# Long-Term Debt, Default Risk, and Policy Transmission during Severe Recessions*

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**Abstract**

A growing empirical literature finds that long-term debt and its maturity reduce firm-level investment, particularly during recessions. I introduce long-term debt maturity, alongside short-term financial savings, into an otherwise standard general equilibrium model in which heterogeneous firms borrow subject to default risk. A stochastic process governs whether all or none of a firm's debt matures in a period. This allows the model to capture the persistent negative effect of maturing debt on investment found in the data.

I apply the model to the 2020 recession to assess how firm investment responds to a severe recession and to evaluate the effects of firm-targeted stimulus at the firm- and aggregate level. Absent stimulus, the model generates a large, brief, contraction, during which maturing debt lowers investment more than in ordinary times. Cash grants boost firm investment, accelerating the recovery. However, this acceleration increases the long-term risk-free interest rate that anchors firm loan rate schedules, making borrowing more expensive. The higher borrowing costs outweigh the benefits of cash grants for the most indebted firms, increasing defaults over the recession's first year. An alternative policy that lowers long-term interest rates is more effective at reducing rollover risk and preventing defaults. However, it has little effect on larger firms' investment demand and therefore has small aggregate effects. When combined, cash grants and rate reductions strengthen the economic recovery and lower the default rate.

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# 1 Introduction

How does long-term debt and its maturity affect firms' investment activities over severe recessions? Empirical studies have found that firms with a large share of long-term debt maturing during the Global Financial Crisis had lower investment than otherwise similar firms (e.g., Almeida et al., 2011; Kalemli-Özcan, Laeven, & Moreno, 2018). I consider this question in a heterogeneous firm model where long-term debt matures probabilistically. This relatively novel approach to modeling long-term debt generates skewness in rollover risk, allowing my framework to replicate salient facts. Firms with maturing debt have significantly weaker investment over the following two years than otherwise identical firms. Additionally, the negative effect of maturity on investment is larger during recessions than in the model's steady state.

To evaluate the firm-level and aggregate implications of long-term debt, I develop a quantitative, general equilibrium environment with persistent heterogeneity across firms. Debt is long-term, noncontingent, and defaultable. Debt matures probabilistically, with either none or all of a firm's debt maturing in a given period. Any firm with non-maturing debt can choose to prepay, subject to a small transaction cost. Hence, while some firms must pay back their debt in a given period, no firm is excluded from the debt market. Moreover, unlike conventional long-term debt models, my probabilistic maturity structure prompts firms to hold financial debt and financial savings simultaneously, allowing an additional relevant element of realism.

When calibrated to U.S. data on firm-level investment rates and leverage, my model predicts large real consequences arise from the noncontingent nature of debt. Even in steady state, the average firm only invests two-thirds of what it would choose absent financial frictions. I apply the model to the U.S. 2020 recession to assess the extent to which these inefficiencies may be amplified over a large downturn, and the extent to which the specific modeling of long-term debt influences the changes in investment and default. This particular recession provides an unprecedented opportunity to assess the effects of firm-targeted stimulus during a severe recession. The Paycheck Protection Program (PPP) directed roughly $850 billion in forgivable loans to small businesses. Meantime, the Federal

Reserve expanded its quantitative easing program, increasing its balance sheet by two-thirds over the first two months of the crisis. My model allows a controlled setting where I can decompose the direct and indirect consequences of these policies for individual firms. I consider various cuts of the distribution, include size and debt maturity status, to examine how firms' disparate responses ultimately affect the shape and speed of the economic recovery. Modeling PPP as targeted cash grants and the expansion of quantitative easing as effectively a reduction in the baseline risk-free interest rate, I find that cash grants have large general equilibrium effects on long-term interests, and that cash grants and rate reductions prevent more defaults when implemented in tandem than when either is implemented alone. Since identifying such effects from data is challenging, quantitative analyses such as this one are a useful complement to the growing empirical literature examining the 2020 recession.

I find a nonmonotonic relationship between a firm's size, as measured by assets, and negatively it responds to the recession. Surprisingly, the largest firms experience steeper declines in investment than smaller firms. High-net worth firms' investment is not reliant on external financing, and is entirely shaped by the direct effects of the pandemic shocks and changes in equilibrium prices. In general, firms with fewer assets are inefficiently small and allocate all available funds into investment. Consequently, their investment decisions are less responsive to the recession.[1] There are two exceptions to this. Firms with debt maturing at the start of the recession reduce their investment by 1.5 percentage points more than otherwise identical firms. This response is 0.4 percentage point larger than in the steady state; maturing debt is more detrimental during the recession. Additionally, when the smallest firms receive large transitory declines in productivity, their investment is persistently weaker than that of similarly sized firms that do not receive the shock. In contrast, investment among the 10 percent largest firms is unaffected by transitory declines in productivity. While many firms with debt overhang have muted responses to the recession, financial frictions can amplify the effects of real shocks and generate larger declines in investment.

[1] This pattern is similar to empirical studies over business cycles (Moscarini & Postel-Vinay, 2012) and is consistent with data on the 2020 recession (Krumann, Lalé, & Ta, 2021). Additionally, this relates to the observation in Ottonello and Winberry (2021) that the investment of firms with high default risk is less responsive to changes in the expected return on capital.

I implement a cash grant policy akin to PPP to evaluate how effective such policies are at stimulating firms' investment during the recession. Cash grants spur a moderately faster recovery, with output over the first year of the recession rising 0.7 percentage point relative to when no stimulus is provided. However, this faster recovery leads to a steeper rebound in aggregate consumption, which puts upward pressure on the risk-free one-period interest rate. Lenders respond to this upward shift in future interest rates by increasing the risk-free borrowing rate at the start of the recession. This exacerbates debt overhang across the most indebted firms. Consequently, the default rate over the first year of the recession is 0.1 percentage point *higher* when cash grants are implemented than when no stimulus is provided. The higher default rate is entirely due to the policy's effects on aggregate prices. In a partial equilibrium counterfactual, I show that cash grants generate an immediate, sustained decline in defaults.

A notable effect of quantitative easing is that it lowers long-term interest rates. To study how firms respond to a reduction in long-term rates, I implement a policy that prevents the risk-free lending rate from rising above its steady state level. Reducing borrowing costs eases the negative effect of maturing debt on investment, returning it to its steady state level. By reducing rollover risk, rate reductions offset about one quarter of the excess defaults generated by the recession, and spur investment across highly leveraged firms. Since firms with high leverage tend to be small, the policy predominately increases the investment of firms with fewer assets. This is consistent with the empirical findings of Lakdawala and Moreland (2019) that smaller firms' investment responds more to monetary policy shocks under quantitative easing than larger firms'. But since rate reductions do not stimulate larger firms' investment, the policy"s aggregate effects are small. This finding suggests that, to the extent that quantitative easing stimulates the real economy, it does not do so by increasing firms' demand for debt.

In practice, PPP and quantitative easing were implemented in tandem during the 2020 recession. In the model, cash grants and rate reductions operate through different channels: the grants increase firms' cash-on-hand, while rate reductions ease borrowing conditions. Consequently, simultaneously implementing the policies prevents more defaults than either policy does on its own. However, since most firms have low default risk, the aggregate recovery is only slightly stronger than when cash grants are implemented alone. The costs of the cash grant policy exceed its benefits. Even when implemented alongside rate reductions, each dollar of grants returns just 64 cents in additional output.

The real side of the model follows the literature standard closely (e.g., Ottonello & Winberry, 2021; Khan, Senga, & Thomas, 2016). I develop a one-sector, real general equilibrium model, in which firms are subject to idiosyncratic, persistent productivity risk. This persistence, combined with firms' use of a decreasing returns to scale technology, generates a nontrivial distribution of firms in equilibrium. Production relies on physical capital and labor; the latter is supplied by a representative household. Firms' assets consist of physical capital and risk-free financial savings.

What distinguishes this environment from those examined in previous quantitative studies is that firms borrow using long-term debt that matures probabilistically.[2] Maturity is probabilistic: every period, each firm receives a shock that determines whether none or all of its debt matures in that period. Firms that do not receive the maturity shock can choose to prepay their debt and borrow anew, subject to a small prepayment cost. This ensures that no firm is restricted from the debt market. The model accounts for several important aspects of firm financial frictions. Under one-period maturity, firms optimally allocate all available funds towards increasing their capital or reducing their debt (Ottonello & Winberry, 2021)). This result does not hold under probabilistically maturing debt, allowing the model to capture more of the dynamic effects of debt overhang. Firms with a high degree of debt overhang reduce their investment and increase their dividends. This increases the share of expected returns going to shareholders, but also increases the firm's default risk, and hence is inefficient at the aggregate level.

Probabilistically maturing debt also allows the model to address empirical findings on the real effects of maturing debt (e.g., Almeida et al., 2011; Kalemli-Özcan, Laeven, & Moreno, 2018) and extend

[2] Geelen (2016) and Chen, Xu, and Yang (2021) consider a similarly styled contract, which they refer to as lumpy maturity. In these settings, firm revenues are exogenous and aggregate prices are exogenously determined. This strand of the literature is more focused on analyzing the optimal maturity structure, which I abstract from here to assess the interactions between debt and investment choices.

on theoretical research exploring its implications (e.g., Chen, Xu, & Yang, 2021; Deng & Fang, 2020; Jungherr et al., 2021). Maturity has competing effects on firm investment in the model. Firms with maturing debt must pay back the entirety of their debt, reducing funds available for investment. However, they can also frictionlessly access the debt market. In contrast, firms with non-maturing debt have lower payments, but can only access the market by paying a cost.[3] Young firms, in particular, are inefficiently small and benefit from opportunities to increase their borrowing and expand more quickly. In contrast with other existing quantitative models, this framework captures the net effect of long-term debt maturity on investment.

Probabilistic maturity also generates a nontrivial distinction between a firm's financial debt and assets; firms optimally hold both long-term debt and one-period, risk-free financial savings. During the recession, firms' aggregate debt and aggregate savings both rise. High-net worth firms borrow to hold precautionary savings, which ensure that their future investment will not be curtailed by losses in profit. Such behavior is consistent with the "borrow to save" phenomenon observed by Xiao (2019). The separation between financial savings and debt also ensures that providing cash grants is not isomorphic to a debt forgiveness policy. This distinction is not feasible in the standard quantitative framework of long-term firm debt, where no firm would simultaneously hold financial savings and debt (e.g., Gomes, Jermann & Schmid, 2016; Bustamante, 2019).[4,5] Additionally, under probabilistic maturity, long-term debt issuance is infrequent, as is true in the data. Consequently, the model will not overstate the impact of lowering long-term interest rates: only firms that take on new debt directly benefit from the policy.

I compare the equilibrium in my model to that in an otherwise identical setting where debt matures geometrically, as is conventional in the long-term debt literature. In my model with probabilistically maturing debt, firms repay their full principal all at once when it matures, and they may take on

[3] I calibrate this cost to match the average prepayment share in the data.

[4] This framework, which extends on Leland (1994, 1995), relies on granular debt maturity; a share of long-term debt matures in every period, and there is no adjustment cost on new debt. Consequently, there is no benefit to holding financial savings and debt simultaneously; the firm would be better off using the savings to reduce its debt.

[5] Khan and Lee (2021) is a prominent exception. In their model, entrepreneurs borrow using contracts that never mature, but can be redeemed at the entrepreneur's discretion. Consequently, this model abstracts from instances when the maturity of debt is detrimental to the entrepreneur.

no new debt in the interim. Thus, each loan is priced only once. When the lender sets the loan schedule it offers a firm, it factors in the fact that it can only discipline that firm's leverage choices at origination. In contrast, with geometrically maturing debt, indebted firms are active in the debt market each period, so their decisions are influenced by loan price schedules throughout the life of a loan. Consequently, lending conditions are more restrictive in my environment than in settings where debt matures geometrically. This greater restrictiveness prevents a substantial number of the defaults that occur under geometric maturity, lowering the average aggregate default rate from 8.7 percent to 1.6 percent annually. At the same time, it also inhibits firms' growth and thereby reduces steady state output by 1.5 percent.

To emulate the 2020 recession, I rely on two sets of shocks. I assume a rise in households' disutility of labor, a reduced-form way of capturing workers' desire to minimize their risk of contracting the virus. This increases the equilibrium wage, consistent with the rise in labor costs observed at the start of the recession. Additionally, a subset of firms is subject to temporary "lockdowns", which operate like negative productivity shocks. For example, a restaurant may only be able to operate at half capacity, or a retailer may need to reduce hours of operation. Lockdowns are independently and identically distributed, akin to how the pandemic was especially severe in certain areas at different times. These lockdown shocks generate a rise in idiosyncratic uncertainty.

I examine two policies, cash grants and a reduction in long-term interest rates. The cash grants are a generalized form of PPP, which allocated $835 billion in forgivable loans to small businesses in the U.S. These loans were only forgiven if the funds were used towards wages, interest payments, or other operating expenses. I generalize on this, and consider a policy that provides grants; firms never need to pay back their receipts.[6] I assume that all eligible firms receive cash grants upon impact date, and that these grants are drawn from financial resources. Consequently, the policy does not directly affect aggregate resources in the economy; all aggregate effects are due to firms' endogenous choices on how to use the funds.

[6] As of August 15, 2021 60 percent of PPP loans have been forgiven, and 95 percent of applications for forgiveness had been accepted (SBA, 2021). (Both measures are by loan volume.)

Since debt is long-term, the model's risk-free borrowing rate is highly responsive to aggregate shocks. Recessions that trigger a slow recovery in consumption lower the equilibrium discount factor for a prolonged time. This reduces the present-discounted value of future debt payments, making a risk-free loan worth less to a lender during a recession than in the steady state. This endogenously tightens credit conditions over and beyond the increased default risk caused by the recession. To study the implications of this tightening, I implement a policy that reduces long-term interest rates. This policy captures what I refer to as the firm-demand channel of quantitative easing. A prominent effect of quantitative easing is that it reduces long-term interest rates on firm debt across the credit spectrum (Krishnamurthy & Vissing-Jorgensen, 2011). I capture how firms' demand for debt responds to this fall in interest rates, and how this changes their investment decisions. This approach does abstract from the implementation of quantitative easing and its effects on the financial sector. But, this is the first assessment of the firm-demand channel under persistent firm heterogeneity. I show that this heterogeneity is essential to capture the channel's aggregate effects: the firms that increase their debt the most are engaging in precautionary savings, and hence their investment response is muted.

The remainder of this paper is organized as follows. Section 2 summarizes the related literature. Section 3 describes the model and defines its equilibrium. Section 4 outlines some key theoretical characteristics of firm behavior in the model. In section 5, I discuss the calibration and firm behavior over the model's steady state. In section 6, I study the model dynamics over the recession and how firms respond to policy intervention. Section 7 examines the channels through which the policies operate. Lastly, section 8 concludes.

## 2 Related Literature

This paper speaks to three branches of the literature. A growing number of papers explore various dimensions of the 2020 recession. Additionally, there has been extensive research on the channels and efficacy of quantitative easing. Lastly, with its focus on financial frictions, the model also relates to the literature on firm debt and default risk.

There are quite a few empirical papers that study how the Paycheck Protection Program (PPP) and other government interventions supported firms' survival prospects and employment. As the pandemic is barely a year old at the time of this writing, the focus to date has been the short- and medium-term effects. Broadly speaking, the results suggest that PPP was more effective at preventing firm closures than at preserving jobs. Bartik et al. (2020) found that firms receiving PPP funds were more optimistic of their survival prospects through the end of 2020, and that this effect was strongest among firms most impacted by COVID-19. Bartlett and Morse (2020) document an important size gradient: across the smallest of firms, PPP recipients were less likely to expect to be closed over the next 6 months. However, PPP had no significant effect among business with more than 20 employees. Estimates of the employment effects are more mixed, perhaps reflecting the fact that different data sources capture different groups of workers (Autor et al., 2020). Using data on private-sector employment across a wide range of firms, Autor et al. (2020) estimate that PPP boosted payrolls by several percentage points. In contrast, using data that focused more on especially low-wage earners, Chetty et al. (2020) find no significant effect of the program on employment. In part, that may be because PPP was not fully used to directly support payrolls. For example, research indicates that PPP recipients strengthened their balance sheets (Chodorow-Reich et al., 2020) and were less likely to be behind on debt payments (Granja et al., 2020).

Complimenting empirical analyses, quantitative research has modeled and quantified the pandemic and its interactions with the real economy. Many incorporate the epidemiological SEIR model into more standard economic frameworks to capture changes in household behavior in response to health risk (e.g., Eichnebaum, Rebelo, and Trabandt, 2020; Glover, Heathcote, Krueger, and Rios-Rull, 2020; Kaplan, Moll, and Violante, 2020). Others abstract from the virus itself to examine the implications of sector-wide shocks to demand or employment (e.g., Baqaee and Farhi, 2020; Guerrieri, Lorenzoni, and Straub, 2020). A smaller number have focused on firm-level heterogeneity and the 2020 recession. These include Gourinchas et al. (2020) and Elenev, Landvoight, and Van Nieurwerburgh (2020), and Khan and Lee (2021).

Gourinchas et al. (2020) examine a multiple-sector business model, in which COVID-19 generates both supply and demand shocks. The analysis focuses on the short run: prices are held fixed, and firms cannot adjust their fixed factors of production during the crisis. Firms are subject to labor constraints, which may prevent them from operating at scale, and fluctuations in the demand for their goods, as well as in their productivity. The model abstracts from access to an external financial sector, so when firms' costs exceed their profits, they cannot operate. Absent government intervention, the model generates sizeable increases in default across small- and medium-sized enterprises, with a large degree of heterogeneity across sectors. Policy interventions can help to curb the rise in bankruptcies, and preserve employment, but non-targeted policies risk inefficiently allocating funds, offsetting some of the gains of intervention. While my model is unable to speak to the cross-sector heterogeneity in Gourinchas et al. (2020), it relaxes the constraints around firms' capital decisions and access to financial markets.

Elenev, Landvogit, and Van Nieurwerburgh (2020) explore the interactions between the production and financial sectors of an economy. In their environment, both firms and their lenders are subject to default risk. These risks can build off one another, amplifying the effects of shocks through endogenously determined prices. Such amplification is present in their results: the pandemic generates a substantial rise in default and a large decline in output. Providing firms with funding, akin to PPP, does help offset some of these adverse effects. To maintain tractability, though, all idiosyncratic shocks are iid, and all intertemporal decisions are effectively made by a representative agent. This implies a large degree of inter-sectoral risk sharing; so long as it doesn't default, realizing an adverse shock has no affect on a firm's (or lender's) future trajectory. Additionally, a timing friction is imposed on the model, preventing firms from taking on new debt to make up for a shortfall of cash on hand. As a consequence, there is no idiosyncratic behavior that affects the likelihood of default; the risk is a function of general equilibrium effects and exogenous shocks. My model assumes away frictions between the financial and household sectors, and hence it cannot account for the ability of the financial sector to amplify shocks. However, by assuming persistent productivity shocks, it generates intertemporal heterogeneity that can substantively influence investment and default decisions.

Khan and Lee (2021) is the paper that most closely relates to this one. The paper also evaluates the effects of fiscal policy during the 2020 recession, in an environment with heterogeneous firms and long-term debt. They focus in particular on how the distribution of leverage affects the aggregate response to the recession, and find that an economy with high leverage has a slower recovery. Additionally, wage subsidies are an effective stimulus policy, but a policy that delays debt payments offsets some of the subsidies' benefits. One dimension of richness that their environment captures that I abstract from is endogenous entry. Their firms are owned by entrepreneurs, who can chose to suspend their business and enter the workforce. This allows the model to capture the cyclicality of firm entry and exit. In contrast, firm entry in my model is exogenous, and the only endogenous form of firm exit is default. The entrepreneurs in Khan and Lee (2021) borrow using consols: the entrepreneur must make a coupon payment until the bond matures, and mature only occurs at the entrepreneur's discretion. So, similar to my model, there are periods in which an entrepreneur will not participate in the debt market, and hence will not be disciplined by contemporaneous financial conditions. However, since their maturity is solely endogenous, debt overhang will not be as prevalent for their entrepreneurs. Rather than have to pay back debt at a non-optimal time, the entrepreneur can always wait and have its debt mature later.

This paper also relates to the literature on how financial frictions can affect firms' investment decisions. Under aggregate uncertainty, financial frictions can constrain individual firms' activities, and influence aggregate trends. In a model with heterogeneous firms and stochastic borrowing constraints, Khan and Thomas (2013) find that the trends observed over the Great Recession are better captured by a shock to financial conditions than a shock to aggregate productivity. Jermann and Quadrini (2012) build a New Keynesian model in which firms borrow subject to a leverage constraint and can issue new equity. They also find a large effect of financial shocks on investment and output-roughly ten times the effect of TFP shocks. Gilchrist, Sim, and Zakrajek (2014) find that uncertainty shocks affect investment decisions largely through their influence on financial frictions, rather than real frictions (e.g., the partial irreversibility of and adjustment costs to physical capital).

A couple papers have also examined the effects of quantitative easing during the 2020 recession. Ebsim, Faria-e-Castro, and Kozlowski (2021) evaluate how the effects of policy vary with the nature of the shock. In their environment, there are four types of firms, which vary over leverage and liquidity. Default is driven by idiosyncratic preference shocks, and intertemporal decisions within a firm type are made by a representative agent. Similar to my approach, they treat quantitative easing as an exogenous increase in bond prices (i.e., a reduction in the effective interest rate). They find that quantitative easing is able to boost output in response to financial aggregate shocks, but has very modest real effects if the recession is triggered by a decline in aggregate productivity. These findings echo those of Elenev et al (2021), which examines the effects of quantitative easing in a DSGE environment that accounts for spillover effects between risk in the real and financial sectors of the economy. On its own, quantitative easing has a moderately stimulative effect during a recession. Only when combined with changes in capital requirements and a higher inflationary target is the policy able to generate large changes in aggregates.

Business cycle models with financial frictions have largely structured debt contracts so that in equilibrium there is no default.[7] An emerging trend among quantitative models is to allow equilibrium default, requiring idiosyncratic firm risk to be endogenously priced. Arellano, Bai, and Kehoe (2019) link default risk and agency risk, and calibrate their model to match moments on firm credit spreads. They find that their model can generate the observed volatility in output, as well as the volatility in aggregate labor, despite only having one element of aggregate uncertainty: shocks to the variance of firms' idiosyncratic productivity. Similarly, Ottonello and Winberry (2020) find that default risk constrains the responsiveness of firm investment. They develop a New Keynesian model and find that the aggregate response to an expansionary monetary policy shock is driven by firms with low default risk. Firms with high default risk are less responsive because they face high marginal costs. Khan, Senga, and Thomas (2019) develop a real business cycle model in which firms are subject to default risk and there is an endogenous entry choice. Potential entrants are less likely to enter

[7] E.g., Gertler & Karadi (2011); Jermann & Quadrini (2012); Khan & Thomas (2013)

the economy during downturns, resulting in a procyclical mass of active firms. With endogenous entry and default risk, negative financial frictions can drive a severe, protracted recession, akin to the trends observed over the Great Recession.

Building off literatures in corporate finance and sovereign debt, Gomes, Jermann, and Schmid (2016) analyze a dynamic stochastic general equilibrium model in which debt is long-term, nominal, and subject to default risk. Long-term debt is paid off geometrically, and refinancing is costless. Compared to a model with one-period debt, Gomes, Jermann, and Schmid find that long-term debt increases the responsiveness of investment and other aggregates to productivity shocks. In particular, endogenous responses are more persistent under long-term debt. Bustamante (2019) extends on Gomes, Jermann, and Schmid (2016) by making idiosyncratic productivity shocks persistent. Hence, in the model equilibrium there is a non-degenerate distribution of firms, which vary across their levels of capital and debt. Bustamante (2019) finds that this heterogeneity is important in matching the trends observed over the Great Recession. Small firms rely on debt to scale up; adverse shocks reduce their revenues, while leaving what they owe unchanged. Bustamante (2019) documents that this debt overhang becomes more pronounced as debt maturity lengthens.

Gomes, Jermann, and Schmid (2016) and Bustamante (2019) rely on consol-style contracts. Each period, a portion of debt obligations mature, and firms can choose a different level of debt. Hence, these papers cannot examine the heterogeneity driven by maturity risk. Empirical evidence suggests this maturity risk can be significant during crises. For example, Almeida et al. (2011) find that firms with a large amount of long-term debt maturing during the 2007 credit crunch subsequently had weaker investment, accounting for 30% of the decline in aggregate investment. Additional studies by Campello, Graham, and Harvey (2010) and Duchin, Ozbas, and Sensoy (2010) also find significant relationships between maturing debt and firm outcomes during the financial crisis. To test whether such forces were also present during the 2020 recession, we need a model with variation in debt maturity across firms.

# 3 Model

I analyze a dynamic general equilibrium model, in which there is no aggregate uncertainty. The code of the model is a unit mass of perfectly competitive, goods-producing firms. Production exhibits decreasing returns to scale technologies, using physical capital and labor as inputs. Decreasing returns to scale, combined with persistent idiosyncratic shocks to productivity, generate a nondegenerate distribution in equilibrium. The representative household supplies labor, owns firms, and saves through a one-period, risk-free asset. Perfectly competitive, risk-neutral financial intermediaries channel household savings into firm borrowing. Ffirms face three dimensions of idiosyncratic risk: their production is subject to persistent productivity shocks, their debt matures stochastically, and each period there is a chance they may be forced to exit the economy permanently.

Ffirms fund their investment decisions through a combination of retained earnings and external borrowing. Borrowing occurs through long-term bonds, which mature with an iid probability. In each period, firms pay a coupon proportional to the bond's face value. When a firm receives a maturity shock, it must also pay back the debt's face value. When a firm does not receive a maturity shock, it can choose to prepay its debt by paying a cost proportional to the face value of its debt. Firms have limited liability, and can default on their debt; defaulting firms receive a value of 0, and permanently exit the economy. Non-defaulting firms purchase physical capital to use in the next period, and can also save in the one-period, risk-free asset. Firms' dividends are bounded below by zero; these firms cannot access equity markets. In addition to the endogenous exit driven by default, firms are subject to an iid exit shock each period. Both defaulting and exiting firms are replaced by new firms, leaving the operating mass of firms constant. I normalize the mass of firms to a unit measure.

Time subscripts $t$ capture the aggregate state. Aggregate TFP can vary, $z_t$, as can the distribution of firms over the idiosyncratic state space, $\mu_t$. All agents know how the firm distribution evolves over time: $\Gamma_t(\mu_t) = \mu_{t+1}$.

A firm's idiosyncratic state is $\psi = \{k, a, b, \varepsilon\}$. The firm's assets consist of physical capital, $k$, and financial savings in the one-period asset, $a$. The face value of its debt is $b$. Lastly, $\varepsilon \in \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$ is the firm's idiosyncratic productivity. $\varepsilon$ follows a first-order Markov process. For every TFP level $\varepsilon_i$, $Pr(\varepsilon_{t+1} = \varepsilon_j | \varepsilon_t = \varepsilon_i) = \pi^{\varepsilon}_{i,j} \geq 0$, where $\sum_{p=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} = 1$. With probability $\rho$, a firm receives an exit shock, and with probability $1 - \rho$ it can continue operating. When it does not receive the exit shock, the firm has a $\lambda$ chance of having its debt mature.

The remainder of this section consists of: a description of the firm's problem, given prices; a discussion of how bonds are priced, given firms' decisions; an overview of the representative household's problem and aggregation; and a definition of the model equilibrium.

### 3.1 Firms' Problem

At the start of a period, the firm observes its $\varepsilon$, whether it will exit in the period, and whether its debt will mature in the period. Firms that do not receive the exit shock decide whether to default or continue to operate. Additionally, firms that do not receive the maturity shock decide whether or not to prepay their debt. Continuing firms then produce, pay their debt obligations, and chose their physical capital and financial savings for next period. Additionally, firms with maturing debt can issue a new bond. Exiting firms pay back their debt and issue any remaining funds as dividends.

Each period, operating firms solve a static production problem. Both physical capital and labor are inputs in production. I denote operating profits as

$$R_t(k, \varepsilon) = \max_{\ell} z_t \varepsilon k_t^{\alpha} \ell^{\eta} - w_t \ell - \kappa(\varepsilon) + (1 - \delta)k \tag{1}$$

Operating profits consist of revenues from production, less wage costs. Additionally, firms pay an operating cost that is a function of their idiosyncratic productivity. I assume operating expenses are weakly increasing in productivity, $\kappa'(\varepsilon) \geq 0$. I assume that $\alpha + \eta < 0$, so that firms operate under decreasing returns to scale.

To minimize notation, let $J_t(\psi)$ denote value of a firm given a realized $\varepsilon$, but before it is known whether the firm will exit, or whether its debt will mature.

$$J_t(\psi) = (1-\rho)\lambda J_t^m(\psi) + (1-\rho)(1-\lambda)J_t^n(\psi) + \rho J_t^e(\psi) \tag{2}$$

$J^m$ is the value of the firm when its debt matures, $J^n$ is the value when its debt does not mature, and $J^e$ is the value when it exits. $J$ is a construct that I use only to represent future expectations. Firm choices are always determined given the realization of the maturity and exit shocks.

#### 3.1.1 Exiting Firms

If an exiting firm can afford to pay back what it owes after production, it produces and issues any remaining funds as dividends. Otherwise, it cedes control to the lender and receives value 0. The lender then decides whether the firm should operate in the period and liquidate after production, or if the firm should go through default proceedings at the start of the period. In either case, a firm receiving an exit shock solves

$$J_t^e(k_t, a_t, b_t, \varepsilon) = \max\{0, R_t(k_t, \varepsilon) + a_t - (1+c)b_t\} \tag{3}$$

The firm's gross assets in the period come from operating profits, non-depreciated physical capital, and returns on last period's financial savings. Its debt obligations consists of the face value of debt, as well as the periodic coupon payment $cb$.

#### 3.1.2 Firms with Maturing Debt

When debt matures, the firm's problem is similar to models with one-period maturity. The firm chooses between defaulting and getting value 0, and continuing and getting value $V^m$. As all firms are owned by the representative household, they use the same stochastic discount factor, $M_{t,t+1}$. When it takes on new debt, the firm receives $q(\cdot)b_{t+1}$ in numeraire. It purchases financial savings at discount price $q^f$.A t debt maturity, the firm solves

$$J_t^m(k_t, a_t, b_t, \varepsilon) = \max\Big\{0, V_t^m(k_t, a_t, b_t, \varepsilon)\Big\} \tag{4}$$

$$V_t^m(k_t, a_t, b_t, \varepsilon_i) = \max_{d_t, k_{t+1}, a_{t+1}, b_{t+1}} d_t + M_{t,t+1} \sum_{p=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon J_{t+1}(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_j)$$

s.t. $d_t \geq 0$ and

$$k_{t+1} + q_t^f a_{t+1} + d_t = R_t(k_t, \varepsilon_i) - cb_t + a_t - b_t + q_t(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_i) b_{t+1}$$

As the firm's $\varepsilon$ and choices of $k_{t+1}$, $a_{t+1}$, and $b_{t+1}$ can all affect default risk, the debt price schedule $q(\cdot)$ varies with both the idiosyncratic and aggregate states. As a technical point, I assume that any firm that chooses $b_{t+1} = 0$ does not face the debt-maturity shocks. Conditional on not exiting, they can freely choose a different $b_{t+1}$ next period.

### 3.1.3 Firms with Non-Maturing Debt

A firm that does not receive the maturity shock has three options: it can default and receive value 0, continue in its debt contract with value $V^n$, or prepay its debt and get value $V^m$. If it chooses to prepay its debt, the firm pays a restructuring cost $\gamma(b)$, $\gamma'(b) \geq 0$. After accounting for the restructuring cost, a firm prepaying its debt faces the same problem as a firm that received the maturity shock.

$$J_t^n(k_t, a_t, b_t, \varepsilon) = \max\Big\{0, V_t^n(k_t, a_t, b_t, \varepsilon), V_t^m(k_t, a_t - \gamma(b_t), b_t, \varepsilon)\Big\} \tag{5}$$

$$V_t^n(k_t, a_t, b_t, \varepsilon_i) = \max_{d_t, k_{t+1}, a_{t+1}} d + M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon J_{t+1}(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_j)$$

$$\text{s.t.} \quad k_{t+1} + q_t^f a_{t+1} + d_t = R_t(k_t, \varepsilon_i) - cb_t + a_t$$

$$d_t \geq 0, \;\; b_{t+1} = b_t$$

When a firm does not pay back its debt, it only pays the coupon and the face value of debt does not change into the next period.

### 3.1.4 Entering Firms

Each exiting firm is replaced by an entrant. Households endow the entrant firms with financial assets $a_e$, which are fraction $\iota \in [0,1]$ of the average capital stock of firms. They are not endowed with any physical capital or any debt, but they can take out debt in the period in which they enter. New firms draw their idiosyncratic productivity from the ergodic distribution of $\varepsilon$. Given these assumptions, the optimization problem of a new firm is a special case of equation (4). In particular, a new firm is indistinguishable from a firm with no physical capital and no debt outstanding.

Let $g_t^{n,k}(\psi),\ g_t^{n,a}(\psi),\ g_t^{m,k}(\psi),\ g_t^{m,a}(\psi),\ g_t^{m,b}(\psi)$ denote the firm's optimal decision rules for $k_{t+1},\ a_{t+1}$, and $b_{t+1}$ given idiosyncratic state $\psi$. Additionally, let $\mathbb{D}^m(\psi)$ and $\mathbb{D}^n(\psi)$ be indicator variables for firms' default decisions, and let $\mathbb{P}(\psi)$ be an indicator variable indicating prepayment.

## 3.2 Default & Bond Pricing

When a firm defaults, its shareholders lose all claims, receiving the outside value of 0. The financial intermediary takes over the firm, forgives its debt, and sells it to new shareholders. This sale allows the intermediary to recover at least part of what was owed prior to default. If the sold firm is worth more than what the defaulting firm owed, the surplus is allocated to the representative household in a lump sum.[8] Since payoffs under default vary with the defaulting firm's state, lenders evaluate risk at the idiosyncratic level, as in Chatterjee et al. (2007).

Selling a firm is costly: only $\chi$ share of physical capital is recoverable by the lender. In addition, the intermediary must spend a cost proportional to the face value of the defaulted debt, $\nu b$. These losses, $(1-\chi)k$ and $\nu b$, are transferred to the representative household lump-sum. This is a conservative

[8]This scenario is possible due to the combination of debt forgiveness and the lender's compensation being the equity value of the new firm, not the scrap value of its assets.

assumption, and ensures that the aggregate effects of default are solely determined by endogenous choices, and not through reductions in aggregate resources. The lender can fully recover a firms' financial assets.

Since default occurs at the beginning of the period, the new firm can produce during that same period. Additionally, I do not prevent the new firm from borrowing into the next period. Hence, as in Gomes, Jermann, and Schmid (2016), default risk influences firm's decisions, but does not affect the mass of firms active in a period.

If firm $\{k, a, b, \varepsilon\}$ defaults in $t$, the value of the new firm is $J_t^m(\chi k, a, 0, \varepsilon)$. The lender's recovery in default is $\Theta^d$.

$$\Theta_t^d(k, a, b, \varepsilon) = \min\{J_t^m(\chi k, a, 0, \varepsilon), (1 + c)b\} - \nu b \tag{6}$$

Similarly, let $\Theta^e$ be the lender's recovery when a firm receives an exogenous exit shock. If the exiting firm can pay back its debt, the lender receives the full $(1 + c)b$ it is owed. Otherwise, the firm cedes control to the lender at the start of the period. The lender determines whether it is optimal to put the firm through default proceedings at the start of the period, or to let it operate and liquidate it at the end of the period. As such a firm receives its outside value, 0, in either case, it is indifferent.

$$\Theta_t^e(k, a, b, \varepsilon) = \max\Big\{\Theta_t^d(k, a, b, \varepsilon), \min\{R_t(k, \varepsilon) + a, (1 + c)b\}\Big\} \tag{7}$$

By construction, under prepayment all of a firm's debt matures. Hence, the prepayment choice weakly increases the likelihood that a firm will pay back the entirety of its debt in the next period. To reduce notation, let $\psi_{t+1} \equiv \{k_{t+1}, a_{t+1}, b_{t+1}\}$ denote the firms' asset choices. Since financial intermediaries are perfectly competitive and risk neutral, the equilibrium price of debt satisfies

$$
\begin{aligned}
&q_t(\psi_{t+1}, \varepsilon_i) b_{t+1} = \\
&\quad + \lambda(1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} (1 - \mathbb{D}^m_{t+1}(\psi_{t+1}, \varepsilon_j))(1+c) b_{t+1} \\
&\quad + (1-\lambda)(1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} (1 - \mathbb{D}^n_{t+1}(\psi_{t+1}, \varepsilon_j) - \mathbb{P}_{t+1}(\psi_{t+1}, \varepsilon_j))[c + q_{t+1}(k_{t+2}, a_{t+2}, b_{t+1}, \varepsilon_j)] b_{t+1} \\
&\quad + (1-\lambda)(1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} \mathbb{P}_{t+1}(\psi_{t+1}, \varepsilon_j)(1+c) b_{t+1} \\
&\quad + (1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} [\lambda \mathbb{D}^m_{t+1}(\psi_{t+1}, \varepsilon_j) + (1-\lambda)\mathbb{D}^n_{t+1}(\psi_{t+1}, \varepsilon_j)] \Theta^d_{t+1}(\psi_{t+1}, \varepsilon_j) \\
&\quad + \rho M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} \Theta^e_{t+1}(\psi_{t+1}, \varepsilon_j)
\end{aligned}
\tag{8}
$$

The price of debt takes into account five potential outcomes next period. The first three rows account for when the firm does not default. If its debt matures and it does not default, the lender receives the full $(1+c)b_{t+1}$. If its debt does not mature and it does not default, the firm either stays in the contract or prepays its debt. If the firm stays in the contract, the lender receives the coupon payment and the expected future value of the contract. Since there is no private information in this economy, lenders use the equilibrium decision rules to account for the default risk of future periods: $k_{t+2} = g^{n,k}_t(\psi_{t+1})$ and $a_{t+2} = g^{n,a}_t(\psi_{t+1})$. if the firm chooses to prepay, the lender's return is the same as if the maturity shock was realized. The fourth row in (8) captures what the lender receives in the event of default. While the probability of default can vary with the realization of the maturity shock, the lender's recovery is unaffected. The last row accounts for the recovery value when the firm receives an exit shock.

This debt-pricing arrangement departs from the literature in two substantial ways. The first is its approach to the maturity structure. While the literature standard also relies on a stochastic process to support long-term maturities, it assumes a constant fraction of debt matures each period: firms always have a combination of debt that matures and does not mature. Hence, in models such as Gomes, Jermann, and Schmid (2016), firms can freely adjust their total debt stock in every period. In contrast, the model here assumes maturity shocks affect the full stock of debt: either all of a firm's debt matures, or none of it does. The inclusion of the prepayment choice means debt maturity is partly endogenous–any firm not receiving the maturity shock can choose to still adjust its debt level. However, since this is an endogenous choice, such firms can also choose to remain in their preexisting contract and avoid current financial conditions.

The second departure I take is with respect to recovery under default. It is common for models to assume that lenders can only recover a fraction of physical capital in default, rather than firm value. But in these models, the lender effectively sells the restructured firm to the representative household. From the representative household's perspective, they have ownership rights to a new firm, and hence the transfer is in terms of firm value. Absent some imposed constraint, the lender would optimally sell the rights to a firm worth $J(\xi k, \cdot)$ for numeraire equal to $J(\xi k, \cdot)$, rather than $\xi k$. Since the new firm has no debt, its value is bounded below by $\xi k$. This is what the firm would be worth if it sold all its assets for dividends, and the continuing value from this action was 0. This is a nontrivial departure in terms of equilibrium outcomes–by increasing the recovery value to lenders under default, debt prices rise overall. This approach broadly corresponds to Chapter 11 of the US bankruptcy code.

## 3.3 Aggregation & Representative Household

### 3.3.1 Aggregation

At the start of the period, the distribution of firms over the idiosyncratic state space is $\mu$. However, exit shocks and default decisions alter this distribution within the period. First, lenders take over defaulting firms, and sell their debt-free replacements to new managers. Production occurs from these replacements, alongside non-defaulting firms. I denote the distribution of producers by $\mu^Y$. Lastly, firms that receive the exit shock and are liquidated after production are replaced by entrants.

These new firms join the remaining producing firms in making investment decisions. Hence, it is this distribution over investors, $\mu^I$, that determines the distribution of firms in the next period, $\mu_{t+1}$. I describe how these distributions are constructed in the appendix.

From $\mu^Y$, aggregate labor demand and output can be measured:

$$N_t = \int \ell_t(k,\varepsilon)\mu_t^Y(d[k \times a \times b \times \varepsilon]) + \pi_s^1 \ell(k_s^1, \varepsilon_1^s) + \pi_s^1 \ell(k_s^2, \varepsilon_2^s) \tag{9}$$

$$Y_t = z_t \int \varepsilon F(k, \ell_t(k,\varepsilon))\mu_t^Y(d[k \times a \times b \times \varepsilon]) + \pi_1^s z_t F(k_1^s, \ell_t(k_1^s, \varepsilon_1^s)) + \pi_2^s z_t F(k, \ell_t(k_2^s, \varepsilon_2^s)) \tag{10}$$

Let aggregate capital be

$$K \equiv \int k\mu(d[k \times a \times b \times \varepsilon]) \tag{11}$$

Since savings and debt are financial variables, they do not affect the aggregate resource constraint. Physical capital only depreciates when it is used in production. When a firm is restructured, it is only the capital from its successor that gets depreciated. Hence, in the right-hand side of the aggregate resource constraint, total physical capital consists of all capital at the start of the period, less the depreciated fraction of capital that was used in production. The representative household consumes $C$. Then the aggregate resource constraint is

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t \tag{12}$$

### 3.3.2 Representative Household

There is a unit mass of infinitely lived, identical households. Each has a unit endowment of time every period, and saves through one-period, risk-free contracts with the financial intermediary, $a^h$. Households are also initially endowed with ownership of all firms. There are no frictions within the household sector; households are exposed to aggregate TFP risk, but are protected from idiosyncratic firm risk. Households gain periodic utility from consumption and leisure, $u(C, 1-n^h)$.

Their subjective discount factor is $\beta \in (0,1)$.

Household wealth comes from four sources. They earn returns on their previous financial savings, $a^h$. Additionally, their ownership of firms provides them dividends and equity value. Their labor supply is endogenous, yielding income $wn$. Lastly, $T$ captures the lump-sum transfers to the household of operating expenses and the losses from default. Each period, households choose their labor supply, consumption, risk-free savings, and shares in firms. I denote the household's shares in firm $\psi$ by $g(\psi)$, and the household's share holdings across all firms by $\mathbf{g}$.

The representative household solves

$$\begin{aligned} H_t(\mathbf{g}_t, a_t^h) &= \max_{n_h, C, \mathbf{g}_{t+1}, a_{t+1}^h} u(C, 1-n_h) + M_{t,t+1} H_{t+1}(\mathbf{g}_{t+1}, a_{t+1}^h) \ s.t. \\ C + q_t^f a_{t+1}^h + \int g_{t+1}(\psi) \hat{J}_t(\psi_t) \mu_t^I d[\psi] &\leq w_t n^h + a_t^h + \int g_t(\psi) J_t(\psi) \mu_t d[\psi] + T_t \end{aligned} \tag{13}$$

where $\hat{J}$ is the ex-dividend value of a firm.

### 3.4 Recursive Competitive Equilibrium

Recursive competitive equilibrium consists of five groups of functions. Firms and households both have value functions: $\{J_t^m(\psi), J_t^n(\psi), J_t^e(\psi), J_t^s(k^s, \varepsilon^s)\}$ and $H_t(\mathbf{g}_t, a_t^h)$. Firms have decision rules for their employment, dividends, future capital, future savings, and future debt: $\ell_t(k, \varepsilon)$, $d_t^x(\psi)$, $g_t^{x,k}(\psi)$, $g_t^{x,a}(\psi)$, and $g_t^{m,b}(\psi)$. ($x$ is a placeholder for $m$ and $n$, which reflect whether the firm's debt matures or not.) The household has decision rules for consumption, labor supply, financial savings, and firm shares: $C_t(\mathbf{g}_t, a_t^h)$, $n_t^h(\mathbf{g}_t, a_t^h)$, $a_{t+1}(\mathbf{g}_t, a_t^h)$, and $g_{t+1}(\psi; \mathbf{g}_t, a_t^h)$. The price functions are for wages, the risk-free asset rate, the firms' stochastic discount factor, and the debt price schedule: $w_t$, $q_t^f$, $M_{t,t+1}$, and $q_t(\psi)$. Lastly, there is a perceived aggregate law of motion: $\mu_{t+1} = \Gamma_t(\mu_t)$, which is observed by all agents.

These functions are such that, for every $\psi = (k, a, b, \varepsilon)$ and every $t$:

(a) Firms:

- $J_t^m(\psi)$ solves (4), $J_t^n(\psi)$ solves (5), and $J_t^e(\psi)$ solves (3)
- $\ell_t(k,\varepsilon)$, $d_t^x(\psi)$, $g_t^{x,k}(\psi)$, $g_t^{x,a}(\psi)$, and $g_t^{m,b}(\psi)$, are the associated decision rules ($x = m, n$)

(b) Households:

- $H_t(\mathbf{g}_t, a_t^h)$ solves (13)
- $C_t(\mathbf{g}_t, a_t^h),\ n_t^h(\mathbf{g}, a_h),\ a_{t+1}^h(\mathbf{g}_t, a_t^h)$, and $g_{t+1}(\psi; \mathbf{g}_t, a_t^h)$ are the associated decision rules

(c) Prices are competitively determined:

$$w_t = \frac{u_n(C_t, 1-n_t^h)}{u_C(C_t, 1-n_t^h)}$$

$$M_{t,t+1} = \beta \frac{u_C(C_{t+1}, 1-n_{t+1}^h)}{u_C(C_t, 1-n_t^h)}$$

$$q_t^f = M_{t,t+1}$$

$$q_t(\psi) \text{ satisfies (8)}$$

(e) Whenever households start the period with $\mathbf{g} = \mathbf{1}$, the labor market clears and households choose to hold all shares of each active firm. The aggregate resource constraint is also satisfied (12). The market for the risk-free asset, in which households and firms intermediate with the financial sector, is cleared by Walras' law.

# 4 Characterization of Equilibrium Actions

Before presenting model results, I briefly describe firms' default decisions and discuss how debt overhang can affect firm decisions when debt is not maturing. To simplify notation, I focus on the model's steady state. All results discussed carry through to the full dynamic model. Since the focus is on the steady state, I use $\beta$ to represent the discount factor, which holds in equilibrium.

## 4.1 Default Decisions

Default is possible in any state a firm finds itself: exit, non-maturing debt, and maturing debt. In each of these cases, the default decision can be characterized analytically. As is common in these models, I assume that any firm that is indifferent between defaulting and continuing chooses to continue. Then Proposition 1 of Ottonello and Winberry (2020) holds in this model environment as well: a firm will default if and only if it cannot satisfy the non-negativity constraint on dividends.

The logic follows that of Ottonello and Winberry (2020). Because default is possible in any state, the value obtained in default puts a lower bound on the firm's value function. In other words, if the maximum value continuing was strictly less than the firm's outside option, the firm would optimally chose to default. Since in this model the firm's value of default is assumed to be 0, 0 is also the lower bound on expected future value for any firm, as the expectations operator is a convex function.[9] Consequently, any firm that has a feasible option to continue will weakly prefer to continue rather than default. A feasible option is one that satisfies the non-negativity constraint on dividends, and therefore one that offers weakly positive flow value. Since future value is also weakly positive, any feasible inside option will weakly dominate default. From this, it is clear that a firm will default if and only if it cannot satisfy the non-negativity constraint on dividends.

Accordingly, firms in the model default when

- Debt matures: $R(k_t, \varepsilon) + a_t - (1+c)b_t + \max_{k_{t+1}, a_{t+1}, b_{t+1}} \{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\} < 0$
- Debt does not mature: $R(k_t, \varepsilon) + a_t - cb_t < 0$
- Firm exits: $R(k_t, \varepsilon) + a_t - (1+c)b_t < 0$

Hence, in each of these cases, there is an analytic solution for default thresholds. I would especially like to highlight the case when debt matures. Let $m$ denote net assets in this case,

[9] More generally, if the outside option was $\Omega$, the lower bound on expected future value would be $\beta\Omega$, to account for the discounting. When the outside option is normalized to 0, discounting is not a factor.

$m(k, a, b, \varepsilon) \equiv R(k_t, \varepsilon) + a_t - (1 + c)b_t$. Then any firm with maturing debt and net assets below $\underline{m}(\varepsilon)$ will default:

$$\underline{m}(\varepsilon) \max_{k_{t+1}, a_{t+1}, b_{t+1}} \{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\} = 0$$

$$\underline{m}(\varepsilon) = - \max_{k_{t+1}, a_{t+1}, b_{t+1}} \{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\}$$

Because I have assumed away capital adjustment costs and other real frictions, the only way the right-hand-side varies with the firm's state is through its productivity, $\varepsilon$. Accordingly, default thresholds under debt maturity only vary with the debt price schedule. The thresholds are even more straightforward when debt does not mature of when the firm exits–in these cases, the only endogenous object is the equilibrium wage.

This greatly simplifies the quantitative solution of the model. For ease of discussion, consider a partial steady state equilibrium, in which the wage is held constant. For a given debt price schedule, firms' choices to default or continue are known with certainty, independently of the value function. The state space can then by partitioned into defaulting and non-defaulting regions, allowing debt prices to be updated. Iterating between these thresholds and the debt price schedule can occur independently of solving for firms' decision rules and value. In this model, with long-term debt and default recovery tied to firm value, further iterations are needed to ensure consistency between debt prices and firms' choices and value. However, in models with one-period debt and default recovery tied to asset value (e.g., Ottonello and Winberry, 2020; Khan, Senga, Thomas, 2018), debt prices and default thresholds can be fully determined before knowing firms' value function or their capital and borrowing decision rules.

### 4.2 Debt Overhang & Endogenous Default Risk

Unlike models with one-period or consol-style debt, where a firm's debt matures each period, in this model debt can go many periods without maturing. As a consequence, firms are not subject to debt-market discipline in every period they are active. This vastly increases the scope for endogenously driven default. In particular, some firms will find it optimal to choose the most extreme form of endogenous driven default: selling off all their assets and issuing the proceeds as dividends. This all but guarantees default in the next period. I refer to this extreme form as strategic default. Ottonello & Winberry (2020) show that in their one-period maturity setting, firms weakly prefer to use available funds to increase capital or reduce debt; no firm would strictly prefer to issue dividends while debt is outstanding. Under probabilistic maturity long-term debt, this result does not hold.

The appeal of strategic default is that it allows the firm to evade paying back its debt. Consider a hypothetical scenario in which a firm's debt is not maturing and it somehow knows with certainty that it will receive the exit shock with period. Its expected future value is

$$\sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} \max\{R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t, 0\}$$

Under strategic default, the firm chooses $k_{t+1} = a_{t+1} = 0$, and issue all funds as dividends, leading to strategic default: without any assets, the firm will not be able to pay back the $(1+c)b_t$ it owes, so it will default in all future states. Then its expected future value is 0, leading to total value of $R(k_t, \varepsilon) + a_t - c_b t$. For this expositional case, strategic default will strictly dominate the optimal continuing strategy whenever

$$R(k_t, \varepsilon) + a_t - cb_t >$$
$$\max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \left\{ R(k_t, \varepsilon) + a_t - cb_t - k_{t+1} - q_f a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} \max\{R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t, 0\} \right\}$$

Suppose this firm's optimal choices conditional on continuing ensure that it would be able to pay off its debt with certainty in the next period.[10] Then the inequality simplifies to

$$0 > \max_{k_{t+1},a_{t+1}|k_{t+1}+a_{t+1}>0} \Big\{ -k_{t+1} - q_f a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} [R(k_{t+1},\varepsilon_j) + a_{t+1} - (1+c)b_t] \Big\}$$

$$\beta(1+c)b_t > \max_{k_{t+1},a_{t+1}|k_{t+1}+a_{t+1}>0} \Big\{ -k_{t+1} - q_f a_{t+1} + \beta a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} R(k_{t+1},\varepsilon_j) \Big\}$$

Lastly, in equilibrium, the discount price on savings equals the household's discount rate. Since $q^f a_{t+1} = \beta a_{t+1}$, the inequality further simplifies to

$$\beta(1+c)b_t > \max_{k_{t+1}|k_{t+1}>0} \Big\{ -k_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} R(k_{t+1},\varepsilon_j) \Big\}$$

This expositional case imposes a high bar on continuing. The expected return on capital must not just exceed the cost of funds, but must also more than compensate for pre-existing debt obligations. As productivity falls and as leverage rises, a firm will be less likely to generate a high enough return to justify continuing, and hence more likely to find strategic default optimal.

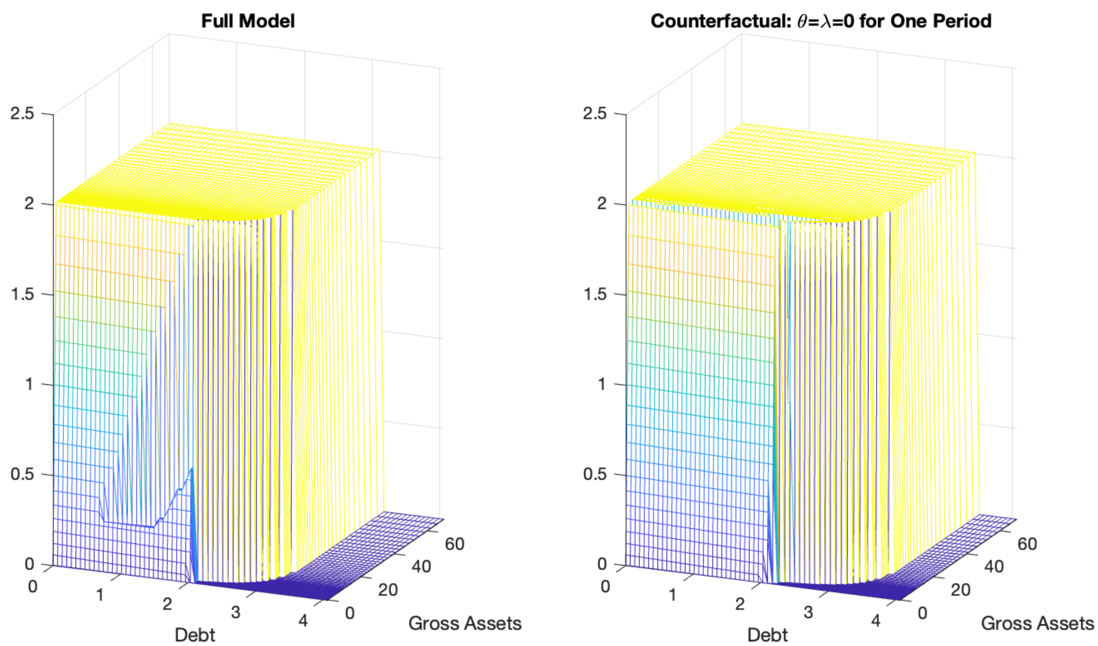
In the full model, the tradeoff is less stark, as the realization of exit shocks is never known ex ante. Nonetheless, the fundamental question is unchanged: does continuing generate sufficiently high expected returns to justify having to pay back the face value of debt, if a maturity or exit shock is realized? In the case of maturity, the ability to take on new debt and continue operating in future periods lifts the firm's inside option. Even so, there are portions of the state space in which firms choose to strategically default.

To illustrate how debt maturity risk affects firms' endogenous default choices, Figure 1 shows the capital decision rules of firms with the lowest productivity that do not have debt maturing. The left panel shows these decisions in the steady state equilibrium. The right panel considers a one-period deviation from this equilibrium, in which the firms know with certainty that in next period they will not receive either the maturity or exit shock, and that the model returns to the steady state

[10] In addition to reducing notation, this increases future value relative to a case in which there is default risk, and hence makes this inside option more appealing.

afterwards. In this counterfactual, firms that want to prepay still have this option, but there is no chance that the firm will be forced to pay back its debt.

Figure 1: Capital Decision Rules for Firms with the Lowest Productivity & Non-Maturing Debt




Focusing on the full model, there are two regions with endogenous strategic default. At the right of the graphs are firms that choose strategic default: their future capital (and savings) choice is 0, even though the firm has plenty of funds to invest, and has no default risk. Following the logic of the expositional case above, over much of the state space strategic default is not driven by default risk, but by debt overhang. A firm with gross assets of 40 would have no problem paying back a debt with face value 4, for example. But this choice is not optimal, because the expected returns on capital are too low. Strategic default is virtually non-existent across the state space for firms with average productivity.

While the counterfactual scenario does not fully eliminate strategic default, it does reduce its prevalence. Whereas in the full model no low-productivity firm with debt of 3.1 would choose to continue, eliminating one period of maturity and exit risk increases this threshold to 3.5. The reduction in risk is actually relatively small: while in the counterfactual a firm can have non-maturing debt with 100% probability, under the full model this probability is still 93%. In addition to reducing strategic default, the counterfactual also eliminates a region of "gambling". Under the full model, firms with a moderate level of debt (from 1 to 2) and low assets (less than 2) do not invest as much in capital as they could, and instead issue dividends. These small, highly leveraged firms would benefit from continuing if possible. However, they face a high degree of default risk, particularly under exit shocks. As such, they hedge their bets: they invest in capital to try to continue, but also issue some dividends to take advantage of this risk-free source of value. The firm could reduce its default risk by not paying dividends, but since it cannot eliminate this risk because of its high leverage, the firm optimally increases its default risk through dividend payments. As this risk is predominately driven by the exit shock, by eliminating this risk for a period, the counterfactual scenario eliminates this "gambling" portion of the state space.

Endogenous default risk is a poor outcome for the lender, as it both reduces the likelihood of being paid in full and reduces the amount of assets available to recover under default. As such, the debt price schedule adapts to take such behavior into account, deterring firms from being in a position where they will resort to this choice. The long horizon between when debt is issued and when it is expected to mature hinders this; the longer the expected horizon, the more sequences of productivity shocks there are that could lead a firm towards endogenous default risk. Since firms are not subject to financial market discipline until their loan matures, the lender must account for as much of this downside risk as possible up front.[11] In contrast, in a consol-style environment (e.g., Gomes, Jerman, & Schmid, 2016), outstanding debt matures fractionally each period and firms are always able to take on new debt costlessly. Since such firms are continuously subject to financial market discipline, current lenders will benefit from the discipline of future lenders. In effect, consol-stlye debt allows for more state-contingent lending conditions, reducing the lender's need to impose tight conditions up front.

[11] As is true of other models with maturity shocks (e.g., Chen, Xu, & Yang, 2021), this model abstracts from loan covenants. This is a common feature of long-term debt contracts that allows the lender to impose discipline on a firm's choices even after maturity. However, Chodrow-Reich and Falato (2020) find that even among large loans (with a commitment of at least $20 million), one in four violated a covenant during 2006 and 2007. It is relatively rare for lenders to take action in response to these violations–even during the 2008-2009 crisis, two out of three covenant violations were waived. If such waivers are anticipated by firms, then they would operate similarly to the firms in my model, which never face such waivers.

While endogenous default risk is present even in the model's steady state, a severe recession, such as the one generated by COVID-19, exacerbates the debt overhang. In particular, the debt's value is non-contingent, even as lockdown shocks generate negative uncertainty shocks, increasing the hazard of default. With expected future returns weakened, more firms will want to reduce their exposure to debt obligations by increasing their default risk. Since most firms' debt does not mature when the recession starts, only a small fraction will be subject to the financial market discipline that might deter them from increasing their default risk. However, lenders will optimally price in this increased risk, along with heightened default risk more generally. This generates a more conservative debt price schedule, further reducing firms' expected future value, and hence endogenously amplifying firms' increased desire to reduce their debt overhang by taking on more default risk.

# 5 Calibration & Steady State Dynamics

## 5.1 Calibration & Model Fit

I evaluate the model at a quarterly frequency. Table 1 presents the model parameters. Roughly half of the parameters are externally set, or correspond one-to-one with data moments (e.g., the labor share). The remaining parameters are internally set to jointly match data moments. Data moments are taken from the United States economy.

The discount factor is set to yield a 4% risk-free rate in the steady state, in line with the average after-tax return on capital from 1954 to 2000 (Gomme, Ravikumar, and Rupert, 2011). I assume households have indivisible labor preferences: $u(C, 1-n) = \ln C + B(1-n)$. I set the leisure utility parameter so that households spend about one third of their time working. The production weight on capital pins down the capital-output ratio. Using data from the Bureau of Economics Analysis (BEA), I find that the capital-output ratio averaged 2.28 from 1954 to 2018. Over that period, the investment to capital ratio averaged 0.067, after accounting for growth.[12] This sets the depreciation rate. Additionally, the labor share of production ensures that 60% of income goes to

[12]This measure of investment includes the purchases of consumer durables.

Table 1: Parameters (Annualized)

| | Parameter | Symbol | Value |
|---|---|---|---|
| Externally Set | Discount factor | $\beta$ | 0.96 |
| | Labor share of output | $\eta$ | 0.6 |
| | Capital depreciation rate | $\delta$ | 0.067 |
| | Exit rate | $\rho$ | 0.085 |
| | Coupon payment | $c$ | $1/\beta - 1$ |
| | Average debt maturity | $\lambda$ | 1/5 |
| | Persistence of idiosyncratic shocks | $\rho_\varepsilon$ | 0.757 |
| | Super Firm Share, Type 1 | – | 0.11% |
| | Super Firm Share, Type 2 | – | 0.18% |
| Internally Set | Leisure utility | $B$ | 2.1 |
| | Capital share of output | $\alpha$ | 0.275 |
| | Variance of idiosyncratic shocks | $\sigma_\varepsilon^2$ | 0.075 |
| | Operating expense | $\kappa$ | 0.12 |
| | Entrant endowment rate | $\iota$ | 0.025 |
| | Capital recovery | $\chi$ | 0.54 |
| | Default cost | $\nu$ | 0.0525 |
| | Debt prepayment cost | $\gamma$ | 0.75% |
| | Super Firm Productivity, Type 1 | – | 1.826 |
| | Super Firm Productivity, Type 2 | – | 1.566 |

workers (Cooley and Prescott, 1995).

The exogenous exit rate is set to match the average of 8.5% from 1984-2006 (Business Dynamics Statistics, "BDS," Census Bureau). Entrants' endowment is 2.5% of firms' average capital, and is set to match new firms' share of total employment. This endowment is smaller than in other papers (e.g., Khan and Thomas, 2013), but entrants are able to borrow into their first period of production. This endogenous funding augments the start-up funds provided by the representative household. The persistence of idiosyncratic productivity is set at 0.757 at an annual frequency, following estimates by Syversyon (2008). The variance of productivity shocks is 0.075 annually, to match the standard deviation in investment rates found by Cooper and Haltiwanger (2006). I assume operating expenses take the form $\kappa\varepsilon^{1/(1-\eta)}$, with $\kappa = 0.12$ at an annual rate. Holding other parameters constant, $\kappa$ helps to govern the mean and variance of firm-level leverage. I take these moments from Dinlersolz et al (2018).

From 1990 to 2017, the recovery rate averaged 42% for bonds and 70% for loans (Moody's, 2018). Flow of Funds data indicate that over that period, nonfinancial corporations had $1.26 dollars in bonds for every $1 in loans. Applying this proportion to the Moody's data, I estimate an overall recovery rate of 54%, and choose this as the value of $\chi$. Given this, I set the the proportional default cost to 5.25%, to target the default rate of 1.65% (Moody's, 2018). This default rate of 1.65% is well below the 3% rate commonly used in the literature (e.g., Bernanke, Gertler, & Gilchrist, 1999; Ottonello & Winberry, 2020). Bernanke, Gertler, & Gilchrist (1999) cite a Duns and Bradstreet survey for their measure, which spans 1984-1994 (Fisher, 1996). While I use Moody's default data to be consistent with the recoveries given loss, according to PayNet, across small businesses the annual default rate from 2012-2018 was 1.70%.[13] PayNet is a subsidiary of Equifax, one of the major credit-rating agencies in the United States.

Following Gomes, Jermann, and Scmid (2016), I set the coupon payment so that the bond price for risk-free debt is normalized to 1 in the steady state.[14] Over the dynamic equilibrium, the coupon rate is held constant so that the bond operates similarly to fixed-rate instruments. Firms have a 5% chance of having their debt mature in any given period, implying an average maturity of 5 years. This matches the average over 2002 to 2012 found in Choi, Hackbarth, and Zechner (2018). A 5-year expected duration is also used in other computational models of long-term corporate debt (e.g., Gomes, Jermann, and Schmid, 2016; Bustamante, 2019). The prepayment cost is set at 0.75% of debt's face value, to match the prepayment rate observed over corporate bonds (Xu, 2017).

Lastly, to capture the extreme skewness of the distribution of employment across firms in the United States, I assume that a small fraction of firms are exogenously unconstrained. These "super" firms are highly productive and can frictionlessly issue equity whenever necessary.[15] I assume there are two types of super firms. According to the BDS, the largest 0.11% of establishments have over 1,000 employees, and employ 14.4% of workers. Another 0.18% of establishments have between 500 and 999 employees, and account for 6.9% of employment. "Type-1" super firms are the largest employers, and these 0.11% of firms have a productivity level of 1.846. (The average ordinary firm

[13]Data taken from https://sbinsights.paynetonline.com/loan-performance/. Last accessed September 1, 2021.

[14]In the steady state, risk-free borrowing has a more favorable price than risk-free saving: $1 > \beta$. The reason for this is that the borrower pays periodic coupons, whereas risk-free savings are a zero-coupon asset. While the prices vary, the expected returns on risk-free debt and risk-free savings equal one another.

[15]These super firms are similar to the "no-constraint" firms in Khan & Thomas (2013).

has a productivity level of 1.) “Type-2” super firms have a productivity level of 1.583, and make up 0.18% of firms. These super firms ensure that in the steady state, 21.3% of employment is concentrated in 0.29% of firms.

Table 2 summarizes the models' fit to targeted empirical moments. For firm-level moments, I draw data from two sources. Dinlersoz et al. (2019) construct a novel dataset that spans both private and public firms in the US (LOCUS). Most panel datasets on firms focus on public firms, for which more information is readily available. However, the vast majority of firms active in the US are private, rather than public. The ideal dataset spans both types of firms, as private firms are more representative of individual-level characteristics, while public firms tend to be larger, and hence have outsized influence on aggregates. The leverage rows ($b/(k+a)$) draw on the private measures of total leverage in Dinlersoz et al. (2019). Across public firms, firms have a higher mean leverage ratio, but a similar standard deviation. I choose total leverage, rather than financial leverage, as my target, because the former is the compliment of a firm's equity ratio. Targeting total leverage is isomorphic to targeting a firm's internal funding share. For capital growth, I draw from Cooper & Latiwanger (2006), which examines the Census Bureau's Longitudinal Research Database (LRD). Lastly, for labor share by firm age, I use Ottonello's & Winberry's (2020) analysis of the Census Bureaus' Longitudinal Business Database (LBD).

Overall, the model's real economy fits the data fairly well, with a reasonable fit across the first two moments of annual capital growth. The model's fit to financial moments is a little weaker, but it still does a fairly decent job capturing the mean and standard deviation of leverage. Consistent with the findings of Xu (2018), prepayment is relatively uncommon in the steady state: 2.4% of aggregate debt is prepaid in a year. The model overshoots that somewhat, with 4.7% of debt being prepaid annually. In a given quarter, 1.4% of firms that do not receive the maturity shock choose to prepay their debt. The model is less successful at targeting the pricing of risk: the average recovery rate across defaulted firms is roughly 20 percentage points (ppts) too high, relative to Moody's data. Accordingly, the average risk premium in the steady state is about 1.5 ppts lower than the long-run average in the data.

Table 2: Steady State Moments (Annualized)

| Moment | Data | Model |
|---|---|---|
| Capital/Output | 2.28 | 2.17 |
| Investment/Capital | 0.069 | 0.069 |
| Share of Time Worked | 1/3 | 0.336 |
| Default rate (%) | 1.65 | 1.61 |
| Share of Debt Prepaid (%) | 2.3 | 4.7 |
| Recovery rate (%) | 54.4 | 76.6 |
| Mean risk premium (%) | 2.65 | 0.96 |
| Mean Annual Capital Growth | 0.122 | 0.118 |
| St. Dev. Annual Capital Growth | 0.337 | 0.363 |
| Mean Debt/Assets | 0.46 | 0.52 |
| St. Dev. Debt/Assets | 0.38 | 0.31 |
| Employment Share, Firms $<1$ year | 0.03 | 0.03 |
| Emp. Share, Firms $\in (1, 10)$ years | 0.21 | 0.28 |

Firm-level model moments winsorized at 1% level

Capital data: Cooper & Haltiwanger (2006): LRD

Leverage data: Dinlersoz et al (2019): LOCUS private firms

Employment data: Ottonello & Winberry (2020): LBD

To assess the model's ability to capture firm-level dynamics, Table 3 compares quarterly employment growth rates from the model steady state distribution to data on establishments from the Business Employment Dynamics dataset from 2001 to 2006 (Davis et al., 2010). Despite being untargeted moments, the model does a fairly reasonable job of matching the distribution of employment growth observed in the data. The largest misses are across the two high-growth groups: the share of firms with growth over 20% is 6.9 ppts higher in the model than in the data (14.5% versus 7.6%). The opposite holds for the share of firms with growth rates between 5% and 20%: just 7.7% of model firms have these growth rates, compared to 16.9% of firms in the data. The model implies a substantial mass of firms with growth rate just above 20%: 6.6% have growth rates between 20% and 21%, for example. While the data may be similarly concentrated, this large mass of firms close to the threshold indicates that the relatively weak fit is not being driven by a large number of firms with extremely high growth rates. Overall, the model is reasonably able to reproduce the dynamics of employment growth across firms.

Table 3: Firm Distribution (%)
over Quarterly Employment Growth

| | Data | Model |
|---|---|---|
| (-2, 0.2] | 7.6 | 5.4 |
| (-0.2,-0.05] | 16.7 | 17.0 |
| (-0.05, 0.05] | 51.2 | 55.5 |
| (0.05, 0.2] | 16.9 | 7.7 |
| (0.2, 2) | 7.6 | 14.4 |

Growth calculated as $2 * (n_{t+1} - n_t)/(n_{t+1} + n_t)$

Data source: Davis et al. (2010)

## 5.2 The Effects of Long-Term Debt Maturity on Firm-Level Investment

One of the primary motivations for this paper is empirical analyses that highlight the substantial effects of long-term debt maturity on firm-level investment (e.g., Almeida et al, 2011; Kalemli-Özcan, Laeven, & Moreno, 2018). Hence, a key test of this model is whether it can replicate the effects found in the data.

To estimate the effects of long-term debt maturity, I use Compustat data on public firms from 1984 through 2020. In Compustat, data on long-term debt maturity are available at an annual frequency. Specifically, I can identify the amount of long-term debt that is due to mature in the following year. This is compatible with an annual version of the model. Sample selection and the choice control variables follow the literature norm closely (e.g., Almeida et al, 2011; Bustamante, 2019; Ottonello & Winberry, 2020), and are described in Appendix B. Investment growth is measured as the log growth in property, plant and equipment, adjusted for inflation. I estimate equations of the following form

$$\Delta \ln k_{i,t+1} = \alpha_i + \alpha_{s,t} + \beta \mathcal{M}_{i,t} + \Gamma' X_{i,t} + \epsilon_{i,t} \tag{14}$$

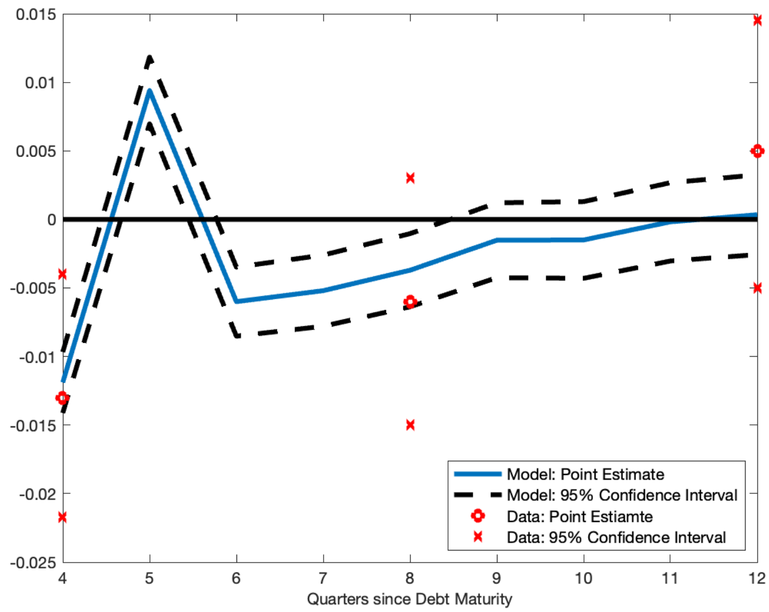
where firm $i$ is in sector $s$ in year $t$. I control for fixed effects at the firm and industry-year levels. $\mathcal{M}$ is the variable on maturity of long-term debt, and $X$ is a vector or controls. Specifically, the controls are the log of the firm's real assets, its lagged real sales growth, its book leverage, and its Tobin's $q$ statistic. In the model, I control for the log of a firm's assets, financial assets as a share of total assets, leverage, and its idiosyncratic productivity. My model sample consists of 1,00,000 firms randomly drawn from the steady state distribution. For both the data and model, my estimates exclude firms without debt.

In the model, maturity is orthogonal to the firm's state and is unpredictable. Accordingly, any association between debt maturity and future investment has strong causal implications. Identification is weaker in the data, as the maturity of long-term debt can be anticipated and could, in principle, be correlated with other decisions made by the firm. For example, a firm could time the maturity of its debt to coincide with the cumulation of an R&D project, so that it can coordinate its future debt and investment needs. The identifying assumption I make, common with the rest of the literature, is that because long-term debt has such a long duration, such coordination is too challenging to implement in practice. Since the typical long-term bond lasts for 5 years, about the same duration as a business cycle, I do not view this as a strong assumption.

My primary proxy for debt maturity is an indicator variable for firms with a high share of long-term debt maturing. This fits well with the nature of the model, where either all or none of a firm's debt matures in a given period. Since in the model, 20% of firms have debt maturing in a year, in the data I identify firms with more than 20 of their debt maturing in the year. This is also the threshold used in Almeida et a. (2011) to identify firms with a high share of maturing debt. For robustness, in Appendix B I consider other specifications in both the data and the model.

Figure 2 tracks the estimated effect of debt maturity on year-over-year capital growth over a three-year horizon, both in the model and in the data. The lines track the model point estimates and 95% confidence interval from quarter to quarter. The points show the point estimates and 95% confidence interval from the data, which are only available once a year. Focusing first on the data, I find that having a large share of long-term debt maturing leads to significantly weaker investment over the following year, with the point estimate implying that maturity generates a 1.3 log-point decline in future capital. In the second and third years, the effects are less precisely estimated; neither coefficient is significantly different from 0. This suggests that debt maturity operates like a growth shock: investment falls for a year, but does not subsequently accelerate.

Figure 2: Estimated Effect of Debt Maturity on Four-Quarter Investment, Model & Data




From estimates of equation (14). Investment measured as the difference in log capital. Maturity in the data: indicator for firms with more than 20% of long-term debt maturing in the year. Maturity in the model: indicator for firms receiving the maturity shock. Regressions include firm-level controls, such as size, as well as fixed effects at the industry-year level for the data. Standard errors clustered at the firm level.

Qualitatively, the effects of debt maturity in the model track those found in the data: maturity triggers an initial decline in investment that eases over time. A perhaps notable exception to this occurs in growth from the first quarter after initiation through the fifth quarter. Over this period, lagged debt maturity is associated with a significant *increase* in investment relative to firms whose debt did not mature in the previous period. However, this additional growth reflects base effects: because investment falls substantially in the quarter of maturity, a firm's capital stock is particularly low in the period after maturity. This rebase effect fades away, and the negative effect of maturity on investment is sustained for several subsequent periods.

Quantitatively, the model finds similar point estimates as in the data over the first two years: a 1.2 log-point decline in the first year, and a 0.4 log-point decline during the second. Over the third year, the model's point estimate is 0.0 log points, and the data estimate of 0.5 log points falls outside the model's 95% confidence interval. However, at three years out the data estimate is highly imprecisely estimated; the model's entire 95% confidence interval falls within the left tail of the data's confidence interval. Overall, I conclude that the model captures the effects of debt maturity on firm investment observed in the data quite well, despite these being untargeted moments.

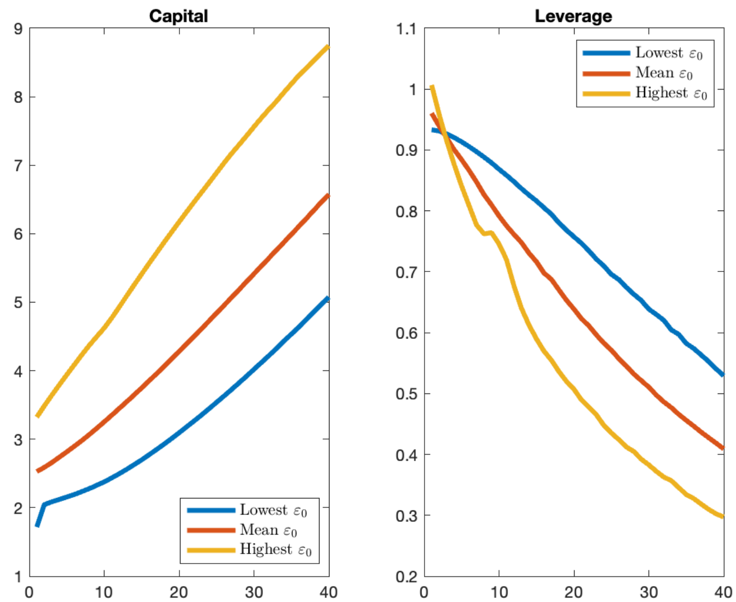
Both in the data and in the model, I show that the maturity of long-term debt is associated with sustained declines in firm-level investment even outside of recessions. This finding contrasts with that of Almeida et al. (2011), which found that maturity generated a large decline in investment during the Great Recession, but had no effect in previous years. Our empirical methodologies differ, though, in that Almeia et al. (2011) used a matching process to connect firms with high shares of long-term debt maturing to similar firms with lower shares of maturing long-term debt. This procedure leads to a very small sample, which makes precision issues more challenging. The panel regression approach employed here examines 25,000 firms, rather than fewer than 100. This paper is also not the first to identify a significant relationship between long-term debt maturity and investment. Using a similar approach, Bustamante (2019) finds that the share of long-term debt maturing is negatively related to investment, even after controlling for the firm's leverage.

### 5.3 Lifecycle Dynamics in the Steady State

To capture lifecycle trends across firms, Figures 3 and Figure 4 track a cohort of firms from entry. The maturity structure makes a firm's level of debt highly persistent: in the absence of prepayment, a firm's debt is expected to mature only once every 5 years. Hence, over the 40-quarter horizon examined here, in expectation a firm will only be able to choose new debt twice. Due to this persistence, I take averages conditioning on initial productivity, as an entrant's initial level of productivity influences its first choice of debt.

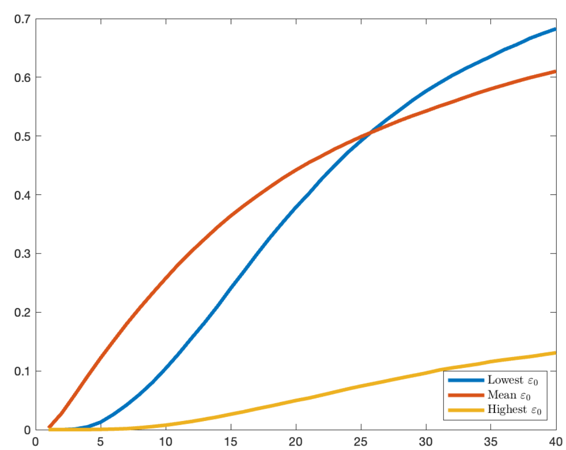
Over the first few years from entry, firms that initially have the highest productivity have fairly linear capital growth, on average, while the least productive firms capital trajectory is far more flat. Less productive firms start with a much smaller capital base, which curbs their growth.

Figure 3: Firm Lifecycle Dynamics in Steady State, by Initial Productivity




*Averages across continuing firms. x-axis: quarters since entry.*

Figure 4: Cumulative Prepayment Probability in Steady State, by Initial Productivity




*Averages across continuing firms. x-axis: quarters since entry.*

Consequently, even while there is mean reversion in productivity levels, the gap in average capital actually widens slightly over time. The gap between the initially average-productive and the initially least-productive firms grows from 27% in the second quarter of production 30% in the fortieth quarter. Similarly, the gap between the initially most-productive and the initially average-productive firm edges up from 31% to 33%. While the firms initially the most productive tend to reduce their debt over time, across the first 40 quarters the average debt at firms that initially had average or the lowest productivity increases. These debt increases allow the firms to scale up more quickly. However, the less-productive firms are hindered from increasing their debt along both the extensive and intensive margins. On the extensive margin, since long-term debt only matures probabilistically and prepayment is costly, opportunities to take on new debt are relatively rare. Additionally, these firms tend to have high leverage, which means that more available funds need to go towards paying back outstanding debt, rather than providing collateral for new borrowing. On net, these forces imply that the initially least productive still have an average leverage ratio over 0.5 after operating for 40 quarters. In contrast, the firms that initially were the most productive have reached an average leverage of 0.3 by that time.

Related to this, Figure 4 tracks the cumulative probability that a firm will choose to prepay at least once. For example, 26% of firms that initially had average productivity have prepaid their debt at least once in their first 10 quarters. Firms that started with the highest level of productivity are much less likely to prepay. Through 40 quarters, just 13% have prepaid at least once, compared to 61% of firms that started with average productivity, and 68% of firms that initially had the lowest level of productivity. Prepayment is more common among firms that receive positive productivity shocks. Since productivity is mean-reverting, in expectation these firms' borrowing conditions are better now than in the future. Prepayment allows firms to more fully take advantage of positive productivity shocks: the median firm that chooses to prepay increases its debt 26.5%, boosting its leverage ratio by 2.5 ppt. This increase in leverage runs counter to the economy-wide trend of firms reducing their leverage over time. The cost of prepayment deters many firms from making this choice–in particular, if a firm waits, there is a chance its debt might mature costlessly next period. The gains from prepayment will tend to be largest for young firms, which are the most below scale. Accordingly, the median firm choosing to prepay is 16 quarters old, roughly half the economy-wide median age of 30 quarters.

# 6 Model Dynamics under Recession

My recession simulations start in the quarter beginning in March 2020.[16] To discipline the shocks, I target changes in output and in wages.[17] Both sets of shocks last for four quarters and get less severe over time. Table 4 shows the aggregate shocks. The labor shock reflects the rise in wages observed since the start of the recession, and falls linearly. The lockdown shocks were chosen so that, absent stimulus, the peak decline in total output slightly exceeds what has been observed in the US.[18] I assume that initially 1/3 of firms are subject to a lockdown that lasts a month (i.e., one third of the quarter). Since output is proportional to idiosyncratic productivity, lockdown shocks can also be viewed as productivity shocks (e.g., a restaurant that can operate throughout the quarter, but at reduced capacity). Over time, fewer firms get locked down and lockdowns are of shorter duration.

Table 4: Exogenous Productivity Shocks

| Quarter | Labor Disutility Shock (%) | Lockdown Share* | Lockdown Duration** |
|---|---|---|---|
| Q1 | 2.5 | 1/3 | 1/3 |
| Q2 | 1.875 | 1/5 | 1/3 |
| Q3 | 1.25 | 1/10 | 1/6 |
| Q4 | 0.625 | 1/10 | 1/6 |
| Q5+ | 0 | 0 | – |

*: Share of firms locked down in a period. **: Share of period a firm cannot operate.

I evaluate two policies: cash grants and reductions in lending rates. Cash grants increase eligible firms' financial savings on impact date. Following the 2020 Paycheck Protection Program, eligible firms receive 2.5 months of their normal wage bill, capped at $2 million. Cash grants are funded through financial transfers, and therefore only affect aggregate resources through endogenous channels. I assume that the fiscal authority borrows from the representative household to finance the grants, which it pays back through lump-sum taxation of the representative household. Conceptu-

[16] In the US, domestic transmission of COVID-19 started around February 2020. Data for the full 2020Q1 smooth over the recession's initial impact. For example, real personal consumption expenditures in the US rose 0.4% in January, edged down 0.1% in February, and dropped 6.5% in March. Across the full first quarter of 2020, consumption fell just 1.8%.

[17] The only source I am aware of on firm lockdowns is a survey by the Census Bureau of small businesses. I choose not to rely on the Census data, as they do not distinguish between temporary closures due to government restrictions and voluntary closures (e.g., because of a lack of demand). As the former are endogenous, calibrating to temporary closures would overstate the magnitude of the exogenous shocks.

[18] In the US, output in 2020Q2 was 10% below its level at the end of 2019.

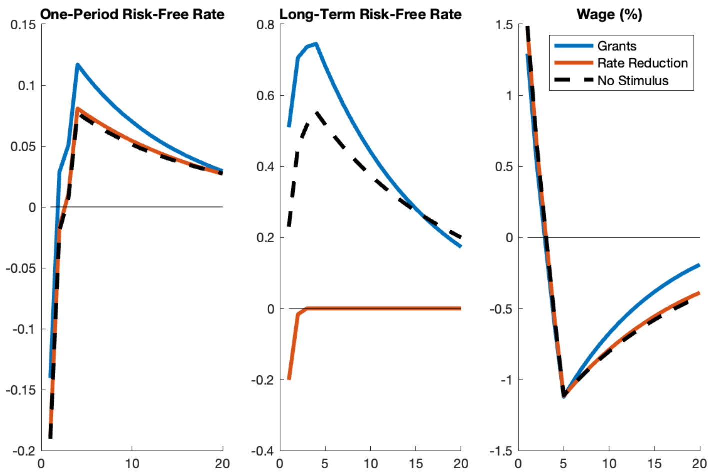
ally, this is similar to approaches taken elsewhere in the literature (e.g., Khan and Lee, 2021), and implies that the financing of cash grants does not affect aggregate prices. This offers an upper bound of the effectiveness of the program, as distortionary taxation would slow down the recovery. In the rate reduction scenario, I impose a floor on lenders' discount rate: when pricing new debt contracts, lenders are at least as patient as they are in steady state. This ensures the risk-free lending rate never exceeds its steady state level, which I view as a plausible upper bound for what policy might seek to implement. It also generates a fall in long-term interest rates that is quantitatively similar to the reductions Krishnamurthy & Vissing-Jorgensen (2011) estimate from the quantitative easing implemented in the aftermath of the Great Recession. Rate reductions last for the duration of the exercise horizon.

The next three subsections discuss the dynamics of the recession over the no-stimulus baseline, and when the two stimulus policies are implemented independently of one another. I then turn to the scenario in which both policies are simultaneously implemented.

### 6.1 Price Dynamics over the Pandemic Recession

To provide context for how agents respond to the recession shocks, Figure 5 tracks the trends in aggregate prices. Figure 5 highlights two key features of long-term debt contracts with fixed coupon payments. Because expected payments are back-dated under long-term lending, the model generates a substantial rise in the risk-free loan rate, which peaks 0.5 ppt above the risk-free one-period rate absent stimulus. Hence, credit conditions tighten far more in environments with long-term debt than those with one-period debt. In particular, the risk-free lending rate moves above its steady state level immediately upon impact date, even as the risk-free one-period rate is lower than its steady state level. So whereas risk-free borrowing is more expensive in this long-term debt environment, in a one-period debt model risk-free borrowing would actually be cheaper at the onset of the recession than in the steady state.

Figure 5: Price Dynamics over Pandemic Recession,
Percentage Point Differences from Steady State

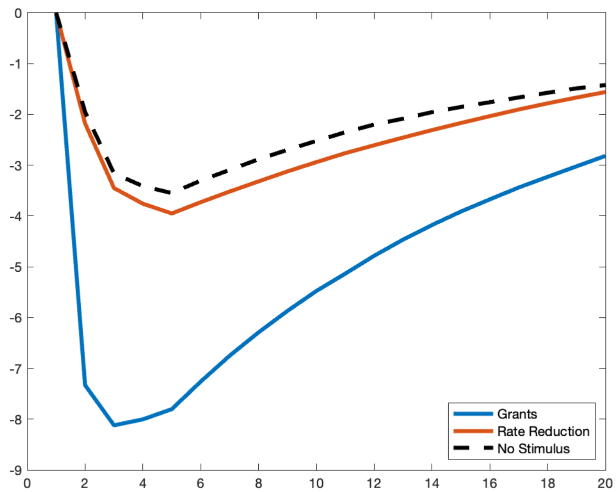



Cash grants increase households' impatience over the first 20 quarters of the recession, leading to larger rises in the risk-free borrowing rate. Through this channel, the policy generates a further tightening of credit conditions for much of the recession. While risk-free borrowing does eventually become cheaper under cash grants than in the no-stimulus case, this occurs well after the economy has started to recover. By construction, under the rate reduction scenario the risk-free lending rate never exceeds its steady state level. Borrowing conditions initially ease, tracking the fall in the risk-free one-period rate. But after the second period, the floor on lenders' discounting binds.

The right panel of Figure 5 shows the trends in wages. There are two offsetting forces influencing the wage rate. For the first four quarters, the labor disutility shock increases wages. However, with consumption falling below its steady state level, households' value of leisure declines, putting downward pressure on the equilibrium wage. The disutility shocks dominate initially, but after their removal wages fall about 1% below their steady state level. Cash grants generate a faster recovery in wages, due to the faster recovery in consumption.

To quantify how these general equilibrium effects influence investment, Figure 6 plots a financially unconstrained firm's capital over the recession. As in Ottonello and Winberry (2021), an unconstrained firm is one whose investment decisions are not influenced by their level of debt or assets. This firm's capital choice only responds to expectations of future lockdown risk, future wages, and the discount factor.[19] Absent stimulus, the capital of an unconstrained firm troughs 3.6% below its steady state level, four quarters after impact date, and recovers half these losses in 11 quarters. Rate reductions slightly reduce patience, amplifying the peak capital decline to 4.0%. Since the discount factor falls even more under cash grants, the peak capital decline rises to 8.1%. However, the half-life of this decline is two quarters shorter under cash grants, because the policy generates a faster decline in the discount factor.

Figure 6: Capital for a Financially Unconstrained Firm over Pandemic Recession, Percent Deviations from Steady State Level

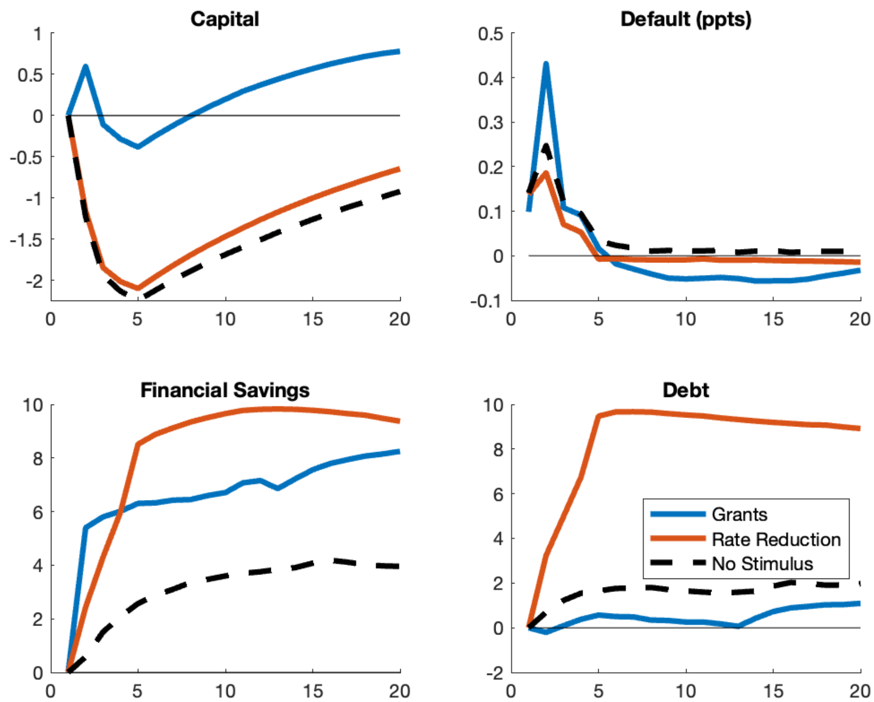



## 6.2 Ordinary Firms' Response to the Pandemic Recession

To assess how financially constrained firms respond to the recession, Figure 7 examines only ordinary firms, which account for over 99.5% of all firms in the economy. Over the no-stimulus scenario, ordinary firms' aggregate capital troughs 2.2% below its steady state level, and requires 12 quarters to make up half this loss. This decline is smaller than that of financially unconstrained firms (Figure 6). This is because financial constraints restrain ordinary firms' size. In the steady state, the average ordinary firm's capital choice is two-thirds what it would chose absent financial constraints.

[19]The equilibrium discount factor is the reciprocal of the one-period risk-free interest rate.

Figure 7: Ordinary Firms' Aggregate Stocks over Pandemic Recession, Percent Deviations from Steady State Level




Absent stimulus, the recession prompts a steady rise in aggregate debt, which peaks 2.3% above its steady state level. These extra funds larger lead to increased financial savings, which rise 2.6% over the first year of the recession and peak 4.2% above their steady state level. Firms without debt on impact date account for about half of the rise in debt. In part, this reflects firms that were unconstrained in the steady state, but become financially constrained due to lockdowns. Additionally, firms engage in precautionary savings, hedging against the risk that future lockdowns may make them constrained. Since the benchmark lending rate rises over the first year of the recession (Figure 5), borrowing on impact date to insure against future risk can be cheaper than waiting and borrowing only upon the realization of a negative shock. The simultaneous rise in in debt and savings is consistent with the "borrow to save" behavior observed by Xiao (2019) in the aftermath of the Great Recession. Additionally, prepayment is procyclical, consistent with the empirical findings of Xu (2018). Over the first year of the recession, 4.3% of firms prepay in the no-stimulus scenario, down from 5.8% in the steady state. As firms that prepay tend to increase their borrowing, so they can scale up, a decline in prepayment activity lowers aggregate debt.

The rate reduction policy generates a larger rise in aggregate debt. 3.8% of firms choose to prepay in the first quarter alone, in response to the relaxation of lending conditions. Similarly, more debt-free firms choose to borrow. This generates a substantial rise in aggregate financial savings, which peak 11.6% above their steady state level. In contrast, reducing the risk-free lending rate has small effects on ordinary firms' aggregate capital, shaving off just 0.14 ppt of the peak decline observed without stimulus. This is because the additional borrowing is largely by firms with a lower risk of becoming financially constrained. The most at-risk firms engage in precautionary borrowing even absent the ceiling on lending rates. As the additional borrowing is by marginally at-risk firms, less of it gets converted into additional investment. While rate reductions have little effect on how much capital falls, they slightly speed up the recovery, shortening the half-life by two quarters.

In contrast to rate reductions, cash grants are effective at shoring up ordinary firms' aggregate capital. Despite the pandemic shocks, the stimulus boosts ordinary firms' investment upon impact date, leading to a 0.6% rise in their aggregate capital. Their aggregate capital subsequently falls to 0.4% below steady state, as lockdown shocks and general equilibrium effects weigh down investment. However, in recovery ordinary firms' aggregate capital overshoots its steady state level eight quarters after impact date. Cash grants also generate a sustained increase in financial savings. In level terms, the increase in financial savings exceeds the rise in ordinary firms' aggregate capital by a factor of 4.4 just after impact date. In contrast, aggregate debt is little changed from the steady state under the cash grant scenario. In part, this reflects the grants' ability to reduce firms' reliance on external financing, by increased their internal funds. However, borrowing is also constrained by the increased risk-free lending rate generated by the cash grants. On impact date, the prepayment rate falls to 0.7%, less than half the steady state level.

In the absence of stimulus, defaults increase modestly from steady state. The default rate rises 0.6 ppt to 2.2% over the first year of the recession.[20] In line with the aggregate recovery, by the second

[20]The short duration of the exogenous shocks to the economy reduces the realization of defaults. For example, each firm has a 65% chance of not being locked down from the second quarter on. Ongoing work examines a more severe, longer recession, to assess how sensitive defaults are to the recession's duration.

year the default rate has largely recovered to the steady state. Rate reductions are moderately successful in preventing defaults, lowering the rate over the first year by 0.16 ppt, or a quarter of excess defaults. By improving borrowing conditions, rate reductions ease the downside risks of maturity shocks. This leads to a fall in strategic default behavior, as reduced rollover risk improves firms' inside option of maximizing continuation value, while having little effect on firms' outside option of issuing dividends and increasing default risk. In contrast, the cash grants actually lead to an 0.12 ppt increase in the default rate over the first year of the recession. While cash grants reduce defaults on impact date, these defaults are simply delayed, rather than prevented. Price effects, such as the tighter debt price schedule, push against the benefits of the stimulus and undermine cash grants' ability to reduce defaults. As these price effects start to ease in the second year, the default rate under cash grants moves consistently lower than the default rate absent stimulus.

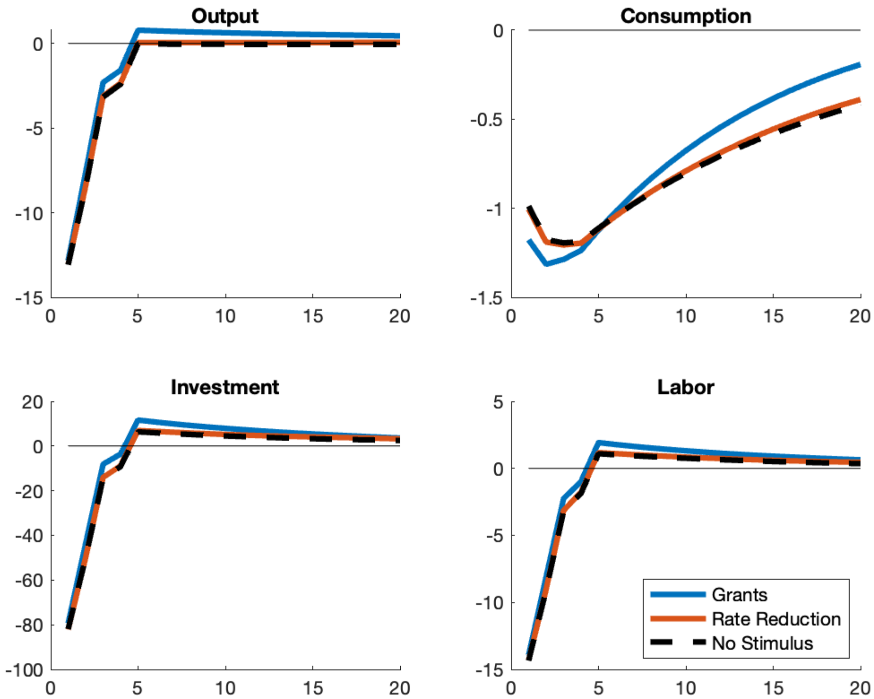
### 6.3 Aggregate Flows over the Pandemic Recession

Figure 8 tracks the responses in aggregate output, consumption, investment, and labor across the first 20 quarters of the recession. Overall, the effects of stimulus tend to be relatively small. For example, over the first year of the recession, aggregate output was 6.8% below its steady state level in the no-stimulus baseline. Rate reductions shave off just 0.1 ppt of this decline. Cash grants have larger effects on aggregate output, reducing the first year's decline by 0.7 ppt. By the second year of the recession, output in the baseline has returned to its steady state level. Under the rate reduction scenario, output over the second year is 0.1% higher than its steady state level, and under cash grants it is 0.7% higher. The additional gains from policy start to diminish starting in the third year after impact date. The trends across total investment and labor qualitatively match those of aggregate output. In both cases, rate reductions provide a slight boost relative to the no-stimulus baseline, while cash grants provide more support.

As is common in complete markets models, the model generates a disproportionately large fall in investment, and a milder fall in consumption.[21] Nonetheless, the model provides useful insights into

[21] An expanded set of shocks would bring the model closer to the data. For example, a fall in households' utility from

Figure 8: Aggregate Flows over Pandemic Recession,
Percent Deviations from Steady State Level




the relative effects of the stimulus policies. Rate reductions have very negligible effects on current consumption; the modest increase in output is invested into future capital. This spurs the longer-term recovery, but does not provide much relief to the household during the crisis. In contrast, cash grants generate a steeper trend in aggregate consumption, falling more upon impact date but rising more quickly after the first year of the recession. Because cash grants stimulate investment more at the start of the recession, firms can subsequently produce more.

### 6.4 Simultaneous Implementation of the Cash Grants & Rate Reductions

During the 2020 recession, the Paycheck Protection Program and quantitative easing were implemented simultaneously. To explore the interactions between these types of policies, I consider a "full policy" scenario that implements both cash grants and rate reductions. Table 5 summarizes the

consumption would help match the steep decline observed in 2020Q2. Conceptually, this could reflect individuals' reduced desire to have restaurant meals or other high-risk activities. Holding aggregate resources constant, the fall in consumption caused by that demand shock would necessitate an increase in aggregate investment.

economy's response to the pandemic in the full policy scenario, relative to the no-stimulus scenario. For comparison, it also examines the cash grant- and rate reduction-only scenarios.

Table 5: Model Dynamics under Stimulus, relative to No-Stimulus Baseline (%)

| Economy-Wide | Output | | | Consumption | | | Investment | | | Employment | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | CG | RR | Both | CG | RR | Both | CG | RR | Both | CG | RR | Both |
| Year 1 | 0.75 | 0.05 | 0.81 | -0.12 | -0.01 | -0.13 | 8.44 | 0.21 | 8.77 | 0.87 | 0.07 | 0.94 |
| Year 2 | 0.77 | 0.09 | 0.87 | 0.04 | 0.00 | 0.04 | 4.39 | 0.60 | 4.85 | 0.73 | 0.08 | 0.83 |
| Year 3 | 0.67 | 0.10 | 0.77 | 0.14 | 0.01 | 0.15 | 3.14 | 0.67 | 3.56 | 0.54 | 0.08 | 0.62 |
| **Ordinary Firms** | **Capital** | | | **Default (ppt)** | | | **Savings** | | | **Debt** | | |
| | CG | RR | Both | CG | RR | Both | CG | RR | Both | CG | RR | Both |
| Year 1 | 1.89 | 0.12 | 2.01 | 0.12 | -0.16 | -0.12 | 3.85 | 3.83 | 9.47 | -1.15 | 5.12 | 6.24 |
| Year 2 | 1.92 | 0.20 | 2.14 | -0.16 | -0.12 | -0.27 | 2.99 | 5.79 | 9.49 | -1.43 | 7.71 | 7.12 |
| Year 3 | 1.91 | 0.25 | 2.16 | -0.25 | -0.08 | -0.32 | 3.28 | 5.84 | 9.56 | -1.35 | 7.73 | 7.19 |

CG: cash grants. RR: rate reductions.

Aggregate capital, savings, and debt across ordinary firms measured at end of year.

The benefits of the full policy scenario are roughly equal to the sum of what each policy generates on its own. For example, under full policy output is 0.81% higher over the first year than in the absence of stimulus. This is 0.06 ppt more than what cash grants generate, effectively the same as the 0.05% boost of the rate reductions-only scenario. These additive effects indicate that the two policies do not substantially alter one another's real economic effects. However, simultaneous implementation does alter financial trends. Notably, the full policy scenario causes a sustained fall in the default rate, in contrast to the initial increase generated by cash grants on their own. Additionally, the full-policy scenario leads to larger rises in both debt and financial savings. Hence, the muted increase in debt in the cash grant-only scenario is not driven by substitution of debt for grants, but is a response to the tighter financial conditions the policy generates.

There are benefits to simultaneously implementing cash grants and rate reductions. Cash grants help minimize the economic downturn and speed up the recovery. Rate reductions are more effective at easing conditions for the most financially constrained firms, reducing the default rate. The two policies target different forms of financial constraints: cash grants improve firms' liquidity positions, while rate reductions ease borrowing constraints and reduce rollover risk. Consequently, the two policies' effects do not substantially interact with one another in the aggregate. This allows the combined policies to generate both a stronger aggregate recovery and a reduction in defaults.

# 7 Channels of Policy Transmission

This section examines the underlying drivers of the aggregate trends discussed in Section 6. I start by quantifying the aggregate implications of cash grants' general equilibrium effects. I then assess how the maturity of long-term debt affects investment during the recession, and how effective stimulus measures are at mitigating this effect. Next, I track how firms of different sizes respond to the recession and to stimulus. Lastly, I measure the long-term benefits of the stimulus policies.

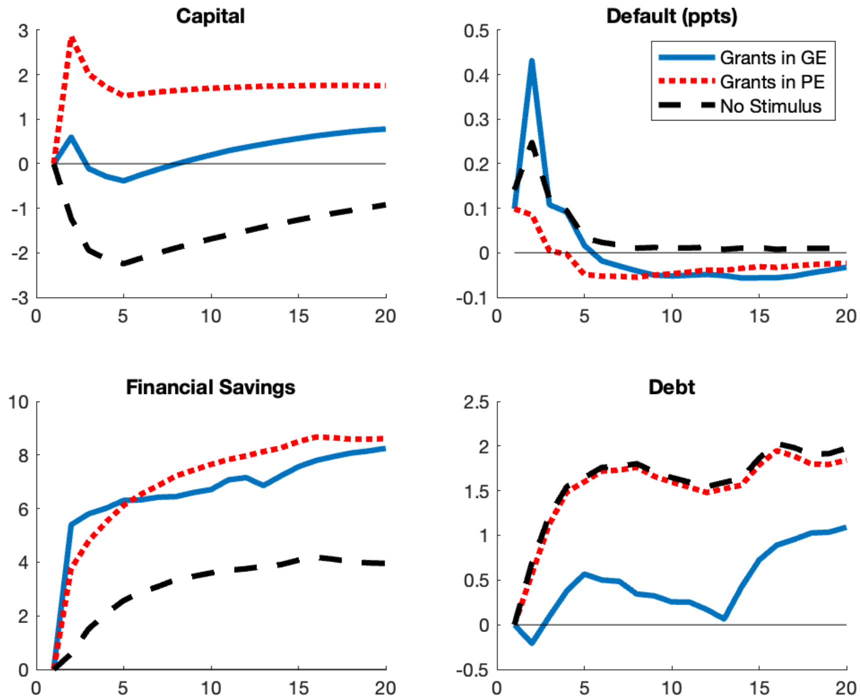
## 7.1 Cash Grants & General Equilibrium Effects

Section 6 identified large general equilibrium effects by cash grants. To quantify the total impact of these effects, I consider a counterfactual scenario in which prices follow their trajectories in the no-stimulus baseline, but firms receive cash grants on impact date. This disentangles the direct effects of the grants from the indirect effects they generate.

Figure 9 compares the aggregate response of ordinary firms under this counterfactual exercise to when the grants are given in general equilibrium and when no stimulus is given. Under the partial equilibrium counterfactual, ordinary firms' capital rises 2.9% in the aggregate, 2.2 ppts more than what cash grants generate under general equilibrium. In contrast, the rise in financial savings in the counterfactual is initially smaller than in general equilibrium. This illustrates the degree to which aggregate price movements lower target capital levels for unconstrained firms, leading to a reallocation of funds from physical capital to financial savings.

Default rates also vary markedly between the partial and general equilibrium scenarios. In the partial equilibrium counterfactual, the default rate over the first year of the recession is 1.8%, compared to 2.3% from transfers under general equilibrium and 2.2% in the no-stimulus baseline.

Figure 9: General Equilibrium Effects & Cash Grants:
Ordinary Firms' Aggregate Stocks, Percent Deviations from Steady State Level




*Grants in PE: Prices follow the no-stimulus trajectory, but funds are given upon impact date. The other two scenarios are unchanged from Figure 6.*

In the second state, transfers under the counterfactual push the default rate below its steady state level. The counterfactual reduces the default rate more than when the transfers are implemented alongside rate reductions in general equilibrium (Table 5). This indicates that the reduction in defaults is not solely due to the relaxation of borrowing conditions. Since cash grants lower the discount rate in general equilibrium (Figure 5), firms have more incentive to gamble and allocate funds towards present dividends, instead of to risky future returns.

Table 6 summarizes aggregate flows in the counterfactual scenario, when no stimulus is given, and when cash grants are given in general equilibrium. Absent general equilibrium effects, cash grants reduce the loss in output by 1.8 ppt, 1.1 ppt more than in general equilibrium. Hence, in terms of output, general equilibrium forces absorb 64% of the benefits of the stimulus over the first year. A similar result holds for employment. The results for aggregate investment are even more striking. Under general equilibrium, cash grants shore up investment over the first recession year by 5.2 ppt, relative to the no-stimulus scenario, but that is still down 33.7% from the steady state. In the absence of general equilibrium effects, cash grants actually drive investment 4.6% above its steady level, despite the severe recession. Aggregate flows are boosted in the counterfactual scenario both by ordinary firms, which enjoy an increase in capital (Figure 8), and by super firms, which invest more because the discount rate is higher under baseline prices (Figure 6).

Table 6: Aggregate Flows across First Recession Year
% Deviations from Steady State

| | No Stimulus | Cash Grants GE | Cash Grants PE |
|---|---|---|---|
| Output | -6.8 | -6.1 | -5.0 |
| Employment | -7.1 | -6.3 | -5.3 |
| Investment | -38.9 | -33.7 | 4.6 |

GE: Cash grants stimulus given in general equilibrium.
PE: Stimulus given; prices follow no-stimulus scenario.

Overall, the model identifies very strong general equilibrium effects for cash grants. These effects, particularly the downward pressure on firms' discounting and tightening in credit conditions, lower capital investment, leading to more funds allocated towards financial savings instead. Additionally, these effects reduce cash grants' ability to prevent firm defaults. Lastly, in the absence of general equilibrium effects, cash grants would boost employment and output over the first year of the recession by another 1 ppt of the steady state level.

## 7.2 Maturity Frictions & Stimulus Programs

While the literature has identified a negative relationship between debt maturity and capital investment during recessions (e.g., Almeida et al, 2011), it is hard to decompose the source of this negative relationship empirically. During downturns, weaker revenues reduce the cash-on-hand firms can use to pay back debt. Additionally, rollover risk increases due to tighter borrowing conditions. Which channel is the primary driver of the weaker investment among firms with maturing long-term debt? Since rate reductions and cash grants cleanly target one channel more than the other, these policies allow me to answer this question. Cash grants increases firms' cash-on-hand, reducing the negative effects of debt payment. In contrast, rate reductions do not directly affect cash-on-hand, but ease rollover risk by lowering the benchmark lending rate.

I randomly draw 1,000,000 firms from the equilibrium distribution and simulate this panel over the first year of the recession across each of the four main scenarios: no-stimulus, cash grants-only, rate reductions-only, and the full policy scenario. Table 7 shows the estimated effect of debt maturity on investment using the same regression approach implemented in my steady state analysis. For comparison purposes, I also include the steady state estimate from Figure 2.

Table 7: Debt Maturity's Effects on Four-Quarter Capital Growth, %

| | | 2020 Recession | | | |
|---|---|---|---|---|---|
| | Steady State | No Stimulus | CG | RR | Both |
| Maturity | $-1.189^{***}$ | $-1.541^{***}$ | $-1.350^{***}$ | $-1.160^{***}$ | $-1.142^{***}$ |
| Shock | (0.114) | (0.120) | (0.122) | (0.121) | (0.123) |

CG: Cash grants. RR: rate reductions.
Base group: firms with non-maturing debt. Firms with 0 debt excluded.
Additional controls: gross assets, leverage, savings, productivity
Standard deviations in parentheses.
See Appendix B for full regression estimates.
$***: \ p < 0.01, \ **: \ p < 0.05, \ *: p < 0.1$

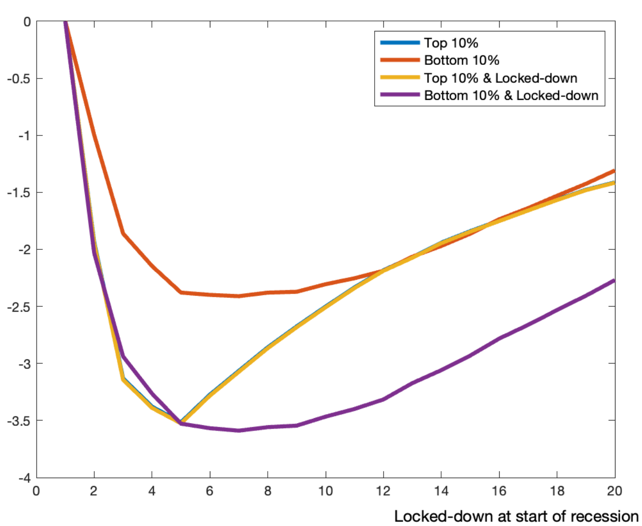
Absent stimulus, the recession increases the maturity friction by 0.4 ppt. This is qualitatively consistent with empirical evidence from the Global Financial Crisis (e.g., Almeida et al, 2011; Bustamante, 2019). While the 2020 recession did not have a financial crisis, the rise in credit risk and the increase in the benchmark lending rate both lead to tighter financing conditions than in the steady state. Stimulus softens the maturity frictions during the recession. Cash grants lower the maturity-shock effect by 0.2 ppt, while rate reductions fully eliminate the increase. This indicates that, during the recession, the larger maturity friction is predominately driven through an increase in rollover risk. The full-stimulus scenario further corroborates this: there is little difference in the effect of maturity on investment across the full-stimulus scenario compared to when rate reductions are implemented alone.

### 7.3 Variation in Firm Responses by Size

The Paycheck Protection Program specifically targeted small businesses, suggesting policymakers were concerned that such firms were particularly at risk of having to reduce employment or close. Consequently, I address two questions of the model in this section. How did the recession affect firms of different sizes? How did responses to stimulus vary with firm size?

I measure size by a firm's gross assets, $k+a$, and identify the smallest and largest 10% of firms at the start of the recession. In Figure 10, I follow these cohorts' aggregate capital over the no-stimulus recession scenario, and benchmark against where they firms would be in the steady state. Such benchmarking is particularly important when considering the smallest firms, which tend to scale up over time. Over the first three years of the recession, the 10% largest firms experience a larger decline in capital than the 10% smallest firms, relative to the steady state. Aggregate capital at the largest firms falls 3.5 ppts below its steady state level, a percentage point larger than the peak decline among smaller firms.

Figure 10: Capital Evolution during Recession among the Smallest and Largest Firms, No-Stimulus Scenario, Percent Deviations from Steady State Level



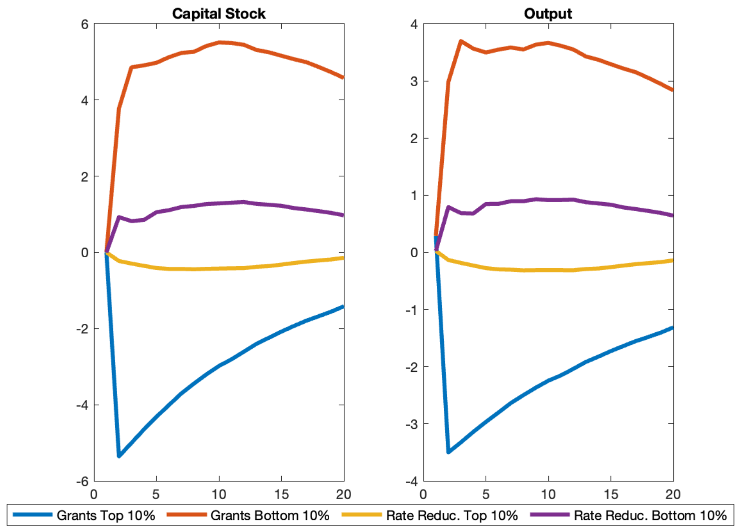

*Size measured by gross assets, $k+a$, on impact date.*

The bigger declines among the larger firms echo the empirical results of Moscarini and Postel-

Vinay (2012), which finds that the largest firms' employment is more closely correlated with the aggregate business cycle than smaller firms' employment. Within the model, these differences are driven by variation in financial frictions across firms. Because the smallest firms tend to be the most constrained, their investment is largely driven by available funds. Future expectations would need to be severely averse to deter these firms from scaling up. In contrast, a financially unconstrained firm's investment will be influenced by expectations over future prices, and hence be negative at the start of a recession. However, the relationship between financial constraints and growth is nonmonotonic over the recession. Figure 10 corroborates this by also showing the trends in capital across the 10% largest and smallest firms that were locked down at the start of the recession. There is a negligible difference between the capital trends of the largest firms by lockdown status. In contrast, when the smallest firms are locked down, they experience a far larger and more persistent decline in capital. Lockdowns generate a large decline in revenue, which amplify the financial constraints among the smallest firms, hindering how much they can invest. These findings may explain why Crouzet & Mehrotra (2020) do not find a systematic relationship between cyclicality and financial frictions. Financial constraints can inhibit business' responses to negative aggregate shocks, because they are already inefficiently small. However, when these aggregate shocks coincide with a large loss in revenue, financial constraints can bind tighter, leading to larger responses.

I next consider how stimulus alters the responses of the largest and smallest 10% of firms during the pandemic recession. The left panel of Figure 11 looks at these firms' aggregate capital stock, while the right panel tracks their aggregate output. In both cases, I compare these aggregates against their levels under the no-stimulus baseline. In the period following impact date, the capital of the largest 10% of firms falls an additional 5.7%, above and beyond the losses sustained in the no-stimulus scenario. In contrast, the policy propels an additional 4.0% increase in the capital of the smallest 10% of firms. In subsequent periods, the largest firms start to recover their additional losses, and converge towards their level in the no-stimulus baseline. In contrast, the smallest firms' gains are sustained for several years following the implementation of the stimulus. Hence, consistent with many empirical studies of the Paycheck Protection Program, the cash grants do generate marked support to the smallest firms (e.g., Barlett & Morse, 2020; Chodorow-Reich et al. 2020; Kurmann, Lalé, and Ta, 2021).

Figure 11: Effects of Policy among the Smallest and Largest Firms:
Percent Deviations from No-Stimulus Scenario




*Size measured by gross assets, $k + a$, on impact date.*

In contrast to the cash grants policy, the rate reduction scenario generates much more modest changes in firm outcomes. For example, the initial gains in capital among the smallest 10% firms are just 0.9%, compared to 4.0% under cash grants. On the other hand, because the rate reduction policy has such small effects on aggregates, the additional fall in large firms' investment is much milder. By reducing rollover risk, the rate reduction policy is able to spur investment among the smallest firms. This suggests that, through the firm demand channel, policies such as quantitative easing are able to increase investment among the most financially constrained firms. The finding that more financially constrained firms respond more to rate reductions is somewhat in contrast with the finding of Ottonello and Winberry (2020) that it is the least constrained firms that respond most to conventional monetary policy. However, Lackdawla and Moreland (2019) show empirically that after the Great Recession, high-leverage firms responded more to monetary policy shocks than low-leverage firms, the opposite of what occurred prior to the recession. This period coincides with when quantitative easing was implemented in the United States.

Kurmann, Lalé, and Ta (2021) find that a year after the 2020 recession started, small business employment had recovered more than large business employment. The model suggests that this may have occurred even without the Paycheck Protection Program. Because small firms are already inefficiently small, they respond less to changes in aggregate prices than larger, less constrained businesses. However, the relationship between financial constraints and response to the recession is nonmonotonic. The lockdown shocks disproportionately affect the smallest firms, which face a weaker recovery than similarly sized firms that are not locked down. Cash grants and, to a lesser extent, reductions in borrowing rates put upward support on small business' investment, leading to sustained gains in their output and employment.

### 7.4 Long-Run Benefits of Stimulus

To measure the net benefits of the stimulus, I calculate the present discounted value (PDV) of aggregate output across each scenario, and use the output of the no-stimulus scenario as a benchmark. Table 8 shows the relative gains in output generated by the stimulus. The "all firms" row shows the gains to aggregate output, while the "memo" row focuses solely on the ordinary firms that received the stimulus. Cash grants boost the PDV of output up by 0.36%, 4.5 times larger than the 0.08% gain under rate reductions. The fully policy scenario boosts PDV output by 3 basis points over cash grants-only. The gains are much larger among the ordinary firms that received the stimulus. Implementing both policies simultaneously increases the PDV of ordinary firms' output by 0.76%, roughly twice the aggregate gain. General equilibrium effects weigh down the investment and output of financially unconstrained firms, including the super firms. Their lower output offsets much of the gains among ordinary firms.

Table 8: % Change in Cumulative Output, from No-Stimulus Scenario
Present-Discounted Value at Start of Recession

| | Grants | Rate Reductions | Both |
|---|---|---|---|
| All Firms | 0.36 | 0.08 | 0.39 |
| *Memo: Ordinary Firms Only* | *0.68* | *0.12* | *0.76* |

Table 9: Cumulative Output Gain, % of Grant Program
Present-Discounted Value at Start of Recession

| | Grants | Rate Reductions | Both |
|---|---|---|---|
| All Firms | 58.5 | – | 63.9 |
| *Memo: Ordinary Firms Only* | *88.9* | – | *98.4* |

To put these cumulative benefits in context, Table 9 takes the ratio of the increased PDV of output to the total cost of the grant program. The grants cost 9.6% of steady state annual output, a higher share than the Paycheck Protection Program (4.2% of 2019 output), but in line with the estimate by Elenev, Landvoight, and Van Nieuwerburgh (2020) of total firm-targeted stimulus as a share of GDP (9.8%).[22] When cash grants are implemented on their own, only 58.5 cents of every dollar is converted into increased output. Even among ordinary firms, every dollar of funding generated 89 cents of increased output. Implementing a rate reduction policy alongside cash grants partially reduces losses. Overall, the full policy scenario boosts returns to aggregate output by 5.4 ppts, but that still means that about one in every three dollars in expenditures is not recovered. However, among the ordinary firms, the full policy scenario roughly breaks even, with a dollar of stimulus returning 98 cents in PDV output gains.

I focus my cost-benefit analysis on output gains, because the model assumes perfect risk-sharing among households. The welfare gains of cash grants would be larger for, say, the proprietors of a small business or communities that faced more severe lockdowns. On the other hand, these cost-benefit analyses rely on the strong assumption that cash grants are funded through lump-sum taxation. If the program was instead funded through distortionary taxes, either on the household or on firms, that would slow down the economic recovery. Accordingly, my results offer an upper bound to the output gains from the cash grant policy. Since this upper bound of output gains is worth only two-thirds of the program's costs, the welfare gains abstracted away from this environment would need to be quite large in order for it to be optimal to implement the program.

[22] There are two primary factors driving up the cost of the cash grant policy relative to the Paycheck Protection Program in the data. First, I assume that all eligible firms receive funding. Schweitzer and Borawski (2021) estimate that only 76% of eligible businesses got funds. Additionally, I assume that all firms receive the maximum amount they are eligible for.

# 8 Conclusion

In this paper, I construct and evaluate a general equilibrium model in which firms' long-term debt matures infrequently. This variation in maturity allows the model to capture the sustained fall in investment that follows a large share of long-term debt maturing. Moreover, it generates several forms of financial frictions, including debt overhang and rollover risk. The principal firm-targeted stimulus measures applied during the 2020 recession target different types of financial frictions. Applying the model to this recession allows me to quantify how responsive firms investments is to policy.

I show that the negative effect of debt maturity on investment is more sensitive to reductions in the risk-free borrowing rate than to increases in firm cash. However, larger firms' investment is less responsive to long-term interest rates, and consequently reducing them has little effect on aggregate investment. In contrast, cash grant policies that improve firm liquidity do increase aggregate investment, even after accounting for general equilibrium effects. However, the policy's general effects tighten borrowing conditions, increasing default risk among the most financially constrained firms. Since cash grants and rate reductions operate through different channels, implementing them simultaneously leads to a faster recovery and a fall in defaults.

The underlying environment examined here captures several salient aspects of firms' financing and investment behavior in the U.S. A natural way to extend on this would be to incorporate a financial sector with its own frictions. Such frictions would allow the model to also capture the feedback effects of increased default risk on aggregate lending conditions (e.g., Elenev, Landvoight, and Van Nieurwerburgh, 2020). Studies on the effects of quantitative easing in the aftermath of the Global Financial Crisis highlight the importance of the banking channel (e.g., Chodorow-Reich, 2014). The current paper isolates the firm debt-demand channel. Accounting for this alongside the banking channel would allow for a more complete assessment of quantitative easing's effects on the economy.

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# Appendix

## A: Aggregation for Definition of Equilibrium

Here, I map the beginning-of-period distribution, $\mu$, into the distributions that determine output and investment during the period, $\mu_Y$ and $\mu_I$. $\mu_Y$ differs from $\mu$ due to the restructuring of defaulted firms, which leads to a loss of $(1-\chi)$ fraction of capital. The restructuring of defaulted firms also leads $\mu_I$ to differ from $\mu$. In addition, $\mu_I$ incorporates new entrants, which replaced firms that received the exit shock and were not restructured.

For ease of notation, let $\mathbb{D}^e$ be an indicator for an exiting firm that lenders put through default. Recall that $\mathbb{D}^n$ and $\mathbb{D}^m$ are indicators for firms that endogenously default under non-maturing and maturing debt. The distributions $\mu$ and $\mu^Y$ are defined on the Borel algebra $\mathcal{A}$, generated by the open subsets of the product space $\mathcal{A} = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{E}$, where $\mathbb{E} \equiv \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$. For any $(A, \varepsilon_m) \in \mathcal{A}$, $\mu^Y$, is measured by

$$
\begin{aligned}
\mu^Y(A,\varepsilon_m) = (1-\lambda)(1-\rho) & \int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{D}_t^n(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \\
+(1-\lambda)(1-\rho) & \int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^n(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\lambda(1-\rho) & \int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{I}_t^m(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\lambda(1-\rho) & \int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^m(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\rho & \int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{I}_t^e(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\rho & \int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^e(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m])
\end{aligned}
\tag{15}
$$

The state space for $\mu^I$ must also account for whether or not a firm's debt matures, as decision rules vary along this margin. I denote this Borel algebra as $\mathcal{A}^I$, which is generated by the open sets of the produce space $\mathcal{A}^I = 2 \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{E}$. Let 1 be an indicator variable for a firm whose debt matures, with 0 reflect a firm with non-maturing debt.

$$\mu_t^I(0, A, \varepsilon_m) = (1-\lambda)(1-\rho)\int_{(k,a,b)|(k,a,b)\in A}(1-\mathbb{D}_t^n(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m]) \tag{16}$$

$$\begin{aligned}\mu_t^I(1, A, \varepsilon_m) = (1-\lambda)(1-\rho)&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_t^n(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m])\\ +\lambda(1-\rho)&\int_{(k,a,b)|(k,a,b)\in A}(1-\mathbb{I}_t^m(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m])\\ +\lambda(1-\rho)&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_t^m(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m])\\ +\rho\mathbb{1}[(0,\iota a_e,0)\in A]\bar{\pi}_m^{\varepsilon}&\int(1-\mathbb{I}_t^e(k,a,b,\varepsilon))\mu(d[k\times a\times b\times\varepsilon])\\ +\rho&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_e(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m])\end{aligned} \tag{17}$$

where $\mathbb{1}$ is an indicator variable for whether the entrants' state is an element of $A$ and $\bar{\pi}^{\varepsilon}$ is the ergodic distribution over firms' idiosyncratic productivities. Entrants' financial assets are a fraction of firms' average physical capital:

$$a_e = \iota\int k\mu(d[k\times a\times b\times\varepsilon]) \tag{18}$$

The aggregate law of motion for the distribution of firms relies on $\mu_I$ as an intermediate distribution. For any $(A, \varepsilon_m) \in \mathcal{A}$, $\mu_{t+1}$ is measured by

$$\begin{aligned}\mu_{t+1}(A,\varepsilon_n) = &\int_{(k,a,b,\varepsilon_m)|(g_k^n(\cdot),g_a^n(\cdot),b)\in A}\pi_{m,n}^{\varepsilon}\mu_I(0,d[k\times a\times b\times\varepsilon_m])\\ +&\int_{(k,a,b,\varepsilon_m)|(g_k^m(\cdot),g_a^m(\cdot),g_b^m(\cdot))\in A}\pi_{m,n}^{\varepsilon}\mu_I(1,d[k\times a\times b\times\varepsilon_m])\end{aligned} \tag{19}$$

where the arguments for $g_k$, $g_a$, $g_b$, and $\mathbb{D}$ are $(k,a,b,\varepsilon)$ and $t$ subscripts are left off to reduce notation.

## B: Empirical Methodology & Additional Regression Results

### B.1 Empirical Sample Selection & Variable Construction

My empirical methodology follows the literature standard closely (e.g., Almeida et al, 2011; Bustamante, 2019; Ottonello & Winberry, 2021). I use the Compustat Fundamental Annual dataset from 1984 to 2020. I exclude several groups of firms. First, I require firms to be incorporated in the United States and to denominate their accounts in U.S. dollars. Additionally, I exclude utilities, financial institutions, and public administrations. Lastly, I impose several financial criteria. Firms with assets less than \$0.5 million are dropped, as are firms with asset or sales growth above 100%. Firms are also dropped if they have long-term debt greater than their total assets, negative short-term debt, or inconsistencies in the long-term debt variables (e.g., more debt maturing in 5 to 10 years than all debt maturing in more than 1 year). Lastly, I require firms to have at least five consecutive years' worth of data, and to have positive debt. All variables are winsorized at the 1% level.

I measure investment as the log change in property, plan, and equipment, adjusted for inflation. To estimate Tobin's $q$, I subtract common equity and deferred taxes and investment tax credit from total assets plus market capitalization, and then take the ratio of this to total assets. Leverage is the ratio of debt in current liabilities plus total long-term debt maturing in more than one year to total assets. Consistent with Almeida et al. (2011), I consider a firm to have a high share of long-term debt maturing if more than 20% of its debt is maturing in the year.

### B.2 Regression Results

Table B1: Debt Maturity & Firm-Level Investment in the Data

| | $\Delta k_{t,t+1}$ | $\Delta k_{t+1,t+2}$ | $\Delta k_{t+2,t+3}$ |
|---|---|---|---|
| Debt Maturity Indicator | -0.013*** | -0.006 | 0.005 |
| | (0.004) | (0.0047) | (0.005) |
| Log(Assets) | -0.031*** | -0.061*** | -0.064*** |
| | (0.011) | (0.012) | (0.012) |
| Log(Assets)$^2$ | 0.001 | 0.001* | 0.001 |
| | (0.001) | (0.001) | (0.001) |
| Sales Growth | 0.079 *** | 0.015 | 0.024** |
| | (0.011) | (0.011) | (0.011) |
| Sales Growth$^2$ | 0.057** | 0.067*** | -0.020 |
| | (0.024) | (0.025) | (0.025) |
| Tobin's $q$ | 0.054*** | 0.045*** | 0.022*** |
| | (0.005) | (0.005) | (0.005) |
| Tobin's $q^2$ | (-0.004)*** | -0.003*** | -0.001** |
| | (0.0005) | (0.0004) | (0.0005) |
| Leverage | -0.152*** | -0.149*** | -0.072* |
| | (0.040) | (0.040) | (0.043) |
| Leverage$^2$ | -0.107* | 0.006 | 0.040 |
| | (0.055) | (0.058) | (0.059) |
| Cash/Assets | -0.321*** | 0.228*** | 0.139** |
| | (0.048) | (0.048) | (0.058) |
| Cash/Assets$^2$ | -0.267*** | -0.207* | -0.060 |
| | (0.098) | (0.104) | (0.126) |
| Constant | 0.095** | 0.248*** | 0.292*** |
| | (0.043) | (0.044) | (0.044) |
| Observations | 25,418 | 22,320 | 19,790 |
| $R^2$ | 0.0986 | 0.0714 | 0.0591 |
| Firm Controls | yes | yes | yes |
| Time/industry FE | yes | yes | yes |

Standard errors in parentheses, and clustered at the firm level.

$***: \ p < 0.01, \ ** \ p < 0.05, \ * \, p < 0.1$

High long-term debt: more than 20% of long-term debt matures in year.

Growth rates and ratios are winsorized at the 1% level.