Pr(the card after first A is also A)

$$= \frac{48! \times \binom{49+3-1}{3} \times \underbrace{4\times3\times2\times1}}{52!}$$

$$= \frac{4}{52}$$

or you can use the reasoning from today and calculate $Pr(\cdots) = 4 \times \frac{1}{52}$ directly.

Stat 394 Probability I

Lecture 4

Richard Li

July 3, 2017

→ arrange 48 non-A cards

→ # of ways to put 3 identical objects into 49 boxes

→ Arrange the 4 A's and consider first two as one object

But $p(AB) = P(AB|C)P(C) + P(AB|C^c)P(C^c)$

$= \frac{1}{4} \times p + 1 \times (1-p)$

$= 1 - \frac{3}{4}p \Rightarrow p(AB) = p(A)p(B)$ only when $p = 1$ or 0

otherwise they are dependent.

Example

Two coins, coin A: regular coin, and coin B: both sides are heads.

A: Randomly pick one coin and toss once, it gives you H.

B: Toss the same coin a second time, it gives you H.

C: The coin you picked is coin A

Conditional on C

$P(A|C) = \frac{1}{2}$

$P(B|C) = \frac{1}{2}$

$P(AB|C) = \frac{1}{4}$

$\Rightarrow$ A and B conditionally indep. given C.

Conditional on C^c

$P(A|C^c) = 1$

$P(B|C^c) = 1$

$P(AB|C^c) = 1$

$\Rightarrow$ A and B conditionally indep. given C^c

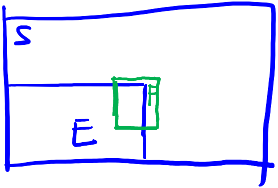
Unconditional: $P(A) = P(A|C)P(C) + P(A|C^c)P(C^c)$, Let $p = P(C)$

$= \frac{1}{2} \times p + 1 \times (1-p) = 1 - p/2$

$P(B) = 1 - p/2$ as well similarly

Example

Does $P(E) \leq P(E|F)$ in general?



Counter-example 1

No! And No for the other direction too

Counter-example 2 Toss 2 coins

E: first toss gives H

F_1: Both tosses give H

F_2: —— " —— T

$P(E) = \frac{1}{2}$

$P(E|F_1) = 1$

$P(E|F_2) = 0$.