1 Introduction

$f(\phi(u,v)) = g(u,v)$

$$\phi(x,0) = (x,0) \tag{1}$$

$\phi = (\phi_1, \phi_2)$
implies the folowing:
$y\, divides\, \phi_2(x,y)$
ie
$\phi_2(x,y) = y.k(x,y)$
and:
$\phi_1(x,y) = x + y.q(x,y)$
for some polynomials q, k.

2 Gk conditions

the Gk are all the partial derivative of order less than k of $f(phi)$ and g are equale.

3 G1 Equation

Because of the 1 the jacobian of ϕ is of the forme:

$$D\phi = \begin{pmatrix} 1 & a(u,v) \\ 0 & b(u,v) \end{pmatrix}$$

a and b are functions of u,v.

and the G1 condition: $\partial_v g = a(u,v)\, \partial_u f(\phi) + b(u,v)\, \partial_v f(\phi)$ with $v = 0$. can also be written:

$$\partial_v g = (\partial_u f(\phi), \partial_v f(\phi)) \begin{pmatrix} a \\ b \end{pmatrix}$$

4 G2 condition

when $v = 0$:

$$\partial_v^2 g = (a,b)(DDf)(\phi) \begin{pmatrix} a \\ b \end{pmatrix} + Df(\phi)\partial_v \begin{pmatrix} a \\ b \end{pmatrix}$$

DDf is the hessian of f.

5 Deduce gluing data from a surface

5.1 Gluing condition

We use the following G^1 conditions: can also be written:

$$c(\partial_v g) = (\partial_u f(\phi), \partial_v f(\phi)) \begin{pmatrix} a \\ b \end{pmatrix}$$

5.2 existantce of a solution

After homogenisation, the polynomial solution exists according to hilbert theorem on syzygies.

5.3 Practical case: R^2 domain

In case of an R^2 domain. we multiply the G1 equation above by $\partial_u f$ we get

$$c(\partial_u f \times \partial_v g) = (\partial_u f \times \partial_v f) b$$

we multiplyu the G1 equation above by $\partial_v f$ we get

$$c(\partial_v f \times \partial_v g) = (\partial_v f \times \partial_u f) a$$

Remark: in the RHS of the two equations $(\partial_v f \times \partial_u f) = -(\partial_u f \times \partial_v f)$.
RMK: one solution will be to take $c = (\partial_u f \times \partial_v f)$ and a,b will follow.

5.4 Practical case: R^3 domain

In case of an R^3 domain, we take $c = LCM(\partial_u f \times \partial_v f)$, and the reste will follow.

The other value of c that can be tested $c = Prod(\partial_u f \times \partial_v f)$ product of all components of the vector.

6 Computing the basis over a corresponding topology

The G1 condition is a polynomial of variable u of degree $k = 3 * deg_u f$ at most.

by evaluating this polynomial at k values, we get the system determining the $G1$ relations.

So we are looking for functions $F = \sum_{i,j} f_{i,j} B^{i,j}$ $G = \sum_{i,j} g_{i,j} B^{i,j}$, such that:

$$c(\partial_v g) = (\partial_u f(\phi), \partial_v f(\phi)) \begin{pmatrix} a \\ b \end{pmatrix}$$

by suing this notation for the CP:

$$\begin{matrix} g_{01} & g_{11} & g_{21} & g_{31} & g_{41} \\ f_{00} & f_{10} & f_{20} & f_{30} & f_{40} \\ f_{01} & f_{11} & f_{21} & f_{31} & f_{41} \end{matrix}$$

we get the folowing equation:

$$c \sum B_n^i (g_{i,1} - f_{i0}) = a \sum B_{n-1}^{i-1} (f_{i0} - f_{i-1,0}) + b \sum B_n^i (f_{i,1} - f_{i0})$$

We can organize a system in the folowing way:

$$M \begin{pmatrix} g_{01} \\ g_{11} \\ g_{21} \\ g_{31} \\ g_{41} \\ f_{00} \\ f_{10} \\ f_{20} \\ f_{30} \\ f_{40} \\ f_{01} \\ f_{11} \\ f_{21} \\ f_{31} \\ f_{41} \end{pmatrix}$$

with M contain k lines (the number of sample points given above), each one given by: $Mi, * = (G_1, F_0, F_1)$ and

$$G_1 = \begin{pmatrix} c(p_k)B_n^0(p_k) & ... & c(p_k)B_n^n(p_k) & \end{pmatrix}$$

$$F_0 = \begin{pmatrix} c(p_k)B_n^0(p_k) + a(p_k)B_{n-1}^0 + b(p_k)B_n^0(p_k) \\ \bigoplus_{i=1..n-1} c(p_k)B_n^i(p_k) - a(p_k)B_{n-1}^{i-1} + a(p_k)B_{n-1}^i + b(p_k)B_n^i(p_k) \\ c(p_k)B_n^n(p_k) - a(p_k)B_{n-1}^{n-1} + b(p_k)B_n^n(p_k) \end{pmatrix}$$

$$G_1 = \begin{pmatrix} -b(p_k)B_n^0(p_k) & ... & -b(p_k)B_n^n(p_k) & \end{pmatrix}$$