

### 3.

For the bisection method

- What does the method do?
  - The method finds the value in the domain so that the value of the image is 0.
- What is required for the method to work?
  - A continuous function on the interval  $[a,b]$
  - Must have a positive value and negative value within the interval  $[a,b]$
- What problem is the fixed point iteration trying to solve?
  - Fixed Point iteration solves  $f(a) = a$  by transforming the problem into a bisection problem.

### 4.

1A) The bisection successfully calculates the  $x = 1$  root.

1B) For  $(-1, 0.5)$  both endpoints are negative so bisection cannot determine if a root exists.

1C) For  $(-1, 2)$  bisection finds the root at  $x = 1$  but not the root at  $x = 0$ .

It is not possible to find the root at  $x = 0$  with the bisection method because it will reach a point where the endpoints  $a$  and  $b$  are both negative with  $x = 0$  in the middle and it will return that a value cannot be found.

2A) It found the  $x = 1$  root so it was successful since the  $x = 3, 5$  roots are outside of the  $(0, 2.4)$  interval. It did achieve the desired accuracy.

2B) Here the bisection method was not successful since  $f(x) \leq 0$  over the entire  $(0, 2)$  interval.

2C)  $\sin(x)$  has a root at  $x = 0$  so on the interval  $(0, 0.1)$  this was a trivial problem as the root was at the initial left endpoint. It does not work on the interval  $(0.5, 3\pi/4)$  because  $\sin(x) \leq 0$  for all  $x$  on the interval.

3A) - 3D)

$f(x) = x$  for  $a-d$  for  $x = 7^{1/5}$

3A, B, C) This results in overflow because  $x = 1$  and the fixed point at  $7^{1/5}$  are in an interval where the derivative is greater than zero.

3D) The fixed point iteration algorithm converges for this function. It works for this function because the behavior follows the rules required for FPI (the area around the fixed point must have a derivative less than 1).