

**STAT 394**  
**Probability I**  
**Summer 2017**

**Problem Session # 1**

*Instructions:* Find the group with the circled number. Start working together on the circled question. Continue with the next question (1 is the next question after the last). Do as many as you have time for.

1. Winthrop, a pre-med student, has been summarily rejected by all 126 US medical schools. Desperate, he sends his transcripts and MCATs to the two least selective foreign schools he can think of, the two branch campuses (N and E ) of Swampwater Tech. Based on the success his friends have had there, he estimates that his probability of being accepted at E is 0.7, and at N, 0.4. He also suspects there is a 75% chance that at least one of his applications will be rejected. What is the probability that at least one of the schools will accept him?

2. Three fair dice are rolled, and the numbers are added together. The outcomes 9 and 10 can both be obtained in six different ways:

$$10 = 1 + 3 + 6 = 1 + 4 + 5 = 2 + 2 + 6 = 2 + 3 + 5 = 2 + 4 + 4 = 3 + 3 + 4$$

$$9 = 1 + 2 + 6 = 1 + 3 + 5 = 1 + 4 + 4 = 2 + 2 + 5 = 2 + 3 + 4 = 3 + 3 + 3$$

Yet, experienced gamblers claim that 10 is more likely than 9. Explain.

3. What is the probability that in a well shuffled deck of cards all four aces are adjacent?
4. In the game of odd man out, each player tosses a fair coin. If all the coins come out the same, except for one, the minority coin is declared "odd man out" and is out of the game. Suppose that three people play odd man out. What is the probability that on the first toss someone will be eliminated? What if they play the same game by rolling a die?
5. Three balls are dropped at random into three boxes. What is the probability that exactly one box is empty?
6. Suppose you draw one card at the time from a deck of 52 cards. What is the probability that the second card drawn is an ace?

7. The probability that there are  $n$  insured losses throughout a year obey the rule  $p_{n+1} = p_n/5$ . What is the probability that there are two or more insured losses?
8. Without calculation, use logic and definition to explain

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

9. Without calculation, use logic and definition to explain

$$k \binom{n}{k} = n \binom{n-1}{k-1}$$

10. A woman has  $n$  keys, of which one will open her door. If she tries the keys at random, discarding those that do not work, what is the probability that she will open the door on her  $k$ th try? What if she does not discard any of the tried keys (i.e., in this case, she might try the same key multiple times in a row)?