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Exercises:

$$g(x) = x - \frac{f(x)}{f'(x)}$$

$$g'(x) = 1 - \frac{f'(x)^2 - f(x)f''(x)}{f'(x)^2}$$

$$|1 - 1 - \frac{-f(x)f''(x)}{f'(x)^2}| < 1$$

$$|\frac{f(x)f''(x)}{f'(x)^2}| < 1$$

1.

```
def bisection(f, df, d2f, a, b, tol, Nmax):
    fa = f(a)
    fb = f(b)
    count = 0

    if fa * fb > 0:
        ier = 1
        astar = a
        return [astar, ier]

    if fa == 0:
        astar = a
        ier = 0
        return [astar, ier]

    if fb == 0:
        astar = b
        ier = 0
        return [astar, ier]

    while count < Nmax:
        c = 0.5 * (a + b)
        fc = f(c)
        if fc == 0:
            astar = c
            ier = 0
            return [astar, ier]
        if fa * fc < 0:
            b = c
```

```
        elif fb * fc < 0:
            a = c
            fa = fc
        else:
            astar = c
            ier = 3
            return [astar, ier]
        if (f(c) * d2f(c)) / (df(c)**2) < 1:
            astar = a
            ier = 0
            return [astar, ier]
        count += 1

    astar = a
    ier = 2
    return [astar, ier]
```

3. Yes, you now need to input the first and second derivatives of your function.

5. Advantages include that if initial conditions for the bisection method are met (f(a)f(b) < 0 for a continuous function f) it is guaranteed to converge, and will do so faster since Newton's method will converge faster than bisection method. A limitation is that you need f to be twice differentiable and also satisfy the initial requirements for the bisection method.

6.

a. the approximate root is 3.000000014901161
   the error message reads: 0
   f(astar) = 1.9371511439381095e-07
b. Found approximate root ~ 3 to tol 10^-8 in 26 iterations
c. The root was ~3 in 7 iterations

The hybrid method worked the quickest, with 7 iterations. The most cost effective method was the hybrid, as it was 8 function evaluations (1 with bisection to find starting point and 7 evaluations with Newton's).