100.76160a_6 + 131.8912a_7 \\ +60.4321280a_8 + 184.9688064a_9 + 204.6820352a_{10} = -35\end{gathered} \tag{3.16}$$

Solving equation (3.9) to (3.16) using MAPPLE software we'll get,

$$a_0 = 4$$

$$a_1 = -7$$

$$a_2 = 11.5$$

$$a_3 = -21.1568095803235$$

$$a_4 = 25.7025825764423$$

$$a_5 = -25.7025825764423$$

$$a_6 = 20.6108772911056$$

$$a_7 = -13.2197430722544$$

$$a_8 = 6.36628644261859$$

$$a_9 = -2.01884761127450$$

$$a_{10} = 0.310192721946573$$

substituting the values of a_0 to a_{10} into equation (3.4), in order to get our Approximate Solution, we have

$$\begin{aligned}y(x) = {} & 4 - 7x + 11.5x^2 - 21.1568095803235x^3 + 25.7025825764423x^4 \\ & -25.7025825764423x^5 + 20.6108772911056x^6 - 13.2197430722544x^7 \\ & +6.36628644261859x^8 - 2.01884761127450x^9 + 0.310192721946573x^{10}\end{aligned} \tag{3.17}$$

3.2 NUMERICAL EXAMPLE 2

Consider the ODE

$$y^{iv} + 13y'' + 36y = 0 \tag{3.18}$$

together with the initial conditions:

$$y(0) = 0,\ y'(0) = -3,\ y''(0) = 5.\ y'''(0) = -3$$

The exact solution is given as

$$y(x) = \cos 2x - 3\sin 2x - \cos 3x + \sin 3x \tag{3.19}$$

Solution

The general form of a power series is

$$y(x) = \sum_{n=0}^{N} a_n\, x^n \tag{3.20}$$

but for this problem, we shall consider, from $n = 0$ to $n = 10$ therefore,

$$y(x) = \sum_{n=0}^{10} a_n\, x^n \tag{3.21}$$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$$y' = \sum_{n=1}^{N} na_n\, x^{n-1} \tag{3.22}$$

$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + 7a_7x^6 + 8a_8x^7 + 9a_9x^8 + 10a_{10}x^9$$

$$y'' = \sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} \tag{3.23}$$

$$y''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + 42a_7x^5 + 56a_8x^6 + 72a_9x^7 + 90a_{10}x^8$$

$$y''' = \sum_{n=3}^{N} n(n-1)(n-2)a_n\, x^{n-3} \tag{3.24}$$

$$y'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + 120a_6x^3 + 210a_7x^4 + 336a_8x^5 + 504a_9x^6 + 720a_{10}x^7$$

$$y^{iv} = \sum_{n=4}^{N} n(n-1)(n-2)(n-3)a_n\, x^{n-4} \tag{3.25}$$

$$y^{iv}(x) = +24a_4 + 120a_5x + 360a_6x^2 + 840a_7x^3 + 1680a_8x^4 + 30240a_9x^5 + 5040a_{10}x^6$$

Now, substituting the expansion of (3.21) to (3.25) into (3.18), and after simplification we have

$$\begin{aligned} 36x^{10}a_{10} + 36a_9x^9 + 36a_9x^8 + 1170a_10x^8 + 360a_7x^7 + 936a_9x^7 \\ +36a_6x^6 + 728a_8x^6 + 5040a_{10}x^6 + 36a_5x^5 + 546a_7x^5 + 3024a_9x^5 \\ +36a_4x^4 + 390a_6x^4 + 1680a_8x^4 + 36a_3x^3 + 260a_5x^3 + 840a_7x^3 \\ +36a_2x^2 + 156a_4x^2 + 360a_6x^2 + 36xa_1 + 78xa_3 + 120xa_5 \\ +36a_0 + 26a_2 + 24a_4 \ = 0 \end{aligned} \tag{3.26}$$

Using the boundary conditions,

$$y(0) = 0; \Longrightarrow a_0 = 0$$

$$y'(0) = -3; \Longrightarrow a_1 = -3$$

$$y''(0) = 5; \Longrightarrow 2a_2 = 5 \Longrightarrow a_2 = 2.5$$

$$y'''(0) = -3; \Longrightarrow 6a_3 = -3 \Longrightarrow a_3 = -0.5$$

Now, collocating (3.26) using

$$x_k \ = \frac{a + (b-a)}{N}$$

$$k = 1, 2, ...[N - (n-1)]$$

When $k = 1, x_1 = \frac{1}{10} = 0.1$

$$\begin{aligned} 36a_0 + 3.6a_1 + 26.36a_2 + 7.836a_3 + 25.5636a_4 + 12.26036a_5 \\ +3.639036a_6 + 0.8454636a_7 + 0.16872836a_8 \\ +0.030333636a_9 + 0.0050517036a_{10} = 0 \end{aligned} \tag{3.27}$$

When $k = 2, x_2 = \frac{2}{10} = 0.2$

$$\begin{aligned} 36a_0 + 7.2a_1 + 27.44a_2 + 15.888a_3 + 30.2976a_4 + 26.09152a_5 \\ +15.026304a_6 + 6.895808a_7 + 2.73468416a_8 \\ +0.979679232a_9 + 0.32555888a_{10} = 0 \end{aligned} \tag{3.28}$$

When $k = 3, x_3 = \frac{3}{10} = 0.3$

$$\begin{aligned} 36a_0 + 10.8a_1 + 29.24a_2 + 24.372a_3 + 38.3316a_4 + 43.10748a_5 \\ +35.585244a_6 + 24.146532a_7 + 14.14107396a_8 \\ +7.553731788a_9 + 3.751136276a_{10} = 0 \end{aligned} \tag{3.29}$$

When $k = 4, x_4 = \frac{4}{10} = 0.4$

$$\begin{aligned} 36a_0 + 14.4a_1 + 31.76a_2 + 33.504a_3 + 49.8816a_4 + 65.00864a_5 \\ +67.731456a_6 + 59.4100224a_7 + 46.01348096a_8 \\ +32.50873958a_9 + 21.4143867a_{10} = 0 \end{aligned} \tag{3.30}$$

When $k = 5, x_5 = \frac{5}{10} = 0.5$

$$\begin{aligned} 36a_0 + 18a_1 + 35a_2 + 43.5a_3 + 65.25a_4 + 93.625a_5 \\ +114.9375a_6 + 122.34375a_7 + 116.515625a_8 \\ +101.8828125a_9 + 83.35546875a_{10} = 0 \end{aligned} \tag{3.31}$$

When $k = 6, x_6 = \frac{6}{10} = 0.6$

$$\begin{aligned} 36a_0 + 21.6a_1 + 38.96a_2 + 54.576a_3 + 84.8256a_4 + 130.95936a_5 \\ +181.823616a_6 + 224.9047296a_7 + 252.2982298a_8 \\ +261.7110467a_9 + 255.0154254a_{10} = 0 \end{aligned} \tag{3.32}$$

When $k = 7, x_7 = \frac{7}{10} = 0.7$

$$\begin{aligned}36a_0 + 25.2a_1 + 43.64a_2 + 66.748a_3 + 109.6836a_4 + 179.23052a_5 \\ +274.274364a_6 + 382.8509748a_7 + 491.0918004a_8 \\ +586.7800347a_9 + 661.4160426a_{10} = 0\end{aligned} \tag{3.33}$$

Solving equation (3.27) to (3.32) using MAPPLE software we'll get:-

$$a_0 = 0$$

$$a_1 = -3$$

$$a_2 = 2.5$$

$$a_3 = -0.5$$

$$a_4 = -2.709300437$$

$$a_5 = 1.230053634$$

$$a_6 = 0.90689472$$

$$a_7 = -0.3207162506$$

$$a_8 = -0.2101073479$$

$$a_9 = 0.09885866202$$

$$a_{10} = -0.008658126837$$

substituting the values of a_0 to a_{10} into equation (3.21), in other to get our Approximate Solution, we have

$$\begin{aligned}y(x) = -3x + 2.5x^2 - 0.5x^3 - 2.709300437x^4 + 1.230053634x^5 \\ +0.90689472x^6 - 0.3207162506x^7 - 0.2101073479x^8 \\ +0.09885866202x^9 - 0.008658126837x^{10}\end{aligned} \tag{3.34}$$

3.3 NUMERICAL EXAMPLE 3

Consider the ODE

$$y^{iv} - 10y'' + 9y = 0 \tag{3.35}$$

together with the initial conditions:

$$y(0) = 5,\, y'(0) = -1,\, y''(0) = 21.\, y'''(0) = -49$$

The exact solution is given as

$$y(x) = 4e^{x} - e^{-x} + 2e^{-3x} \tag{3.36}$$

Solution

The general form of a power series is

$$y(x) = \sum_{n=0}^{N} a_n\, x^n \tag{3.37}$$

but for this problem, we shall consider, from $n = 0$ to $n = 10$ therefore,

$$y(x) = \sum_{n=0}^{10} a_n\, x^n \tag{3.38}$$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$$y' = \sum_{n=1}^{N} na_n\, x^{n-1} \tag{3.39}$$

$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + 7a_7x^6 + 8a_8x^7 + 9a_9x^8 + 10a_{10}x^9$$

$$y'' = \sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} \tag{3.40}$$

$$y''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + 42a_7x^5 + 56a_8x^6 + 72a_9x^7 + 90a_{10}x^8$$

$$y''' = \sum_{n=3}^{N} n(n-1)(n-2)a_n\, x^{n-3} \tag{3.41}$$

$$y'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + 120a_6x^3 + 210a_7x^4 + 336a_8x^5 + 504a_9x^6 + 720a_{10}x^7$$

$$y^{iv} = \sum_{n=4}^{N} n(n-1)(n-2)(n-3)a_n\, x^{n-4} \tag{3.42}$$

$$y^{iv}(x) = 24a_4 + 120a_5x + 360a_6x^2 + 840a_7x^3 + 1680a_8x^4 + 30240a_9x^5 + 5040a_{10}x6$$

Now, substituting the expansion of (3.38) to (3.42) into (3.37), and after simplification we have

$$\begin{aligned} 9x^{10}a_{10} + 9a_9x^9 + 9a_8x^8 - 900a_10x^8 + 9a_7x^7 - 720a_9x^7 \\ +9a_6x^6 - 560a_8x^6 + 5040a_{10}x^6 + 9a_5x^5 - 420a_7x^5 \\ +3024a_9x^5 + 9a_4x^4 - 300a_6x^4 - 1680a_8x^4 + 9a_3x^3 \\ -200a_5x^3 + 840a_7x^3 + 9a_2x^2 - 120x^2a_4 + 360x^2a_6 \\ +9xa_1 - 60xa_3 + 120xa_5 + 9a_0 - 20a_2 + 24a_4 \ = 0 \end{aligned} \tag{3.43}$$

Using the boundary conditions,

$$y(0) = 5; \Longrightarrow a_0 = 5$$

$$y'(0) = -1; \Longrightarrow a_1 = -1$$

$$y''(0) = 21; \Longrightarrow 2a_2 = 21 \Longrightarrow a_2 = 10.5$$

$$y'''(0) = -49; \Longrightarrow 6a_3 = -49 \Longrightarrow a_3 = -8.167$$

Now, collocating (3.43) using

$$x_k \ = \ \frac{a + (b-a)}{N}$$

$$k = 1, 2, ...[N - (n-1)]$$

When $k = 1, x_1 = \frac{1}{10} = 0.1$

$$\begin{aligned} 9a_0 + 0.9a_1 - 19.91a_2 - 5.991a_3 + 22.8009a_4 + 11.80009a_5 \\ +3.570009a_6 + 0.8358009a_7 + 0.16744009a_8 \\ +0.030168009a_9 + 0.005310009a_{10} = 0 \end{aligned} \tag{3.44}$$

When $k = 2, x_2 = \frac{2}{10} = 0.2$

$$\begin{aligned} 9a_0 + 1.81a_1 - 19.64a_2 - 11.928a_3 + 19.2144a_4 + 22.40288a_5 \\ +13.920576a_6 + 6.5887152a_7 + 2.65218304a_8 \\ +0.95868608a_9 + 0.3202569216a_{10} = 0 \end{aligned} \tag{3.45}$$

When $k = 3, x_3 = \frac{3}{10} = 0.3$

$$\begin{aligned} 9a_0 + 2.7a_1 - 19.19a_2 - 17.757a_3 + 13.2729a_4 + 30.62187a_5 \\ +29.976561a_6 + 21.6613683a_7 + 13.20035049a_8 \\ +7.1910331470a_9 + 3.615164144a_{10} = 0 \end{aligned} \tag{3.46}$$

When $k = 4, x_4 = \frac{4}{10} = 0.4$

$$\begin{aligned} 9a_0 + 3.6a_1 - 18.56a_2 - 23.424a_3 + 5.0304a_4 + 35.29216a_5 \\ +49.956864a_6 + 49.4739456a_7 + 40.72013824a_8 \\ +29.78847130a_9 + 20.05495972a_{10} = 0 \end{aligned} \tag{3.47}$$

When $k = 5, x_5 = \frac{5}{10} = 0.5$

$$\begin{aligned} 9a_0 + 4.5a_1 - 17.75a_2 - 28.875a_3 - 5.4375a_4 + 35.28125a_5 \\ +71.390625a_6 + 91.9453125a_7 + 96.28515625a_8 \\ +88.89257812a_9 + 75.24316406a_{10} = 0 \end{aligned} \tag{3.48}$$

When $k = 6, x_6 = \frac{6}{10} = 0.6$

$$\begin{aligned} 9a_0 + 5.4a_1 - 16.76a_2 - 34.056a_3 - 18.0336a_4 + 29.49984a_5 \\ +91.139904a_6 + 149.0327424a_7 + 191.7518054a_8 \\ +215.0815473a_9 + 220.0841156a_{10} = 0 \end{aligned} \tag{3.49}$$

When $k = 7, x_7 = \frac{7}{10} = 0.7$

$$
\begin{aligned}
9a_0 + 6.3a_1 - 15.59a_2 - 38.913a_3 - 32.6391a_4 + 16.91263a_5 \\
+105.428841a_6 + 218.2717887a_7 + 338.0033921a_8 \\
+449.3117665a_9 + 541.3219787a_{10} = 0
\end{aligned}
\tag{3.50}
$$

Solving equation (3.44) to (3.50) using MAPPLE software we'll get:-

$$a_0 = 5$$

$$a_1 = -1$$

$$a_2 = 10.5$$

$$a_3 = -8.167$$

$$a_4 = 6.8774474066$$

$$a_5 = -4.0055733$$

$$a_6 = 2.019571593$$

$$a_7 = -0.8448066490$$

$$a_8 = 0.2914647531$$

$$a_9 = -0.07375814569$$

$$a_{10} = 0.01002659277$$

substituting the values of a_0 to a_{10} into equation (3.37), in other to get our Approximate Solution, we have

$$
\begin{aligned}
y(x) = 5 - x + 10.5x^2 - 8.167x^3 + 6.8774474066x^4 - 4.0055733x^5 \\
+2.019571593x^6 - 0.8448066490x^7 + 0.2914647531x^8 \\
-0.07375814569x^9 + 0.01002659277x^{10}
\end{aligned}
\tag{3.51}
$$

Chapter 4

DEMONSTRATION OF THE COEFFICIENT COMPARISON METHOD

4.1 NUMERICAL EXAMPLE 1

Consider the ODE

$$y''' + 4y'' - 5y' = 0 \tag{4.1}$$

together with the initial conditions:

$$y(0) = 4,\ y'(0) = -7,\ y''(0) = 23.$$

The exact solution is given as

$$y(x) = 5 - 2e + e^{5x} \tag{4.2}$$

Solution

The general form of a power series is

$$y(x) = \sum_{n=0}^{N} a_n\, x^n \tag{4.3}$$

but for this problem, we shall consider, from $n = 0$ to $n = 10$ therefore,

$$y(x) = \sum_{n=0}^{10} a_n\, x^n \tag{4.4}$$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$$y' = \sum_{n=1}^{N} na_n\, x^{n-1} \tag{4.5}$$

$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + 7a_7x^6 + 8a_8x^7 + 9a_9x^8 + 10a_{10}x^9$$

$$y'' = \sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} \tag{4.6}$$

$$y''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + 42a_7x^5 + 56a_8x^6 + 72a_9x^7 + 90a_{10}x^8$$

$$y''' = \sum_{n=3}^{N} n(n-1)(n-2)a_n\, x^{n-3} \tag{4.7}$$

$$y'''(x) = 6a_3x + 24a_4x + 60a_5x^2 + 120a_6x^3 + 210a_7x^4 + 336a_8x^5 + 504a_9x^6 + 720a_{10}x^7$$

Now, substituting the expansion of (4.5) to (4.7) into (4.4), and after simplification we have

$$\begin{aligned}
-50a_{10}x^9 - 45a_9x^8 + 360a_10x^8 - 40a_8x^7 + 288a_9x^7 + 720a_10x^7 \\
-35a_7x^6 + 224a_8x^6 + 504a_9x^6 - 30a_6x^5 + 618a_7x^5 + 336a_x^5 - 25a_5x^4 \\
+120a_6x^4 + 210a_7x^4 - 20a_4x^3 + 80a_5x^3 + 120a_6x^3 - 15a_3x^2 + 48a_4x^2 \\
+60a_5x^2 - 10a_2x + 24a_3x + 24a_4x + -5a_1 + 8a_2 + 6a_3 \ = 0
\end{aligned} \tag{4.8}$$

Using the boundary conditions,

$$y(0) = 4; \Longrightarrow a_0 = 4$$

$$y'(0) = -7; \Longrightarrow a_1 = -7$$

$$y''(0) = 23; \Longrightarrow 2a_2 = 23 \Longrightarrow a_2 = 11.5$$

Next, we have to collect like terms

$$127 + 6a_3 = 0 \tag{4.9}$$

$$-115x + 24a_3x + 24a_4x = 0 \tag{4.10}$$

$$-15a_3x^2 + 48a_4x^2 + 60a_5x^2 = 0 \tag{4.11}$$

$$-20a_4x^3 + 80a_5x^3 + 120a_6x^3 = 0 \tag{4.12}$$

$$-25a_5x^4 + 120a_6x^4 + 210a_7x^4 = 0 \tag{4.13}$$

$$-30a_6x^5 + 168a_7x^5 + 336a_85^4 = 0 \tag{4.14}$$

$$-35a_7x^6 + 224a_8^6 + 504a_9x^6 = 0 \tag{4.15}$$

$$-40a_8x^7 + 288a_9x^7 + 720a_{10}x^7 = 0 \tag{4.16}$$

We now have 8 equations, We need to get $a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ Solving equation (4.49) to (4.16) using MAPPLE, we have:

$$a_0 = 4$$

$$a_1 = -7$$

$$a_2 = 11.5$$

$$a_3 = -21.1667$$

$$a_4 = 25.9583$$

$$a_5 = -26.0583$$

$$a_6 = 21.6986$$

$$a_7 = -15.501389$$

$$a_8 = 9.688070437$$

$$a_9 = -5.382294422$$

$$a_{10} = 2.691143804$$

substituting the values of a_0 to a_{10} into equation (4.5), in other to get our Approximate Solution, we have

$$\begin{aligned} y(x) = 4 - 7x + 11.5x^2 - 21.1667x^3 + 25.9583x^4 - 26.0583x^5 \\ +21.6986x^6 - 15.501389x^7 + 9.688070437x^8 \\ -5.382294422x^9 + 2.691143804x^{10} \end{aligned} \tag{4.17}$$

4.2 NUMERICAL EXAMPLE 2

Consider the ODE

$$y^{iv} + 13y'' + 36y = 0 \tag{4.18}$$

together with the initial conditions:

$$y(0) = 0,\ y'(0) = -3,\ y''(0) = 5.\ y'''(0) = -3$$

The exact solution is given as

$$y(x) = \cos 2x - 3\sin 2x - \cos 3x + \sin 3x \tag{4.19}$$

Solution

The general form of a power series is

$$y(x) = \sum_{n=0}^{N} a_n\, x^n \tag{4.20}$$

but for this problem, we shall consider, from $n = 0$ to $n = 10$ therefore,

$$y(x) = \sum_{n=0}^{10} a_n\, x^n \tag{4.21}$$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$$y' = \sum_{n=1}^{N} na_n\, x^{n-1} \tag{4.22}$$

$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + 7a_7x^6 + 8a_8x^7 + 9a_9x^8 + 10a_{10}x^9$$

$$y'' = \sum_{n=2}^{N} n(n-1)a_n\, x^{n-2} \tag{4.23}$$

$$y''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + 42a_7x^5 + 56a_8x^6 + 72a_9x^7 + 90a_{10}x^8$$

$$y''' = \sum_{n=3}^{N} n(n-1)(n-2)a_n\, x^{n-3} \tag{4.24}$$

$$y'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + 120a_6x^3 + 210a_7x^4 + 336a_8x^5 + 504a_9x^6 + 720a_{10}x^7$$

$$y^{iv} = \sum_{n=4}^{N} n(n-1)(n-2)(n-3)a_n\, x^{n-4} \tag{4.25}$$

$$y^{iv}(x) = +24a_4 + 120a_5x + 360a_6x^2 + 840a_7x^3 + 1680a_8x^4 + 30240a_9x^5 + 5040a_{10}x6$$

Now, substituting the expansion of (4.19) to (4.23) into (4.18), and after simplification we have

$$\begin{aligned} 36x^{10}a_{10} + 36a_9x^9 + 36a_9x^8 + 1170a_{10}x^8 + 360a_7x^7 + 936a_9x^7 \\ +36a_6x^6 + 728a_8x^6 + 5040a_{10}x^6 + 36a_5x^5 + 546a_7x^5 + 3024a_9x^5 \\ +36a_4x^4 + 390a_6x^4 + 1680a_8x^4 + 36a_3x^3 + 260a_5x^3 + 840a_7x^3 \\ +36a_2x^2 + 156a_4x^2 + 360a_6x^2 + 36xa_1 + 78xa_3 + 120xa_5 \\ +36a_0 + 26a_2 + 24a_4 \ = 0 \end{aligned} \tag{4.26}$$

Using the boundary conditions,

$$y(0) = 0; \Longrightarrow a_0 = 0$$

$$y'(0) = -3; \Longrightarrow a_1 = -3$$

$$y''(0) = 5; \Longrightarrow 2a_2 = 5 \Longrightarrow a_2 = 2.5$$

$$y'''(0) = -3; \Longrightarrow 6a_3 = -3 \Longrightarrow a_3 = -0.5$$

Next, we have to collect like terms

$$36a_0 + 26a_2 + 24a_4 = 0 \tag{4.27}$$

$$36xa_1 + 76xa_3 + 120xa_5 = 0 \tag{4.28}$$

$$36x^2a_2 + 156x^2a_4 + 360x^2a_6 = 0 \tag{4.29}$$

$$36x^3a_3 + 260x^3a_5 + 840x^3a_7 = 0 \tag{4.30}$$

$$36x^4a_4 + 390x^4a_6 + 1680x^4a_8 = 0 \tag{4.31}$$

$$36x^5a_5 + 546x^5a_7 + 3024x^5a_9 = 0 \tag{4.32}$$

$$36x^6a_6 + 278x^6a_8 + 5040x^6a_{10} = 0 \tag{4.33}$$

We now have 7 equations, We need to get $a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ Solving this equation (4.27) to (4.33) using MAPPLE soft ware we'll get:-

$$a_0 = 0$$

$$a_1 = -3$$

$$a_2 = 2.5$$

$$a_3 = -0.5$$

$$a_4 = -2.70833$$

$$a_5 = 1.2250$$

$$a_6 = 0.92361$$

$$a_7 = -0.3577380952$$

$$a_8 = -0.1563740079$$

$$a_9 = 0.0500082672$$

$$a_{10} = 0.015599013448$$

substituting the values of a_0 to a_{10} into equation (4.20), in other to get our Approximate Solution, we have,

$$\begin{aligned} y(x) = -3x + 2.5x^2 - 0.5x^3 - 2.70833x^4 + 1.2250x^5 \\ +0.92361x^6 - 0.3577380952x^7 - 0.1563740079x^8 \\ +0.0500082672x^9 + 0.015599013448x^{10} \end{aligned} \tag{4.34}$$

4.3 NUMERICAL EXAMPLE 3

Consider the ODE

$$y^{iv} - 10y'' + 9y = 0 \tag{4.35}$$

together with the initial conditions:

$$y(0) = 5,\ y'(0) = -1,\ y''(0) = 21.\ y'''(0) = -49$$

The exact solution is given as

$$y(x) = 4e^{x} - e^{-x} + 2e^{-3x} \tag{4.36}$$

Solution

The general form of a power series is

$$y(x) = \sum_{n=0}^{N} a_n\,(x)^n \tag{4.37}$$

but for this problem, we shall consider, from $n = 0$ to $n = 10$ therefore,

$$y(x) = \sum_{n=0}^{10} a_n\,(x)^n \tag{4.38}$$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$$y' = \sum_{n=1}^{N} na_n\,(x)^{n-1} \tag{4.39}$$

$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + 7a_7x^6 + 8a_8x^7 + 9a_9x^8 + 10a_{10}x^9$$

$$y'' = \sum_{n=2}^{N} n(n-1)a_n\,(x)^{n-2} \tag{4.40}$$

$$y''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + 42a_7x^5 + 56a_8x^6 + 72a_9x^7 + 90a_{10}x^8$$

$$y''' = \sum_{n=3}^{N} n(n-1)(n-2)a_n\,(x)^{n-3} \tag{4.41}$$

$$y'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + 120a_6x^3 + 210a_7x^4 + 336a_8x^5 + 504a_9x^6 + 720a_{10}x^7$$

$$yiv = \sum_{n=4}^{N} n(n-1)(n-2)(n-3)a_n\,(x)^{n-4} \tag{4.42}$$

$$y^{iv}(x) = 24a_4 + 120a_5x + 360a_6x^2 + 840a_7x^3 + 1680a_8x^4 + 30240a_9x^5 + 5040a_{10}x6$$

Now, substituting the expansion of (4.34) to (4.38) into (4.34), we have

$$\begin{aligned} 9x^{10}a_{10} + 9a_9x^9 + 9a_8x^8 - 900a_10x^8 + 9a_7x^7 - 720a_9x^7 \\ +9a_6x^6 - 560a_8x^6 + 5040a_{10}x^6 + 9a_5x^5 - 420a_7x^5 \\ +3024a_9x^5 + 9a_4x^4 - 300a_6x^4 - 1680a_8x^4 + 9a_3x^3 \\ -200a_5x^3 + 840a_7x^3 + 9a_2x^2 - 120x^2a_4 + 360x^2a_6 \\ +9xa_1 - 60xa_3 + 120xa_5 + 9a_0 - 20a_2 + 24a_4 = 0 \end{aligned} \tag{4.43}$$

Using the boundary conditions,

$$y(0) = 5; \Longrightarrow a_0 = 5$$

$$y'(0) = -1; \Longrightarrow a_1 = -1$$

$$y''(0) = 21; \Longrightarrow 2a_2 = 21 \Longrightarrow a_2 = 10.5$$

$$y'''(0) = -49; \Longrightarrow 6a_3 = -49 \Longrightarrow a_3 = -8.167$$

Next, we have to collect like terms

$$9a_0 - 20a_2 + 24a_4 = 0 \tag{4.44}$$

$$9xa_1 - 60xa_3 + 120xa_5 = 0 \tag{4.45}$$

$$9x^2a_2 - 120x^2a_4 + 360x^2a_6 = 0 \tag{4.46}$$

$$9x^3a_3 - 200x^3a_5 + 840x^3a_7 = 0 \tag{4.47}$$

$$9x^4a_4 - 300x^4a_6 + 1680x^4a_8 = 0 \tag{4.48}$$

$$9x^5a_5 - 420x^5a_7 + 3024x^5a_9 = 0 \tag{4.49}$$

$$9x^6a_6 - 560x^6a_8 + 5040x^6a_{10} = 0 \tag{4.50}$$

We now have 7 equations, We need to get $a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ Solving this equations using MAPPLE soft ware we'll get:-

$$a_0 = 5$$

$$a_1 = -1$$

$$a_2 = 10.5$$

$$a_3 = -8.167$$

$$a_4 = 6.875$$

$$a_5 = 4.1585$$

$$a_6 = 2.02916667$$

$$a_7 = 0.9026154762$$

$$a_8 = 0.325520833$$

$$a_9 = 0.112986667725$$

$$a_{10} = 0.03254546958$$

substituting the values of a_0 to a_{10} into equation (4.34), in other to get our Approximate Solution, we have

$$\begin{aligned} y(x) = 5 - x + 10.5x^2 - 8.167x^3 + 6.875x^4 + 4.1585x^5 \\ +2.02916667x^6 + 0.9026154762x^7 + 0.325520833x^8 \\ +0.112986667725x^9 + 0.03254546958x^{10} \end{aligned} \tag{4.51}$$

Chapter 5

TABLE OF RESULTS

5.1 Table of results for example 1

X	Exact Solution	Standard method	Coefficient method	Error of Standard	Error of Coefficient
0	4	4	4	0	0
0.1	3.3961882400	3.3961936394	3.3961882442	4.8150×10^{-6}	0
0.2	2.9250739252	2.9250958601	2.9250739423	2.1935×10^{-5}	1.7000×10^{-8}
0.3	2.5234125443	2.5234608702	2.5234144469	4.4832×10^{-5}	1.9020×10^{-6}
0.4	2.1516858874	2.1517679253	2.1517297931	8.2038×10^{-5}	4.3906×10^{-5}
0.5	1.7846424572	1.7847644971	1.7851355820	1.2204×10^{-4}	4.9313×10^{-4}
0.6	1.4055494682	1.4057173954	1.4090882924	1.6793×10^{-4}	3.5388×10^{-3}
0.7	1.0026919691	1.0029116101	1.0213400934	2.1964×10^{-4}	1.8648×10^{-2}
0.8	0.5672337829	0.5675112024	0.6456377230	2.7742×10^{-4}	7.8404×10^{-2}
0.9	0.0919027745	0.0922484972	0.3693719610	3.4572×10^{-4}	2.7747×10^{-1}
1.0	-0.4298257090	-0.4293860864	-0.4274754682	4.3962×10^{-4}	8.5730×10^{-1}

5.2 Table results for example 2

X	Exact Solution	Standard method	Coefficient method	Error of Standard	Error of Coefficient
0	0	0	0	0	0
0.1	-0.2757576970	-0.2757575693	-0.2757575697	1.9872×10^{-5}	0
0.2	-0.5078871746	-0.5078878156	-0.5078871743	6.4100×10^{-7}	3.0000×10^{-10}
0.3	-0.7068748634	-0.7068772081	-0.7068748558	2.3447×10^{-6}	7.6000×10^{-9}
0.4	-0.8856802320	-0.8856858516	-0.8856800355	5.6196×10^{-6}	1.9650×10^{-7}
0.5	-1.0573528634	-1.0573635721	-1.0573505371	1.6709×10^{-5}	2.3260×10^{-6}
0.6	-1.2327097782	-1.2327273731	-1.2326922062	1.7595×10^{-5}	1.7572×10^{-5}
0.7	-1.4183265753	-1.4183525750	-1.4182293832	2.5997×10^{-5}	9.7192×10^{-5}
0.8	-1.6150634351	-1.6150988043	-1.6146356190	3.5374×10^{-5}	4.2782×10^{-4}
0.9	-1.8172929663	-1.8173375924	-1.8157117010	4.4626×10^{-5}	1.5813×10^{-3}
1.0	-2.0129266112	-2.0129752191	-2.0078359243	4.8608×10^{-5}	5.0907×10^{-3}

5.3 Table of result for example 3

X	Exact Solution	Standard method	Coefficient method	Error of Standard	Error of Coefficient
0	5	5	5	0	0
0.1	4.9974826954	4.9974826631	4.9974823610	3.2000×10^{-8}	3.4459×10^{-4}
0.2	5.1645035512	5.1645031902	5.1645008323	3.6100×10^{-7}	2.7190×10^{-6}
0.3	5.4717563303	5.4717549361	5.4717469310	1.3940×10^{-6}	9.3990×10^{-6}
0.4	5.8993671704	5.8993635662	5.8993644063	3.6040×10^{-6}	2.3110×10^{-5}
0.5	6.4346147440	6.4346071590	6.4345714261	7.5850×10^{-6}	4.3318×10^{-5}
0.6	7.0702613402	7.0702472350	7.0702032883	1.4105×10^{-5}	5.8052×10^{-5}
0.7	7.8033383813	7.8033141933	7.8033419790	2.4188×10^{-5}	3.5980×10^{-6}
0.8	8.6342706554	8.6342314632	8.6346711233	3.9192×10^{-5}	4.0047×10^{-4}
0.9	9.5662538102	9.5661930080	9.5681608773	6.0802×10^{-5}	1.9071×10^{-3}
1.0	10.604822013	10.604732491	10.611358960	8.9500×10^{-5}	6.5369×10^{-3}

5.4 Discussion of result

From the 3 tables above, it is clearly shown that the Standard Collocation Method performed better than the Coefficient comparison method. We assume our Approximate solution with Power Series of degree 10.

5.5 Summary

The Power series solution method is mainly used to find power series solution of differential equations whose solution cannot be expressed in term of familiar functions such as polynomials, exponential or trigonometric functions.for example

$$y'' + y = 0 \tag{5.1}$$

We know that using the general method we obtain

$$y(x) = A\cos x \ and \ B\sin x$$

but with power series we obtain a series solution, for example
$a_0\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right) + a_1\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$ In this project, I compared the standard collocation method with the coefficient comparison method, for the numerical solution of ordinary differential equations.

5.6 Conclusion

It was observed from the numerical examples that both the standard collocation method and the coefficient comparison method produced results which are almost closed to the exact solution. Out of the two methods considered, it is observed that the standard collocation method is much more efficient and accurate than the coefficient comparison method.

5.7 Recommendation

Based on the result obtained, I am recommending the Standard Collocation method to Ordinary Linear Differential equations.

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