STAT 394
Probability I
Summer 2017

Problem Session # 3

Instructions: Find the group with the circled number. Start working together on the circled question. Continue with the next question (1 is the next question after the last). Do as many as you have time for.

1. Consider a horse race with 6 horses, which are numbered 1 through 6. Suppose any particular order of the 6 horses finishing the race are equally likely, and ignore the possibility of ties. Let event A denote the event that number-1 horse finish among the top three, and event B denote the event that number-2 horse finish the second place. Calculate the following probability: $P(A \cup B)$, $P(A|B)$, and $P(B|A)$.

2. In 1990 - 1993, the National Basketball Association (NBA) draft lottery involves the 11 teams that had the worst wonlost records during the year. A total of 66 balls are placed in an urn. Each of these balls is inscribed with the name of a team: Eleven have the name of the team with the worst record, 10 have the name of the team with the second-worst record, 9 have the name of the team with the third-worst record, and so on (with 1 ball having the name of the team with the 11th-worst record). A ball is then chosen at random, and the team whose name is on the ball is given the first pick in the draft of players about to enter the league. Another ball is then chosen, and if it "belongs" to a team different from the one that received the first draft pick, then the team to which it belongs receives the second draft pick. (If the ball belongs to the team receiving the first pick, then it is discarded and another one is chosen; this continues until the ball of another team is chosen.) Finally, another ball is chosen, and the team named on the ball (provided that it is different from the previous two teams) receives the third draft pick. The remaining draft picks 4 through 11 are then awarded to the 8 teams that did not win the lottery, in inverse order of their wonlost records. For instance, if the team with the worst record did not receive any of the 3 lottery picks, then that team would receive the fourth draft pick. Let X denote the draft pick of the team with the worst record. Find the probability mass function of X.

3. Matt and Jimmy play a series of games. Each game is independently won by Matt with probability p, and by Jimmy with probability $1 - p$. They stop when the total number of wins of one player is two greater than that of the

other player, and the player with greater number of wins is declared the winner. What is the probability that Matt wins the game? (Hint: if the outcome of the first two games are one win each for each player, what is the probability of winning for Matt then?)

4. Denote by d the dominant gene and by r the recessive gene at a single locus. Then dd is called the pure dominant genotype, dr is called the hybrid, and rr the pure recessive genotype. The two genotypes with at least one dominant gene, dd and dr, result in the phenotype of the dominant gene, while rr results in a recessive phenotype. Assuming that both parents are hybrid and have n children, what is the probability that at least two will have the recessive phenotype?

5. You and your opponent both roll a fair die. If you both roll the same number, the game is repeated, otherwise whoever rolls the larger number wins. Let X be the number of times the two dice have to be rolled before the game is decided. (a) Determine the probability mass function of X. (b) Compute the probability of you winning the game. (d) Assume that you get paid \$10 for winning in the first round, \$1 for winning in any other round, and nothing otherwise. Compute your expected winnings.