Long-Term Debt, Default Risk, and Policy Transmission during Severe Recessions*

Ben Lidofsky[†]

November 21, 2021

Latest Version Available Here

Abstract

A growing empirical literature finds that long-term debt and its maturity reduce firm-level investment, particularly during recessions. I introduce long-term debt maturity, alongside short-term financial savings, into a dynamic general equilibrium model in which heterogeneous firms borrow subject to default risk. A stochastic process governs whether all or none of a firm's debt matures in a period. This allows the model to capture the persistent negative effect of maturing debt on investment found in the data. Notably, probabilistic maturity exacerbates capital misallocation during financial crises, weakening the recovery beyond what is implied by the standard approach to long-term debt.

I apply the model to the 2020 recession to assess how firm investment responds to a severe recession and to evaluate the effects of firm-targeted stimulus at the firm- and aggregate level. Targeted cash grants, akin to the Paycheck Protection Program, boost firm investment, accelerating the recovery. However, the grants also lead to a rise in defaults over the first year of the recession. The stronger recovery increases the long-term risk-free interest rate that anchors firm loan rate schedules, making borrowing more expensive. Higher borrowing costs outweigh the benefits of cash grants for the most indebted firms. An alternative policy that lowers long-term interest rates, similar to quantitative easing, is more effective at reducing rollover risk and preventing defaults. However, it has little effect on larger firms' investment demand and therefore has small aggregate effects. When combined, cash grants and rate reductions strengthen the economic recovery and lower the default rate.

*I thank my advisor, Julia Thomas, for her guidance and support, and Kyle Dempsey and Aubhik Khan for their advice. I would also like to thank David Baqaee, Mariacristina De Nardi, Jan Eeckhout, Ludwig Straub, and participants of the 2021 North American Summer Meeting of the Econometric Society, the 16th Economics Graduate Students' Conference at WUSTL, and the Dempsey-Khan-Thomas Workshop at Ohio State for their helpful comments. I thank Vanessa Ordoñez and Rohan Shah for their generous help.

†Department of Economics, Ohio State University. Email: lidofsky.1@osu.edu

1 Introduction

How does long-term debt and its maturity affect firms' investment activities over severe recessions? Empirical studies have found that firms with a large share of long-term debt maturing during the Global Financial Crisis had lower investment than otherwise similar firms (e.g., Almeida et al., 2011; Kalemli-Özcan, Laeven, & Moreno, 2018). I consider this question in a heterogeneous firm model where long-term debt matures probabilistically. This relatively novel approach to modeling long-term debt generates skewness in rollover risk, allowing my framework to replicate salient facts. Firms with maturing debt have significantly weaker investment over the following two years than otherwise identical firms. Additionally, the negative effect of maturity on investment is larger during recessions than in the model's steady state. I apply the model to two large recessions in recent U.S. history, the 2007 and 2020 recessions, to assess how probabilistically maturing debt influences firm- and macro-level responses to different types of aggregate shocks.

To evaluate the firm-level and aggregate implications of the rollover risk and debt overhang implied by long-term debt, I develop a quantitative, general equilibrium environment in which firms are subject to idiosyncratic, persistent productivity risk. This persistence, combined with firms' decreasing returns to scale technology, generates a nontrivial distribution of firms in equilibrium. Production relies on physical capital and labor; the latter is supplied by a representative household. Firms' assets consist of physical capital and one-period financial savings.

What distinguishes this environment from those examined in previous quantitative studies is that firms borrow using long-term debt that matures probabilistically.[1] Every period, each firm receives a shock that determines whether none or all of its debt matures in that period. Any firm with non-maturing debt can choose to prepay, subject to a small transaction cost. Hence, while some firms must pay back their debt in a given period, no firm is excluded from the debt market.

Another feature of probabilistic maturity is that it generates a nontrivial distinction between a

[1] Geelen (2016) and Chen, Xu, and Yang (2021) consider a similar style of maturity, which they refer to as lumpy maturity. In these settings, firm revenues are exogenous and aggregate prices are exogenously determined. This strand of the literature studies the optimal maturity structure, which I abstract from here to assess the interactions between debt and investment choices.

firm's financial debt and assets. In my model, firms optimally hold both long-term debt and one-period, risk-free financial savings. Consequently, they can borrow for precautionary savings, as well as for current investment. Precautionary savings are particularly important over recessions, as they allow firms to hedge against future shocks that could render a firm unable to maintain its investment solely through internal funds, requiring it to take on debt. By acting prior to the realization of a negative shock, the firm can borrow under more favorable conditions. Such "borrow to save" behavior is consistent with empirical evidence from the Global Financial Crisis (Xiao, 2019).[2] The presence of precautionary savings further distinguishes my environment from the standard quantitative framework with long-term firm debt (e.g., Gomes, Jermann & Schmid, 2016; Bustamante, 2019), as firms in such models never simultaneously hold savings and debt.[3,4]

When calibrated to U.S. data on firm-level investment rates and leverage, my model predicts large real consequences arising from noncontingent debt. Even in steady state, the average firm only invests two-thirds of what it would choose absent financial frictions. These aggregate effects are not simply due to the long-term nature of debt. I compare the equilibrium in my model to that in an otherwise identical setting where debt matures geometrically, as is conventional in the long-term debt literature.[5] Steady state output is 1.5 percent smaller in my setting with probabilistic maturity. Under probabilistically maturing debt, firms cannot take on new debt until they have repaid their outstanding debt in full, so each loan is priced only once. This makes the lender conservative when it sets the loan schedule it offers a firm, factoring in the fact that it can only discipline that firm's leverage choices at origination. In contrast, with geometrically maturing debt, indebted firms are active in the debt market each period, so loan price schedules influence their decisions throughout

[2]Xiao (2019) is also able to generate simultaneous risky borrowing and savings in a quantitative model. Firms are subject to demand shocks, and use risk-free savings to hedge against these. In contrast to Xiao (2019), debt in my model is long-term, rather than one-period lived. I find that the additional financial frictions embedded in long-term debt affect the model's steady state and its aggregate dynamics.

[3]This framework, which extends on Leland (1994, 1995), relies on granular debt maturity; a share of long-term debt matures in every period, and there is no adjustment cost on new debt. Consequently, firms do not benefit from holding financial savings and debt simultaneously; they would be better off using the savings to reduce their debt.

[4]Khan and Lee (2021) are a prominent exception. In their model, entrepreneurs borrow using contracts that never mature, but retire at the entrepreneur's discretion. Consequently, their model abstracts from instances when the maturity of debt is detrimental to the entrepreneur.

[5]In both models, the expected duration of a newly originated loan is five years, consistent with empirical data from Choi, Hackbarth, and Zechner (2018). Five-year expected maturity is also the standard in the quantitative literature on long-term firm debt (e.g., Gomes, Jermann, & Schmid, 2016; Bustamante, 2019).

the life of a loan. Consequently, lending conditions are more restrictive in my environment than in settings where debt matures geometrically. This greater restrictiveness inhibits firms' growth, reducing the economy's size. At the same time, it prevents a substantial number of the defaults that occur under geometric maturity, lowering the steady-state default rate from 10.3 percent to 1.7 percent annually, far more consistent with the average corporate default rate in the U.S.

Probabilistic maturity also exacerbates the real consequences of financial crises. I consider a one-time balance-sheet shock that unexpectedly raises all firms' financial operating costs by the equivalent of 5 percent of steady state output. The resulting model recession is qualitatively similar to the U.S. 2007 recession, with steep declines and weak recoveries in both aggregate investment and consumption. In both of these respects, probabilistically maturing debt worsens the response to the financial shock relative to the outcomes obtained under geometrically maturing long-term debt. The decline in GDP is larger, and the series requires an additional two years to recover half its losses. The more severe recession is partially due to an increase in capital misallocation. Under probabilistic maturity, firms cannot frictionlessly adjust their debt, skewing their investment responses. Consequently, financial shocks generate larger declines in endogenous productivity under probabilistic maturity than under geometric maturity. The frictions in debt maturity also dampen firms' adjustments in the periods after the shock, leading to a more prolonged, slower recovery.

I also apply my model to the U.S. 2020 recession to consider how the default risk and rollover risk associated with discretely maturing debt influences the changes in investment and default over recessions not driven by financial shocks. This particular event provides an unprecedented opportunity to evaluate the effects of firm-targeted stimulus during a severe recession. The Paycheck Protection Program directed roughly $850 billion in forgivable loans to small businesses. Meantime, the Federal Reserve expanded its quantitative easing program, increasing its balance sheet by two-thirds over the first two months of the pandemic.

My model allows a controlled setting where I can decompose the direct and indirect consequences of these 2020 stimulus policies for individual firms and in the aggregate. Since identifying such effects from data is challenging, quantitative analyses such as this one are a useful complement to the growing empirical literature examining the 2020 recession. I consider various cuts of the distribution, including size and debt maturity status, to examine how firms' disparate responses ultimately affect the shape and speed of the economic recovery following the dramatic downturn brought on by widespread pandemic shutdowns.

To emulate the 2020 recession in my model, I confront it with two sets of shocks. A rise in households' disutility of labor captures workers' desire to minimize their risk of contracting the virus. Additionally, temporary "lockdowns" reduce the operating hours (productive capacity) of a random subset of firms. Surprisingly, and in contrast to the responses following a financial shock, my model predicts steeper investment declines among firms large in assets versus smaller firms over the pandemic recession. This happens because the investment activities of firms with high net worth are not reliant on external financing, and are instead shaped entirely by the direct effects of the pandemic shocks and changes in equilibrium prices. In general, firms with fewer assets have inefficiently small capital stocks, so they allocate all available funds into investment. Consequently, their investments are less affected by the recession.[6] However, although most indebted firms have muted responses to the recession, financial frictions can amplify the effects of real shocks and generate larger declines in investment among some firms. The usual detrimental effects of debt maturity are exacerbated by the recession, so firms with debt maturing at the start of the recession reduce their investment more than otherwise identical firms, and by a greater margin relative to the maturity effect seen in steady state.

Implementing a targeted cash grant policy akin to the Paycheck Protection Program to evaluate its ability to stimulate firms' investment during the recession, I find that cash grants soften the fall in output and promote a moderately faster recovery. However, the hastened recovery has an unintended consequence. Anticipating a steeper rebound in aggregate consumption, which will drive up risk-free one-period interest rates in coming periods, lenders respond by raising the risk-free borrowing rate at the start of the recession. That, in turn, raises interest rates on all risky

[6] This pattern is similar to empirical studies over business cycles (e.g., Moscarini & Postel-Vinay, 2012) and is consistent with data on the 2020 recession (Krumann, Lalé, & Ta, 2021). Additionally, this relates to the observation in Ottonello and Winberry (2021) that the investment of firms with high default risk is less responsive to changes in the expected return on capital.

loans. By making borrowing costlier, the policy increases default risk among the most indebted firms. Consequently, the default rate over the first year of the recession is 0.1 percentage point *higher* when cash grants are implemented than when no stimulus is provided. The higher default rate is entirely due to the policy's effects on aggregate prices. In a partial equilibrium counterfactual, I show that cash grants generate an immediate, sustained decline in defaults.

A hallmark feature of quantitative easing is that it puts downward pressure on long-term interest rates (Krishnamurthy & Vissing-Jorgensen, 2011). To study how firms respond to this form of stimulus over the recession, I implement a policy preventing the risk-free lending rate from rising above its steady state level.[7] By lowering borrowing costs for all firms relative to the no-policy recession, the policy prevents the negative effect of debt maturity on investment from worsening relative to the steady state. By reducing rollover risk, the rate reductions prevent about one quarter of the excess defaults generated by the recession and stimulate highly leveraged firms' investment.[8] However, the aggregate effects of these rate reductions are small, because the policy has little effect on larger firms' investment demand. To the extent that quantitative easing stimulates the real economy, this finding suggests that it does not do so by increasing firms' demand for debt.

I also consider the consequences of these two policies when they are implemented in tandem, as occurred during the 2020 recession. In the model, cash grants and rate reductions operate through different channels; grants increase firms' cash-on-hand, while rate reductions directly improve borrowing conditions. While the two policies together prevent more defaults than either policy on its own, the aggregate recovery is only slightly stronger than when cash grants are implemented alone, because most firms have low default risk. Thus, the costs of implementing the Paycheck Protection Program in my model exceed its benefits, even when implemented alongside rate reductions; each dollar of grants returns just 64 cents in additional output.

[7] This approach focuses specifically on the "firm demand" channel of quantitative easing. Chodorow-Reich (2014a) shows quantitative easing improves conditions in the financial sector. I abstract from this channel to assess how changes in firms' demand for debt affect firm-level and aggregate investment.

[8] Since firms with high leverage tend to be small, the rate reductions predominately increase the investment of firms with fewer assets. This is consistent with the empirical findings of Lakdawala and Moreland (2019) that smaller firms' investment responds more to monetary policy shocks under quantitative easing than larger firms'.

The remainder of this paper is organized as follows. Section 2 summarizes the related literature. Section 3 describes the model and defines its equilibrium. Section 4 outlines some key theoretical characteristics of firm-level decision-making therein. In section 5, I discuss model calibration and firms' behavior in the steady state. Section 6 evaluates the economy's response to a financial crisis. In section 7, I study model dynamics over the recession and how firms respond to policy intervention. Section 8 examines the channels through which the policies operate, and section 9 concludes.

2 Related Literature

This paper contributes to two branches of the literature. First, it builds on studies examining how noncontingent debt affects firms' investment and default risk. Second, it adds to a growing set of papers exploring many unique dimensions of the 2020 recession.

Debt, Default & Investment

Business cycle models with financial frictions traditionally have structured debt contracts so that there is no default in equilibrium.[9] An emerging trend among quantitative models is to allow equilibrium default, requiring idiosyncratic firm risk to be endogenously priced. Arellano, Bai, and Kehoe (2019) link one-period debt, default, and agency risk and calibrate their model to match moments on firm credit spreads. Their model generates the observed volatilities of output and aggregate labor, despite only having one element of aggregate uncertainty: shocks to the variance of firms' idiosyncratic productivity. Similarly, Ottonello and Winberry (2020) find that default risk constrains the responsiveness of firm investment. They develop a New Keynesian model and find firms with low default risk drive the aggregate response to an expansionary monetary policy shock. In contrast, firms with high default risk are less responsive to monetary policy, because they face high marginal costs to investment. Khan, Senga, and Thomas (2019) develop a real business cycle model where defaulting firms exit and potential entrants are less likely to enter the economy during

[9] See, for example, Gertler & Karadi (2011), Jermann & Quadrini (2012), and Khan & Thomas (2013).

downturns, resulting in a procyclical number of active firms. They find that negative financial shocks worsen two misallocation channels linked to one-period, noncontingent debt in their model, which can drive severe, protracted recessions.

Building on literatures in corporate finance and sovereign debt, Gomes, Jermann, and Schmid (2016) analyze a dynamic stochastic general equilibrium model in which debt is long-term, nominal, and subject to default risk. In contrast to my long-term debt setting, the firm-level debt there matures geometrically and refinancing is costless. Compared to a corresponding model with one-period maturity, Gomes, Jermann, and Schmid find that long-term debt increases the responsiveness of investment and other aggregates to both productivity and deflationary shocks. Without the debt-over hang channel operationalized by long-term maturity, they find no persistence in the response to a temporary shock in their model, given the lack of persistent heterogeneity.

Bustamante (2019) extends Gomes, Jermann, and Schmid (2016) by making idiosyncratic productivity shocks persistent. As a result, the model equilibrium features a non-degenerate distribution of firms, which varies over capital and debt. He finds that this heterogeneity is important in matching trends observed over the Great Recession. Small firms rely on debt to scale up; adverse shocks reduce their revenues, while leaving what they owe unchanged. Bustamante (2019) documents that this debt overhang becomes more pronounced as debt maturity lengthens.

Like Gomes, Jermann, and Schmid (2016), Bustamante (2019) assume consol-style debt contracts. Each period, a portion of debt obligations mature, and firms can choose a different level of debt. Hence, these papers cannot examine the heterogeneous behavior driven by maturity risk. Empirical evidence suggests this maturity risk can be significant during crises. For example, Almeida et al. (2011) find that firms with a large amount of long-term debt maturing during the 2007 credit crunch had significantly weaker investment subsequently, relative to their otherwise similar counterparts with less debt maturity. They estimate that the effects of long-term debt maturity on firm-level investment account for 30 percent of the decline in aggregate investment. Additional studies by Campello, Graham, and Harvey (2010) and Duchin, Ozbas, and Sensoy (2010) also find significant relationships between maturing debt and firm investment and production during the

2007 recession.

In Jungherr and Schott (2021), firms borrow using a combination of short- and long-term debt. Long-term debt matures geometrically, while short-term debt matures at the end of the period in which it was originated. Their model highlights offsetting effects of long-term debt on firms. Debt origination is costly, so relying on long-term debt reduces origination costs. However, long-term debt also exposes the firm to default risk, because of debt overhang and debt dilution. Consequently, the debt price schedule tends to be more conservative for long-term loans than short-term ones. Thanks to these competing effects, the model replicates the positive relationship between firm size and reliance on long-term debt observed in the data. Jungherr et al. (2021) extend this model to include nominal frictions and costs to equity issuance. They show that firms in the model with more maturing debt are more responsive to monetary policy shocks, and they confirm this relationship also holds in the data. Unlike my study, Jungherr and Schott (2021) and Jungherr et al. (2021) have heterogeneity in debt maturity because firms borrow both short- and long-term debt; for a given debt choice, there is no variation in maturity across firms.

The Pandemic Recession

A separate literature examines the 2020 recession, including a number of empirical papers that study how the Paycheck Protection Program (PPP) and other government interventions supported firms' survival prospects and employment. As the pandemic is still ongoing at the time of writing, the focus to date has been the short- and medium-term effects. Broadly speaking, the results suggest that PPP was more effective at preventing firm closures than at preserving jobs. Bartik et al. (2020) find that firms receiving PPP funds were more optimistic of their survival prospects through the end of 2020, and that this effect was strongest among firms most impacted by COVID-19. Bartlett and Morse (2020) document that PPP had no significant effect on the expectations of businesses with more than 20 employees, whereas the smallest PPP recipients lowered their expectations of being closed in six months. Estimates of the policy's employment effects are more mixed, perhaps because different data sources capture different groups of workers (Autor et al., 2020). Using data on private-sector employment across a wide range of firms, Autor et al. (2020) estimate that PPP increased payrolls by several percentage points. In contrast, using data that focused more on especially low-wage earners, Chetty et al. (2020) find no significant employment effect of the program. Some studies suggest this may have been because PPP was not fully used to support payrolls directly. Chodorow-Reich et al. (2020) find that PPP recipients strengthened their balance sheets, while Granja et al. (2020) find that they were less likely to be behind on debt payments.

These empirical analyses are complemented by a body of quantitative research modeling and quantifying the pandemic and its interactions with the real economy. A number of studies incorporate the epidemiological SEIR model into otherwise standard economic frameworks to study changes in household behavior in response to health risk (e.g., Eichenbaum, Rebelo, & Trabandt, 2020; Glover, Heathcote, Krueger, & Ríos-Rull, 2020; Kaplan, Moll, & Violante, 2020). Others abstract from the virus itself to examine the implications of sector-wide shocks to demand or employment induced by the pandemic (e.g., Baqaee & Farhi, 2020; Guerrieri, Lorenzoni, & Straub, 2020). My work on the topic joins a small set of papers focusing on the recession's disparate effects on the economy's producers. Below, I discuss the two such papers closest to mine.

Khan and Lee (2021) evaluate the effects of fiscal policy during the 2020 recession in a heterogeneous-firm environment with long-term debt, emphasizing how the distribution of leverage affects the aggregate downturn and recovery. They find that wage subsidies are an effective stimulus policy, while a policy postponing debt payments offsets some of the subsidies' benefits. One dimension of richness in their environment absent in mine is endogenous entry. While my endogenously defaulting firms exit the economy, each one is replaced by an entrant to preserve the measure of firms. In contrast, their firms are owned by entrepreneurs, who can choose to suspend their business and enter the workforce. However, whereas my firms face rollover risk, due to the probabilistic maturity of debt, entrepreneurs in Khan and Lee (2021) borrow using consols, making coupon payments each period until they *choose* to retire the debt. Like the firms in my model, their entrepreneurs have periods when they do not participate in the debt market and so are not disciplined by contemporaneous lending terms. However, since maturity is a choice, Khan and Lee's entrepreneurs are never forced to repay their debts at an inopportune time. Thus, they are largely shielded against the default risk and corresponding debt overhang confronting firms in my model.

Elenev, Landvogit, and Van Nieurwerburgh (2020) explore an environment where both firms and their lenders are subject to default risk, and find that these risks can reinforce one another through endogenous price changes, thereby amplifying responses to aggregate shocks. Subjecting their economy to a pandemic shock, they show that this amplifying interaction generates a substantial rise in default and a large decline in output. They also find that funding stimulus akin to PPP can offset some of these adverse effects. In contrast to my setting, the effects of an adverse shock are very short-lived for any individual firm that does not immediately default, because firm-level shocks are iid and a representative agent makes all intertemporal decisions. Additionally, a timing friction there prevents firms from taking on new debt to make up for a shortfall of cash on hand, so any firm's likelihood of default is entirely a function of general equilibrium effects and exogenous shocks. While my model abstracts from frictions *inside* the financial sector, it has no such timing friction and allows for persistent productivity shocks. Thus, firms' decisions and idiosyncratic shocks persistently influence their individual lending terms, default probabilities, and investments, and this evolving intertemporal heterogeneity is an additional source of propagation following aggregate shocks.

A few papers have also examined the effects of quantitative easing during the 2020 recession. Ebsim, Faria-e-Castro, and Kozlowski (2021) evaluate how the policy's effects differ with the nature of the shock. Their environment has four types of firms, varying over leverage and liquidity, with default driven by idiosyncratic preference shocks and intertemporal decisions within a firm type made by a representative agent. They treat quantitative easing as an exogenous increase in bond prices (i.e., a reduction in the effective interest rate), as do I here, and find that quantitative easing boosts output during financial crises, but has very modest real effects during recessions driven by aggregate productivity shocks. These findings echo those of Elenev et al. (2021), who examine the effects of quantitative easing in a New Keynesian environment that accounts for spillover effects between risk in the real and financial sectors of the economy. They find that quantitative easing has a moderately stimulative effect during a recession on its own; the policy generates large changes in aggregates only when combined with changes in capital requirements and a raised inflation target.

3 Model

I analyze a dynamic general equilibrium model. At the core lies a unit mass of perfectly competitive, goods-producing firms. Production exhibits decreasing returns to scale technologies, using physical capital and labor as inputs. Decreasing returns to scale, combined with persistent idiosyncratic shocks to productivity, generate a nondegenerate distribution in equilibrium. The representative household supplies labor, owns firms, and saves through a one-period, risk-free asset. Perfectly competitive, risk-neutral financial intermediaries channel household savings into firm borrowing. Firms face three dimensions of idiosyncratic risk: their production is subject to persistent productivity shocks, their debt matures stochastically, and each period there is a chance they will exit the economy permanently.

Firms fund their investment decisions through a combination of retained earnings and external borrowing. Borrowing occurs through long-term bonds, which mature with an iid probability. In each period, firms pay a coupon proportional to the bond's face value. When a firm receives a maturity shock, it must also pay back the debt's face value. When a firm does not receive a maturity shock, it can choose to prepay all of its debt by paying a cost proportional to the debt's face value. Firms have limited liability, and can default on their debt; defaulting firms receive a value of 0, and permanently exit the economy. Non-defaulting firms purchase physical capital to use in the next period, and can also save in the one-period, risk-free asset. Firms' dividends are bounded below by zero; these firms cannot access equity markets. In addition to the endogenous exit driven by default, firms are subject to an iid exit shock each period. Both defaulting and exiting firms are replaced by new firms, leaving the operating mass of firms constant. I normalize the mass of firms to a unit measure.

Time subscripts t capture the aggregate state. Aggregate TFP can vary over time, z_t, as can the distribution of firms over the idiosyncratic state space, μ_t. All agents know how the firm distribution evolves over time: $\Gamma_t(\mu_t) = \mu_{t+1}$.

A firm's idiosyncratic state is $\psi = \{k, a, b, \varepsilon\}$. The firm's assets consist of physical capital, k, and financial savings in the one-period asset, a. The face value of its debt is b. Lastly, $\varepsilon \in \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$ is the firm's idiosyncratic productivity. ε follows a first-order Markov process. For every TFP level ε_i, $Pr(\varepsilon_{t+1} = \varepsilon_j | \varepsilon_t = \varepsilon_i) = \pi^{\varepsilon}_{i,j} \geq 0$, where $\sum_{p=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} = 1$. With probability ρ, a firm receives an exit shock, and with probability $1 - \rho$ it can continue operating. When it does not receive the exit shock, the firm has a λ chance of having its debt mature.

The remainder of this section consists of: a description of the firm's problem, given prices; a discussion of how bonds are priced, given firms' decisions; an overview of the representative household's problem and aggregation; and a definition of the model equilibrium.

3.1 Firms' Problem

At the start of a period, the firm observes its ε, whether it will exit in the period, and whether its debt will mature in the period. Firms that do not receive the exit shock decide whether to default or continue to operate. Additionally, firms that do not receive the maturity shock decide whether or not to prepay their debt. Continuing firms then produce, pay their debt obligations, and chose their physical capital and financial savings for next period. Additionally, firms with maturing debt can issue a new bond. Exiting firms pay back their debt and issue any remaining funds as dividends.

Each period, operating firms solve a static production problem. Both physical capital and labor are inputs in production. I denote operating profits as

$$R_t(k, \varepsilon) = \max_{\ell} z_t \varepsilon k_t^{\alpha} \ell^{\eta} - w_t \ell - \kappa(\varepsilon) + (1 - \delta) k \tag{1}$$

Operating profits consist of revenues from production, less wage costs. Additionally, firms pay an operating cost that is a function of their idiosyncratic productivity. I assume operating expenses are weakly increasing in productivity, $\kappa'(\varepsilon) \geq 0$. I assume that $\alpha + \eta < 0$, so that firms operate under decreasing returns to scale.

To minimize notation, let $J_t(\psi)$ denote value of a firm given a realized ε, but before it is known whether the firm will exit, or whether its debt will mature.

$$J_t(\psi) = (1-\rho)\lambda J_t^m(\psi) + (1-\rho)(1-\lambda)J_t^n(\psi) + \rho J_t^e(\psi) \tag{2}$$

J^m is the value of the firm when its debt matures, J^n is the value when its debt does not mature, and J^e is the value when it exits. J is a construct that I use only to represent future expectations. Firm choices are always determined given the realization of the maturity and exit shocks.

3.1.1 Exiting Firms

If an exiting firm can afford to pay back what it owes after production, it produces and issues any remaining funds as dividends. Otherwise, it cedes control to the lender and receives value 0. The lender then decides whether the firm should operate in the period and liquidate after production, or if the firm should go through default proceedings at the start of the period. In either case, a firm receiving an exit shock solves

$$J_t^e(k_t, a_t, b_t, \varepsilon) = \max\{0, R_t(k_t, \varepsilon) + a_t - (1+c)b_t\} \tag{3}$$

The firm's gross assets in the period come from operating profits, non-depreciated physical capital, and returns on last period's financial savings. Its debt obligations consists of the face value of debt, as well as the periodic coupon payment cb.

3.1.2 Firms with Maturing Debt

When debt matures, the firm's problem is similar to models with one-period maturity. The firm chooses between defaulting and getting value 0, and continuing and getting value V^m. As all firms are owned by the representative household, they use the same discount factor, $M_{t,t+1}$. When it takes on new debt, the firm receives $q_t(\cdot)b_{t+1}$ in numeraire, where $q_t(\cdot)$ is the discount price of the firm's debt. The discount price varies with the firm's ε and its choices of k_{t+1}, a_{t+1}, and b_{t+1}, as each of these are predictors of future default risk. Lastly, the purchases financial savings at discount price q^f. At debt maturity, the firm solves

$$J_t^m(k_t, a_t, b_t, \varepsilon) = \max\Big\{0, V_t^m(k_t, a_t, b_t, \varepsilon)\Big\} \tag{4}$$

$$V_t^m(k_t, a_t, b_t, \varepsilon_i) = \max_{d_t, k_{t+1}, a_{t+1}, b_{t+1}} d_t + M_{t,t+1} \sum_{p=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon J_{t+1}(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_j)$$

s.t. $d_t \geq 0$ and

$$k_{t+1} + q_t^f a_{t+1} + d_t = R_t(k_t, \varepsilon_i) - cb_t + a_t - b_t + q_t(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_i) b_{t+1}$$

3.1.3 Firms with Non-Maturing Debt

A firm that does not receive the maturity shock has three options: it can default and receive value 0, continue in its debt contract with value V^n, or prepay its debt and get value V^m. If it chooses to prepay its debt, the firm pays a restructuring cost $\gamma(b)$, $\gamma'(b) \geq 0$. After accounting for the restructuring cost, a firm prepaying its debt faces the same problem as a firm that received the maturity shock.

$$J_t^n(k_t, a_t, b_t, \varepsilon) = \max\Big\{0, V_t^n(k_t, a_t, b_t, \varepsilon), V_t^m(k_t, a_t - \gamma(b_t), b_t, \varepsilon)\Big\} \tag{5}$$

$$V_t^n(k_t, a_t, b_t, \varepsilon_i) = \max_{d_t, k_{t+1}, a_{t+1}} d + M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} J_{t+1}(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_j)$$

$$\text{s.t.} \quad k_{t+1} + q_t^f a_{t+1} + d_t = R_t(k_t, \varepsilon_i) - cb_t + a_t$$

$$d_t \geq 0, \; b_{t+1} = b_t$$

When a firm does not pay back its debt, it only pays the coupon and the face value of debt does not change into the next period.

3.1.4 Entering Firms

Each exiting firm is replaced by an entrant. Households endow the entrant firms with financial assets a_e, which are fraction $\iota \in [0,1]$ of the average capital stock of firms. They are not endowed with any physical capital or any debt, but they can take out debt in the period in which they enter. New firms draw their idiosyncratic productivity from the ergodic distribution of ε. Given these assumptions, the optimization problem of a new firm is a special case of equation (4). In particular, a new firm is indistinguishable from a firm with no physical capital and no debt outstanding.

Let $g_t^{n,k}(\psi)$, $g_t^{n,a}(\psi)$, $g_t^{m,k}(\psi)$, $g_t^{m,a}(\psi)$, $g_t^{m,b}(\psi)$ denote the firm's optimal decision rules for k_{t+1}, a_{t+1}, and b_{t+1} given idiosyncratic state ψ. Additionally, let $\mathbb{D}^m(\psi)$ and $\mathbb{D}^n(\psi)$ be indicator variables for firms' default decisions, and let $\mathbb{P}(\psi)$ be an indicator variable indicating prepayment.

3.2 Default & Bond Pricing

When a firm defaults, its shareholders lose all claims, receiving the outside value of 0. The financial intermediary takes over the firm, forgives its debt, and sells it to new shareholders. This sale allows the intermediary to recover at least part of what was owed before default. If the sold firm is worth more than what the defaulting firm owed, the surplus is allocated to the representative household in a lump sum.[10] Since payoffs under default vary with the defaulting firm's state, lenders evaluate

[10] This situation is possible because of debt forgiveness and the lender's receiving the equity value of the new firm, not the scrap value of the old firm's assets.

risk at the idiosyncratic level, as in Chatterjee et al. (2007).

Selling a firm is costly: the lender recovers only χ share of physical capital. In addition, restructuring incurs a cost proportional to the face value of the defaulted debt, νb. These losses, $(1-\chi)k$ and νb, are transferred to the representative household lump-sum. This is a conservative assumption, and ensures that the aggregate effects of default are solely determined by endogenous choices, and not through reductions in aggregate resources. The lender can fully recover a firms' financial assets.

Since default occurs at the beginning of the period, the new firm can produce during that same period. Additionally, I do not prevent the new firm from borrowing into the next period. Hence, as in Gomes, Jermann, and Schmid (2016), default risk influences firms' decisions, but does not affect the mass of firms active in a period.

If firm $\{k, a, b, \varepsilon\}$ defaults in t, the value of the new firm is $J_t^m(\chi k, a, 0, \varepsilon)$. The lender's recovery in default is Θ^d.

$$\Theta_t^d(k, a, b, \varepsilon) = \min\{J_t^m(\chi k, a, 0, \varepsilon), (1+c)b\} - \nu b \tag{6}$$

Similarly, let Θ^e be the lender's recovery when a firm receives an exogenous exit shock. If the exiting firm can pay back its debt, the lender receives the full $(1+c)b$ it is owed. Otherwise, the firm cedes control to the lender at the start of the period. The lender determines whether it is optimal to put the firm through default proceedings at the start of the period, or to let it operate and liquidate it at the end of the period. As such a firm receives its outside value, 0, in either case, it is indifferent.

$$\Theta_t^e(k, a, b, \varepsilon) = \max\left\{\Theta_t^d(k, a, b, \varepsilon), \min\{R_t(k, \varepsilon) + a, (1+c)b\}\right\} \tag{7}$$

By construction, under prepayment all of a firm's debt matures. Hence, the prepayment choice weakly increases the likelihood that a firm will pay back the entirety of its debt in the next period. To reduce notation, let $\psi_{t+1} \equiv \{k_{t+1}, a_{t+1}, b_{t+1}\}$ denote the firms' asset choices. Since financial intermediaries are perfectly competitive and risk neutral, the equilibrium price of debt satisfies

$$
\begin{aligned}
&q_t(\psi_{t+1},\varepsilon_i)b_{t+1} = \\
&\quad + \lambda(1-\rho)M_{t,t+1}\sum_{j=1}^{N_\varepsilon}\pi^{\varepsilon}_{i,j}(1-\mathbb{D}^m_{t+1}(\psi_{t+1},\varepsilon_j))(1+c)b_{t+1} \\
&\quad + (1-\lambda)(1-\rho)M_{t,t+1}\sum_{j=1}^{N_\varepsilon}\pi^{\varepsilon}_{i,j}(1-\mathbb{D}^n_{t+1}(\psi_{t+1},\varepsilon_j)-\mathbb{P}_{t+1}(\psi_{t+1},\varepsilon_j))[c+q_{t+1}(k_{t+2},a_{t+2},b_{t+1},\varepsilon_j)]b_{t+1} \\
&\quad + (1-\lambda)(1-\rho)M_{t,t+1}\sum_{j=1}^{N_\varepsilon}\pi^{\varepsilon}_{i,j}\mathbb{P}_{t+1}(\psi_{t+1},\varepsilon_j)(1+c)b_{t+1} \\
&\quad + (1-\rho)M_{t,t+1}\sum_{j=1}^{N_\varepsilon}\pi^{\varepsilon}_{i,j}[\lambda\mathbb{D}^m_{t+1}(\psi_{t+1},\varepsilon_j)+(1-\lambda)\mathbb{D}^n_{t+1}(\psi_{t+1},\varepsilon_j)]\Theta^d_{t+1}(\psi_{t+1},\varepsilon_j) \\
&\quad + \rho M_{t,t+1}\sum_{j=1}^{N_\varepsilon}\pi^{\varepsilon}_{i,j}\Theta^e_{t+1}(\psi_{t+1},\varepsilon_j)
\end{aligned} \tag{8}
$$

The price of debt takes into account five potential outcomes next period. Recall that $\mathbb{D}$ si an indicator for firm default. The first three rows of (8) account for when the firm does not default. If its debt matures and it does not default, the lender receives the full $(1+c)b_{t+1}$. If its debt does not mature and it does not default, the firm either stays in the contract or prepays its debt. If the firm stays in the contract, the lender receives the coupon payment and the expected future value of the contract.[11] If the firm chooses to prepay, the lender's return is the same as if the maturity shock was realized. The fourth row in (8) captures what the lender receives in the event of default. While default risk varies with the realization of the maturity shock, the lender's recovery is unaffected. The last row accounts for the recovery value when the firm receives an exit shock.

This debt-pricing arrangement departs from the literature in two substantive ways. The first is its approach to the maturity structure. While the literature standard also relies on a stochastic process to support long-term maturities, it assumes a constant fraction of debt matures each period: firms always have a combination of debt that matures and does not mature. Hence, in models such as

[11] Since there is no private information in this economy, lenders use the equilibrium decision rules to account for the default risk of future periods: $k_{t+2} = g^{n,k}_t(\psi_{t+1})$ and $a_{t+2} = g^{n,a}_t(\psi_{t+1})$.

Gomes, Jermann, and Schmid (2016), firms can freely adjust their total debt stock in every period. In contrast, the model here assumes maturity is discrete: either all of a firm's debt matures, or none of it does.

The second departure I take is with respect to recovery under default. It is common for models to assume that lenders can only recover a fraction of physical capital in default, rather than firm value. But in these models, the lender effectively sells the restructured firm to the representative household. From the representative household's perspective, they have ownership rights to a new firm, and hence the transfer is in terms of firm value. Absent some imposed constraint, the lender would optimally sell the rights to a firm worth $J(\xi k, \cdot)$ for numeraire equal to $J(\xi k, \cdot)$, rather than ξk. Since the new firm has no debt, its value is bounded below by ξk. This is what the firm would be worth if it sold all its assets for dividends, and the continuing value from this action was 0. This is a nontrivial departure in terms of equilibrium outcomes–by increasing the recovery value to lenders under default, debt prices rise overall. This approach broadly corresponds to Chapter 11 of the US bankruptcy code.

3.3 Aggregation & Representative Household

3.3.1 Aggregation

At the start of the period, the distribution of firms over the idiosyncratic state space is μ. However, exit shocks and default decisions alter this distribution within the period. First, lenders take over defaulting firms, and sell their debt-free replacements to new managers. Production occurs from these replacements, alongside non-defaulting firms. I denote the distribution of producers by μ^Y. Lastly, firms that receive the exit shock and are liquidated after production are replaced by entrants. These new firms join the remaining producing firms in making investment decisions. Hence, it is this distribution over investors, μ^I, that determines the distribution of firms in the next period, μ_{t+1}. I describe how these distributions are constructed in the appendix.

From μ^Y, aggregate labor demand and output can be measured:

$$N_t = \int \ell_t(k,\varepsilon)\mu_t^Y(d[k \times a \times b \times \varepsilon]) + \pi_s^1 \ell(k_s^1, \varepsilon_1^s) + \pi_s^1 \ell(k_s^2, \varepsilon_2^s) \tag{9}$$

$$Y_t = z_t \int \varepsilon F(k, \ell_t(k,\varepsilon))\mu_t^Y(d[k \times a \times b \times \varepsilon]) + \pi_1^s z_t F(k_1^s, \ell_t(k_1^s, \varepsilon_1^s)) + \pi_2^s z_t F(k, \ell_t(k_2^s, \varepsilon_2^s)) \tag{10}$$

Let aggregate capital be

$$K \equiv \int k\mu(d[k \times a \times b \times \varepsilon]) \tag{11}$$

Since savings and debt are financial variables, they do not affect the aggregate resource constraint. Physical capital only depreciates when it is used in production. When a firm is restructured, it is only the capital from its successor that gets depreciated. Hence, on the right-hand side of the aggregate resource constraint, total physical capital consists of all capital at the start of the period, less the depreciated fraction of capital used in production. The representative household consumes C. Then the aggregate resource constraint is

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t \tag{12}$$

3.3.2 Representative Household

There is a unit mass of infinitely lived, identical households. Each has a unit endowment of time every period, and saves through one-period, risk-free contracts with the financial intermediary, a^h. Households are also initially endowed with ownership of all firms. There are no frictions within the household sector; households are exposed to aggregate risk, but are protected from idiosyncratic firm risk. Households gain periodic utility from consumption and leisure, $u(C, 1-n^h)$. Their subjective discount factor is $\beta \in (0,1)$.

Household wealth comes from four sources. They earn returns on their previous financial savings, a^h. Additionally, their ownership of firms provides them dividends and equity value. Their labor supply is endogenous, yielding income wn. Lastly, T captures the lump-sum transfers to the household of operating expenses and the losses from default. Each period, households choose their labor supply, consumption, risk-free savings, and shares in firms. I denote the household's shares in firm ψ by $g(\psi)$, and the household's share holdings across all firms by $\mathbf{g}$.

The representative household solves

$$\begin{aligned} H_t(\mathbf{g}_t, a_t^h) = & \max_{n_h, C, \mathbf{g}_{t+1}, a_{t+1}^h} u(C, 1-n_h) + M_{t,t+1} H_{t+1}(\mathbf{g}_{t+1}, a_{t+1}^h) \ s.t. \\ C + q_t^f a_{t+1}^h + & \int g_{t+1}(\psi) \hat{J}_t(\psi_t) \mu_t^I d[\psi] \leq w_t n^h + a_t^h + \int g_t(\psi) J_t(\psi) \mu_t d[\psi] + T_t \end{aligned} \tag{13}$$

where $\hat{J}$ is the ex-dividend value of a firm.

3.4 Recursive Competitive Equilibrium

Recursive competitive equilibrium consists of five groups of functions. Firms and households both have value functions: $\{J_t^m(\psi), J_t^n(\psi), J_t^e(\psi), J_t^s(k^s, \varepsilon^s)\}$ and $H_t(\mathbf{g}_t, a_t^h)$. Firms have decision rules for their employment, dividends, future capital, future savings, and future debt: $\ell_t(k, \varepsilon)$, $d_t^x(\psi)$, $g_t^{x,k}(\psi)$, $g_t^{x,a}(\psi)$, and $g_t^{m,b}(\psi)$. (x is a placeholder for m and n, which reflect whether the firm's debt matures or not.) The household has decision rules for consumption, labor supply, financial savings, and firm shares: $C_t(\mathbf{g}_t, a_t^h)$, $n_t^h(\mathbf{g}_t, a_t^h)$, $a_{t+1}(\mathbf{g}_t, a_t^h)$, and $g_{t+1}(\psi; \mathbf{g}_t, a_t^h)$. The price functions are for wages, the risk-free asset rate, the firms' stochastic discount factor, and the debt price schedule: w_t, q_t^f, $M_{t,t+1}$, and $q_t(\psi)$. Lastly, there is a perceived aggregate law of motion: $\mu_{t+1} = \Gamma_t(\mu_t)$, which is observed by all agents.

These functions are such that, for every $\psi = (k, a, b, \varepsilon)$ and every t:

(a) Firms:

- $J_t^m(\psi)$ solves (4), $J_t^n(\psi)$ solves (5), and $J_t^e(\psi)$ solves (3)
- $\ell_t(k, \varepsilon)$, $d_t^x(\psi)$, $g_t^{x,k}(\psi)$, $g_t^{x,a}(\psi)$, and $g_t^{m,b}(\psi)$, are the associated decision rules ($x = m, n$)

(b) Households:

- $H_t(\mathbf{g}_t, a_t^h)$ solves (13)
- $C_t(\mathbf{g}_t, a_t^h),\ n_t^h(\mathbf{g}, a_h),\ a_{t+1}^h(\mathbf{g}_t, a_t^h)$, and $g_{t+1}(\psi; \mathbf{g}_t, a_t^h)$ are the associated decision rules

(c) Prices are competitively determined:

$$w_t = \frac{u_n(C_t, 1 - n_t^h)}{u_C(C_t, 1 - n_t^h)}$$

$$M_{t,t+1} = \beta \frac{u_C(C_{t+1}, 1 - n_{t+1}^h)}{u_C(C_t, 1 - n_t^h)}$$

$$q_t^f = M_{t,t+1}$$

$$q_t(\psi) \text{ satisfies (8)}$$

(d) Whenever households start the period with $\mathbf{g} = \mathbf{1}$, the labor market clears and households choose to hold all shares of each active firm. The aggregate resource constraint is also satisfied (12). The market for the risk-free asset, in which households and firms intermediate with the financial sector, is cleared by Walras' law.

4 Characterization of Equilibrium Actions

Before presenting model results, I briefly describe firms' default decisions and discuss how debt overhang can affect firm decisions when debt is not maturing. To simplify notation, I focus on the model's steady state. All results discussed carry through to the full dynamic model. Since the focus is on the steady state, I use β to represent the discount factor, which holds in equilibrium.

4.1 Default Decisions

Default is possible in any state a firm finds itself: exit, non-maturing debt, and maturing debt. In each of these cases, the default decision can be characterized analytically. Following the literature, I assume that any firm indifferent between defaulting and continuing chooses to continue. Then Proposition 1 of Ottonello and Winberry (2020) holds in this model environment as well: a firm will default if and only if it cannot satisfy the non-negativity constraint on dividends.

The logic follows that of Ottonello and Winberry (2020). Because default is possible in any state, the value obtained in default puts a lower bound on the firm's value function. In other words, if the maximum value continuing was strictly less than the firm's outside option, the firm would optimally choose to default. Since in this model the firm's value of default is assumed to be 0, 0 is also the lower bound on expected future value for any firm, as the expectations operator is a convex function.[12] Consequently, any firm that can continue will weakly prefer to do so rather than default. A feasible option satisfies the non-negativity constraint on dividends, and therefore one that offers weakly positive flow value. Since future value is also weakly positive, any feasible inside option will weakly dominate default. Accordingly, a firm will default if and only if it cannot satisfy the non-negativity constraint on dividends.

Accordingly, firms in the model default only in the following cases:

- If debt matures: $R(k_t, \varepsilon) + a_t - (1+c)b_t + \max_{k_{t+1},a_{t+1},b_{t+1}}\{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\} < 0$
- If debt does not mature: $R(k_t, \varepsilon) + a_t - cb_t < 0$
- If the firm exits: $R(k_t, \varepsilon) + a_t - (1+c)b_t < 0$

Hence, in each of these cases, there is an analytic solution for default thresholds. Focusing on the case when debt matures, let m denote net assets: $m(k, a, b, \varepsilon) \equiv R(k_t, \varepsilon) + a_t - (1+c)b_t$. Then any firm with maturing debt and net assets below $\underline{m}(\varepsilon)$ will default:

$$\underline{m}(\varepsilon) \max_{k_{t+1},a_{t+1},b_{t+1}}\{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\} = 0$$

$$\underline{m}(\varepsilon) = -\max_{k_{t+1},a_{t+1},b_{t+1}}\{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\}$$

[12]More generally, if the outside option was Ω, the lower bound on expected future value would be $\beta\Omega$, to account for the discounting. When the outside option is normalized to 0, discounting is not a factor.

The only way the right-hand side varies with the firm's state is through its productivity, ε. Accordingly, default thresholds under debt maturity only vary with the debt price schedule. The thresholds are even more straightforward when debt does not mature or when the firm exits; in these cases, the only endogenous object is the equilibrium wage.

This greatly simplifies the quantitative solution of the model. For ease of discussion, consider a partial equilibrium, in which the wage is held constant. For a given debt price schedule, firms' choices to default or continue are known with certainty, independently of the value function. The state space can then be partitioned into defaulting and non-defaulting regions, allowing debt prices to be updated. Iterating between these thresholds and the debt price schedule can occur independently of solving for firms' decision rules and value. In this model, with long-term debt and default recovery tied to firm value, further iterations are needed to ensure consistency between debt prices and firms' choices and value. However, in models with one-period debt and default recovery tied to asset value (e.g., Ottonello and Winberry, 2020; Khan, Senga, Thomas, 2018), debt prices and default thresholds can be fully determined before knowing firms' value function or their capital and borrowing decision rules.

4.2 Debt Overhang & Endogenous Default Risk

Unlike models with one-period maturity or consol-style debt, where a firm's debt matures each period, in this model debt can go many periods without maturing. As a consequence, firms are not subject to debt-market discipline in every period they are active. This vastly increases the scope for endogenously driven default. In particular, some firms will find it optimal to choose the most extreme form of endogenous driven default: selling off all their assets and issuing the proceeds as dividends. This all but guarantees default in the next period. I refer to this extreme form as strategic default. Ottonello & Winberry (2020) show that in their one-period maturity setting, firms weakly prefer to use available funds to increase capital or reduce debt; no firm would strictly prefer to issue dividends while debt is outstanding. Under probabilistic maturity long-term debt, this result does not hold.

The appeal of strategic default is that it allows the firm to evade paying back its debt. Consider a hypothetical scenario in which a firm's debt is not maturing and it happens to know with certainty that it will receive the exit shock next period. Its expected future value is

$$\sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} \max\{R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t, 0\}$$

Under strategic default, the firm chooses $k_{t+1} = a_{t+1} = 0$, and issue all funds as dividends, leading to strategic default: without any assets, the firm cannot pay back the $(1+c)b_t$ it owes, so it will default in all future states. Then its expected future value is 0, leading to total value of $R(k_t, \varepsilon) + a_t - c_b t$. For this expositional case, strategic default will strictly dominate the optimal continuing strategy whenever

$$R(k_t, \varepsilon) + a_t - cb_t >$$

$$\max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \Big\{ R(k_t, \varepsilon) + a_t - cb_t - k_{t+1} - q_f a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} \max\{R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t, 0\} \Big\}$$

Suppose this firm's optimal choices conditional on continuing ensure that it could pay off its debt with certainty in the next period.[13] Then the inequality simplifies to

$$0 > \max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \Big\{ - k_{t+1} - q_f a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} [R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t] \Big\}$$

$$\beta(1+c)b_t > \max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \Big\{ - k_{t+1} - q_f a_{t+1} + \beta a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} R(k_{t+1}, \varepsilon_j) \Big\}$$

Lastly, in equilibrium, the discount price on savings equals the household's discount rate. Since $q^f a_{t+1} = \beta a_{t+1}$, the inequality further simplifies to

$$\beta(1+c)b_t > \max_{k_{t+1} | k_{t+1} > 0} \Big\{ - k_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} R(k_{t+1}, \varepsilon_j) \Big\}$$

[13] This assumption is without loss of generality, as it increases future value relative to a case in which there is default risk, making this inside option more appealing.

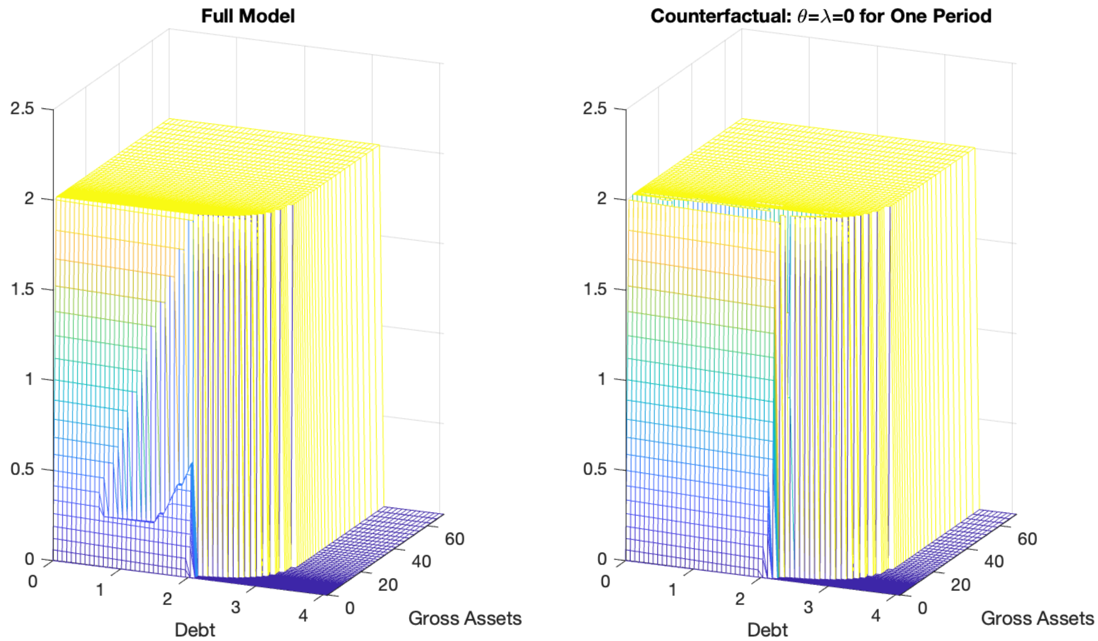
This expositional case imposes a high bar on continuing. The expected return on capital must both exceed the cost of funds and compensate for pre-existing debt obligations. As productivity falls and as leverage rises, a firm will be less likely to generate a high enough return to justify continuing, reducing the opportunity cost of strategic default.

In the full model, the tradeoff is less stark, as the realization of exit shocks is never known ex ante. Nonetheless, the fundamental question is unchanged: does continuing generate sufficiently high expected returns to justify paying back the face value of debt, if a maturity or exit shock is realized? In the case of maturity, the ability to take on new debt and continue operating in future periods lifts the firm's inside option. Even so, there are portions of the state space in which firms choose to strategically default.

To illustrate how debt maturity risk affects firms' endogenous default choices, Figure 1 shows the capital decision rules of firms with the lowest productivity that do not have debt maturing. The left panel shows these decisions in the steady state equilibrium. The right panel considers a one-period deviation from this equilibrium, in which firms know with certainty that in the next period they will not receive either the maturity or exit shock, and that the model returns to the steady state afterward. In this counterfactual, firms that want to prepay still have this option, but there is no chance that the firm will have to pay back its debt.

In the full model, there are two types with endogenous default risk. The first is illustrated at the right of the graphs. Here, firms choose strategic default: their future capital (and savings) choice is 0, even though the firm has plenty of funds to invest, and could continue without risking default. Following the logic of the expositional case above, across much of the state space strategic default is not driven by default risk, but by debt overhang. A firm with gross assets of 40 could easily pay back debt with a face value 4, for example. But this choice is not optimal, because the expected returns on capital are too low. Strategic default is virtually non-existent across the state space for firms with average productivity.

Figure 1: Capital Decision Rules for Firms with the Lowest Productivity & Non-Maturing Debt



While the counterfactual scenario does not fully eliminate strategic default, it does reduce its prevalence. While in the full model no low-productivity firm with debt above 3.1 would choose to continue, eliminating one period of maturity and exit risk increases this threshold to 3.5. The reduction in risk is actually relatively small: while in the counterfactual a firm can have non-maturing debt with 100% probability, under the full model this probability is still 93%.

The second type of default risk is due to "gambling". Under the full model, firms with a moderate level of debt (say, 1.5) and low assets (less than 2) do not invest as much in capital as they could, and instead issue dividends. These small, highly leveraged firms have position continuation value. However, they face a high degree of default risk, particularly under exit shocks. As such, they hedge their bets: they invest in capital to try to continue, but also issue some dividends to take advantage of this risk-free source of value. The firm could reduce its default risk by not paying dividends, but it is too leveraged to eliminate this risk. Consequently, the firm optimally increases its default risk through dividend payments. When the risk of debt maturing or the firm exiting is relaxed for a period, this "gambling" behavior is eliminated.

Endogenous default risk is a poor outcome for the lender, as it reduces both the likelihood of being paid in full and the amount of assets available to recover under default. As such, the debt price schedule adapts to take such behavior into account, deterring firms from being in a position where they will resort to this choice. The long horizon between when debt is issued and when it is expected to mature hinders this; the longer the horizon, the more sequences of productivity shocks there are that could lead a firm towards endogenous default risk. Since firms are not subject to financial market discipline until their loan matures, the lender must account for as much of this downside risk as possible up front.[14] This is not the case in consol-style environments (e.g., Gomes, Jermann, & Schmid, 2016). There, outstanding debt matures fractionally each period and firms can always take on new debt. Since firms are continuously subject to financial market discipline under that contract, current lenders benefit from the discipline of future lenders. In effect, consol-style debt allows for more state-contingent lending conditions, reducing the lender's need to impose tight conditions up front.

While endogenous default risk is present even in the model's steady state, a severe recession, such as the one generated by COVID-19, exacerbates the debt overhang. In particular, the debt's value is non-contingent, even as lockdown shocks generate negative uncertainty shocks, increasing the hazard of default. With expected future returns weakened, more firms will want to reduce their exposure to debt obligations by increasing their default risk. Since most firms' debt does not mature when the recession starts, only a small fraction will be subject to the financial market discipline that might deter them from increasing their default risk. However, lenders will optimally price in this increased risk, along with heightened default risk more generally. This generates a more conservative debt price schedule, further reducing firms' expected future value, and hence endogenously amplifying firms' incentive to evade debt overhang by taking on more default risk.

[14]As is true of other models with maturity shocks (e.g., Chen, Xu, & Yang, 2021), this model abstracts from loan covenants. Covenants are common in long-term debt contracts, and allow the lender to discipline a firm's choices after origination. However, Chodrow-Reich and Falato (2020) find that even among large loans (with a commitment of at least $20 million), one in four had a covenant violation during 2006 and 2007. It is relatively rare for lenders to take action in response to these violations–even during the 2008-2009 crisis, two out of three covenant violations were waived. If firms anticipate such waivers, they would operate similarly to the firms in my model.

5 Calibration & Steady State Dynamics

5.1 Calibration & Model Fit

I evaluate the model at a quarterly frequency. Table 1 presents the model parameters. Roughly half of the parameters are externally set, or correspond one-to-one with data moments (e.g., the labor share). The remaining parameters are internally set to jointly match data moments. Data moments are taken from the United States economy.

Table 1: Parameters (Annualized)

	Parameter	Symbol	Value
Externally Set	Discount factor	β	0.96
	Labor share of output	η	0.6
	Capital depreciation rate	δ	0.067
	Exit rate	ρ	0.085
	Persistence of idiosyncratic shocks	ρ_ε	0.757
	Coupon payment	c	$1/\beta - 1$
	Average debt maturity	λ	1/5
Internally Set	Leisure utility	B	2.1
	Capital share of output	α	0.275
	Entrant endowment share	ι	0.025
	Standard deviation of idiosyncratic shocks	σ_ε	0.075
	Operating expense	κ	0.12
	Capital recovery	χ	0.54
	Default cost	ν	0.0525
	Debt prepayment cost	γ	0.75%

Starting with the exogenously determined parameters, the discount factor, β, is set to yield a 4% risk-free rate in the steady state, in line with the average after-tax return on capital from 1954 to 2000 (Gomme, Ravikumar, and Rupert, 2011). The labor share of production, η, ensures that 60% of income goes to workers (Cooley and Prescott, 1995). Using data from the Bureau of Economics Analysis (BEA), I find that from 1945-2018, the investment to capital ratio averaged 0.067, after accounting for growth.[15] This sets the depreciation rate, δ. The exogenous exit rate, ρ, is set to match the average of 8.5% from 1984-2006 (Business Dynamics Statistics, "BDS," Census Bureau). Lastly, the persistence of idiosyncratic productivity, ρ_ϵ, is set at 0.757 at an annual frequency, following estimates by Syversyon (2008).

[15] This measure of investment includes the purchases of consumer durables.

Following Gomes, Jermann, and Schmid (2016), I set the coupon payment, c, so that the bond price for risk-free debt is normalized to 1 in the steady state.[16] Over the dynamic equilibrium, the coupon rate is held constant so that the bond operates similarly to fixed-rate instruments. Firms have a 5% chance of having their debt mature in any given period (λ), implying an average maturity of 5 years. This matches the average over 2002 to 2012 found in Choi, Hackbarth, and Zechner (2018). A 5-year expected duration is also used in other computational models of long-term corporate debt (e.g., Gomes, Jermann, and Schmid, 2016; Bustamante, 2019).

Turning to the internally set parameters, I assume households have indivisible labor preferences: $u(C, 1-n) = \ln C + B(1-n)$. I set the leisure utility parameter, B, so that households spend about one third of their time working. The production weight on capital, α, pins down the capital-output ratio. From the BEA data, I find that the capital-output ratio averaged 2.28 from 1954 to 2018. Entrants' endowment, ι, is 2.5% of firms' average capital, so that the model matches new firms' share of total employment.[17] The standard deviation of productivity shocks, σ_ε, is 0.075 annualized, to match the standard deviation in investment rates found by Cooper and Haltiwanger (2006). I assume operating expenses take the form $\kappa\varepsilon^{1/(1-\eta)}$, with $\kappa = 0.12$ at an annual rate. Holding other parameters constant, κ helps to govern the mean and variance of firm-level leverage. I take these moments from Dinlersolz et al. (2018).

From 1990 to 2017, the recovery rate averaged 42% for bonds and 70% for loans (Moody's, 2018). Flow of Funds data indicate that over that period, nonfinancial corporations had \$1.26 dollars in bonds for every \$1 in loans. Applying this proportion to the Moody's data, I estimate an overall recovery rate of 54%, and choose this as the value of χ. Given this, I set the the proportional default cost, ν, to 5.25%, to target the default rate of 1.65% (Moody's, 2018). This default rate of 1.65% is lower than the 3% rate commonly used in the literature (e.g., Bernanke, Gertler, & Gilchrist, 1999; Ottonello & Winberry, 2020). Bernanke, Gertler, & Gilchrist (1999) cite a Duns and Bradstreet

[16] In the steady state, risk-free borrowing has a more favorable price than risk-free saving: $1 > \beta$. The reason for this is that the borrower pays periodic coupons, while risk-free savings are a zero-coupon asset. While the prices vary, the expected returns on risk-free debt and risk-free savings equal one another.

[17] This endowment is smaller than in other papers (e.g., Khan and Thomas, 2013), but entrants are able to borrow into their first period of production. Borrowing offers entrants another source of funds with which to invest.

survey for their measure, which spans 1984-1994 (Fisher, 1996). I use Moody's default data to be consistent with the recoveries given loss. However data from PayNet indicates that across small businesses the annual default rate from 2012-2018 was 1.70%.[18] Lastly, the prepayment cost, γ, is set at 0.75% of debt's face value, to match the prepayment rate observed over corporate bonds (Xu, 2017).

Finally, to capture the extreme skewness of the distribution of employment across firms in the United States, I assume that a small fraction of firms are exogenously unconstrained. These "super" firms are highly productive and can frictionlessly issue equity whenever necessary.[19] I assume there are two types of super firms. According to the BDS, the largest 0.11% of establishments have over 1,000 employees, and employ 14.4% of workers. Another 0.18% of establishments have between 500 and 999 employees, and account for 6.9% of employment. "Type-1" super firms are the largest employers, and these 0.11% of firms have a productivity level of 1.846. (The average ordinary firm has a productivity level of 1.) "Type-2" super firms have a productivity level of 1.583, and make up 0.18% of firms. These super firms ensure that in the steady state, 21.3% of employment is concentrated in 0.29% of firms.

Table 2 summarizes the models' fit to targeted empirical moments. For firm-level moments, I draw data from two sources. Dinlersoz et al. (2019) construct a novel dataset that spans both private and public firms in the US (LOCUS). Most panel datasets on firms focus on public firms, for which more information is readily available. However, the vast majority of firms active in the US are private, rather than public. The ideal dataset spans both types of firms, as private firms are more representative of individual-level characteristics, while public firms have an outsized influence on aggregates. The leverage rows (debt/assets) draw on the private firms' measures of total leverage in Dinlersoz et al. (2019).[20] I choose total leverage, rather than financial leverage, as my target, because the former is the complement of a firm's equity ratio. Targeting total leverage is isomorphic to targeting a firm's internal funding share. For capital growth, I draw from Cooper & Haltiwanger

[18]Data taken from https://sbinsights.paynetonline.com/loan-performance/. Last accessed September 1, 2021. PayNet is a subsidiary of Equifax, one of the major credit-rating agencies in the United States.

[19]Super firms in this model are similar to the "no-constraint" firms in Khan & Thomas (2013).

[20]Public firms have a higher mean leverage ratio but a similar standard deviation.

(2006), which examines the Census Bureau's Longitudinal Research Database (LRD). Lastly, for labor share by firm age, I use Ottonello's & Winberry's (2020) analysis of the Census Bureaus' Longitudinal Business Database (LBD).

Table 2: Steady State Moments (Annualized)

Moment	Data	Model
Capital/Output	2.28	2.17
Investment/Capital	0.069	0.069
Share of Time Worked	1/3	0.336
Default rate (%)	1.65	1.61
Share of Debt Prepaid (%)	2.3	4.7
Recovery rate (%)	54.4	76.6
Mean risk premium (%)	2.65	0.96
Mean Annual Capital Growth	0.122	0.118
St. Dev. Annual Capital Growth	0.337	0.363
Mean Debt/Assets	0.46	0.52
St. Dev. Debt/Assets	0.38	0.31
Employment Share, Firms < 1 year	0.03	0.03
Emp. Share, Firms $\in (1, 10)$ years	0.21	0.28

Firm-level model moments winsorized at 1% level

Capital data: Cooper & Haltiwanger (2006): LRD

Leverage data: Dinlersoz et al. (2019): LOCUS private firms

Employment data: Ottonello & Winberry (2020): LBD

Overall, the model's real economy fits the data closely, with a reasonable fit across the first two moments of annual capital growth. The model's fit to financial moments is less tight, but the model still does a fairly decent job capturing the mean and standard deviation of leverage. Consistent with the findings of Xu (2018), prepayment is relatively uncommon in the steady state: 2.4% of aggregate debt is prepaid in a year. The model overshoots that somewhat, with 4.7% of debt prepaid annually. In a given quarter, 1.4% of firms that do not receive the maturity shock choose to prepay their debt. The model is less successful at targeting debt prices: the average recovery rate across defaulted firms is roughly 20 percentage points (ppts) too high, relative to Moody's data. Accordingly, the average risk premium in the steady state is about 1.5 ppts lower than the long-run average in the data.

To assess the model's ability to capture firm-level dynamics, Table 3 compares quarterly employment growth rates from the model steady state distribution to data on establishments from the Business

Employment Dynamics dataset from 2001 to 2006 (Davis et al., 2010). The model does a fairly reasonable job matching the distribution of employment growth observed in the data, even though these moments are untargeted. The largest misses are across the two high-growth groups: the share of firms with growth rates over 20% is 6.9 ppts higher in the model than in the data (14.5% versus 7.6%). The opposite holds for the share of firms with growth rates between 5% and 20%: just 7.7% of model firms have these growth rates, compared to 16.9% of firms in the data. The model implies a substantial mass of firms with growth rates just above 20%: 6.6% have growth rates between 20% and 21%, for example. While firms may be similarly concentrated in the data, this large mass of firms close to the threshold indicates that the relatively weak fit is not due to a large number of firms with extremely high growth rates. Overall, the model is reasonably able to reproduce the dynamics of employment growth across firms.

Table 3: Firm Distribution (%) over Quarterly Employment Growth

	Data	Model
(-2, 0.2]	7.6	5.4
(-0.2,-0.05]	16.7	17.0
(-0.05, 0.05]	51.2	55.5
(0.05, 0.2]	16.9	7.7
(0.2, 2)	7.6	14.4

Growth calculated as $2 * (n_{t+1} - n_t)/(n_{t+1} + n_t)$

Data source: Davis et al. (2010)

5.2 The Effects of Long-Term Debt Maturity on Firm-Level Investment

One of the primary motivations for this paper is empirical analyses that highlight the substantial effects of long-term debt maturity on firm-level investment (e.g., Almeida et al., 2011; Kalemli-Özcan, Laeven, & Moreno, 2018). Hence, a key test of this model is whether it can replicate the effects found in the data.

To estimate the effects of long-term debt maturity, I use Compustat data on public firms from 1984 through 2020. In Compustat, data on long-term debt maturity are available at an annual frequency. Specifically, I can identify the amount of long-term debt that will mature in the following year. This is compatible with an annual version of the model. Sample selection and the choice control variables follow the literature norm closely (e.g., Almeida et al., 2011; Bustamante, 2019; Ottonello & Winberry, 2020), and are described in Appendix B. Investment growth is measured as the log growth in property, plant, and equipment, adjusted for inflation. I estimate equations of the following form

$$\Delta \ln k_{i,t+1} = \alpha_i + \alpha_{s,t} + \beta \mathcal{M}_{i,t} + \Gamma' X_{i,t} + \epsilon_{i,t} \tag{14}$$

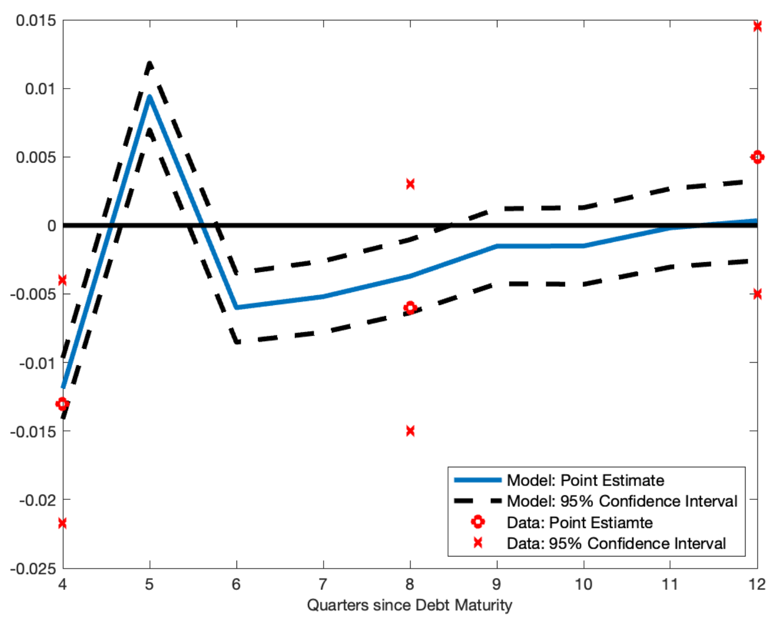
where firm i is in sector s in year t. I control for fixed effects at the firm and industry-year levels. $\mathcal{M}$ captures the firm's exposure to maturing long-term debt, and X is a vector of control variables. Specifically, the controls are the log of the firm's real assets, its lagged real sales growth, its book leverage, and its Tobin's q statistic. In the model, I control for the log of a firm's assets, financial assets as a share of total assets, leverage, and idiosyncratic productivity level. My model sample consists of 1,00,000 firms randomly drawn from the steady state distribution. For both the data and model, my estimates exclude firms without debt.

In the model, maturity is orthogonal to the firm's state and is unpredictable. Accordingly, any association between debt maturity and future investment has strong causal implications. Identification is weaker in the data, as the maturity of long-term debt can be anticipated and could, in principle, be correlated with other decisions made by the firm. For example, a firm could time the maturity of its debt to coincide with the culmination of an R&D project to coordinate its future debt and investment needs. The identifying assumption I make, common in the literature, is that because long-term debt has such a long duration, such coordination is too challenging to implement in practice. Since the typical long-term bond lasts for 5 years, about the same duration as a business cycle, I do not view this as a strong assumption.

My primary proxy for debt maturity is an indicator variable for firms with a high share of long-term debt maturing, as it is consistent with the discrete maturity in the model. Since 20% of firms have debt maturing in a year in the model, in the data I identify firms with more than 20 of their debt maturing in the year. This is also the threshold used in Almeida et a. (2011) to identify firms with a high share of maturing debt. For robustness, in Appendix B I consider other specifications in both the data and the model.

Figure 2 tracks the estimated effect of debt maturity on year-over-year capital growth over a three-year horizon in both the model and the data. The lines follow the model point estimates and 95% confidence interval from quarter to quarter. The points show the point estimates and 95% confidence interval from the data, which are only available once a year. Focusing first on the data, I find that having a large share of long-term debt maturing leads to significantly weaker investment over the following year. The point estimate implies that maturity generates a 1.3 log-point decline in future capital. In the second and third years, the effects are less precisely estimated; neither coefficient is significantly different from 0 at the 5% level. This suggests that debt maturity operates like a growth shock: investment falls for a year, but does not subsequently accelerate.

Figure 2: Estimated Effect of Debt Maturity on Four-Quarter Investment, Model & Data



From estimates of equation (14). Investment measured as the difference in log capital. Maturity in the data: indicator for firms with more than 20% of long-term debt maturing in the year. Maturity in the model: indicator for firms receiving the maturity shock. Regressions include firm-level controls, such as size, as well as fixed effects at the industry-year level for the data. Standard errors clustered at the firm level.

Qualitatively, the effects of debt maturity in the model track those found in the data: maturity triggers an initial decline in investment that eases over time. A notable exception to this occurs in growth from the first quarter after maturity through the fifth quarter. Over this period, lagged debt maturity is associated with a significant *increase* in investment relative to firms whose debt did not mature in the previous period. However, this additional growth reflects base effects: because investment falls substantially in the quarter of maturity, a firm's capital stock is particularly low in the period after maturity. This base effect fades away, while the negative effect of maturity on investment remains for several subsequent periods.

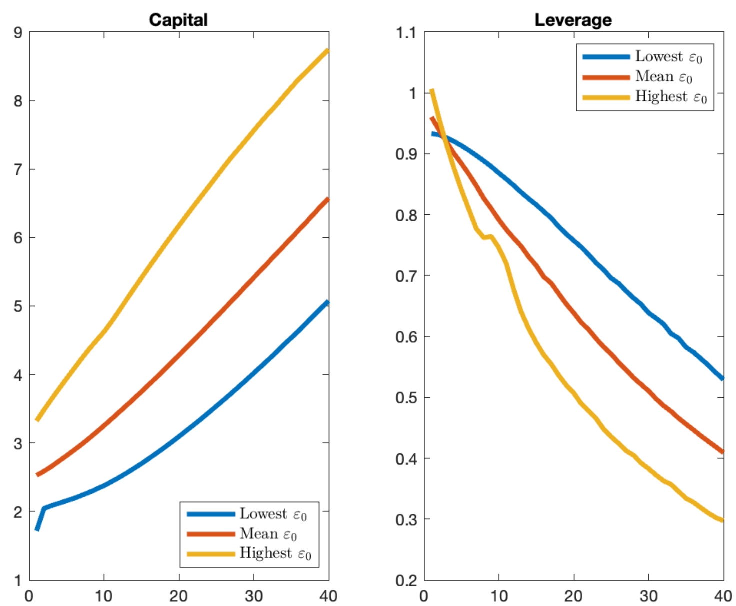
Quantitatively, the model finds similar point estimates as in the data over the first two years: a 1.2 log-point decline in the first year, and a 0.4 log-point decline during the second. Over the third year, the model's point estimate is 0.0 log points, and the data estimate of 0.5 log points falls outside the model's 95% confidence interval. However, at three years out the data estimate is highly imprecisely estimated; the model's entire 95% confidence interval falls within the left tail of the data's confidence interval. Overall, I conclude that the model captures the effects of debt maturity on firm investment observed in the data quite well, despite these being untargeted moments.

In both the data and the model, I show that the maturity of long-term debt is associated with sustained declines in firm-level investment even outside of recessions. This finding contrasts with that of Almeida et al. (2011), which found that maturity generated a large decline in investment during the Great Recession, but had no effect in previous years. Our empirical methodologies differ, though, in that Almeia et al. (2011) used a matching process to connect firms with high shares of long-term debt maturing to similar firms with lower shares of maturing long-term debt. This procedure leads to a very small sample, which makes precision issues more challenging. The panel regression approach employed here examines 25,000 firms, rather than fewer than 100. This paper is also not the first to identify a significant relationship between long-term debt maturity and investment. Using a similar approach, Bustamante (2019) finds that the share of long-term debt maturing is negatively related to investment, even after controlling for the firm's leverage.

5.3 Lifecycle Dynamics in the Steady State

To capture lifecycle trends across firms, Figures 3 and 4 track a cohort of firms from entry. The maturity structure makes a firm's level of debt highly persistent: in the absence of prepayment, a firm's debt is expected to mature only once every 5 years. Hence, over the 40-quarter horizon examined here, in expectation a firm can only choose new debt twice. Due to this persistence, I take averages conditioning on initial productivity, as an entrant's initial level of productivity influences its first choice of debt.

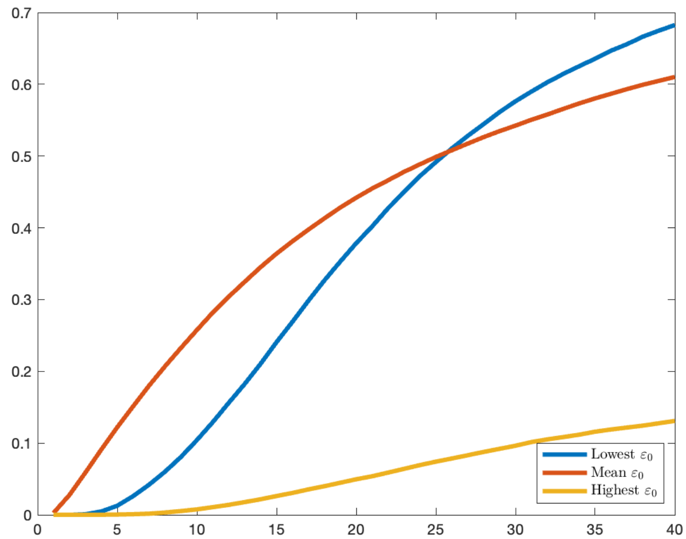
Figure 3: Firm Lifecycle Dynamics in Steady State, by Initial Productivity



Averages across continuing firms. x-axis: quarters since entry.

Over the first few years from entry, firms with the highest productivity initially have fairly linear capital growth, on average, while the least productive firms' capital trajectory is far flatter. Less productive firms start with a much smaller capital base, which curbs their growth. Consequently, even though productivity is mean-reverting, the gap in average capital actually widens slightly over time. The gap between the initially average-productive and the initially least-productive firms

Figure 4: Cumulative Prepayment Probability in Steady State, by Initial Productivity



Averages across continuing firms. x-axis: quarters since entry.

grows from 27% in the second quarter of production to 30% in the fortieth quarter. Similarly, the gap between the initially most-productive and the initially average-productive firm edges up from 31% to 33%. While the initially most-productive firms tend to reduce their debt over time, across the first 40 quarters the average debt at firms that initially had average or the lowest productivity increases. These debt increases allow the firms to grow more quickly. However, the less-productive firms are hindered from increasing their debt along both the extensive and intensive margins. On the extensive margin, since long-term debt only matures probabilistically and prepayment is costly, opportunities to take on new debt are relatively rare. Additionally, these firms tend to have high leverage, which means they need to allocate more funds towards paying back outstanding debt, rather than providing collateral for new borrowing. On net, these forces imply that the initially least productive still have an average leverage ratio over 0.5 after operating for 40 quarters. In contrast, across the firms that had the highest productivity initially, average leverage is 0.3 then.

Related to this, Figure 4 tracks the cumulative probability that a firm will choose to prepay at least once. For example, 26% of firms that initially had average productivity have prepaid their debt at least once in their first 10 quarters. Firms that started with the highest level of productivity are much less likely to prepay. Through 40 quarters, just 13% have prepaid at least once, compared to 61% of firms that started with average productivity, and 68% of firms that initially had the lowest level of productivity. Prepayment is more common among firms that receive positive productivity shocks. Since productivity is mean-reverting, in expectation these firms' borrowing conditions are better now than in the future. Prepayment allows firms to take advantage of positive productivity shocks more fully; the median firm that chooses to prepay increases its debt 26.5%, boosting its leverage ratio by 2.5 ppt. This increase in leverage runs counter to the economy-wide trend of firms reducing their leverage over time. Though the transactional cost of prepayment is small, it nonetheless deters most firms from making this choice. In particular, firms know that if they choose not to prepay this period, there is a chance their debt might mature costlessly next period. The gains from prepayment must exceed the option value of weighting. Accordingly, prepayment is a young firm's game; the median firm choosing to prepay is 16 quarters old, half the economy-wide median age. The gains from prepayment tend to be highest for young firms, which are the most below scale.

6 Financial Crises under Probabilistic Maturity

In this section, I compare the model economy under probabilistic maturity to an economy with the standard long-term debt contract (e.g., Gomes, Jermann, & Schmid, 2016). The two economies are identically parameterized, ensuring that all differences in outcomes are solely attributable to the differences in the debt contracts. In this section only, parameterization is at an annual frequency. As in the core model, average debt maturity is 5 years. Probabilistic maturity restricts the size of the steady state economy, and leads to more protracted recessions during financial crises.

6.1 Steady State Comparison

Table 4 compares the models' steady states. Probabilistic maturity reduces steady state output by 1.5% and steady state consumption by 1.1% relative to the economy under consol debt. The smaller economy is due to the more conservative debt price schedule under probabilistic maturity. Across firms borrowing new debt, the average debt price is 0.986 under probabilistic maturity, compared to 0.900 under consols. Even though firms' choices yield higher debt prices under probabilistic maturity, they are more constrained than firms in the consol-debt economy. The average firm's capital choice is only 45% of the unconstrained optimal level, about 5 ppts less than under consol debt. The tighter constraints under probabilistic maturity reflect lenders' inability to influence firms' decisions after originating a loan. Accordingly, discipline is front-loaded, leading to a broader tightening in financial conditions. As a result, default is far less frequent under probabilistic maturity, with an annual default rate of 1.7%, compared to 10.3% under consol debt.[21]

Table 4: Steady State Comparison

	Consol	Probabilistic
Relative Output	–	0.985
Relative Consumption	–	0.989
Mean $k'/k^*(\varepsilon)$	0.509	0.454
Average Debt Price	0.900	0.986
Default Rate (%)	10.3	1.70

$k^*(\varepsilon)$: unconstrained optimal capital choice
Models are at an annual frequency.

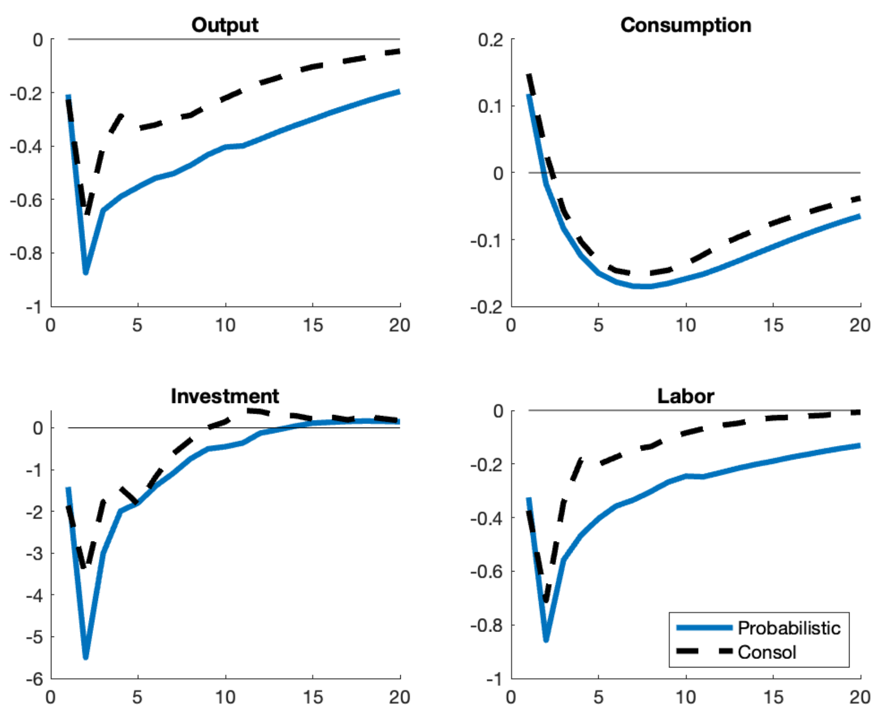
6.2 Model Dynamics under a Financial Crisis

To simulate a severe financial crisis, I examine an unexpected rise in firms' financial operating expenses. I assume expenses rise by 5% of steady state output, and last for one period. Because the cost is financial, the crisis does not directly lead to any losses in aggregate resources. This approach is isomorphic to a balance-sheet shock that reduces the value of a firm's assets but leaves its productive capacity unaffected. In models without financial frictions, the real economy is unresponsive to such shocks. In contrast, models with financial frictions can generate large real downturns to financial crises.

[21] One reason for the higher default rate under consols is that the model permits debt dilution. Under consol debt, the firm's net debt issuance is $q(\cdot)(b' - (1-\lambda)b) - (\lambda + c)b$. This makes corner solutions potentially appealing. For example, if choosing $k' = 0$ implies a bond price of 0, a firm can set itself up for endogenous default and wipe out $1 - \lambda$ fraction of its outstanding debt obligations. The longer is debt maturity, the larger is this fraction. The equilibrium debt price schedule will factor this in, reducing the likelihood that a firm would choose such a corner solution. However, in general this behavior will not be eliminated in equilibrium.

Figure 5 compares aggregate flows for 20 years after the financial shock in both the economy under probabilistic maturity and the one under consol maturity. Both economies generate protracted declines in aggregate consumption and investment, echoing the trends of the U.S. 2007 recession. However, under probabilistic maturity, the economy responds more to the financial crisis. Compared to the consol economy, probabilistic maturity generates a peak decline in output that is 0.2 ppt larger and a half-life in output that is 2 years longer.

Figure 5: Aggregate Flows over Financial Recession by Maturity Type,
Percent Deviations from Steady State Level



Probabilistically maturing debt increases capital misallocation during the financial crisis, weakening the recovery. Large firms have sufficient assets to cover the unexpected cost without reducing their investment. In fact, because the recession generates a rise in the equilibrium discount rate, large firms actually increase their investment during the recession, even as investment declines in the aggregate. Under probabilistic maturity, the financial crisis generates both a larger rise in big firms' investment and a larger decline in small firms' investment. For example, across the smallest one-third of firms, as measured by assets, the average investment rate on impact date is -1.8% under probabilistic maturity compared to 2.8% under consol debt. Effectively, the financial shock operates as a growth shock for these firms, rendering them persistently smaller.

Under probabilistic maturity, the financial shock has slightly larger negative effects for firms with maturing debt. The mean investment rate across firms with maturing debt is 0.8 ppt lower than the average among firms with non-maturing debt. This gap is 0.3 ppt larger than it is in steady state.[22] While they are spared rollover risk, misallocation still occurs among firms with non-maturing debt, because their investment responses are constrained by their inability to costlessly adjust their borrowing at the shock's onset. Across all indebted firms, measured TFP falls 0.3% under probabilistic maturity on impact date, compared to a 0.1% decline in the consol economy. This endogenous decline in TFP, combined with the larger fall in investment under probabilistic maturity, weighs down output and slows its recovery.

7 Model Dynamics under a Pandemic Recession

My pandemic recession starts in the quarter beginning in March 2020.[23] To generate the recession, I increase households' disutility of labor and impose lockdown shocks, which I discipline to match changes in output and in wages.[24] As shown in Table 5, both sets of shocks last for four quarters and get less severe over time. The labor shock reflects the rise in wages observed since the start of the recession, and falls linearly. The lockdown shocks were chosen so that, absent stimulus, the peak decline in total output slightly exceeds what has been observed in the US.[25] I assume that initially 1/3 of all firms are subject to a lockdown that lasts a month (i.e., one third of the quarter).

[22]This gap would be larger, were it not for general equilibrium effects. The equilibrium risk-free lending rate falls about 40 basis points at the start of the recession, reducing the cost of new borrowing.

[23]In the US, domestic transmission of COVID-19 started around February 2020. Data for the full 2020Q1 smooth over the recession's initial impact. For example, real personal consumption expenditures in the US rose 0.4% in January, edged down 0.1% in February, and dropped 6.5% in March. Across the full first quarter of 2020, consumption fell just 1.8%.

[24]The only source I am aware of on firm lockdowns is a survey by the Census Bureau of small businesses. I choose not to rely on the Census data, as they do not distinguish between temporary closures due to government restrictions and voluntary closures (e.g., because of a lack of demand). As the latter are endogenous, calibrating to temporary closures would overstate the magnitude of the exogenous shocks.

[25]In the US, output in 2020Q2 was 10% below its level at the end of 2019.

Since output is proportional to idiosyncratic productivity, lockdown shocks can also be viewed as productivity shocks (e.g., a restaurant that can operate throughout the quarter, but at reduced capacity). Over time, fewer firms get locked down and lockdowns are of shorter duration.

Table 5: Exogenous Productivity Shocks

Quarter	Labor Disutility Shock (%)	Lockdown Share*	Lockdown Duration**
Q1	2.5	1/3	1/3
Q2	1.875	1/5	1/3
Q3	1.25	1/10	1/6
Q4	0.625	1/10	1/6
Q5+	0	0	–

$*$: Share of firms locked down in a period. $**$: Share of period a firm cannot operate.

I evaluate the effects of two policies: cash grants and reductions in lending rates. Cash grants increase eligible firms' financial savings on impact date. Following the 2020 Paycheck Protection Program, eligible firms receive 2.5 months of their normal wage bill, capped at $2 million. Cash grants are funded through financial transfers, and therefore do not directly affect aggregate resources. I assume that the fiscal authority borrows from the representative household to finance the grants, which it pays back through lump-sum taxation of the representative household. Conceptually, this is similar to approaches taken elsewhere in the literature (e.g., Khan and Lee, 2021), as it implies that the financing of cash grants does not affect aggregate prices. This offers an upper bound of the program's effectiveness, as distortionary taxation would slow down the recovery.

To implement rate reductions, I impose a floor on lenders' discount rate; when pricing new debt contracts, lenders are at least as patient as they are in steady state. This ensures the risk-free lending rate never exceeds its steady state level, which I view as a plausible upper bound for what policy might seek to implement.[26] Rate reductions last for the duration of the exercise horizon.

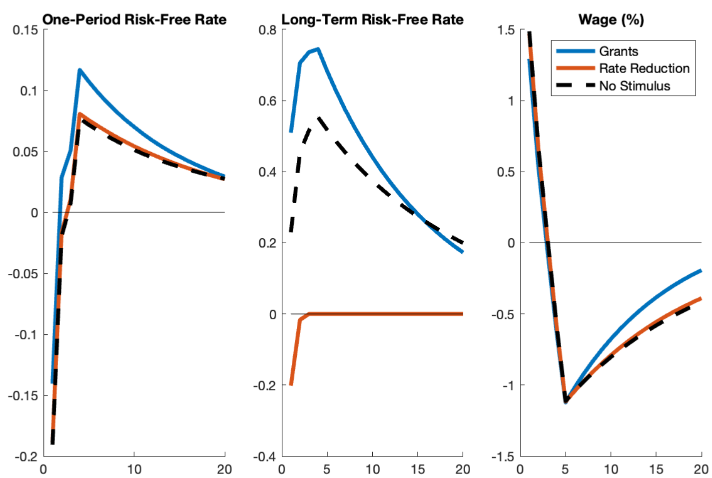
The next three subsections discuss the dynamics of the pandemic recession over the no-stimulus baseline, and when the two stimulus policies are implemented independently of one another. Lastly, I discuss how the pandemic recession affects firms of different sizes.

[26]The resulting fall in long-term interest rates is also quantitatively similar to the reductions Krishnamurthy & Vissing-Jorgensen (2011) estimate from the quantitative easing implemented in the aftermath of the Great Recession.

7.1 Price Dynamics over the Pandemic Recession

To provide context for how agents respond to the recession shocks, Figure 6 tracks the trends in aggregate prices. It illustrates two key features of long-term debt contracts with fixed coupon payments. Because expected payments are back-dated under long-term lending, the model generates a substantial rise in the risk-free loan rate, which peaks 0.5 ppt above the risk-free one-period rate absent stimulus. Hence, credit conditions tighten far more in environments with long-term debt than those with one-period maturity. Additionally, the risk-free lending rate moves above its steady state level immediately upon impact date, even as the risk-free one-period rate falls below its steady state level. So while risk-free borrowing is more expensive in this long-term debt environment, under one-period maturity risk-free borrowing would actually be cheaper at the onset of the recession.

Figure 6: Price Dynamics over Pandemic Recession,
Percentage Point Differences from Steady State



Cash grants increase households' impatience over the first 20 quarters of the recession, leading to larger rises in the risk-free borrowing rate. Through this channel, the policy generates a further tightening of credit conditions for much of the recession. While risk-free borrowing does eventually become cheaper under cash grants than in the no-stimulus case, this occurs well after the economy has started to recover. By construction, under the rate reduction scenario the risk-free lending rate never exceeds its steady state level. Borrowing conditions initially ease, tracking the fall in the risk-free one-period rate. But after the second period, the floor on lenders' discounting binds.

The right panel of Figure 6 shows the trends in wages. Two offsetting forces influence the wage rate. For the first four quarters, the labor disutility shock increases wages. However, with consumption falling below its steady state level, households' value of leisure declines, putting downward pressure on the equilibrium wage. The disutility shocks dominate initially, but after their removal wages fall about 1% below their steady state level. Cash grants generate a faster recovery in wages, due to the faster recovery in consumption.

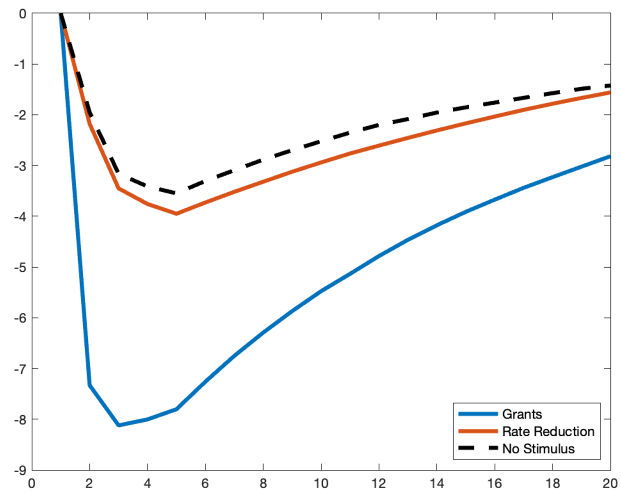
To quantify how these general equilibrium effects influence investment, Figure 7 plots a financially unconstrained firm's capital over the recession. As in Ottonello and Winberry (2021), an unconstrained firm is one whose investment decisions are independent of its level of debt or assets. This firm's capital choice only responds to expectations of future lockdown risk, future wages, and the discount factor.[27] Absent stimulus, the capital of an unconstrained firm troughs 3.6% below its steady state level, four quarters after impact date, and recovers half these losses in 11 quarters. Rate reductions slightly reduce patience, amplifying the peak capital decline to 4.0%. Since the discount factor falls even more under cash grants, the peak capital decline rises to 8.1%. However, the half-life of this decline is two quarters shorter under cash grants, because the policy generates a faster decline in the discount factor.

7.2 Aggregate Flows over the Pandemic Recession

Figure 8 tracks the responses in aggregate output, consumption, investment, and labor across the first 20 quarters of the recession. Overall, the effects of stimulus tend to be relatively small. For example, aggregate output was 6.8% below its steady state level in the no-stimulus baseline over the first year of the recession. Rate reductions shave off just 0.1 ppt of this decline. Cash grants are more effective, reducing the first year's decline by 0.7 ppt. By the second year of the recession, output in

[27] The equilibrium discount factor is the reciprocal of the one-period risk-free interest rate.

Figure 7: Capital for a Financially Unconstrained Firm over Pandemic Recession, Percent Deviations from Steady State Level

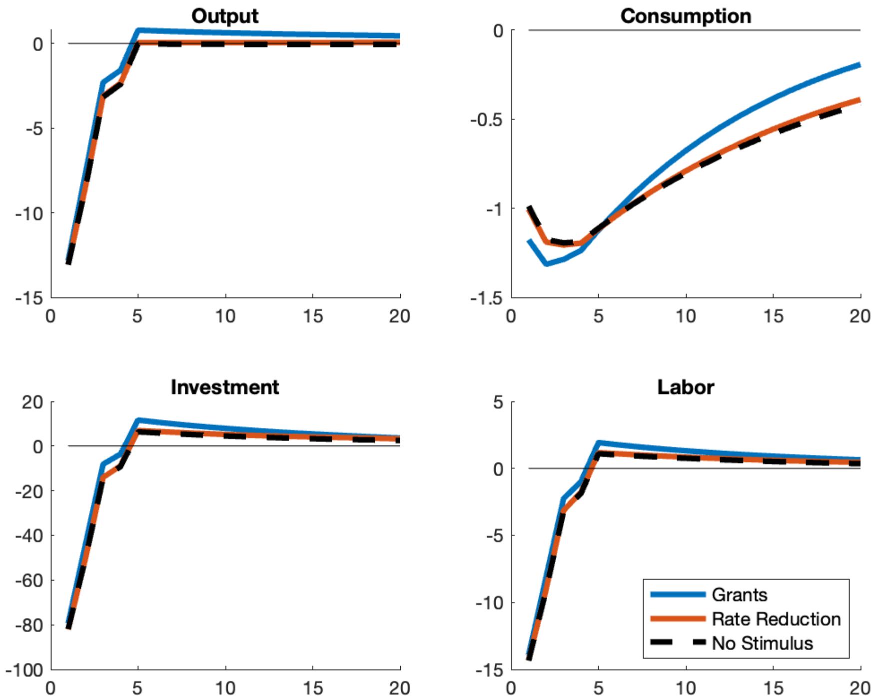


the baseline has returned to its steady state level. Both policies continue to increase output, but these gains start to diminish in the third year after impact date. The trends across total investment and labor qualitatively match those of aggregate output. In both cases, rate reductions provide a slight boost relative to the no-stimulus baseline, while cash grants provide more support.

As is common in complete markets models, the model generates a disproportionately large fall in investment, and a milder fall in consumption.[28] Nonetheless, the model provides useful insights into the relative effects of the stimulus policies. Rate reductions have very negligible effects on current consumption; the modest increase in output is invested into future capital. This spurs the longer-term recovery, but does not provide much relief to the household during the crisis. In contrast, cash grants generate a steeper trend in aggregate consumption, falling more upon impact date but rising more quickly after the first year of the recession. Because cash grants stimulate investment more at the start of the recession, firms can subsequently produce more.

[28] An expanded set of shocks would bring the model closer to the data. For example, a fall in households' utility from consumption would help match the steep decline observed in 2020Q2. Conceptually, this could reflect individuals' reduced desire to have restaurant meals or other high-risk activities. Holding aggregate resources constant, the fall in consumption caused by that demand shock would necessitate an increase in aggregate investment.

Figure 8: Aggregate Flows over Pandemic Recession,
Percent Deviations from Steady State Level

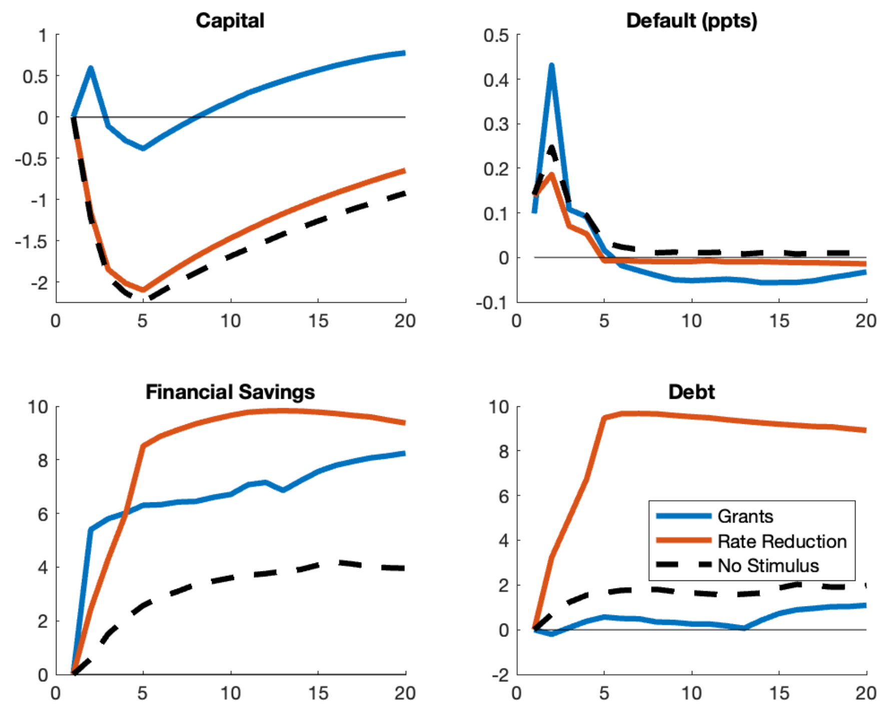


7.3 Ordinary Firms' Response to the Pandemic Recession

To assess how financially constrained firms respond to the recession, Figure 9 examines only ordinary firms, which account for over 99.5% of all firms in the economy. Over the no-stimulus scenario, ordinary firms' aggregate capital troughs 2.2% below its steady state level, and requires 12 quarters to make up half this loss. The peak decline is smaller than that of financially unconstrained firms (Figure 7), because financial constraints restrain ordinary firms' size. In the steady state, the average ordinary firm's capital choice is two-thirds of what it would choose absent financial constraints.

Absent stimulus, the recession prompts a steady rise in firms' aggregate debt and aggregate financial savings. Firms that were debt-free on impact date account for about half of the rise in debt. Some of these firms were unconstrained in the steady state, but became financially constrained due to lockdowns. The rest engage in precautionary savings, hedging against the risk that future lockdowns

Figure 9: Ordinary Firms' Aggregate Stocks over Pandemic Recession, Percent Deviations from Steady State Level



may make them constrained. Since the benchmark lending rate rises over the first year of the recession (Figure 6), waiting and borrowing only upon the realization of a negative shock can be more expensive than hedging against risk at the start of the recession. The simultaneous rise in debt and savings is consistent with the "borrow to save" behavior observed by Xiao (2019) in the aftermath of the Great Recession.

Debt rises even more under the rate reduction policy. The relaxation in lending conditions prompts 3.8% of firms to prepay in the first quarter alone, more than double the prepayment rate in the steady state.[29] Similarly, more debt-free firms borrow. This generates a substantial rise in aggregate financial savings, which peak 11.6% above their steady state level. In contrast, reducing the risk-free lending rate has small effects on ordinary firms' aggregate capital, shaving off just 0.1 ppt of the peak decline observed without stimulus. This is because the additional borrowing stimulated by rate reductions is predominately for precautionary savings. In particular, the policy prompts

[29]In contrast, in the absence of stimulus prepayment falls. The procyclicality of prepayments is consistent with empirical evidence from Xu (2018).

borrowing by firms with a lower risk of becoming financially constrained; the most at-risk firms borrow even without the policy. As the additional borrowing is by firms with less risk, only a small fraction ends up becoming invested.

In contrast to rate reductions, cash grants are effective at shoring up ordinary firms' aggregate capital. Despite the pandemic shocks, the stimulus boosts ordinary firms' investment upon impact date, leading to a 0.6% rise in their aggregate capital. While their aggregate capital subsequently falls below steady state, the decline is brief and the policy ultimately generates a sustained expansion. Cash grants also generate a prolonged increase in financial savings. In level terms, the increase in financial savings exceeds the rise in capital by a factor of 4.4 just after impact date. At the same time, aggregate debt is little changed from the steady state under the cash grant scenario. This partly reflects the grants' ability to reduce firms' reliance on external financing, by increasing their internal funds. However, cash grants also increase the risk-free lending, which deters borrowing. On impact date, the prepayment rate falls to 0.7%, less than half the steady state level.

In the absence of stimulus, defaults increase modestly from steady state. The default rate rises 0.6 ppt to 2.2% over the first year of the recession.[30] In line with the aggregate recovery, the default rate has largely recovered to the steady state by the second year. Rate reductions are moderately successful at reducing default risk, lowering the default rate over the first year by 0.2 ppt. By improving borrowing conditions, rate reductions ease the downside risks of maturity shocks. Easing risk improves firms' inside option of continuing, prompting a fall in endogenous default risk. In contrast, cash grants actually lead to an 0.1 ppt increase in the default rate over the first year of the recession. While cash grants reduce defaults on impact date, these defaults are delayed, not prevented. General equilibrium effects, such as the tighter debt price schedule, push against the benefits of the stimulus and inhibit cash grants' ability to reduce defaults. As these price effects start to ease in the second year, the default rate under cash grants moves consistently below the default rate absent stimulus.

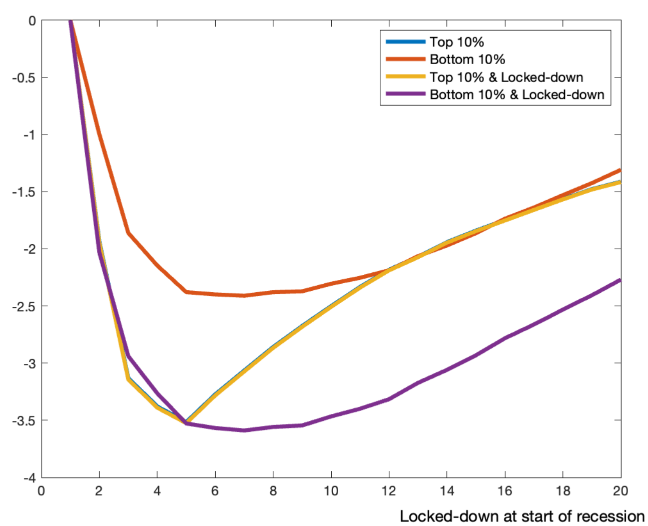
[30] The short duration of the pandemic reduces the realization of defaults. For example, each firm has just a 35% chance being locked down at least once from the second quarter on. Ongoing work examines a longer pandemic recession.

7.4 Variation in Firms' Responses by Size

Kurmann, Lalé, and Ta (2021) find that a year after the 2020 recession started, small business employment had recovered more than large business employment. In the model, small firms similarly have milder declines than large firms, in the aggregate. Moreover, this result holds even without stimulus. This section examines the drivers behind these findings.[31]

I measure size by a firm's gross assets, $k + a$, and identify the smallest and largest 10% of firms at the start of the recession. In Figure 10, I follow these cohorts' aggregate capital over the no-stimulus scenario, and benchmark against where the firms would be in the steady state. Such benchmarking is particularly important for the smallest firms, as these firms grow the fastest in the steady state. Over the first three years of the recession, aggregate capital among the largest 10% of firms falls 3.5 ppt below its steady state level, a percentage point larger than the peak decline among the smallest 10% of firms.

Figure 10: Capital Evolution during Recession among the Smallest and Largest Firms, No-Stimulus Scenario, Percent Deviations from Steady State Level



Size measured by gross assets, $k + a$, on impact date.

The bigger declines among the larger firms are consistent with Moscarini and Postel-Vinay (2012),

[31] Appendix C conducts a similar exercise to show the differential effects of stimulus policy by firm size

who find that the largest firms' employment is more closely correlated with the aggregate business cycle than smaller firms' employment. In the model, small firms are less responsive to the pandemic recession *because* they are more financially constrained. These constraints impede their growth during the steady state, leaving them inefficiently small. Even though the pandemic recession lowers the unconstrained firm's optimal size, most firms remain inefficiently small. Accordingly, most small firms still invest as much as they can. Their expansion is only slowed down by the fall in profits brought on by the recession. In contrast, a financially unconstrained firm's investment will be solely influenced by expectations over future prices and risk. Both channels prompt a decline in large firms' capital at the start of the recession.

However, the relationship between financial constraints and growth is nonmonotonic over the pandemic recession. Figure 10 illustrates this by including the trends in capital across the 10% largest and smallest firms that were locked down at the start of the recession. There is a negligible difference between the capital trends of the largest firms by lockdown status. In contrast, when the smallest firms are locked down, they experience a far larger and more persistent decline in capital. Lockdowns generate a large fall in profit, which amplifies the financial constraints among the smallest firms and limits how much they can invest. These findings may explain why Crouzet and Mehrotra (2020) do not find a systematic relationship between cyclicality and financial frictions. Financial constraints can inhibit business' responses to negative aggregate shocks, because they are already inefficiently small. However, when these aggregate shocks coincide with a large loss in revenue, financial constraints can bind tighter, leading to larger responses.

Following this logic, recall that investment dynamics were markedly different over the financial recession examined in Section 6. Consistent with empirical evidence from the 2007 recession (e.g., Chodorow-Reich, 2014b; Siemer, 2019), smaller, more financially constrained firms suffer more under the balance-sheet shock. The direct effect of the shock erodes small firms' profits, requiring them to sell capital to remain active. Through general equilibrium effects, the financial shock increases firms' discounting, prompting a rise in investment by firms large enough to afford it. Financial frictions render more constrained firms more responsive to financial shocks. These same frictions render more constrained firms less responsive to productivity shocks, such as the lockdowns of the 2020 recession. Consequently, financial frictions can account for trends observed over both the the long-run business cycle (e.g., Moscarini & Postel-Vinay, 2012; Crouzet & Mehrotra, 2020) and the Global Financial Crisis (e.g., Chodorow-Reich, 2014b; Siemer, 2019).

8 Policy Transmission during the Pandemic Recession

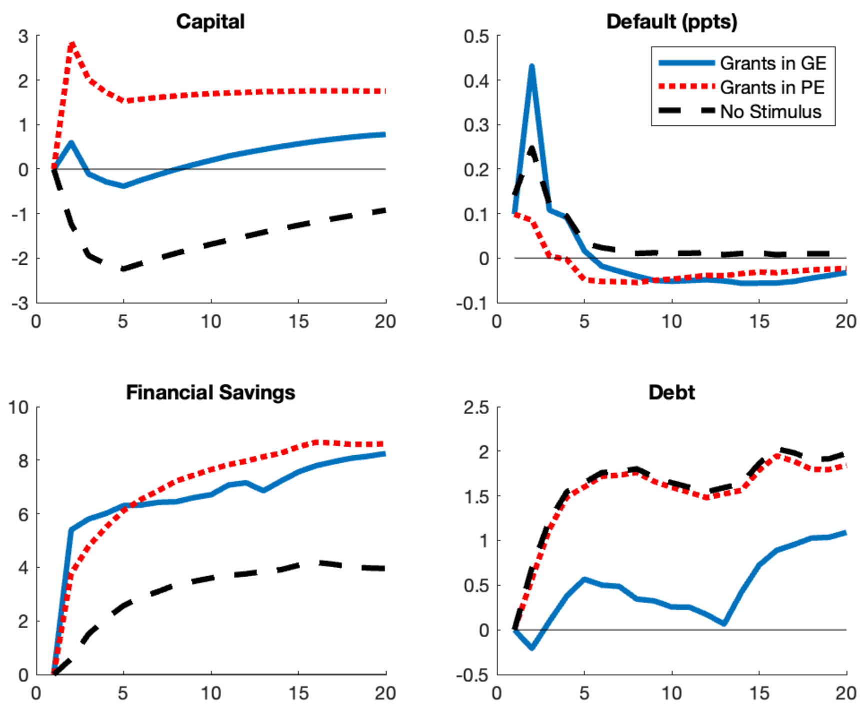
This section examines the underlying drivers of the aggregate trends discussed in Section 7. I start by quantifying the aggregate implications of cash grants' general equilibrium effects. I then assess how the maturity of long-term debt affects investment during the recession, and how effective stimulus measures are at mitigating this effect. Lastly, I study the simultaneous implementation of cash grants and rate reductions, and measure the stimulus policies' long-term benefits.

8.1 Cash Grants & General Equilibrium Effects

Section 7 showed that cash grants generate large general equilibrium effects. To quantify the total impact of these effects, I consider a counterfactual scenario in which prices follow their trajectories in the no-stimulus baseline, but firms receive cash grants on impact date. This disentangles the direct effects of the grants from the indirect effects they generate.

Figure 11 compares the aggregate response of ordinary firms under this counterfactual exercise to when the grants are given in general equilibrium and when no stimulus is given. Under the partial equilibrium counterfactual, ordinary firms' capital rises 2.9% in the aggregate, 2.2 ppts more than what cash grants generate under general equilibrium. In contrast, the rise in financial savings in the counterfactual is initially smaller than in general equilibrium. This illustrates the degree to which aggregate price movements lower target capital levels for unconstrained firms, leading to a reallocation of funds from physical capital to financial savings. Default rates also vary markedly between the partial and general equilibrium scenarios. In the partial equilibrium counterfactual, the default rate over the first year of the recession is 1.8%, compared to 2.3% from transfers under general equilibrium and 2.2% in the no-stimulus baseline. In the second year, transfers push the default rate below its steady state level under partial equilibrium.

Figure 11: General Equilibrium Effects & Cash Grants:
Ordinary Firms' Aggregate Stocks, Percent Deviations from Steady State Level



Grants in PE: Prices follow the no-stimulus trajectory, but funds are given upon impact date.
The other two scenarios are unchanged from Figure 8.

Table 7 summarizes aggregate flows in the counterfactual scenario, when no stimulus is given, and when cash grants are given in general equilibrium. Absent general equilibrium effects, cash grants reduce the loss in output by 1.8 ppt, 1.1 ppt more than in general equilibrium. Hence, in terms of output, general equilibrium forces absorb 64% of the benefits of the stimulus over the first year. A similar result holds for employment. The results for aggregate investment are even more striking. Under general equilibrium, cash grants shore up investment over the first recession year by 5.2 ppt, relative to the no-stimulus scenario, but investment remains 33.7% below its steady state level. In the absence of general equilibrium effects, cash grants actually drive investment 4.6% above its steady level, despite the severe recession. Under partial equilibrium, aggregate flows are boosted by both ordinary firms, which increase their investment (Figure 11), and super firms, which reduce their invest less because the discount rate is higher under baseline prices (Figure 7).

Table 7: Aggregate Flows across First Recession Year
% Deviations from Steady State

	No Stimulus	Cash Grants GE	Cash Grants PE
Output	-6.8	-6.1	-5.0
Employment	-7.1	-6.3	-5.3
Investment	-38.9	-33.7	4.6

GE: Cash grants stimulus given in general equilibrium.
PE: Stimulus given; prices follow no-stimulus scenario.

8.2 Debt Maturity, Investment, & Stimulus Programs

While the literature has identified a negative relationship between debt maturity and capital investment during recessions (e.g., Almeida et al., 2011), it is hard to decompose the source of this negative relationship empirically. During downturns, weaker revenues reduce the cash-on-hand firms can use to pay back debt. Additionally, rollover risk increases due to tighter borrowing conditions. Which channel is the primary driver of the weaker investment among firms with maturing long-term debt? Since rate reductions and cash grants directly target one channel more than the other, I use these policies to address this question. Cash grants increase firms' cash-on-hand, reducing the negative effects of debt payment. In contrast, rate reductions do not directly affect cash-on-hand, but ease rollover risk by lowering the benchmark lending rate.

I randomly draw 1,000,000 firms from the equilibrium distribution and simulate this panel over the first year of the recession across each of the three main scenarios: no-stimulus, cash grants only, rate reductions only, and the full policy scenario. Additionally, I consider a scenario in which cash grants and rate reductions are implemented in tandem. Table 8 shows the estimated effect of debt maturity on investment using the same regression approach implemented in my steady state analysis. For comparison purposes, I also include the steady state estimate from Figure 2.

Table 8: Debt Maturity's Effects on Four-Quarter Capital Growth, %

		2020 Recession			
	Steady State	No Stimulus	CG	RR	Both
Maturity Shock	-1.189^{***} (0.114)	-1.541^{***} (0.120)	-1.350^{***} (0.122)	-1.160^{***} (0.121)	-1.142^{***} (0.123)

CG: Cash grants. RR: rate reductions.

Base group: firms with non-maturing debt. Firms with 0 debt excluded.

Additional controls: gross assets, leverage, savings, productivity

Standard deviations in parentheses.

See Appendix B for full regression estimates.

$***: \; p < 0.01, \; **: \; p < 0.05, \; *: p < 0.1$

Absent stimulus, the recession increases the maturity friction by 0.4 ppt. This is qualitatively consistent with empirical evidence from the Global Financial Crisis (e.g., Almeida et al., 2011; Bustamante, 2019). While the 2020 recession did not have a financial crisis, the rise in credit risk and the increase in the benchmark lending rate both lead to tighter financing conditions than in the steady state. The effect of debt maturity on investment is 0.2 ppt milder under cash grants, while rate reductions fully eliminate the increase brought on by the recession. This indicates that, during the recession, the larger effect of maturity is predominately due to an increase in rollover risk. The full-stimulus scenario corroborates this: there is little difference in the effect of maturity on investment across the full-stimulus scenario compared to when rate reductions are implemented alone.

8.3 Simultaneous Implementation of Stimulus & Long-Run Benefits

During the 2020 recession, the Paycheck Protection Program and quantitative easing were implemented simultaneously. To explore the interactions between these types of policies, I consider a "full policy" scenario that implements both cash grants and rate reductions. Table 6 summarizes the economy's response to the pandemic in the full policy scenario, relative to the no-stimulus scenario. For comparison, it also examines the cash grant and rate reduction only scenarios.

The benefits of the full policy scenario are roughly equal to the sum of what each policy generates on its own. For example, under full policy, output is 0.81% higher over the first year than in the

Table 6: Model Dynamics under Stimulus, relative to No-Stimulus Baseline (%)

Economy-Wide	Output CG	Output RR	Output Both	Consumption CG	Consumption RR	Consumption Both	Investment CG	Investment RR	Investment Both	Employment CG	Employment RR	Employment Both
Year 1	0.75	0.05	0.81	-0.12	-0.01	-0.13	8.44	0.21	8.77	0.87	0.07	0.94
Year 2	0.77	0.09	0.87	0.04	0.00	0.04	4.39	0.60	4.85	0.73	0.08	0.83
Year 3	0.67	0.10	0.77	0.14	0.01	0.15	3.14	0.67	3.56	0.54	0.08	0.62
Ordinary Firms	**Capital CG**	**Capital RR**	**Capital Both**	**Default (ppt) CG**	**Default (ppt) RR**	**Default (ppt) Both**	**Savings CG**	**Savings RR**	**Savings Both**	**Debt CG**	**Debt RR**	**Debt Both**
Year 1	1.89	0.12	2.01	0.12	-0.16	-0.12	3.85	3.83	9.47	-1.15	5.12	6.24
Year 2	1.92	0.20	2.14	-0.16	-0.12	-0.27	2.99	5.79	9.49	-1.43	7.71	7.12
Year 3	1.91	0.25	2.16	-0.25	-0.08	-0.32	3.28	5.84	9.56	-1.35	7.73	7.19

CG: cash grants. RR: rate reductions.

Aggregate capital, savings, and debt across ordinary firms measured at end of year.

absence of stimulus. This is 0.06 ppt more than what cash grants generate, effectively the same as the 0.05% boost from rate reductions. This additivity indicates that the two policies do not substantially alter one another's real economic effects. However, the full policy scenario generates a sustained fall in the default rate, larger than what rate reductions achieve alone and in contrast to the initial increase under cash grants. Additionally, the full policy scenario leads to larger rises in both debt and financial savings. Hence, the muted increase in aggregate debt under cash grants is not driven by substitution of debt for grants, but is a response to the tighter borrowing conditions the policy generates.

Table 9 measures the net benefits of the stimulus. I calculate the present discounted value (PDV) of aggregate output across each scenario, and use the output of the no-stimulus scenario as a benchmark. The "all firms" row shows the gains to aggregate output, while the "memo" row focuses solely on the ordinary firms that received the stimulus. Cash grants boost the PDV of output by 0.36%, 4.5 times larger than the 0.08% gain under rate reductions. When the two policies are implemented in tandem, the PDV of output is just 3 basis points larger than what cash grants achieve alone. Not surprisingly, the gains are much larger among the ordinary firms that received the stimulus. Implementing both policies simultaneously increases the PDV of ordinary firms' output by 0.76%, roughly twice the aggregate gain. General equilibrium effects weigh down the investment and output of financially unconstrained firms, including the super firms. Their lower output offsets much of the gains among ordinary firms.

Table 9: % Change in Cumulative Output, from No-Stimulus Scenario
Present-Discounted Value at Start of Recession

	Grants	Rate Reductions	Both
All Firms	0.36	0.08	0.39
Memo: Ordinary Firms Only	*0.68*	*0.12*	*0.76*

Table 10: Cumulative Output Gain, % of Grant Program
Present-Discounted Value at Start of Recession

	Grants	Rate Reductions	Both
All Firms	58.5	–	63.9
Memo: Ordinary Firms Only	*88.9*	–	*98.4*

To put these cumulative benefits in context, Table 10 takes the ratio of the increased PDV of output to the total cost of the grant program. The grants cost 9.6% of steady state annual output, a higher share than the Paycheck Protection Program (4.2% of 2019 output), but in line with the estimate by Elenev, Landvoight, and Van Nieuwerburgh (2020) of total firm-targeted stimulus as a share of GDP (9.8%).[32] When cash grants are implemented on their own, every dollar yields only 59 cents of increased output. Even among ordinary firms the policy generates 89 cents per dollar. Implementing a rate reduction policy alongside cash grants improves net benefits. PDV as a share of grant costs are 5.4 ppts higher under the full policy scenario, but that still means that about one in every three dollars in expenditures is not recovered. Among the ordinary firms, the full policy scenario roughly breaks even, with a dollar of grants returning 98 cents in PDV output gains.

I focus my cost-benefit analysis on output gains, because the model assumes perfect risk-sharing among households. The welfare gains of cash grants would be larger for, say, the proprietors of a small business or communities that faced more severe lockdowns. On the other hand, my cost-benefit analyses rely on the strong assumption that cash grants are funded through lump-sum taxation. If the program was instead funded through distortionary taxes, either on the household or on firms,

[32] Two factors are primarily responsible for the higher cost of the cash grant policy relative to the Paycheck Protection Program in the data. First, I assume that all eligible firms receive funding. Schweitzer and Borawski (2021) estimate that only 76% of eligible businesses got funds. Additionally, I assume that all firms receive the maximum amount they are eligible for.

that economic recovery would be slower. Accordingly, my results offer an upper bound to the output gains from the cash grant policy. Since this upper bound is worth only two-thirds of the program's costs, the welfare gains abstracted away from this environment would need to be quite large for it to be optimal to implement the program.

9 Conclusion

In this paper, I construct and evaluate a general equilibrium model in which firms' long-term debt matures infrequently. This variation in maturity allows the model to capture the sustained fall in investment that follows a large share of long-term debt maturing. By capturing the high rollover risk associated with maturity, the economy under probabilistic maturity is smaller than an economy with the standard long-term debt contract. Additionally, probabilistic maturity generates larger, longer recessions in response to financial crises.

To quantify how responsive firms' investment is to policy, I evaluate the 2020 recession, in which several firm-targeted stimulus measures were implemented. I show that the negative effect of debt maturity on investment is more sensitive to reductions in the risk-free borrowing rate than to increases in firm cash. But, since larger firms' investment is less responsive to long-term interest rates, reducing them has little effect on aggregate investment. In contrast, targeted cash grant policies improve firm liquidity and increase aggregate investment, even after accounting for general equilibrium effects. However, these general equilibrium effects tighten borrowing conditions, increasing default risk among the most financially constrained firms. Simultaneously implementing cash grants and rate reductions leads to a faster recovery and a fall in defaults.

The underlying environment examined here captures several salient aspects of firms' financing and investment behavior in the U.S. A natural way to extend on this would be to incorporate a financial intermediary subject to its own frictions, which would allow the model to capture the feedback effects of increased default risk on aggregate lending conditions (e.g., Elenev, Landvoight, and Van Nieurwerburgh, 2020). Studies on the effects of quantitative easing in the aftermath of the Global

Financial Crisis highlight the importance of the banking channel (e.g., Chodorow-Reich, 2014a). The current paper isolates the firm debt-demand channel. Accounting for this alongside the banking channel would allow a more robust assessment of quantitative easing.

Works Cited

Almeida, Heitor; Murillo Campello; Brune Laranjeira; and Scott Weisbenner. 2011. "Corporate Debt Maturity and the Real Effects of the 2007 Credit Crisis". *Critical Finance Review.* 1:3-58.

Altig, David; Scott Baker, Jose Maria Barrero; Nicholas Bloom; Philip Bunn; Scarlet Chen; Steven Davis; Julia Leather; Brent Myer; Email Mihaylov; Paul Mizen; Nicholas Parker; Thomas Renault; Pawel Smietanka; and Gregory Thwaites. 2020. "Economic Uncertainty Before and During the COVID-19 Pandemic." *Journal of Public Economics.* 191(104274): 1-13.

Arellano, Cristina; Yan Bai; and Patrick Kehoe. 2019. "Financial Frictions and Fluctuations in Volatility." *Journal of Political Economy.* 127(5): 2049-2103.

Autor, David; David Cho; Leland Crane; Mita Goldar; Byron Lutz; Joshua Montes; William Peterman; David Ratner; Daniel Villar; Ahu Yildirmaz. 2020. "An Evaluation of the Paycheck Protection Program using Administrative Payroll Microdata". *Working paper.*

Baqaee, David and Emmanuel Farhi. 2020. "Nonlinear Production Networks with an Application to the COIVD-19 Crisis". *NBER Working Paper 27281.*

Bartik, Alecander; Zoe Cullen, Edward Glaeser, Michael Lucas, Christopher Stanton, and Adi Sunderam. 2020. "The Targeting and Impact of Paycheck Protection Program Loans to Small Businesses". *NBER Working Paper 27623.*

Bartlettt, Robert and Adair Morse. 2020. "Small Business Survival Capabilities and Policy Effectiveness: Evidence from Oakland". *NBER Working Paper 27629.*

Brunnermeir, Markus and Arvind Krishnamurthy. 2020. "Corporate Debt Overhang and Credit Policy". *Brookings Papers on Economic Activity.* Summer: 447-488.

Bustamante, Christian. 2019. "Debt Overhang, Monetary Policy, and Economic Recoveries after Large Recessions". *Working paper.*

Campello, Murillo; John Graham; and Campbell Harvey. 2010. "The Real Effects of Financial Constraints: Evidence from a Financial Crisis." *Journal of Financial Economics.* 97: 470-487.

Chatterjee, Satyajit; Dean Corbae; Majoto Nakajima; and Ríos-Rull. 2007. "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default." *Econometrica.* 75(6): 1525-1589.

Chen, Hui; Yu Xu; and Jun Yang. 2021. "Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads". *Journal of Financial Economics.* 139: 770-799.

Chetty, Raj; John Friedman; Nathaniel Hendren; Michael Stepner; and The Opportunity Insights Team. 2020. "How Did COVID-19 and Stabilization Policies Affect Spending and Employment?

A New Real-Time Economic Tracker Based on Private Sector Data". *NBER Working Paper 27431.*

Chodorow-Reich, Gabriel. 2014a. "Effects of Unconventional Monetary Policy on Financial Institutions." *Brookings Papers on Economic Activity.* Spring 2014: 155-221.

Chodorow-Reich, Gabriel. 2014b. "The Employment Effects of Credit Market Disruptions: Firm-Level Evidence from the 2008-2009 Financial Crisis". *Quarterly Journal of Economics.* 129(1): 1-59.

Chodorow-Reich, Gabriel; Olivier Darmouni; Stephan Luck; and Matthew Plosser. 2020. "Bank Liquidity Provision across the Firm Size Distribution". *NBER Working Paper 27945.*

Chodorow-Reich, Gabriel and Antonio Falato. 2020. "The Loan Covenant Channel: How Bank Health Transmits to the Real Economy." *NBER Working Paper 23879.*

Choi, Jaewon; Dirk Hackbarth, and Josef Zechner. 2018. "Corporate Debt Maturity Profiles". *Journal of Financial Economics.* 130: 484-502.

Cooley, Thomas and Edward Prescott. 1995. "Economic Growth and Business Cycles". In *Frontiers of Business Cycle Research*, edited by Thomas Cooley. Princeton University Press: Princeton, New Jersey.

Crouzet, Nicolas. 2017. "Default, Debt Maturity and Investment Dynamics". *Working paper.*

Crouzet, Nicolas and Neil Mehrotra. 2020. "Small and Large Firms over the Business Cycle". *American Economic Review.* 110 (11): 3549-3601.

Davis, Steven and John Haltiwanger. 1922. "Gross Job Creation, Gross Job Destruction, and Employment Reallocation". *Quarterly Journal of Economics.* 107(3): 19-63.

Davis, Steven; R. Jason Faberman; John Haltiwanger; and Ian Rucker. 2010. "Adjusted Estimates of Worker Flows and Job Openings in JOLTS." *Labor in the New Economy.* University of Chicago Press.

Deng, Minjie and Mia Fang. 2020. "Lumpy Debt, Monetary Policy, and Investment". *Working paper.*

Dinlersoz, Emin; Sebnem Kalemli-Özcan; Henry Hyatt; and Veronika Penciakova. 2018. "Leverage over the Life Cycle and Implications for Firm Growth and Shock Responsiveness". *NBER Working Paper 25226.*

Duchin, Ran; Oguzhan Ozbas; and Berk Sensoy. 2010. "Costly External Finance, Corporate Investment, and the Subprime Mortgage Crisis." *Journal of Financial Economics.* 97: 418-435.

Eichenbaum, Martin; Sergio Rebelo; and Mathias Trabandt. 2020. "The Macroeconomics of Pandemics". *NBER Working Paper 26882.*

Elenev, Vadim, Tim Landvoight, and Stijn Van Nieuwerburgh. 2020. "Can the COVID Bailouts Save the Economy?" *NBER Working Paper 27207.*

Elenev, Vadim; Tim Landvoigt; Patrick Shultz; and Stijn Van Nieuwerburgh. 2021. "Can Monetary Policy Create Fiscal Capacity?" *NBER Working Paper 29129.*

Geelen, Thomas. 2016. "Debt Maturity and Lumpy Debt." *Working paper.*

Gertler, Mark and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy". *Journal of Monetary Economics.* 58: 17-34.

Glover, Andrew, Jonathan Heathcote, Dirk Krueger, and José-Víctor Ríos-Rull. 2020. "Health versus Wealth: On the Distributional Effects of Controlling a Pandemic." 2020.

Gomes, João; Urban Jermann, and Lukas Schmid. 2016. "Sticky Leverage." *American Economic Review.* 106(12): 3800-3828.

Gomme, Paul; B. Ravikumar; and Peter Rupert. 2011. "The Return to Capital and the Business Cycle". *Review of Economic Dynamics.* 14(2): 262-278.

Granja, João, Christos Makridis, Constantine Yannelis, and Eric Zwick. 2020. "Did the Paycheck Protection Program Hit the Target?" *NBER Working Paper 27095.*

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning. 2020. "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?" *NBER Working Paper 26918.*

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks". *American Economic Review.* 124(4): 1403-1448.

Jungherr, Joachim, Matthias Meier, Time Reinelt, and Immo Schott. 2021. "Corporate Debt Maturity Matters for Monetary Policy". *Working paper.*

Jungherr, Joachim and Schott, Immo. 2021. "Optimal Debt Maturity and Firm Investment." *Review of Economic Dynamics.* 42: 110-132.

Kalemli-Özcan, Şebnem, Luc Laeven, and David Moreno. 2018. "Debt Overhang, Rollover Risk, and Corporate Investment: Evidence from the European Crisis". *Working paper.*

Kaplan, Greg, Benjamin Moll, and Giovanni Violante. 2020. "The Great Lockdown and the Big Stimulus: Tracing the Pandemic Possibility Frontier for the U.S." *NBER Working Paper 27794.*

Khan, Aubhik and Soyoung Lee. 2021. "Firm Heterogeneity, Leverage and the Aftermath of the Pandemic." *Working paper.*

Khan, Aubhik; Tatsuro Senga; and Julia Thomas. 2019. "Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity". *Working paper.*

Khan, Aubhik and Julia Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity". *Journal of Political Economy.* 121(6): 1055-1107.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen. "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy." *Brookings Papers on Economic Activity.* 42(2): 215-287.

Kurmann, André, Etienne Lalé, and Lien Ta. 2021. "The Impact of COVID-19 on Small Business Dynamics and Employment: Real-time Estimates with Homebase Data". *Working paper.*

Moody's Investor Service. 2018, February 15. "Annual Data Study: Corporate Default and Recovery Rates, 1920-2017".

Myers, Stewart. 1977. "Determinants of Corporate Borrowing". *Journal of Financial Economics.* 5: 147-175.

Lackdawala, Aeimit and Timothy Moreland. 2019. "Monetary Policy and Firm Heterogeneity: The Role of Leverage Since the Financial Crisis." *Working paper.*

Ottonello, Pablo and Thomas Winberry. 2020. "Financial Heterogeneity and the Investment Channel of Monetary Policy." *Econometrica.* 88(6): 2473-2502.

Schweitzer, Mark and Garrett Borawski. 2021. "Which Industries Received PPP Loans?" Federal Reserve Bank of Cleveland. *Economic Commentary*: 2021-08.

Siemer, Michael. 2019. "Employment Effects of Financial Constraints During the Great Recession." *Review of Economics and Statistics.* 101(1): 16-29.

Small Business Administration. 2021. "PPP Data". https://www.sba.gov/funding-programs/loans/covid-19-relief-options/paycheck-protection-program/ppp-data. Last Updated: August 15, 2021. Last Accessed: August 26, 2021.

Syverson, Chad. 2011. "What Determines Productivity?" *Journal of Economic Literature.* 49 (2): 326 365.

Xiao, Jasmine. 2019. "Borrowing to Save and Investment Dynamics". *Working paper.*

Xu, Qiping. 2018. "Kicking Maturity Down the Road: Early Refinancing and Maturity Management in the Corporate Bond Market". *The Review of Financial Studies.* 31(8): 3061-3097.

Appendix

A: Aggregation for Definition of Equilibrium

Here, I map the beginning-of-period distribution, μ, into the distributions that determine output and investment during the period, μ_Y and μ_I. μ_Y differs from μ due to the restructuring of defaulted firms, which leads to a loss of $(1-\chi)$ fraction of capital. The restructuring of defaulted firms also leads μ_I to differ from μ. In addition, μ_I incorporates new entrants, which replaced firms that received the exit shock and were not restructured.

For ease of notation, let $\mathbb{D}^e$ be an indicator for an exiting firm that lenders put through default. Recall that $\mathbb{D}^n$ and $\mathbb{D}^m$ are indicators for firms that endogenously default under non-maturing and maturing debt. The distributions μ and μ^Y are defined on the Borel algebra $\mathcal{A}$, generated by the open subsets of the product space $\mathcal{A} = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{E}$, where $\mathbb{E} \equiv \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$. For any $(A, \varepsilon_m) \in \mathcal{A}$, μ^Y, is measured by

$$\begin{aligned}
\mu^Y(A,\varepsilon_m) = (1-\lambda)(1-\rho) &\int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{D}_t^n(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \\
+(1-\lambda)(1-\rho) &\int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^n(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\lambda(1-\rho) &\int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{I}_t^m(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\lambda(1-\rho) &\int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^m(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\rho &\int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{I}_t^e(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \\
+\rho &\int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^e(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m])
\end{aligned} \tag{15}$$

The state space for μ^I must also account for whether or not a firm's debt matures, as decision rules vary along this margin. I denote this Borel algebra as $\mathcal{A}^I$, which is generated by the open sets of the produce space $\mathcal{A}^I = 2 \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{E}$. Let 1 be an indicator variable for a firm whose debt matures, with 0 reflect a firm with non-maturing debt.

$$\mu_t^I(0, A, \varepsilon_m) = (1-\lambda)(1-\rho)\int_{(k,a,b)|(k,a,b)\in A}(1-\mathbb{D}_t^n(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m]) \tag{16}$$

$$\begin{aligned}
\mu_t^I(1, A, \varepsilon_m) = (1-\lambda)(1-\rho)&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_t^n(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\lambda(1-\rho)&\int_{(k,a,b)|(k,a,b)\in A}(1-\mathbb{I}_t^m(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\lambda(1-\rho)&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_t^m(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\rho\mathbb{1}[(0,\iota a_e,0)\in A]\bar{\pi}_m^{\varepsilon}&\int(1-\mathbb{I}_t^e(k,a,b,\varepsilon))\mu(d[k\times a\times b\times\varepsilon]) \\
+\rho&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_e(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m])
\end{aligned} \tag{17}$$

where $\mathbb{1}$ is an indicator variable for whether the entrants' state is an element of A and $\bar{\pi}^{\varepsilon}$ is the ergodic distribution over firms' idiosyncratic productivities. Entrants' financial assets are a fraction of firms' average physical capital:

$$a_e = \iota\int k\mu(d[k\times a\times b\times\varepsilon]) \tag{18}$$

The aggregate law of motion for the distribution of firms relies on μ_I as an intermediate distribution. For any $(A, \varepsilon_m)\in\mathcal{A}$, μ_{t+1} is measured by

$$\begin{aligned}
\mu_{t+1}(A,\varepsilon_n) = &\int_{(k,a,b,\varepsilon_m)|(g_k^n(\cdot),g_a^n(\cdot),b)\in A}\pi_{m,n}^{\varepsilon}\mu_I(0, d[k\times a\times b\times\varepsilon_m]) \\
+&\int_{(k,a,b,\varepsilon_m)|(g_k^m(\cdot),g_a^m(\cdot),g_b^m(\cdot))\in A}\pi_{m,n}^{\varepsilon}\mu_I(1, d[k\times a\times b\times\varepsilon_m])
\end{aligned} \tag{19}$$

where the arguments for g_k, g_a, g_b, and $\mathbb{D}$ are (k,a,b,ε) and t subscripts are left off to reduce notation.

B: Empirical Methodology & Additional Regression Results

B.1 Empirical Sample Selection & Variable Construction

My empirical methodology follows the literature standard closely (e.g., Almeida et al., 2011; Bustamante, 2019; Ottonello & Winberry, 2021). I use the Compustat Fundamental Annual dataset from 1984 to 2020. I exclude several groups of firms. First, I require firms to be incorporated in the United States and to denominate their accounts in U.S. dollars. Additionally, I exclude utilities, financial institutions, and public administrations. Lastly, I impose several financial criteria. Firms with assets less than $0.5 million are dropped, as are firms with asset or sales growth above 100%. Firms are also dropped if they have long-term debt greater than their total assets, negative short-term debt, or inconsistencies in the long-term debt variables (e.g., more debt maturing in 5 to 10 years than all debt maturing in more than 1 year). Lastly, I require firms to have at least five consecutive years' worth of data, and to have positive debt. All variables are winsorized at the 1% level.

I measure investment as the log change in property, plan, and equipment, adjusted for inflation. To estimate Tobin's q, I subtract common equity and deferred taxes and investment tax credit from total assets plus market capitalization, and then take the ratio of this to total assets. Leverage is the ratio of debt in current liabilities plus total long-term debt maturing in more than one year to total assets. Consistent with Almeida et al. (2011), I consider a firm to have a high share of long-term debt maturing if more than 20% of its debt is maturing in the year.

B.2 Regression Results

Table B1: Debt Maturity & Firm-Level Investment in the Data

	$\Delta k_{t,t+1}$	$\Delta k_{t+1,t+2}$	$\Delta k_{t+2,t+3}$
Debt Maturity Indicator	-0.013***	-0.006	0.005
	(0.004)	(0.0047)	(0.005)
Log(Assets)	-0.031***	-0.061***	-0.064***
	(0.011)	(0.012)	(0.012)
Log(Assets)^2	0.001	0.001*	0.001
	(0.001)	(0.001)	(0.001)
Sales Growth	0.079 ***	0.015	0.024**
	(0.011)	(0.011)	(0.011)
Sales Growth^2	0.057**	0.067***	-0.020
	(0.024)	(0.025)	(0.025)
Tobin's q	0.054***	0.045***	0.022***
	(0.005)	(0.005)	(0.005)
Tobin's q^2	(-0.004)***	-0.003***	-0.001**
	(0.0005)	(0.0004)	(0.0005)
Leverage	-0.152***	-0.149***	-0.072*
	(0.040)	(0.040)	(0.043)
Leverage^2	-0.107*	0.006	0.040
	(0.055)	(0.058)	(0.059)
Cash/Assets	-0.321***	0.228***	0.139**
	(0.048)	(0.048)	(0.058)
Cash/Assets^2	-0.267***	-0.207*	-0.060
	(0.098)	(0.104)	(0.126)
Constant	0.095**	0.248***	0.292***
	(0.043)	(0.044)	(0.044)
Observations	25,418	22,320	19,790
R^2	0.0986	0.0714	0.0591
Firm Controls	yes	yes	yes
Time/industry FE	yes	yes	yes

Standard errors in parentheses, and clustered at the firm level.

$***:\ p<0.01,\ **\ p<0.05,\ *\ p<0.1$

High long-term debt: more than 20% of long-term debt matures in year.

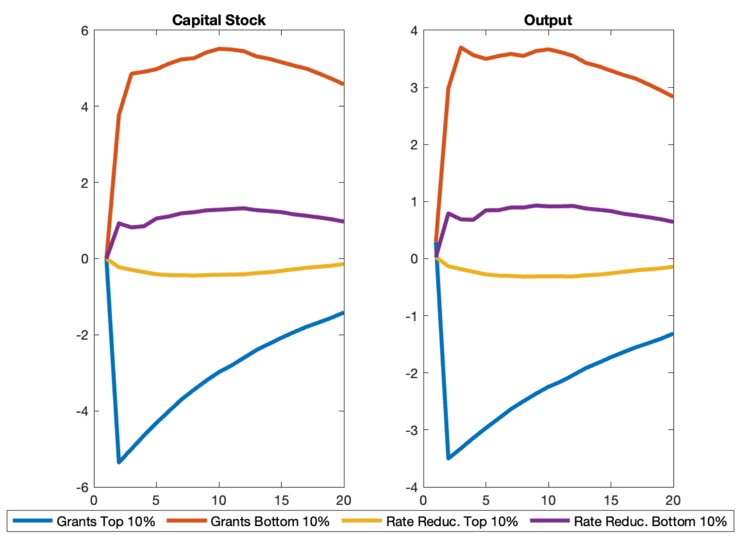
Growth rates and ratios are winsorized at the 1% level.

C: Additional Quantitative Findings

C1: Effects of Stimulus on Firm Investment over Pandemic Recession

This section illustrates how stimulus alters the responses of the largest and smallest 10% of firms during the pandemic recession. The left panel of Figure C1 looks at these firms' aggregate capital stock, while the right panel tracks their aggregate output. In both cases, I compare these aggregates against their levels under the no-stimulus baseline. In the period following impact date, the capital of the largest 10% of firms falls an additional 5.7%, above and beyond the losses sustained in the no-stimulus scenario. In contrast, the policy propels an additional 4.0% increase in the capital of the smallest 10% of firms. In subsequent periods, the largest firms start to recover their additional losses, and converge towards their level in the no-stimulus baseline. In contrast, the smallest firms' gains are sustained for several years following the implementation of the stimulus. Hence, consistent with many empirical studies of the Paycheck Protection Program, the cash grants generate marked support to the smallest firms (e.g., Bartlett & Morse, 2020; Chodorow-Reich et al. 2020; Kurmann, Lalé, and Ta, 2021).

Figure C1: Effects of Policy among the Smallest and Largest Firms:
Percent Deviations from No-Stimulus Scenario



Size measured by gross assets, $k + a$, on impact date.

In contrast to the cash grants policy, rate reductions generate much more modest changes in firm outcomes. For example, the initial gains in capital among the smallest 10% firms are just 0.9%, compared to 4.0% under cash grants. On the other hand, because the rate reduction policy has such small effects on aggregates, the additional fall in large firms' investment is much milder. By reducing rollover risk, the rate reduction policy spurs investment among the smallest firms. This suggests that, through the firm demand channel, policies such as quantitative easing increase investment among the most financially constrained firms. The finding that more financially constrained firms respond more to rate reductions is somewhat in contrast with the finding of Ottonello and Winberry (2020) that it is the least constrained firms that respond most to conventional monetary policy. However, Lackdawla and Moreland (2019) show empirically that after the Great Recession, high-leverage firms responded more to monetary policy shocks than low-leverage firms, the opposite of what occurred prior to the recession. This period coincides with when quantitative easing was implemented in the United States.