Pre-Lab 7

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The Vandermande matrix is constructed using powers of a vector. The matrix is constructed such that each column is a power of the corresponding element in the vector. Below is an example of how we can use this matrix and an A vector to get the data points Y:

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

and let vectors A and Y be:

$$A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

So, we have the equation $VA = Y$, so we can solve for A with $A = V^{-1}Y$. To solve for this in code, we would use np.vander(x, n -1) for our X vector and n nodes. We would then use np.linalgsolve(V, Y) to solve for A.

The lab explores interpolation techniques for the function $f(x) = \frac{1}{1+(10x)^2}$ on the interval (-1, 1). Three interpolation methods, monomial expansion, Lagrange polynomials, and Newton-Divided Differences, are employed and evaluated at 1000 points. The experiments involve comparing the approximations, analyzing errors for different values of N, and addressing the Runge phenomena by modifying interpolation nodes. The stability and behavior of the interpolation techniques, along with their performance on the function numpy.sinc(5x), are examined and compared.