STAT 394
Probability I
Summer 2017

Final Exam
July 19, 2017

This is a closed-book exam, you may refer to the formulas on the second page.
Please show your work!

Name: ______________________________

- There are 16 questions, a total of 80 points.
- There are two extra credit questions, each worth 5 points. You may select one of them and the bonus points will be added to your total score (but no more than 80 points in total).
- If you did both extra credit questions, only the one with the higher score will be counted.

Students: Please leave this table blank!!!

	Total Points	Points Awarded
Q1	3	
Q2	3	
Q3	4	
Q4	4	
Q5	4	
Q6	4	
Q7	5	
Q8	5	
Q9	5	
Q10	5	
Q11	5	
Q12	6	
Q13	6	
Q14	6	
Q15	6	
Q16	8	
Extra 1	*5*	
Extra 2	*5*	
Total	80	

Formula sheet

- Binomial distribution (n, p):

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}, E(X) = np, Var(X) = np(1-p)$$

- Poisson distribution (λ):

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}, E(X) = \lambda, Var(X) = \lambda$$

- Geometric distribution (p):

$$P(X=k) = p(1-p)^{k-1}, E(X) = \frac{1}{p}, Var(X) = \frac{1-p}{p^2}$$

- Negative binomial distribution (r, p):

$$P(X=k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, E(x) = \frac{r}{p}, Var(X) = \frac{r(1-p)}{p^2}$$

- Geometric series: $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$
- Exponential function: $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!} = lim_{n\to\infty}(1+(\frac{x}{n}))^n$

1. *(3 points)* 4 different awards are to be distributed among 10 students. How many distinct results are possible if no student is to receive more than one award?

2. *(3 points)* 4 identical candies are to be distributed among 10 students. How many distinct results are possible if no student is to receive more than one candy?

3. *(4 points)* Calculate $P(X \leq 3)$ for:

   (a) $X \sim Bin(3, 0.7)$

   (b) $X \sim Poi(1)$

4. *(4 points)* If $X \sim Bern(p)$, write down $Var(X)$ and $Var(2X)$.

5. *(4 points)* Three independent flips of a fair coin are made. Let X denote the number of heads obtained. Plot the probability mass function and the cumulative distribution function of the random variable $X - 1$.

6. *(4 points)* State and prove the inclusion-exclusion formula for two events E and F (only describing the Venn diagram does NOT count as proof).

7. *(5 points)* Denote by d the dominant gene and by r the recessive gene at a single locus. Then dd is called the pure dominant genotype, dr is called the hybrid, and rr the pure recessive genotype. The two genotypes with at least one dominant gene, dd and dr, result in the phenotype of the dominant gene, while rr results in a recessive phenotype. Assuming that both parents are hybrid and have n children, what is the probability that at least three will have the recessive phenotype?

8. *(5 points)* In the game of odd man out, each player tosses a fair coin. If all the coins come out the same, except for one, the minority coin is declared "odd man out" and is out of the game. Suppose that 5 people play odd man out. What is the probability that on the first toss someone will be eliminated?

9. *(5 points)* A deck of 52 cards is shuffled and the cars are then turned over one at a time. Given that after turning over 20 cards, you haven't seen any ace,

   (a) what is the probability that the next card is an ace?

   (b) what is the probability that the following two cards are both aces?

   (c) what is the probability that the last card of the deck is an ace?

10. *(6 points)* For three events A, B, and C. State if the following statements are true or false. No proof or counterexamples are required in this question.

    (a) Suppose A is independent of B, then A is independent of B^c.

    (b) Suppose A and B are independent events and $P(C) > 0$, then $P(AB|C) = P(A|C)P(B|C)$.

    (c) For three events A, B, and C. If $P(A|C) > P(B|C)$, and $P(A|C^c) > P(B|C^c)$, then $P(A) > P(B)$.

11. *(5 points)* A multiple choice exam has 4 choices per question. On 80% of the questions you think that you know the answers, while on the remaining 20% you guess randomly. When you think you know the answer you are right 75% of the time.

    (a) What is the probability of getting an arbitrary question right?

    (b) If you do get a question right, what is the probability that it was a lucky guess?

12. *(6 points)* Suppose students enter the Statistics Learning Center for tutoring at a rate of 2 people per minute.

    (a) What is the probability that no one enters between 10:30 and 11:00?

    (b) Suppose the Statistics Learning Center has 10 hour-long sessions each day, i.e., 8:00 - 9:00, 9:00 - 10:00, ..., 17:00 - 18:00, each hosted by a different tutor. What is the probability that during a day, exactly 4 out of these 10 tutors helped exactly 9 people each? You may assume the number of people entering different sessions is independent of each other.

13. *(6 points)* In some population, 45% of the people love cheese, 20% love wine and 5% love both cheese and wine. According to a study related to food trends, 10% of cheese lovers are fond of French food, this rate is 20% for wine lovers and 30% for lovers of both cheese and wine. If a randomly selected person loves at least cheese or wine, what is the probability that this person is fond of French food?

14. *(6 points)* Each of the 1000 members of a criminal organization independently has probability $1/10^3$ of being an undercover FBI agent.

   (a) What is the approximate probability that there exist at least two FBI agents in the organization?

   (b) Suppose you know that an undercover FBI agent, James Bourne, is among the 1000 members under an alias, but you don't know which member is him. What do you think is the probability that there are at least two agents in the organization?

15. *(6 points)* Consider a box containing 4 balls labeled 1 to 4. You randomly draw the balls with replacement 10 times (i.e., you take out a ball, read the number on it, put it back, and then draw the next one).

   (a) Compute the probability that at least one number is drawn exactly 6 times.

   (b) Compute the probability that at least one number is drawn exactly once. *Hint: inclusion-exclusion formula may be helpful.*

16. *(8 points)* Throw a die two times and let X be the number of even-numbered results: 2, 4 or 6, in these two throws. Throw it two more times and let Y be the quantity of the dice (among the last two dice, not the total of four dice) which are equal to 6. For example, if we have the sequence $\{3, 6, 6, 5\}$ of four dice, then $X = 1$ and $Y = 1$. Calculate $E(X - Y)$, and $Var(X - Y)$.

Extra credit questions

1. *(5 points)* Each day, you independently decide, with probability p, to flip a fair coin. Otherwise, you do nothing.

   (a) What is the probability of getting exactly 10 Heads in the first 20 days?

   (b) What is the probability of getting 10 Heads before 5 Tails?

2. *(5 points)* Marshawn is watching a movie with two bowls of skittles (a type of candy). Each time he wants to eat skittles, he is equally likely to take one piece from either bowl. Consider the moment when Marshawn reaches his hand into a bowl and discovers that it is empty. If both bowls initially contained N pieces of skittles, what is the probability that there are exactly k pieces of skittles, $k = 1, 2, ..., N$ in the other bowl?