New York City College of Technology
MAT 3770/D676 - Fall 2019
Homework 2 - Due on 10/21 at 10 am

Instructions: I will be collecting this homework on CoCalc. Go to our course project (MAT 3770 - Fall 2019) and look for the Homework 2 folder where you should upload either a pdf or a Jupiter notebook file. Include the coding part and a technical report (as if you were a consultant) addressing the questions in (a)-(d). One paragraph for each item.

Exercise 1, Section 2.4

Ecologists use the following model to represent the growth process of two competing species, x and y:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\left(1-\frac{x}{K_1}\right)-\alpha_1 xy \\ \frac{dy}{dt} &= r_2 y\left(1-\frac{y}{K_2}\right)-\alpha_2 xy. \end{aligned}$$

The variables x and x represent the number in each population; the parameters r_i represent the intrinsic growth rates of each species; K_i represents the maximum sustainable population in the absence of competition; and α_i represents the effects of competition. Studies of the blue whale and fin whale populations have determined the following parameter values (t in years):

	Blue	Fin
r	0.05	0.08
K	150,000	400,000
α	10^{-8}	10^{-8}

(a) Determine the population levels x and y that maximize the number of new whales born each year. Use the five-step method, and model as an unconstrained optimization problem.

(b) Examine the sensitivity of the optimal population levels to the intrinsic growth rates r_1 and r_2.

(c) Examine the sensitivity of the optimal population levels to the environmental carrying capacities K_1 and K_2.

(d) Assuming that $\alpha_1 = \alpha_2 = \alpha$, is it ever optimal for one species to become extinct?