Stat 394 Probability I

Lecture 1

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Counting

A silly example:

- There are 4 weeks in the summer quarter.
- Each week there are 3 lectures of Stat 394.
- How many lectures there are in total?

Principles of counting

The basic principle of counting

2 experiments   1 : m possible outcomes
2 : n possible outcomes

Total : $(m \times n)$ possible outcome.

The generalized basic principle of counting

r experiments: i-th experiment has n_i possible outcomes

together $(n_1 \times n_2 \times n_3 \cdots n_r)$ possible outcomes

Example: counting

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- What if each number ~~of~~ or letter can only be used once?
  - $26 \times 25 \times 10 \times 9 \times 8 \times \cancel{10}\ 7 = \cancel{468,000}$ 3,276,000

Permutation

- In a basketball game, 5 players from the starting lineup are introduced before the game.
- How many possible ordered arrangements could there be?

1st 5 players

2nd 4

3rd 3

4th 2

5th 1

$5 \times 4 \times 3 \times 2 \times 1 = 5!$

Permutation

In general, Arrange the order of n objects

there are $n!$ different orderings

$$\left(n! = n \times (n-1) \times \cdots \times 3 \times 2 \times 1 \right)$$

By convention $0! = 1$

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$\frac{5!}{2!2!}$

FF G_1 G_2 C

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$\underbrace{\text{FFGGC}}$

$\underbrace{\text{exp1}}$ $\underbrace{\text{exp2}}$ $\underbrace{\text{exp3}}$

$x \times 2! \times 2!$

$= 5!$

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Permutation

In general, for n object, of which n_1 are alike, n_2 are alike, ..., n_r are alike, the total number of different permutations is

$$\underbrace{\frac{n!}{n_1!n_2!\cdots n_r!} = \binom{n}{n_1, n_2, \dots, n_r}}$$

This relates to "combinations"
in the next few slides

Combinations

- Now, consider again counting the permutations of *GGFFF*
- An alternative view:

Combinations

- Now, consider again counting the permutations of *GGFFF*
- An alternative view:
  - find 2 out of 5 slots to put G.
  - $\binom{5}{2} = \frac{5!}{3! \times 2!}$
- In general, Find r slots from n of them

Define $\binom{n}{r} = \frac{\boxed{n!}}{\boxed{(n-r)!r!}} = \frac{n \times (n-1) \times \cdots \times (n-r+1) \times \cancel{\boxed{(n-r) \times \cdots \times 1}}}{\cancel{\boxed{(n-r) \times \cdots \times 1}} \times r \times (r-1) \times \cdots \times 1}$

$= \frac{n \times (n-1) \times \cdots (n-r+1)}{r \times (r-1) \times \cdots \times 1}$

r terms

also r terms

Combinations

Some special cases or convention:

- $\binom{n}{0} = 1$
- $\binom{n}{n} = 1$
- $\binom{n}{k} = 0$, if $k < 0$ or $k > n$

Example

You are at a Poke place, from 5 kinds of fish and 10 choices of toppings, how many different combinations consisting of 3 fish and 5 toppings can be formed (assume you can't choose the same fish more than once)

fish $\binom{5}{3}$

topping $\binom{10}{5}$

combination = $\binom{5}{3}\binom{10}{5} = \frac{5\times4\times3}{3\times2\times1}\times\frac{10\times\cdots\times6}{5\times\cdots1}$

$= 2520$

Example

What if 2 of the toppings are very spicy, and you don't want to have them both together?

Multinomial coefficients

A set of n distinct items is to be divided into r distinct groups of size $n_1, n_2, \ldots, n_r$, where $\sum_{i=1}^{r} n_i = n$. How many different divisions are possible?

Example

In a knockout tournament involving 64 players. In each round, the players are divided into pairs, and the winners go on the next round. How many possible outcomes are there for the first round?

Example

Example

Summary

- Permutation and combination are strongly related, as we have seen in examples.
- We will see more of these in the chapters to come.
- Read Chapter 1 on your own (especially multinomial theorem in Sec 1.5).
- Homework 1 due next Monday.