Exercise 3.2

Residual using LU Factorization for N = 100: 5.36243467359147e-14
Solution Time - Part 1 0.0044515132904052734
Solution Time - Part 2 0.0
Solution Time - Total 0.0044515132904052734

Residual using direct solve for N = 100: 5.36243467359147e-14
Solution Time 0.007314443588256836

Residual using LU Factorization for N = 500: 1.5980984187903108e-12
Solution Time - Part 1 0.6560473442077637
Solution Time - Part 2 0.0015835762023925781
Solution Time - Total 0.6576309204101562

Residual using direct solve for N = 500: 1.5980984187903108e-12
Solution Time 0.2643711566925049

Residual using LU Factorization for N = 1000: 3.0656822914098485e-12
Solution Time - Part 1 0.033106088638305664
Solution Time - Part 2 0.0
Solution Time - Total 0.033106088638305664

Residual using direct solve for N = 1000: 3.0656822914098485e-12
Solution Time 0.035530805587768555

Residual using LU Factorization for N = 2000: 8.931577024511557e-11
Solution Time - Part 1 0.10448622703552246
Solution Time - Part 2 0.001001596450805664
Solution Time - Total 0.10548782348632812

Residual using direct solve for N = 2000: 8.931577024511557e-11
Solution Time 0.15457916259765625

Residual using LU Factorization for N = 4000: 4.3326678216247574e-11
Solution Time - Part 1 0.5753633975982666

Solution Time - Part 2 0.0
Solution Time - Total 0.5753633975982666

Residual using direct solve for N = 4000: 4.3326678216247574e-11
Solution Time 0.665536642074585

Residual using LU Factorization for N = 5000: 1.4052855622028965e-10
Solution Time - Part 1 0.9105446338653564
Solution Time - Part 2 0.009062528610229492
Solution Time - Total 0.9196071624755859

Residual using direct solve for N = 5000: 1.4052855622028965e-10
Solution Time 1.576916217803955

The LU Factorization is better for larger N which makes sense because after the original matrix is decomposed, then the cost for solving is relatively low compared to the system the built-in Python function uses.

Exercises 3.4
1. We validated the answers by computing the L2 norm and comparing the residuals. The QR factorization did better than the normal equations when comparing the residuals because it was more accurate (the norm was smaller). For large N, the normal equations solving method was faster.
2. By decreasing the smallest entry in the matrix, solving through the two methods became less accurate however, QR Factorization was the more accurate out of the two.