Long-Term Debt, Default Risk, and Policy Transmission during Severe Recessions*

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Abstract

The COVID-19 pandemic has caused economic disruption of an all-but unprecedented nature. Accordingly, policymakers in the United States, and elsewhere, implemented novel stimulus measures. Fiscal policies like the Paycheck Protection Program offered small businesses grants at the onset of the crisis. Additionally, the Federal Reserve substantially expanded its quantitative easing program. To evaluate these policies, I study a general equilibrium model in which heterogeneous firms borrow using defaultable, long-term debt. The model endogenously generates a term premium, making quantitative easing a relevant policy instrument. A recession akin to the one triggered by COVID-19 generates a large, protracted fall in aggregate capital and several quarters of elevated default rates. Cash grants are effective at accelerating the economic recovery, but generate strong general equilibrium forces that increase default risk during the recession. Quantitative easing generates mild stimulative effects at the aggregate level, but is able to reduce financial constraints and thereby counteracts some of the downside risks generated by policies such as the Paycheck Protection Program. The simultaneous implementation of these two programs reduces defaults and accelerates the recovery.

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1 Introduction

"No business, whatever its size, will be exposed to the risk of collapse". French President Macron's promise, in March 2020, was one of many signs that the recession triggered by COVID-19 would be unusual. As paradoxical as a no-default recession may sound, governments provided hundreds of billions of dollars in fiscal support directly to firms. Just one policy, the Paycheck Protection Program, allocated nearly $800 billion in forgivable loans to small businesses in the United States. At the same time, central banks extended the scope of unconventional monetary policy, with quantitative easing pushing the Federal Reserve's balance sheet up by two-thirds over the course of just two months. The simultaneous implementation of fiscal stimulus and quantitative easing, against the backdrop of the COVID-19 pandemic, provides an unprecedented opportunity to assess the real effects of firm financial frictions during recessions, and the ability of stimulus policies to alleviate such frictions. How effective are cash transfers or quantitative easing at reducing financial frictions and increasing investment? Do the general equilibrium effects generated by these policies amplify or undermine efficacy? Was the Paycheck Protection Program more impactful for having occurred alongside quantitative easing, or did the two polices conflict with one another?

The COVID-19 recession exacerbated financial frictions through three primary channels. First, many firms were subject to unexpected expenses for protective equipment, increased wage pressure, and a fall in revenue–all of which eroded available funds. Additionally, credit conditions tightened as financial markets factored in worsening aggregate conditions and increased future idiosyncratic default risk. If these were the only channels present, evaluating the effects of the stimulus policies would be straightforward. Forgivable loans, such as those provided by the Paycheck Protection Program (PPP), can offset the funds lost due to the pandemic. Similarly, quantitative easing reduces firm interest rates across the credit spectrum (Krishnamurthy & Vissing-Jorgensen, 2011), and hence relaxes financing conditions. However, there is also a third channel: COVID-19 generated a large rise in idiosyncratic uncertainty (e.g., Altig et. al, 2020; Brunnermeir & Krishnamurthy, 2020). Local lockdowns and other health measures constrained firms' operations in an irregular manner; a firm's ability to operate varied with both time and location. Future lockdowns were inherently unpredictable, as they depended on where the virus was surging at any given time. This idiosyncratic uncertainty amplifies debt overhang. Firms' pre-existing debt obligations dilute their benefits from investment, as debt holders are owed a portion of future returns. The future lockdown risk generated by COVID-19 independently reduces expected returns on investment. The coincidence of these two phenomena further dampens a firm's incentive to invest or avoid default, making it all the more challenging for the financial support provided by PPP and quantitative easing to translate into real economic improvement.

Accounting for each of these channels necessitates a quantitative, general-equilibrium environment that captures the heterogeneity of financial frictions across firms. Since some firms are highly financially constrained, whereas others are not, stimulus policies will differentially affect firms. For example, I find that PPP is partially a victim of its own success. PPP is able to mitigate the severity of the recession and speed up the recovery. But this recovery pushes up the equilibrium wage and the benchmark risk-free loan rate, reducing firm profitability and increasing financing constraints. These general equilibrium forces are so strong that on its own PPP leads to an *increase* in the default rate over the first year of the recession, relative to a counterfactual scenario in which PPP had not been implemented. It would be impossible to assess the overall effects of stimulus without accounting for the heterogeneity in firm financial frictions and general equilibrium forces.

The real side of the model follows the literature standard closely (e.g., Ottonello & Winberry, 2021; Khan, Senga, & Thomas, 2016). I construct and analyze a one-sector, real general equilibrium model, in which firms are subject to idiosyncratic, persistent productivity risk. This persistence, combined with firms' use of a decreasing returns to scale technology, generates a nontrivial distribution of firms in equilibrium. Production relies on physical capital and labor as inputs; the latter is supplied by a representative household. Firms' assets consist of physical capital and risk-free financial savings.

What distinguishes this environment from previous quantitative studies is that firms borrow using long-term debt with lumpy maturity.[1] Debt maturity is stochastic: every period each firm receives a shock that determines whether none or all of its debt matures in that period. Firms that do not receive the maturity shock can choose to prepay their debt and borrow anew, subject to a prepayment cost. Hence, no firm is unable to take on a new loan, but some firms have no choice but to pay back their debt that period. This contract allows me to account for several important aspects of firm financial frictions. Ottonello and Winberry (2021) show that in their one-period maturity setting, firms optimally allocate all available funds towards increasing their capital or reducing their debt. This result does not hold under lumpy-maturity debt, allowing the model to more fully account for the dynamic effects of debt overhang. In the environment explored here, highly financially constrained firms will optimally reduce their investment and increase their dividends. While this increases the share of expected returns going to shareholders, it also increases the firm's default risk, and hence is inefficient at the aggregate level.

Additionally, lumpy-maturity debt allows the model to address empirical analyses exploring the real effects of firm debt maturity (e.g., Almeida et al., 2011; Kalemli-Özcan, Laeven, & Moreno, 2018) and extend on theoretical research exploring the implications of debt maturity (e.g., Chen, Xu, & Yang, 2021; Deng & Fang, 2020; Jungherr et al., 2021). The maturity shocks in the model have competing effects on firm investment. Firms receiving the maturity shock must pay the entirety of their debt back, reducing funds available for investment. However, they can also frictionlessly access the debt market. In contrast, firms that do not receive the maturity shock can enjoy lower debt payments, but firms that would benefit from new borrowing can only access the market by paying a cost. Lumpy maturity allows the model to cleanly isolate the financial frictions generated by debt maturity, and quantify how much PPP and quantitative easing alleviate these frictions.

Lumpy-maturity debt is also a critical to realistically capturing the effects of PPP and quantitative easing. The maturity friction generates a nontrivial distinction between a firm's financial debt and its financial assets, ensuring that PPP is not isomorphic to a debt forgiveness policy; the

[1]Geelen (2016) and Chen, Xu, and Yang (2021) have also analyzed models with lumpy maturity, but in these settings firm revenues are exogenous and aggregate prices are exogenously determined. This strand of the literature focuses more on analyzing the optimal maturity structure, which I abstract from here.

funds increase cash, and do not directly reduce debt. This separation is not feasible in the standard quantitative framework of long-term firm debt, in which no firm would simultaneously hold financial savings and debt in equilibrium (e.g., Gomes, Jermann & Schmid, 2016; Bustamante, 2019).[2,3] Additionally, under lumpy-maturity debt, the majority of firms do not take on new borrowing in any given period, consistent with the fact that most firms infrequently take on new long-term debt. This ensures that the model does not overstate the impact of quantitative easing: only firms that take on new debt will directly benefit from the policy.

To evaluate how the economy responds to the COVID-19 recession, I solve the model under perfect foresight. The recession is triggered by two sets of shocks. I assume a rise in households' disutility of labor, a reduced-form way of capturing workers' desire to not put themselves or their families at risk of contracting the virus. This puts upward pressure on the equilibrium wage, consistent with the rise in labor costs observed at the start of the recession. Additionally, a subset of firms are subject to temporary "lockdowns", which operate like negative productivity shocks. For example, a restaurant may only be able to operate at 50% capacity, or a retailer may need to reduce hours of operation. Lockdowns are independently and identically distributed, akin to how the pandemic was especially severe in certain areas at different times (e.g., New York City in the spring of 2020 versus Los Angeles in late 2020). These lockdown shocks generate the rise in idiosyncratic uncertainty.

I examine two policies, PPP and quantitative easing. In practice, PPP offered loans that were forgivable so long as the funds were used towards wages, interest payments, or other operating expenses. I generalize on this, and consider a policy that provides grants; firms never need to pay back their PPP receipts.[4] Akin to how PPP was implemented in practice, I give small firms 2.5 months' worth of their steady state wage bill, capped at $2 million. For tractability, I assume that all eligible firms receive the money upon impact date. Since the policy I consider is effectively a cash

[2]This framework, which extends on Leland (1994, 1995), relies on granular debt maturity. A share of long-term debt matures in every period, and firms do not face an adjustment cost to take on new debt. Consequently, there is no benefit to holding financial savings and debt simultaneously; the firm would be better off using the savings to reduce its debt.

[3]Khan and Lee (2021) is a prominent exception. In their model, entrepreneurs take on consol-style contracts, which mature at the firm's discretion.

[4]As of August 15, 2021 60% of PPP loans have been forgiven, and 95% of applications for forgiveness had been accepted (SBA, 2021). (Both measures are by loan volume.)

transfer, I assume the PPP funds are drawn from financial resources. Consequently, PPP does not directly affect aggregate resources in the economy; all aggregate effects are due to firms' endogenous choices on how to use the funds.

The long-term nature of debt in the model endogenously generates a term premium during recessions. When current consumption is low relative to future consumption, the equilibrium discount factor falls: a marginal increase in future consumption is less valuable than in the steady state, because future consumption is already relatively high. Accordingly, even in the absence of default risk, the present-discounted value of future debt payments is lower at the start of a recession than it is in the steady state. This endogenous term premium leads to a tightening in credit conditions over and beyond the increased default risk caused by the recession. As quantitative easing lowers long-term interest rates in practice, I consider a policy that targets this endogenous term premium. Specifically, I impose a floor on lenders' discount factor, which I set at the steady state level. This creates a wedge between the discounting of lenders and other economic agents, and eliminates the term premium. To date, the literature has focused on evaluating quantitative benefits through channels within the financial sector (e.g., Chodorow-Reich, 2014; Elenev et al., 2021). I abstract from this channel, and instead examine how firm debt demand is affected by quantitative easing.

In the absence of stimulus, the COVID-19 recession generates a sharp, swift decline in output. Output is 6.8% below its steady state level during the first year of the recession, but slightly overshoots the steady state in the second year. This swift recovery is driven by a rise in aggregate labor; even with the labor disutility shocks, for most of the recession the equilibrium wage is below its steady state level. In contrast, the aggregate capital stock has a far more sluggish recovery; by the fifth year of the recession it has recovered only half its peak losses. This prolonged recovery implies a long period in which output is disproportionately allocated towards investment, slowing down the recovery in aggregate consumption. This slow consumption response implies many periods in which the effective discount factor is above its steady state level. Equivalently, consumers, and the firms and lenders that they own, are relatively impatient until consumption has fully recovered. This impatience is what generates the term premium in the model. At its peak, the term premium is about 0.5 percentage points, whereas it does not exist in the steady state.

The high term premium in the absence of stimulus means that firms that receive the maturity shock will be in an especially disadvantaged position. Not only has the recession reduced revenues, making it harder to pay debt back, but it has also made new borrowing more expensive. Accordingly, firms that receive the debt maturity shock on impact date have an annual investment rate that is 1.5 percentage points lower than an otherwise identical firm with non-maturing debt, half a percentage point larger than the effect over the steady state. By eliminating the term premium, quantitative easing is successful at reducing the investment frictions driven by debt maturity. Quantitative easing reduces the debt maturity effect on investment, bringing it nearly back to its steady state level. PPP is less successful at reducing this debt maturity friction, in part because PPP generates substantial general equilibrium effects. By allocating funds to firms, the policy reduces the decline in aggregate capital and boosts output over the first year of the recession by 0.7 percentage point. This leads to a faster, steeper, rebound in household consumption, which means that the term premium is even higher under PPP than in the no-stimulus scenario. Hence, PPP poses a tradeoff to firms: while the funds they receive can improve their ability to invest, PPP also makes borrowing more expensive.

PPP's general equilibrium effects also affect the policy's efficacy at the aggregate level. When PPP is implemented under general equilibrium, aggregate capital across eligible firms has a peak decline of about 0.3% of its steady state level, a marked improvement from the 2.5% decline generated by the recession in the absence of stimulus. In a counterfactual exercise, I allocate PPP under the no-stimulus scenario, which allows me to account for the price dynamics caused by the recession but excludes PPP's general equilibrium effects. Abstracting away from PPP's general equilibrium effects transforms the 0.3% peak decline in eligible firms' capital into a 2.0% increase. Additionally, under this counterfactual, PPP funds prevent twice as many defaults as when PPP is implemented in general equilibrium. The general equilibrium effects generated by PPP constrain all firms, but these constraints most severely affect the firms most in need of relief.

In contrast with PPP, quantitative easing has modest aggregate effects at the start of the recession.

While PPP boosted output across the recession's first year by 0.7 percentage point, quantitative easing raises it by just 0.1 percentage point. Quantitative easing does spur a rise in borrowing–aggregate debt jumps 9.5% over the first year of the recession–but most of these funds are held in financial savings, rather than physical capital. Unconstrained firms, particularly those that initially had no debt, are largely responsible for the rise in aggregate debt. These firms do not need to borrow to service their present investment needs, but future lockdown risk does create an incentive for precautionary savings. Quantitative easing offers these firms additional funds at favorable terms, but since they are hedging against a tail risk, only a fraction of the savings will ultimately translate into higher investment.

Quantitative easing's aggregate effects are also muted by the fact that only a small fraction of firms borrow in any given period. Accordingly, when the recession is most severe, the majority of firms only benefit because quantitative easing relaxes future financing constraints. This channel is particularly beneficial to the most financially constrained firms, which absent quantitative easing suffer from both the term premium and increased default risk. Relaxing future constraints helps make quantitative easing twice as effective as PPP at preventing defaults. Additionally, relative to the scenario in which no stimulus is provided, quantitative easing increases investment more at firms with higher leverage. This is particularly true across leveraged firms that receive the maturity shock. This is consistent with the empirical analysis by Lackdawala and Moreland (2019), who found that monetary policy shocks after the Great Recession affected investment more at firms with higher leverage. PPP also generates more investment at firms with higher leverage, but this effect does not vary significantly between firms that received debt maturity shocks and firms with non-maturing debt.

These results point to both quantitative easing and PPP being effective policies to combat the COVID-19 recession in their own right. However, each has its limitations. The model indicates that quantitative easing provides only modest stimulus in the short-run, while PPP further tightens borrowing conditions and is less effective at preventing firm default. When implemented simultaneously, though, the two policies compliment one another, allowing for an improvement in aggregate flows and a further reduction in defaults.

The remainder of this paper is organized as follows. Section 2 summarizes the related literature. Section 3 describes the model and defines its equilibrium. Section 4 outlines some key theoretical characteristics of the model equilibrium. In section 5, I discuss my parameterization strategy and the model fit in steady state. In section 6, I study the model dynamics over a recession and how firms respond to policy intervention. Section 7 highlights the channels through which policy intervention operates. Lastly, section 8 concludes.

2 Related Literature

This paper speaks to three branches of the literature. A growing number of papers explore various dimensions of the COVID-19 recession. Additionally, there has been extensive research on the channels and efficacy of quantitative easing. Lastly, with its focus on financial frictions, the model also relates to the literature on firm debt and default risk.

There are quite a few empirical papers that study how the Paycheck Protection Program (PPP) and other government interventions supported firms' survival prospects and employment. As the pandemic is barely a year old at the time of this writing, the focus to date has been the short- and medium-term effects. Broadly speaking, the results suggest that PPP was more effective at preventing firm closures than at preserving jobs. Bartik et al. (2020) found that firms receiving PPP funds were more optimistic of their survival prospects through the end of 2020, and that this effect was strongest among firms most impacted by COVID-19. Bartlett and Morse (2020) document an important size gradient: across the smallest of firms, PPP recipients were less likely to expect to be closed over the next 6 months. However, PPP had no significant effect among business with more than 20 employees. Estimates of the employment effects are more mixed, perhaps reflecting the fact that different data sources capture different groups of workers (Autor et al., 2020). Using data on private-sector employment across a wide range of firms, Autor et al. (2020) estimate that PPP boosted payrolls by several percentage points. In contrast, using data that focused more on especially low-wage earners, Chetty et al. (2020) find no significant effect of the program on employment. In part, that may be because PPP was not fully used to directly support payrolls. For example, research indicates that PPP recipients strengthened their balance sheets (Chodorow-Reich et al., 2020) and were less likely to be behind on debt payments (Granja et al., 2020).

Complimenting empirical analyses, quantitative research has modeled and quantified the pandemic and its interactions with the real economy. Many incorporate the epidemiological SEIR model into more standard economic frameworks to capture changes in household behavior in response to health risk (e.g., Eichnebaum, Rebelo, and Trabandt, 2020; Glover, Heathcote, Krueger, and Rios-Rull, 2020; Kaplan, Moll, and Violante, 2020). Others abstract from the virus itself to examine the implications of sector-wide shocks to demand or employment (e.g., Baqaee and Farhi, 2020; Guerrieri, Lorenzoni, and Straub, 2020). A smaller number have focused on firm-level heterogeneity and the COVID-19 recession. These include Gourinchas et al. (2020) and Elenev, Landvoight, and Van Nieurwerburgh (2020), and Khan and Lee (2021).

Gourinchas et al. (2020) examine a multiple-sector business model, in which COVID-19 generates both supply and demand shocks. The analysis focuses on the short run: prices are held fixed, and firms cannot adjust their fixed factors of production during the crisis. Firms are subject to labor constraints, which may prevent them from operating at scale, and fluctuations in the demand for their goods, as well as in their productivity. The model abstracts from access to an external financial sector, so when firms' costs exceed their profits, they cannot operate. Absent government intervention, the model generates sizeable increases in default across small- and medium-sized enterprises, with a large degree of heterogeneity across sectors. Policy interventions can help to curb the rise in bankruptcies, and preserve employment, but non-targeted policies risk inefficiently allocating funds, offsetting some of the gains of intervention. While my model is unable to speak to the cross-sector heterogeneity in Gourinchas et al. (2020), it relaxes the constraints around firms' capital decisions and access to financial markets.

Elenev, Landvogit, and Van Nieurwerburgh (2020) explore the interactions between the production and financial sectors of an economy. In their environment, both firms and their lenders are subject to default risk. These risks can build off one another, amplifying the effects of shocks through endogenously determined prices. Such amplification is present in their results: the pandemic generates a substantial rise in default and a large decline in output. Providing firms with funding, akin to PPP, does help offset some of these adverse effects. To maintain tractability, though, all idiosyncratic shocks are iid, and all intertemporal decisions are effectively made by a representative agent. This implies a large degree of inter-sectoral risk sharing; so long as it doesn't default, realizing an adverse shock has no affect on a firm's (or lender's) future trajectory. Additionally, a timing friction is imposed on the model, preventing firms from taking on new debt to make up for a shortfall of cash on hand. As a consequence, there is no idiosyncratic behavior that affects the likelihood of default; the risk is a function of general equilibrium effects and exogenous shocks. My model assumes away frictions between the financial and household sectors, and hence it cannot account for the ability of the financial sector to amplify shocks. However, by assuming persistent productivity shocks, it generates intertemporal heterogeneity that can substantively influence investment and default decisions.

Khan and Lee (2021) is the paper that most closely relates to this one. The paper also evaluates the effects of fiscal policy during the COVID-19 recession, in an environment with heterogeneous firms and long-term debt. They focus in particular on how the distribution of leverage affects the aggregate response to the recession, and find that an economy with high leverage has a slower recovery. Additionally, wage subsidies are an effective stimulus policy, but a policy that delays debt payments offsets some of the subsidies' benefits. One dimension of richness that their environment captures that I abstract from is endogenous entry. Their firms are owned by entrepreneurs, who can chose to suspend their business and enter the workforce. This allows the model to capture the cyclicality of firm entry and exit. In contrast, firm entry in my model is exogenous, and the only endogenous form of firm exit is default. The entrepreneurs in Khan and Lee (2021) borrow using consols: the entrepreneur must make a coupon payment until the bond matures, and mature only occurs at the entrepreneur's discretion. So, similar to my model, there are periods in which an entrepreneur will not participate in the debt market, and hence will not be disciplined by contemporaneous financial conditions. However, since their maturity is solely endogenous, debt overhang will not be as prevalent for their entrepreneurs. Rather than have to pay back debt at a non-optimal time, the entrepreneur can always wait and have its debt mature later.

This paper also relates to the literature on how financial frictions can affect firms' investment decisions. Under aggregate uncertainty, financial frictions can constrain individual firms' activities, and influence aggregate trends. In a model with heterogeneous firms and stochastic borrowing constraints, Khan and Thomas (2013) find that the trends observed over the Great Recession are better captured by a shock to financial conditions than a shock to aggregate productivity. Jermann and Quadrini (2012) build a New Keynesian model in which firms borrow subject to a leverage constraint and can issue new equity. They also find a large effect of financial shocks on investment and output-roughly ten times the effect of TFP shocks. Gilchrist, Sim, and Zakrajek (2014) find that uncertainty shocks affect investment decisions largely through their influence on financial frictions, rather than real frictions (e.g., the partial irreversibility of and adjustment costs to physical capital).

Quantitative easing

A couple papers have also examined the effects of quantitative easing during the COVID-19 recession. Ebsim, Faria-e-Castro, and Kozlowski (2021) evaluate how the effects of policy vary with the nature of the shock. In their environment, there are four types of firms, which vary over leverage liquidity. Default is driven by idiosyncratic preference shocks, and intertemporal decisions within a firm type are made by a representative agent. Similar to my approach, they treat quantitative easing as an exogenous increase in bond prices (i.e., a reduction in the effective interest rate). They find that quantitative easing is about to boost output in response to financial aggregate shocks, but has very modest real effects if the recession is triggered by a decline in aggregate productivity. These findings echo those of Elenev et al (2021), which examines the effects of quantitative easing in a DSGE environment. ELABORATE

Business cycle models with financial frictions have largely structured debt contracts so that in equilibrium there is no default.[5] An emerging trend among quantitative models is to allow equilibrium default, requiring idiosyncratic firm risk to be endogenously priced. Arellano, Bai, and Kehoe (2019) link default risk and agency risk, and calibrate their model to match moments on firm credit spreads. They find that their model can generate the observed volatility in output, as well as the volatility in aggregate labor, despite only having one element of aggregate uncertainty: shocks to the variance of firms' idiosyncratic productivity. Similarly, Ottonello and Winberry (2020) find that default risk constrains the responsiveness of firm investment. They develop a New Keynesian model and find that the aggregate response to an expansionary monetary policy shock is driven by firms with low default risk. Firms with high default risk are less responsive because they face high marginal costs. Khan, Senga, and Thomas (2019) develop a real business cycle model in which firms are subject to default risk and there is an endogenous entry choice. Potential entrants are less likely to enter the economy during downturns, resulting in a procyclical mass of active firms. With endogenous entry and default risk, negative financial frictions can drive a severe, protracted recession, akin to the trends observed over the Great Recession.

Building off literatures in corporate finance and sovereign debt, Gomes, Jermann, and Schmid (2016) analyze a dynamic stochastic general equilibrium model in which debt is long-term, nominal, and subject to default risk. Long-term debt is paid off geometrically, and refinancing is costless. Compared to a model with one-period debt, Gomes, Jermann, and Schmid find that long-term debt increases the responsiveness of investment and other aggregates to productivity shocks. In particular, endogenous responses are more persistent under long-term debt. Bustamante (2019) extends on Gomes, Jermann, and Schmid (2016) by making idiosyncratic productivity shocks persistent. Hence, in the model equilibrium there is a non-degenerate distribution of firms, which vary across their levels of capital and debt. Bustamante (2019) finds that this heterogeneity is important in matching the trends observed over the Great Recession. Small firms rely on debt to scale up; adverse shocks affect their optimal capital size, while leaving what they owe unchanged. Bustamante (2019) documents that this debt overhang becomes more pronounced as debt maturity lengthens.

[5] E.g., Gertler & Karadi (2011); Jermann & Quadrini (2012); Khan & Thomas (2013)

Gomes, Jermann, and Schmid (2016) and Bustamante (2019) rely on consol-style contracts. Each period, a portion of debt obligations mature, and firms can choose a different level of debt. Hence, these papers cannot examine the heterogeneity driven by maturity risk. Empirical evidence suggests this maturity risk can be significant during crises. For example, Almeida et al. (2011) find that firms with a large amount of long-term debt maturing during the 2007 credit crunch subsequently had weaker investment, accounting for 30% of the decline in aggregate investment. Additional studies by Campello, Graham, and Harvey (2010) and Duchin, Ozbas, and Sensoy (2010) also find significant relationships between debt maturity and firm outcomes during the financial crisis. In order to test whether such forces were also present during the COVID-19 recession, we need a model with variation in debt maturity across firms.

3 Model

I analyze a dynamic general equilibrium model, in which there is no aggregate uncertainty. The model consists of two types of goods-producing firms: "ordinary" and "super". The vast majority of firms are "ordinary", and are subject to persistent idiosyncratic productivity shocks and financial constraints. In contrast, "super" firms face no financial constraints, and are highly productive. The inclusion of super firms allows the model to tractably account for the extreme skewness in the distribution of firms over labor. Ordinary and super firms are perfectly competitive, and their output is perfectly substitutable. Production exhibits decreasing returns to scale technologies, using physical capital and labor as inputs. The representative household supplies labor, owns firms, and saves through a one-period, risk-free asset. Perfectly competitive, risk-neutral financial intermediaries channel household savings into firm borrowing. Ordinary firms face three dimensions of idiosyncratic risk: their production is subject to persistent productivity shocks, their debt matures stochastically, and each period there is a chance they may be forced to exit the economy permanently.

Ordinary firms fund their investment decisions through a combination of retained earnings and external borrowing. Borrowing occurs through long-term bonds, which mature with an iid probability. In each period, firms pay a coupon proportional to the bond's face value. When a firm receives a maturity shock, it must also pay back the debt's face value. When a firm does not receive a maturity shock, it can choose to prepay its debt by paying a cost proportional to the face value of its debt. Firms have limited liability, and can default on their debt; defaulting firms receive a value of 0, and permanently exit the economy. Non-defaulting firms purchase physical capital to use in the next period, and can also save in the one-period, risk-free asset. Ordinary firms' dividends are bounded below by zero; these firms cannot access equity markets. In addition to the endogenous exit driven by default, firms are subject to an iid exit shock each period. Both defaulting and exiting firms are replaced by new firms, leaving the operating mass of firms constant. I normalize the mass of firms to a unit measure.

As I solve the dynamic model using perfect foresight, time subscripts t capture the aggregate state. Aggregate TFP can vary, z_t, as can the distribution of firms over the idiosyncratic state space, μ_t. All agents know how the firm distribution evolves over time: $\Gamma_t(\mu_t) = \mu_{t+1}$.

An ordinary firm's idiosyncratic state is $\psi = \{k, a, b, \varepsilon\}$. The firm's assets consist of physical capital, k, and financial savings in the one-period asset, a. The face value of its debt is b. Lastly, $\varepsilon \in \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$ is the firm's idiosyncratic productivity. ε follows a first-order Markov process. For every TFP level ε_i, $Pr(\varepsilon_{t+1} = \varepsilon_j | \varepsilon_t = \varepsilon_i) = \pi^{\varepsilon}_{i,j} \geq 0$, where $\sum_{p=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} = 1$. With probability ρ, a firm receives an exit shock, and with probability $1 - \rho$ it can continue operating. When it does not receive the exit shock, the firm has a λ chance of having its debt mature.

The remainder of this section consists of: a description of the ordinary firm's problem, given prices; a discussion of how bonds are priced, given firms' decisions; an overview of the representative household's problem and aggregation; and a definition of the model equilibrium.

3.1 Ordinary Firms' Problem

At the start of a period, the firm observes its ε, whether it will exit in the period, and whether its debt will mature in the period. Firms that do not receive the exit shock decide whether to default or continue to operate. Additionally, firms that do not receive the maturity shock decide whether or not to prepay their debt. Continuing firms then produce, pay their debt obligations, and chose their physical capital and financial savings for next period. Additionally, firms with maturing debt can issue a new bond. Exiting firms pay back their debt and issue any remaining funds as dividends.

Each period, operating firms solve a static production problem. Both physical capital and labor are inputs in production. I denote operating profits as

$$R_t(k, \varepsilon) = \max_{\ell} z_t \varepsilon k_t^{\alpha} \ell^{\eta} - w_t \ell - \kappa(\varepsilon) + (1 - \delta)k \tag{1}$$

Operating profits consist of revenues from production, less wage costs. Additionally, firms pay an operating cost that is a function of their idiosyncratic productivity. I assume operating expenses are weakly increasing in productivity, $\kappa'(\varepsilon) \geq 0$. I assume that $\alpha + \eta < 0$, so that firms operate under decreasing returns to scale.

To minimize notation, let $J_t(\psi)$ denote value of a firm given a realized ε, but before it is known whether the firm will exit, or whether its debt will mature.

$$J_t(\psi) = (1 - \rho)\lambda J_t^m(\psi) + (1 - \rho)(1 - \lambda)J_t^n(\psi) + \rho J_t^e(\psi) \tag{2}$$

J^m is the value of the firm when its debt matures, J^n is the value when its debt does not mature, and J^e is the value when it exits. J is a construct that I use only to represent future expectations. Firm choices are always determined given the realization of the maturity and exit shocks.

3.1.1 Exiting Firms

If an exiting firm can afford to pay back what it owes after production, it produces and issues any remaining funds as dividends. Otherwise, it cedes control to the lender and receives value 0. The lender then decides whether the firm should operate in the period and liquidate after production, or if the firm should go through default proceedings at the start of the period. In either case, a firm receiving an exit shock solves

$$J_t^e(k_t, a_t, b_t, \varepsilon) = \max\{0, R_t(k_t, \varepsilon) + a_t - (1+c)b_t\} \tag{3}$$

The firm's gross assets in the period come from operating profits, non-depreciated physical capital, and returns on last period's financial savings. Its debt obligations consists of the face value of debt, as well as the periodic coupon payment cb.

3.1.2 Firms with Maturing Debt

When debt matures, the firm's problem is similar to models with one-period maturity. The firm chooses between defaulting and getting value 0, and continuing and getting value V^m. As all firms are owned by the representative household, they use the same stochastic discount factor, $M_{t.t+1}$. When it takes on new debt, the firm receives $q(\cdot)b_{t+1}$ in numeraire. It purchases financial savings at discount price q^f.A t debt maturity, the firm solves

$$J_t^m(k_t, a_t, b_t, \varepsilon) = \max\Big\{0, V_t^m(k_t, a_t, b_t, \varepsilon)\Big\} \tag{4}$$

$$V_t^m(k_t, a_t, b_t, \varepsilon_i) = \max_{d_t, k_{t+1}, a_{t+1}, b_{t+1}} d_t + M_{t,t+1} \sum_{p=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon J_{t+1}(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_j)$$

s.t. $d_t \geq 0$ and

$$k_{t+1} + q_t^f a_{t+1} + d_t = R_t(k_t, \varepsilon_i) - cb_t + a_t - b_t + q_t(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_i)b_{t+1}$$

As the firm's ε and choices of k_{t+1}, a_{t+1}, and b_{t+1} can all affect default risk, the debt price schedule $q(\cdot)$ varies with both the idiosyncratic and aggregate states. As a technical point, I assume that any firm that chooses $b_{t+1} = 0$ does not face the debt-maturity shocks. Conditional on not exiting, they can freely choose a different b_{t+1} next period.

3.1.3 Firms with Non-Maturing Debt

A firm that does not receive the maturity shock has three options: it can default and receive value 0, continue in its debt contract with value V^n, or prepay its debt and get value V^m. If it chooses to prepay its debt, the firm pays a restructuring cost $\gamma(b)$, $\gamma'(b) \geq 0$. After accounting for the restructuring cost, a firm prepaying its debt faces the same problem as a firm that received the maturity shock.

$$J_t^n(k_t, a_t, b_t, \varepsilon) = \max \Big\{0, V_t^n(k_t, a_t, b_t, \varepsilon), V_t^m(k_t, a_t - \gamma(b_t), b_t, \varepsilon)\Big\} \tag{5}$$

$$\begin{aligned} V_t^n(k_t, a_t, b_t, \varepsilon_i) = \max_{d_t, k_{t+1}, a_{t+1}} \; & d + M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^{\varepsilon} J_{t+1}(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon_j) \\ \text{s.t.} \quad & k_{t+1} + q_t^f a_{t+1} + d_t = R_t(k_t, \varepsilon_i) - c b_t + a_t \\ & d_t \geq 0, \; b_{t+1} = b_t \end{aligned}$$

When a firm does not pay back its debt, it only pays the coupon and the face value of debt does not change into the next period.

3.1.4 Entering Firms

Each exiting firm is replaced by an entrant. Households endow the entrant firms with financial assets a_e, which are fraction $\iota \in [0, 1]$ of the average capital stock of firms. They are not endowed with any physical capital or any debt, but they can take out debt in the period in which they enter. New firms draw their idiosyncratic productivity from the ergodic distribution of ε. Given these assumptions, the optimization problem of a new firm is a special case of equation (4). In particular, a new firm is indistinguishable from a firm with no physical capital and no debt outstanding.

Let $g_t^{n,k}(\psi),\ g_t^{n,a}(\psi),\ g_t^{m,k}(\psi),\ g_t^{m,a}(\psi),\ g_t^{m,b}(\psi)$ denote the firm's optimal decision rules for $k_{t+1},\ a_{t+1}$, and b_{t+1} given idiosyncratic state ψ. Additionally, let $\mathbb{D}^m(\psi)$ and $\mathbb{D}^n(\psi)$ be indicator variables for firms' default decisions, and let $\mathbb{P}(\psi)$ be an indicator variable indicating prepayment.

3.2 Default & Bond Pricing

When a firm defaults, its shareholders lose all claims, receiving the outside value of 0. The financial intermediary takes over the firm, forgives its debt, and sells it to new shareholders. This sale allows the intermediary to recover at least part of what was owed prior to default. If the sold firm is worth more than what the defaulting firm owed, the surplus is allocated to the representative household in a lump sum.[6] Since payoffs under default vary with the defaulting firm's state, lenders evaluate risk at the idiosyncratic level, as in Chatterjee et al. (2007).

Selling a firm is costly: only χ share of physical capital is recoverable by the lender. In addition, the intermediary must spend a cost proportional to the face value of the defaulted debt, νb. These losses, $(1-\chi)k$ and νb, are transferred to the representative household lump-sum. This is a conservative assumption, and ensures that the aggregate effects of default are solely determined by endogenous choices, and not through reductions in aggregate resources. The lender can fully recover a firms' financial assets.

Since default occurs at the beginning of the period, the new firm can produce during that same period. Additionally, I do not prevent the new firm from borrowing into the next period. Hence, as in Gomes, Jermann, and Schmid (2016), default risk influences firm's decisions, but does not affect the mass of firms active in a period.

If firm $\{k, a, b, \varepsilon\}$ defaults in t, the value of the new firm is $J_t^m(\chi k, a, 0, \varepsilon)$. The lender's recovery in

[6] This scenario is possible due to the combination of debt forgiveness and the lender's compensation being the equity value of the new firm, not the scrap value of its assets.

default is Θ^d.

$$\Theta_t^d(k, a, b, \varepsilon) = \min\{J_t^m(\chi k, a, 0, \varepsilon), (1+c)b\} - \nu b \tag{6}$$

Similarly, let Θ^e be the lender's recovery when a firm receives an exogenous exit shock. If the exiting firm can pay back its debt, the lender receives the full $(1+c)b$ it is owed. Otherwise, the firm cedes control to the lender at the start of the period. The lender determines whether it is optimal to put the firm through default proceedings at the start of the period, or to let it operate and liquidate it at the end of the period. As such a firm receives its outside value, 0, in either case, it is indifferent.

$$\Theta_t^e(k, a, b, \varepsilon) = \max\left\{\Theta_t^d(k, a, b, \varepsilon), \min\{R_t(k, \varepsilon) + a, (1+c)b\}\right\} \tag{7}$$

By construction, under prepayment all of a firm's debt matures. Hence, the prepayment choice weakly increases the likelihood that a firm will pay back the entirety of its debt in the next period. To reduce notation, let $\psi_{t+1} \equiv \{k_{t+1}, a_{t+1}, b_{t+1}\}$ denote the firms' asset choices. Since financial intermediaries are perfectly competitive and risk neutral, the equilibrium price of debt satisfies

$$\begin{aligned}
&q_t(\psi_{t+1}, \varepsilon_i) b_{t+1} = \\
&\quad + \lambda(1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon (1 - \mathbb{D}_{t+1}^m(\psi_{t+1}, \varepsilon_j))(1+c) b_{t+1} \\
&\quad + (1-\lambda)(1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon (1 - \mathbb{D}_{t+1}^n(\psi_{t+1}, \varepsilon_j) - \mathbb{P}_{t+1}(\psi_{t+1}, \varepsilon_j))[c + q_{t+1}(k_{t+2}, a_{t+2}, b_{t+1}, \varepsilon_j)] b_{t+1} \\
&\quad + (1-\lambda)(1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon \mathbb{P}_{t+1}(\psi_{t+1}, \varepsilon_j)(1+c) b_{t+1} \\
&\quad + (1-\rho) M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon [\lambda \mathbb{D}_{t+1}^m(\psi_{t+1}, \varepsilon_j) + (1-\lambda)\mathbb{D}_{t+1}^n(\psi_{t+1}, \varepsilon_j)] \Theta_{t+1}^d(\psi_{t+1}, \varepsilon_j) \\
&\quad + \rho M_{t,t+1} \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon \Theta_{t+1}^e(\psi_{t+1}, \varepsilon_j)
\end{aligned} \tag{8}$$

The price of debt takes into account five potential outcomes next period. The first three rows account for when the firm does not default. If its debt matures and it does not default, the lender receives the full $(1+c)b_{t+1}$. If its debt does not mature and it does not default, the firm either stays in the contract or prepays its debt. If the firm stays in the contract, the lender receives the coupon payment and the expected future value of the contract. Since there is no private information in this economy, lenders use the equilibrium decision rules to account for the default risk of future periods: $k_{t+2} = g_t^{n,k}(\psi_{t+1})$ and $a_{t+2} = g_t^{n,a}(\psi_{t+1})$. if the firm chooses to prepay, the lender's return is the same as if the maturity shock was realized. The fourth row in (8) captures what the lender receives in the event of default. While the probability of default can vary with the realization of the maturity shock, the lender's recovery is unaffected. The last row accounts for the recovery value when the firm receives an exit shock.

This debt-pricing arrangement departs from the literature in two substantial ways. The first is its approach to the maturity structure. While the literature standard also relies on a stochastic process to support long-term maturities, it assumes a constant fraction of debt matures each period: firms always have a combination of debt that matures and does not mature. Hence, in models such as Gomes, Jermann, and Schmid (2016), firms can freely adjust their total debt stock in every period. In contrast, the model here assumes maturity shocks affect the full stock of debt: either all of a firm's debt matures, or none of it does. The inclusion of the prepayment choice means debt maturity is partly endogenous–any firm not receiving the maturity shock can choose to still adjust its debt level. However, since this is an endogenous choice, such firms can also choose to remain in their preexisting contract and avoid current financial conditions.

The second departure I take is with respect to recovery under default. It is common for models to assume that lenders can only recover a fraction of physical capital in default, rather than firm value. But in these models, the lender effectively sells the restructured firm to the representative household. From the representative household's perspective, they have ownership rights to a new firm, and hence the transfer is in terms of firm value. Absent some imposed constraint, the lender would optimally sell the rights to a firm worth $J(\xi k, \cdot)$ for numeraire equal to $J(\xi k, \cdot)$, rather than

ξk. Since the new firm has no debt, its value is bounded below by ξk. This is what the firm would be worth if it sold all its assets for dividends, and the continuing value from this action was 0. This is a nontrivial departure in terms of equilibrium outcomes–by increasing the recovery value to lenders under default, debt prices rise overall. Loosely speaking, this approach corresponds to Chapter 11 of the US bankruptcy code.

3.3 Super Firms

Super firms operate the same general technological process as ordinary firms. Hence, their revenues from production are the same as (1):

$$R_t(k, \varepsilon^s) = \max_{\ell} z_t \varepsilon^s k_t^{\alpha} \ell^{\eta} - w_t \ell - \kappa(\varepsilon^s) + (1-\delta)k$$

In the steady state, super firms face no uncertainty. Over the COVID recession, however, they are subject to the idiosyncratic lockdown risk. In either case, since they are not subject to financial constraints, their future capital choice is only influenced by aggregate prices and expectations over lockdown risk. For simplicity, I assume that super firms do not face exit shocks. The inclusion of exogenous entry and exit would not affect their problem, though, as entrants could always receive enough equity injections to obtain as much capital as they need.

$$J_t^s(k_t^s, \varepsilon_t^s) = \max_{k_{t+1}^s} R_t(k_t^s, \varepsilon_t^s) + M_{t,t+1} J_{t+1}^s(k_{t+1}^s, \varepsilon_{t+1}^s) \tag{9}$$

3.4 Aggregation & Representative Household

3.4.1 Aggregation

At the start of the period, the distribution of ordinary firms over the idiosyncratic state space is μ. However, exit shocks and default decisions alter this distribution within the period. First, lenders take over defaulting firms, and sell their debt-free replacements to new managers. Production occurs from these replacements, alongside non-defaulting firms. I denote the distribution of producers by μ^Y. Lastly, firms that receive the exit shock and are liquidated after production are replaced by entrants. These new firms join the remaining producing firms in making investment decisions. Hence, it is this distribution over investors, μ^I, that determines the distribution of firms in the next period, μ_{t+1}. I describe how these distributions are constructed in the appendix. I include two types of super firms, which have mass π_1^s and π_2^s.

From μ^Y, aggregate labor demand and output can be measured:

$$N_t = \int \ell_t(k,\varepsilon)\mu_t^Y(d[k \times a \times b \times \varepsilon]) + \pi_s^1 \ell(k_s^1, \varepsilon_1^s) + \pi_s^1 \ell(k_s^2, \varepsilon_2^s) \tag{10}$$

$$Y_t = z_t \int \varepsilon F(k, \ell_t(k,\varepsilon))\mu_t^Y(d[k \times a \times b \times \varepsilon]) + \pi_1^s z_t F(k_1^s, \ell_t(k_1^s, \varepsilon_1^s)) + \pi_2^s z_t F(k, \ell_t(k_2^s, \varepsilon_2^s)) \tag{11}$$

Let aggregate capital be

$$K \equiv \int k\mu(d[k \times a \times b \times \varepsilon]) + \pi_1^s k_1^s + \pi_2^s k_2^s \tag{12}$$

Since savings and debt are financial variables, they do not affect the aggregate resource constraint. Physical capital only depreciates when it is used in production. When a firm is restructured, it is only the capital from its successor that gets depreciated. Hence, in the right-hand side of the aggregate resource constraint, total physical capital consists of all capital at the start of the period, less the depreciated fraction of capital that was used in production. The representative household consumes C. Then the aggregate resource constraint is

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t \tag{13}$$

3.4.2 Representative Household

There is a unit mass of infinitely lived, identical households. Each has a unit endowment of time every period, and saves through one-period, risk-free contracts with the financial intermediary, a^h. Households are also initially endowed with ownership of all firms. There are no frictions within the household sector; households are exposed to aggregate TFP risk, but are protected from idiosyncratic firm risk. Households gain periodic utility from consumption and leisure, $u(C, 1-n^h)$. Their subjective discount factor is $\beta \in (0,1)$.

Household wealth comes from four sources. They earn returns on their previous financial savings, a^h. Additionally, their ownership of firms provides them dividends and equity value. Their labor supply is endogenous, yielding income wn. Lastly, T captures the lump-sum transfers to the household of operating expenses and the losses from default. Each period, households choose their labor supply, consumption, risk-free savings, and shares in firms. I denote the household's shares in firm ψ by $g(\psi)$, and the household's share holdings across all firms by $\mathbf{g}$.

The representative household solves

$$\begin{aligned} H_t(\mathbf{g}_t, a_t^h) &= \max_{n_h, C, \mathbf{g}_{t+1}, a_{t+1}^h} u(C, 1-n_h) + M_{t,t+1} H_{t+1}(\mathbf{g}_{t+1}, a_{t+1}^h) \; s.t. \\ C + q_t^f a_{t+1}^h + \int g_{t+1}(\psi)\hat{J}_t(\psi_t)\mu_t^I d[\psi] &\leq w_t n^h + a_t^h + \int g_t(\psi) J_t(\psi)\mu_t d[\psi] + T_t \end{aligned} \tag{14}$$

where $\hat{J}$ is the ex-dividend value of a firm.

3.5 Recursive Competitive Equilibrium

Recursive competitive equilibrium consists of five groups of functions. Firms and households both have value functions: $\{J_t^m(\psi), J_t^n(\psi), J_t^e(\psi), J_t^s(k^s, \varepsilon^s)\}$ and $H_t(\mathbf{g}_t, a_t^h)$. Ordinary firms have decision rules for their employment, dividends, future capital, future savings, and future debt: $\ell_t(k, \varepsilon)$, $d_t^x(\psi)$, $g_t^{x,k}(\psi)$, $g_t^{x,a}(\psi)$, and $g_t^{m,b}(\psi)$. (x is a placeholder for m and n, which reflect whether the firm's debt matures or not.) Super firms have a decision rules for employment and future capital, $\ell_t(k^s, \varepsilon^s)$

and g_t^s. The household has decision rules for consumption, labor supply, financial savings, and firm shares: $C_t(\mathbf{g}_t, a_t^h)$, $n_t^h(\mathbf{g}_t, a_t^h)$, $a_{t+1}(\mathbf{g}_t, a_t^h)$, and $g_{t+1}(\psi; \mathbf{g}_t, a_t^h)$. The price functions are for wages, the risk-free asset rate, the firms' stochastic discount factor, and the debt price schedule: w_t, q_t^f, $M_{t,t+1}$, and $q_t(\psi)$. Lastly, there is a perceived aggregate law of motion: $\mu_{t+1} = \Gamma_t(\mu_t)$, which is observed by all agents.

These functions are such that, for every $\psi = (k, a, b, \varepsilon)$ and every t:

(a) Ordinary firms:

- $J_t^m(\psi)$ solves (4), $J_t^n(\psi)$ solves (5), and $J_t^e(\psi)$ solves (3)
- $\ell_t(k, \varepsilon)$, $d_t^x(\psi)$, $g_t^{x,k}(\psi)$, $g_t^{x,a}(\psi)$, and $g_t^{m,b}(\psi)$, are the associated decision rules ($x = m, n$)

(b) Super firms

- $J_t^s(k^s, \varepsilon^s)$ solves (9), and the associated decision rules are $\ell_t(k^s, \varepsilon^s)$ and g_t^s

(c) Households:

- $H_t(\mathbf{g}_t, a_t^h)$ solves (14)
- $C_t(\mathbf{g}_t, a_t^h)$, $n_t^h(\mathbf{g}, a_h)$, $a_{t+1}^h(\mathbf{g}_t, a_t^h)$, and $g_{t+1}(\psi; \mathbf{g}_t, a_t^h)$ are the associated decision rules

(d) Prices are competitively determined:

$$w_t = \frac{u_n(C_t, 1 - n_t^h)}{u_C(C_t, 1 - n_t^h)}$$

$$M_{t,t+1} = \beta \frac{u_C(C_{t+1}, 1 - n_{t+1}^h)}{u_C(C_t, 1 - n_t^h)}$$

$$q_t^f = M_{t,t+1}$$

$$q_t(\psi) \text{ satisfies (8)}$$

(e) Whenever households start the period with $\mathbf{g} = \mathbf{1}$, the labor market clears and households choose to hold all shares of each active firm. The aggregate resource constraint is also satisfied (13). The market for the risk-free asset, in which households and firms intermediate with the financial sector, is cleared by Walras' law.

4 Characterization of Equilibrium Actions

Before presenting model results, I briefly describe ordinary firms' default decisions and discuss how debt overhang can affect firm decisions when debt is not maturing. To simplify notation, I focus on the model's steady state. All results discussed carry through to the full dynamic model. Since the focus is on the steady state, I use β to represent the discount factor, which holds in equilibrium. Throughout this section, I will refer to ordinary firms simply as firms.

4.1 Default Decisions

Default is possible in any state a firm finds itself: exit, non-maturing debt, and maturing debt. In each of these cases, the default decision can be characterized analytically. As is common in these models, I assume that any firm that is indifferent between defaulting and continuing chooses to continue. Then Proposition 1 of Ottonello and Winberry (2020) holds in this model environment as well: a firm will default if and only if it cannot satisfy the nonnegativity constraint on dividends.

The logic follows that of Ottonello and Winberry (2020). Because default is possible in any state, the value obtained in default puts a lower bound on the firm's value function. In other words, if the maximum value continuing was strictly less than the firm's outside option, the firm would optimally chose to default. Since in this model the firm's value of default is assumed to be 0, 0 is also the lower bound on expected future value for any firm, as the expectations operator is a convex function.[7] Consequently, any firm that has a feasible option to continue will weakly prefer to continue rather than default. A feasible option is one that satisfies the non-negativity constraint on dividends, and therefore one that offers weakly positive flow value. Since future value is also weakly positive, any

[7]More generally, if the outside option was Ω, the lower bound on expected future value would be $\beta\Omega$, to account for the discounting. Normalizing the outside option to 0 makes discounting a nonissue.

feasible inside option will weakly dominate default. From this, it is clear that a firm will default if and only if it cannot satisfy the non-negativity constraint on dividends.

Accordingly, firms in the model default when

- Debt matures: $R(k_t, \varepsilon) + a_t - (1+c)b_t + \max_{k_{t+1}, a_{t+1}, b_{t+1}}\{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\} < 0$
- Debt does not mature: $R(k_t, \varepsilon) + a_t - cb_t < 0$
- Firm exits: $R(k_t, \varepsilon) + a_t - (1+c)b_t < 0$

Hence, in each of these cases, there is an analytic solution for default thresholds. I would especially like to highlight the case when debt matures. Let m denote net assets in this case, $m(k, a, b, \varepsilon) \equiv R(k_t, \varepsilon) + a_t - (1+c)b_t$. Then any firm with maturing debt and net assets below $\underline{m}(\varepsilon)$ will default:

$$\underline{m}(\varepsilon) \max_{k_{t+1}, a_{t+1}, b_{t+1}}\{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\} = 0$$

$$\underline{m}(\varepsilon) = -\max_{k_{t+1}, a_{t+1}, b_{t+1}}\{q(k_{t+1}, a_{t+1}, b_{t+1}, \varepsilon)b_{t+1} - k_{t+1} - q^f a_{t+1}\}$$

Because I have assumed away capital adjustment costs and other real frictions, the only way the right-hand-side varies with the firm's state is through its productivity, ε. Accordingly, default thresholds under debt maturity only vary with the debt price schedule. The thresholds are even more straightforward when debt does not mature of when the firm exits–in these cases, the only endogenous object is the equilibrium wage.

This greatly simplifies the quantitative solution of the model. For ease of discussion, consider a partial steady state equilibrium, in which the wage is held constant. For a given debt price schedule, firms' choices to default or continue are known with certainty, independently of the value function. The state space can then by partitioned into defaulting and non-defaulting regions, allowing debt prices to be updated. Iterating between these thresholds and the debt price schedule can occur independently of solving for firms' decision rules and value. In this model, with long-term debt and default recovery tied to firm value, further iterations are needed to ensure consistency between debt prices and firms' choices and value. However, in models with one-period debt and default recovery tied to asset value (e.g., Ottonello and Winberry, 2020; Khan, Senga, Thomas, 2018), debt prices and default thresholds can be fully determined before knowing firms' value function or their capital and borrowing decision rules.

4.2 Debt Overhang & Endogenous Default Risk

Unlike models with one-period or consol-style debt, where a firm's debt matures each period, in this model debt can go many periods without maturing. As a consequence, firms are not subject to debt-market discipline in every period they are active. This vastly increases the scope for endogenously driven default. In particular, some firms will find it optimal to choose the most extreme form of endogenous driven default: selling off all their assets and issuing the proceeds as dividends. This all but guarantees default in the next period. I refer to this extreme form as strategic default. Ottonello & Winberry (2020) show that in their one-period maturity setting, firms weakly prefer to use available funds to increase capital or reduce debt; no firm would strictly prefer to issue dividends while debt is outstanding. Under lumpy maturity, long-term debt, this result does not hold.

The appeal of strategic default is that it allows the firm to evade paying back its debt. Consider a hypothetical scenario in which a firm's debt is not maturing and it somehow knows with certainty that it will receive the exit shock with period. Its expected future value is

$$\sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} \max\{R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t, 0\}$$

Under strategic default, the firm chooses $k_{t+1} = a_{t+1} = 0$, and issue all funds as dividends, leading to strategic default: without any assets, the firm will not be able to pay back the $(1+c)b_t$ it owes, so it will default in all future states. Then its expected future value is 0, leading to total value of $R(k_t, \varepsilon) + a_t - c_b t$. For this expositional case, strategic default will strictly dominate the optimal continuing strategy whenever

$$R(k_t, \varepsilon) + a_t - cb_t >$$

$$\max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \Big\{ R(k_t, \varepsilon) + a_t - cb_t - k_{t+1} - q_f a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} \max\{R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t, 0\} \Big\}$$

Suppose this firm's optimal choices conditional on continuing ensure that it would be able to pay off its debt with certainty in the next period.[8] Then the inequality simplifies to

$$0 > \max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \Big\{ - k_{t+1} - q_f a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} [R(k_{t+1}, \varepsilon_j) + a_{t+1} - (1+c)b_t] \Big\}$$

$$\beta(1+c)b_t > \max_{k_{t+1}, a_{t+1} | k_{t+1} + a_{t+1} > 0} \Big\{ - k_{t+1} - q_f a_{t+1} + \beta a_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} R(k_{t+1}, \varepsilon_j) \Big\}$$

Lastly, in equilibrium, the discount price on savings equals the household's discount rate. Since $q^f a_{t+1} = \beta a_{t+1}$, the inequality further simplifies to

$$\beta(1+c)b_t > \max_{k_{t+1} | k_{t+1} > 0} \Big\{ - k_{t+1} + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{i,j} R(k_{t+1}, \varepsilon_j) \Big\}$$

This expositional case imposes a high bar on continuing. The expected return on capital must not just exceed the cost of funds, but must also more than compensate for pre-existing debt obligations. As productivity falls and as leverage rises, a firm will be less likely to generate a high enough return to justify continuing, and hence more likely to find strategic default optimal.

In the full model, the tradeoff is less stark, as the realization of exit shocks is never known ex ante. Nonetheless, the fundamental question is unchanged: does continuing generate sufficiently high expected returns to justify having to pay back the face value of debt, if a maturity or exit shock is realized? In the case of maturity, the ability to take on new debt and continue operating in future periods lifts the firm's inside option. Even so, there are portions of the state space in which

[8] In addition to reducing notation, this increases future value relative to a case in which there is default risk, and hence makes this inside option more appealing.

firms choose to strategically default.

To illustrate how debt maturity risk affects firms' endogenous default choices, Figure 1 shows the capital decision rules of firms with the lowest productivity that do not have debt maturing. The left panel shows these decisions in the steady state equilibrium. The right panel considers a one-period deviation from this equilibrium, in which the firms know with certainty that in next period they will not receive either the maturity or exit shock, and that the model returns to the steady state afterwards. In this counterfactual, firms that want to prepay still have this option, but there is no chance that the firm will be forced to pay back its debt.

Focusing on the full model, there are two regions with endogenous strategic default. Towards the right of the panel are firms that choose strategic default: their future capital (and savings) choice is 0, even though the firm has plenty of funds to invest, and has no default risk. Following the logic of the expositional case above, over much of the state space strategic default is not driven by default risk, but by debt overhang. A firm with gross assets of 40 would have no problem paying back a debt with face value 4, for example. But this choice is not optimal, because the expected returns on capital are too low. Strategic default is virtually non-existent across the state space for firms with average productivity.

While the counterfactual scenario does not fully eliminate strategic default, it does reduce its prevalence. Whereas in the full model no low-productivity firm with debt of 3.1 would choose to continue, eliminating one period of maturity and exit risk increases this threshold to 3.5. The reduction in risk is actually relatively small: while in the counterfactual a firm can have non-maturing debt with 100% probability, under the full model this probability is still 93%. In addition to reducing strategic default, the counterfactual also eliminates a region of "gambling". Under the full model, firms with a moderate level of debt (from 1 to 2) and low assets (less than 2) do not invest as much in capital as they could, and instead issue dividends. These small, highly leveraged firms would benefit from continuing if possible. However, they face a high degree of default risk, particularly under exit shocks. As such, they hedge their bets: they invest in capital to try to continue, but also issue some dividends to take advantage of this risk-free source of value. The firm could reduce its default risk by not paying dividends, but since it cannot eliminate this risk because of its high leverage, the firm optimally increases its default risk through dividend payments. As this risk is predominately driven by the exit shock, by eliminating this risk for a period, the counterfactual scenario eliminates this "gambling" portion of the state space.

Endogenous default risk is a poor outcome for the lender, as it both reduces the likelihood of being paid in full and reduces the amount of assets available to recover under default. As such, the debt price schedule adapts to take such behavior into account, deterring firms from being in a position where they will resort to this choice. The long horizon between when debt is issued and when it is expected to mature hinders this; the longer the expected horizon, the more sequences of productivity shocks there are that could lead a firm towards endogenous default risk. Since firms are not subject to financial market discipline until their loan matures, the lender must account for as much of this downside risk as possible up front.[9] In contrast, in a consol-style environment (e.g., Gomes, Jerman, & Schmid, 2016), outstanding debt matures fractionally each period and firms are always able to take on new debt costlessly. Since such firms are continuously subject to financial market discipline, current lenders will benefit from the discipline of future lenders. In effect, consol-stlye debt allows for more state-contingent lending conditions, reducing the lender's need to impose tight conditions up front.

While endogenous default risk is present even in the model's steady state, a severe recession, such as the one generated by COVID-19, exacerbates the debt overhang. In particular, the debt's value is non-contingent, even as lockdown shocks generate negative uncertainty shocks, increasing the hazard of default. With expected future returns weakened, more firms will want to reduce their exposure to debt obligations by increasing their default risk. Since most firms' debt does not mature when the recession starts, only a small fraction will be subject to the financial market discipline that might

[9] As is true of other models with lumpy maturity debt (e.g., Chen, Xu, & Yang, 2021), this model abstracts from loan covenants. This is a common feature of long-term debt contracts that allows the lender to impose discipline on a firm's choices even after maturity. However, Chodrow-Reich and Falato (2020) find that even among large loans (with a commitment of at least $20 million), one in four violated a covenant during 2006 and 2007. It is relatively rare for lenders to take action in response to these violations–even during the 2008-2009 crisis, two out of three covenant violations were waived. If such waivers are anticipated by firms, then they would operate similarly to the firms in my model, which never face such waivers.

deter them from increasing their default risk. However, lenders will optimally price in this increased risk, along with heightened default risk more generally. This generates a more conservative debt price schedule, further reducing firms' expected future value, and hence endogenously amplifying firms' increased desire to reduce their debt overhang by taking on more default risk.

5 Parameterization & Model Fit

5.1 Parameterization

I evaluate the model at a quarterly frequency. Table 1 presents the model parameters. Roughly half of the parameters are externally set, or correspond one-to-one with data moments (e.g., the labor share). The remaining parameters are internally set to jointly match data moments. Data moments are taken from the United States economy.

The discount factor is set to yield a 4% risk-free rate in the steady state, in line with the average after-tax return on capital from 1954 to 2000 (Gomme, Ravikumar, and Rupert, 2011). I assume households have indivisible labor preferences: $u(C, 1-n) = \ln C + B(1-n)$. I set the leisure utility parameter so that households spend about one third of their time working. The production weight on capital pins down the capital-output ratio. Using data from the Bureau of Economics Analysis (BEA), I find that the capital-output ratio averaged 2.28 from 1954 to 2018. Over that period, the investment to capital ratio averaged 0.067, after accounting for growth.[10] This sets the depreciation rate. Additionally, the labor share of production ensures that 60% of income goes to workers (Cooley and Prescott, 1995).

The exogenous exit rate is set to match the average of 8.5% from 1984-2006 (Business Dynamics Statistics, "BDS," Census Bureau). Entrants' endowment is 2.5% of ordinary firms' average capital, and is set to match new firms' share of total employment. This endowment is smaller than in other papers (e.g., Khan and Thomas, 2013), but entrants are able to borrow into their first period of

[10] This measure of investment includes the purchases of consumer durables.

production. This endogenous funding augments the start-up funds provided by the representative household. The persistence of idiosyncratic productivity is set at 0.757 at an annual frequency, following estimates by Syversyon (2008). The variance of productivity shocks is 0.075 annually, to match the standard deviation in investment rates found by Cooper and Haltiwanger (2006). I assume operating expenses take the form $\kappa\varepsilon^{1/(1-\eta)}$, with $\kappa = 0.12$ at an annual rate. Holding other parameters constant, κ helps to govern the mean and variance of firm-level leverage. I take these moments from Dinlersolz et al (2018).

From 1990 to 2017, the recovery rate averaged 42% for bonds and 70% for loans (Moody's, 2018). Flow of Funds data indicate that over that period, nonfinancial corporations had \$1.26 dollars in bonds for every \$1 in loans. Applying this proportion to the Moody's data, I estimate an overall recovery rate of 54%, and choose this as the value of χ. Given this, I set the the proportional default cost to 5.25%, to target the default rate of 1.65% (Moody's, 2018). This default rate of 1.65% is well below the 3% rate commonly used in the literature (e.g., Bernanke, Gertler, & Gilchrist, 1999; Ottonello & Winberry, 2020). Bernanke, Gertler, & Gilchrist (1999) cite a Duns and Bradstreet survey for their measure, which spans 1984-1994 (Fisher, 1996). While I use Moody's default data to be consistent with the recoveries given loss, according to PayNet, across small businesses the annual default rate from 2012-2018 was 1.70%.[11] PayNet is a subsidiary of Equifax, one of the major credit-rating agencies in the United States.

Following Gomes, Jermann, and Scmid (2016), I set the coupon payment so that the bond price for risk-free debt is normalized to 1 in the steady state.[12] Over the dynamic equilibrium, the coupon rate is held constant so that the bond operates similarly to fixed-rate instruments. Firms have a 5% chance of having their debt mature in any given period, implying an average maturity of 5 years. This matches the average over 2002 to 2012 found in Choi, Hackbarth, and Zechner (2018). A 5-year expected duration is also used in other computational models of long-term corporate debt (e.g., Gomes, Jermann, and Schmid, 2016; Bustamante, 2019). The prepayment cost is set at 0.75%

[11] Data taken from https://sbinsights.paynetonline.com/loan-performance/. Last accessed September 1, 2021.

[12] In the steady state, risk-free borrowing has a more favorable price than risk-free saving: $1 > \beta$. The reason for this is that the borrower pays periodic coupons, whereas risk-free savings are a zero-coupon asset. While the prices vary, the expected returns on risk-free debt and risk-free savings equal one another.

of debt's face value, to match the prepayment rate observed over corporate bonds (Xu, 2017).

Lastly, I assume there are two types of super firms. According to the BDS, the largest 0.11% of establishments have over 1,000 employees, and account for 14.4% of total employment. Another 0.18% of establishments have between 500 and 999 employees, and account for another 6.9% of employment. "Type-1" super firms account for establishments with over 1,000 employees, and these 0.11% of firms have a productivity level of 1.846. (The average ordinary firm has a productivity level of 1.) "Type-2" super firms have a productivity level of 1.583, and make up 0.18% of firms. These super firms ensure that the model matches the fact that 21.3% of steady state employment is at firms that are not eligible for PPP funds.

5.2 Steady State Moments

Table 2 summarizes the models' fit to key empirical moments. For firm-level moments, I draw data from two sources. Dinlersoz et al. (2019) construct a novel dataset that spans both private and public firms in the US (LOCUS). Most panel datasets on firms focus on public firms, for which more information is readily available. However, the vast majority of firms active in the US are private, rather than public. The ideal dataset spans both types of firms, as private firms are more representative of individual-level characteristics, while public firms tend to be larger, and hence have outsized influence on aggregates. In Table 3, the leverage rows ($b/(k+a)$) draw on the private measures of total leverage in Dinlersoz et al. (2019). Across public firms, firms have a higher mean leverage ratio, but a similar standard deviation. I choose total leverage, rather than financial leverage, as my target, because the former is the compliment of a firm's equity ratio. Targeting total leverage is isomorphic to targeting a firm's internal funding share. For capital growth, I draw from Cooper & Latiwanger (2006), which examines the Census Bureau's Longitudinal Research Database (LRD). Lastly, for labor share by firm age, I use Ottonello's & Winberry's (2020) analysis of the Census Bureaus' Longitudinal Business Database (LBD).

Overall, the model's real economy fits the data fairly well, with a reasonable fit across the first two moments of annual capital growth. The model's fit to financial moments is a little weaker, but it still does a fairly decent job capturing the mean and standard deviation of leverage. Consistent with the findings of Xu (2018), prepayment is relatively uncommon in the steady state: 2.4% of aggregate debt is prepaid in a year. The model overshoots that somewhat, with 4.7% of debt being prepaid annually. In a given quarter, 1.4% of firms that do not receive the maturity shock choose to prepay their debt. The model is less successful at targeting risk prices: the average recovery rate across defaulted firms is roughly 20 percentage points (ppts) too high, relative to Moody's data. Accordingly, the average risk premium in the steady state is about 1.5 ppts lower than the long-run average in the data.

Table 3 compares quarterly employment growth rates from the model steady state distribution to data on establishments from the Business Employment Dynamics dataset from 2001 to 2006 (Davis et al., 2010). Despite being untargeted moments, the model does a fairly reasonable job of matching the distribution of employment growth observed in the data. The largest misses are across the two high-growth groups: the share of firms with growth over 20% is 6.9 ppts higher in the model than in the data about twice as high in the model as in the data (14.5% versus 7.6%). However, the opposite holds for the share of firms with growth rates between 5% and 20%: just 7.7% of model firms have these growth rates, compared to 16.9% of firms in the data. The model implies a substantial mass of firms with growth rate just above 20%: 6.6% have growth rates between 20% and 21%, for example. While the data may be similarly concentrated, this large mass of firms close to the threshold indicates that the relatively weak fit is not being driven by a large number of firms with extremely high growth rates.

Empirical analyses have found that long-term debt maturity does not have a significant effect on firm investment growth during "normal times", but reduces growth during financial crises (e.g., Almeida et al., 2011). Table 4 explores the effects of debt maturity on annual capital growth over the steady state through a series of regressions. Each regression draws from the same sample of firms randomly selected from the steady state distribution, and controls for size, leverage, and savings as quadratics, and the firm's idiosyncratic productivity draw. Recall that in the model, the maturity outcomes are stark. Debt is paid back in a given period if the firm either draws a maturity shock or if it chooses to prepay. In either case, all debt is paid back. Pooling across these two events, the model replicates the empirical result: over the steady state debt maturity has no statistically significant effect on a firm's capital growth.

However, this headline result masks underlying heterogeneity. When the maturity effect is estimated separately for firms receiving the shock and firms choosing to prepay, maturity has statistically and economically significant effects on capital growth. Firms receiving the maturity shock grow 1 ppt less on average, all else equal, while firms choosing to prepay grow 3.7 ppts more. In the steady state, the negative effect of maturity on growth is not correlated with size or leverage, but is strongly related to the holding of savings. Only one in four firms simultaneously hold positive debt and positive savings. The remaining firms are sufficiently small that, given their current productivity, the marginal expected return on capital exceeds the return on risk-free savings. These firms are relatively more financially constrained, making maturity shocks more onerous, on average. In contrast, among firms with positive savings, the firm with the median asset share grows 0.5 ppt faster if it receives a maturity shock. Such a shock allows the firm to scale up its debt without paying the prepayment cost. Overall, there is a quadratic relationship between the savings rate and the maturity effect, but an increasing in savings only lowers this effect once the savings rate reaches 73% of assets. Fewer than 10% of indebted firms have such a high savings rate. In contrast with the maturity shock, there is a strong size gradient associated with debt prepayment's affects on firm growth. Conditional on choosing to prepay debt, prepayment boosts growth at a median-sized firm by 4.9 ppts. The benefits of debt prepayment in terms of growth is falling in size, in part because small firms tend to use debt prepayment opportunities to increase their debt level, which allows them to scale up more readily.

In this subsection, I have shown that the model is able to match targeted moments fairly well. In addition, it is consistent with both real and financial moments that were untargeted. The model is reasonably successful at capturing the quarterly employment distribution observed in the data. Additionally, it is consistent with the overall insignificant effect of debt payment on capital growth.

However, debt prepayment is associated with higher capital growth rates, while maturity shocks are associated with slower growth. The slow growth due to maturity shocks relates to financial constraints: firms without savings grow more slowly if they get the shock, while firms with savings can expand faster under the maturity shock.

5.3 Lifecycle Dynamics in the Steady State

To capture lifecycle trends across firms, Figures 2 and 3 track a cohort of firms from entry. The maturity structure makes a firm's level of debt highly persistent: in the absence of prepayment, a firm's debt is expected to mature only once every 5 years. Hence, over the 40-quarter horizon examined here, in expectation a firm will only be able to choose new debt twice. Due to this persistence, I take averages conditioning on initial productivity, as an entrant's initial level of productivity influences its first choice of debt.

Over the first few years from entry, firms that initially have the highest productivity have fairly linear capital growth, on average, while the least productive firms capital trajectory is far more flat. Less productive firms start with a much smaller capital base, which curbs their growth. Consequently, even while there is mean reversion in productivity levels, the gap in average capital actually widens slightly over time. The gap between the initially average-productive and the initially least-productive firms grows from 27% in the second quarter of production 30% in the fortieth quarter. Similarly, the gap between the initially most-productive and the initially average-productive firm edges up from 31% to 33%. While the firms initially the most productive tend to reduce their debt over time, across the first 40 quarters the average debt at firms that initially had average or the lowest productivity increases. These debt increases allow the firms to scale up more quickly. However, the less-productive firms are hindered from increasing their debt along both the extensive and intensive margins. On the extensive margin, since long-term debt only matures probabilistically and prepayment is costly, opportunities to take on new debt are relatively rare. Additionally, these firms tend to have high leverage, which means that more available funds need to go towards paying back outstanding debt, rather than providing collateral for new borrowing. On net, these forces imply that the initially least productive still have an average leverage ratio over 0.5 after operating for 40 quarters. In contrast, the firms that initially were the most productive have reached an average leverage of 0.3 by that time.

Related to this, Figure 3 tracks the cumulative probability that a firm will choose to prepay at least once. For example, 26% of firms that initially had average productivity have prepaid their debt at least once in their first 10 quarters. Firms that started with the highest level of productivity are much less likely to prepay. Through 40 quarters, just 13% have prepaid at least once, compared to 61% of firms that started with average productivity, and 68% of firms that initially had the lowest level of productivity. Prepayment is more common among firms that receive positive productivity shocks. Since productivity is mean-reverting, in expectation these firms' borrowing conditions are better now than in the future. Prepayment allows firms to more fully take advantage of positive productivity shocks: the median firm that chooses to prepay increases its debt 26.5%, boosting its leverage ratio by 2.5 ppt. This increase in leverage runs counter to the economy-wide trend of firms reducing their leverage over time. The cost of prepayment deters many firms from making this choice–in particular, if a firm waits, there is a chance its debt might mature costlessly next period. The gains from prepayment will tend to be largest for young firms, which are the most below scale. Accordingly, the median firm choosing to prepay is 16 quarters old, roughly half the economy-wide median age of 30 quarters.

6 Model Dynamics under Recession

In the US, domestic transmission of COVID-19 started around February 2020, and strengthened through March. As such, quarterly measures from January to March smooth over some of the recession's initial impact. For example, real personal consumption expenditures in the US rose 0.4% in January, edged down 0.1% in February, and dropped 6.5% in March. Across the full first quarter of 2020, consumption fell just 1.8%. In my recession simulations, I effectively start in the quarter beginning in March, and examine a downturn that is most severe at its onset. This assumption implies agents know that the exogenous forces imposed on the economy will consistently improve after impact date. Therefore, no divestment or strategic default behavior is driven by expectations of a worsening external aggregate outlook as the recession progresses.

Aside from survey data collected by the Census Bureau on small businesses, I am not aware of any source that tracks the extent to which firms were locked down during the recession. I choose not to rely on the Census data, as it does not distinguish between temporary closures due to government restrictions and voluntary closures (e.g., because of a lack of demand). Ideally, I would be able to isolate between these two reasons, and calibrate solely to mandatory closures.

My simulated recession optimistically assumes a short duration of the pandemic: both sets of shocks last for four quarters. Mapped to real time, this outlook would imply that the exogenous shocks to the US economy ended by March 2021. Moreover, since I use perfect foresight, agents know with certainty that the pandemic will last no longer. Table 5 shows the aggregate shocks. The labor shock was set to match the rise in wages observed since the start of the recession, and falls linearly. The lockdown shocks were chosen so that, absent stimulus, the peak decline in total output slightly exceeds what has been observed in the US. (In the US, output in 2020Q2 was 10% below its level at the end of 2019.) For lockdowns, I assume that initially 1/3 of firms are subject to a lockdown that lasts a month (i.e., one third of the quarter). Since output is proportional to idiosyncratic productivity, this "lockdown" shock can also be viewed as a productivity shock (e.g., a restaurant that can operate throughout the quarter, but at reduced capacity). Over time, these lockdown shocks get milder both in terms of the share of firms affected and the extent to which they reduce productivity.

I examine the model dynamics under three scenarios: no-stimulus, QE (quantitative easing), PPP, and QE and PPP simultaneously provided. In the QE scenario, I impose a floor on lenders' discount rate. Specifically, when pricing new debt contracts, lenders are at least as patient as they are in steady state. This floor fully eliminates the term premium observed during recessions. QE lasts for the duration of the perfect foresight horizon. In the PPP scenario, each ordinary firms' financial savings is increased upon impact date. Following the stimulus provided in 2020, eligible firms receive 2.5 months of their normal wage bill, capped at $2 million. PPP is funded through financial transfers, and therefore only affects aggregate resources through endogenous channels. I assume that the fiscal authority raises debt from the representative household to finance PPP. The fiscal authority subsequently receives revenue through lump-sum taxation, which is returned back to the representative household. Conceptually, this is similar to the approach taken elsewhere in the literature (e.g., Khan and Lee, 2021), and implies that the financing of PPP does not affect aggregate prices.

The next three subsections discuss the dynamics of the recession over the no-stimulus baseline, and when QE and PPP are implemented independently of one another. This allows for the effects of each policy to be more clearly identified. I then turn to the scenario in which both policies are simultaneously implemented.

6.1 Price Dynamics over the Pandemic Recession

To provide context for how agents respond to the recession shocks, I first examine the trends in aggregate prices, illustrated in Figure 4. Due to the assumption of indivisible labor preferences, aggregate prices are direct functions of the representative household's consumption. Consumption follows an inverse-hump shape, falling for the first couple of quarters before rebounding. Consequently, the one-period risk-free rate is below its steady state level, before moving sharply above for the remainder of the recession. Abstracting from price movements, at the start of the recession firms' investment choices are weighed down by expectations over future lockdown shocks. This puts upward pressure on aggregate consumption initially. Price movements and stimulus affect the magnitude of this inverse-hump shape, but do not alter the qualitative trajectory.

The middle panel of Figure 4 charts the long-term risk-free rate, relative to its steady state level. This is the rate the lender would charge a firm that presents no default risk, and would not prepay for the duration of the loan. Comparing the left and middle panels of Figure 4 illustrates two key features of long-term debt contracts with fixed coupon payments. First, because expected payments are back-dated under long-term loans, the model generates a substantial term premium, with the risk-free lending rate peaking about 0.5 ppt higher than the risk-free savings rate in the no-stimulus baseline. This points to credit conditions tightening far more in environments with long-term debt relative to those with one-period debt, in which, absent other frictions, there would be no wedge between the risk-free lending and savings rates. A related observation is that the risk-free lending rate moves above its steady state level immediately upon impact date, even as the risk-free savings rate is lower than its steady state level. Hence, in this long-term debt environment, firms' borrowing conditions immediately tighten, whereas under one-period debt risk-free borrowing at the onset of the recession would actually be cheaper than it is in the steady state.

Under PPP, households are more impatient than in the baseline over the first 20 quarters; they require a higher one-period rate in order to hold savings. Consequently, the risk-free borrowing rate is higher under PPP than in the baseline. Through this channel, PPP generates a further tightening of credit conditions for firms upon impact date and throughout the recession. As the economy's recovery advances, risk-free borrowing does become cheaper under PPP. But these benefits arise well after the rise in risk driven by COVID-19 has dissipated. By construction, under the QE scenario there is no term premium. Borrowing conditions initially ease, tracking the fall in the risk-free savings rate. But after the second period, the floor on lenders' discounting binds, ensuring that the risk-free borrowing rate never exceeds its steady state level.

The right panel of Figure 4 shows the trends in wages. There are two offsetting forces influencing the wage rate. For the first four quarters, the labor disutility shock puts upward pressure on wages. However, with consumption falling below its steady state level, indivisible labor preferences generates a negative effect on the wage. The labor disutility shocks are calibrated to ensure that these dominate the endogenous effect, matching the positive trend in wages observed through 2020. Once these exogenous shocks are removed, though, wages fall about 1% below their steady state level before starting to recover. There is little difference in the wage trends without stimulus and under QE, whereas the wage recovers more quickly under PPP. This is a consequence of the faster consumption recovery supported by PPP.

To illustrate the consequences of these general equilibrium effects, Figure 5 plots the response of super firms' capital across the three recession scenarios. The differences in super firms' capital trajectories are solely driven by the differences in the risk-free savings rate and wage. (In equilibrium, firms' discount factor is the reciprocal of the risk-free rate.) In the no-stimulus baseline, super firms' capital troughs 3.6% below its steady state level, four quarters after impact date. The half life of this peak decline is 11 quarters. Households and firms are slightly less patient under QE than in the baseline, amplifying the fall in super firms' capital. Under QE, the peak decline is 4.0% below steady state, and also occurs 4 quarters after impact date. Since households and firms are even more impatient under PPP than in either the QE or baseline scenarios, the decline in super firms' capital is larger still. Under PPP, super firms' capital falls 8.1% below its steady state level, but the half-life is 2 quarters shorter. This faster recovery in super firms' capital is driven by a faster recovery in effective discount factor under PPP, as illustrated by the left panel of Figure 4. Because super firms only response to aggregate prices, the trends in their capital response are identical to those of financially unconstrained ordinary firms. Consequently, Figure 5 also shows how general equilibrium effects influence ordinary firms' target capital choices across the three scenarios.

6.2 Ordinary Firms' Response to the Pandemic Recession

Having discussed super firms' trends over the recession, in Figure 6 I shift the focus to ordinary firms, which account for over 99.5% of firms in the economy. Over the no-stimulus baseline, ordinary firms' aggregate capital troughs 2.2% below its steady state level, and requires 12 quarters to make up half the maximum loss. Ordinary firms' capital falls less than super firms' because the former are quite small relative to their unconstrained optimal size. In the steady state, the typical ordinary firm's capital stock is about two-thirds what it would chose absent financial constraints. As illustrated in Figure 5, the unconstrained capital stock falls 3.6% under the baseline, much smaller than the one-third decline that would be needed to make the typical ordinary firm want to reduce its capital stock.

Absent stimulus, the decline in ordinary firms' capital coincides with a steady rise in aggregate debt, peaking 2.3% above its steady state level. This additional debt predominately goes towards increased holdings of financial savings, which rise 2.6% over the first year of the recession and peak 4.2% above their steady state level. Firms that had no debt on impact date account for about half of the rise in debt. In part, this reflects firms that were unconstrained in the steady state, but become financially constrained due to lockdown shocks. Additionally, firms engage in precautionary savings, hedging against the risk that future lockdown shocks may make them financially constrained. Since the term premium rises over the first year of the recession (Figure 4), firms may prefer to borrow upon impact date to insure against future risk, rather than wait and borrow only upon the realization of a negative shock. Another group of firms responsible for the rising debt is firms that choose to prepay. Over the first year of the recession, 4.3% of firms prepay in the baseline, down from 5.8% in the steady state. This countercyclicality in prepayments is consistent with the empirical findings of Xu (2018). As a share of gross assets, prepaying firms increased their debt by 14.1% on average, twice the 6.8% increase across firms that switch from being debt-free to borrowing.

In the absence of stimulus, the default rate increases modestly from steady state. Across the first year of the recession, 2.2% of firms default, 0.6 percentage point above the steady state level. After the exogenous shocks have dissipated, the default rate has largely recovered to the steady state; across the second year of the recession, the default rate is 1.7%. QE is moderately successful in reducing defaults, lowering the rate by 0.16 ppt, or a quarter of excess defaults. By improving borrowing conditions, QE reduces the downside risks of maturity shocks. But since only a small fraction of firms' debt matures in any given period, the reduction in defaults under QE is not due solely to the firms receiving a maturity shock. Instead, there is a fall in the number of firms that strategically default, relative to the no-stimulus scenario. In other words, QE has strong effects on firms' inside option of trying to maximize continuation value, whereas it has a smaller, indirect effect on firms' outside option of issuing dividends and increasing default risk. In contrast to QE, PPP actually increases the default rate over the first year of the recession, pushing it 0.12 ppt higher than when there is no stimulus. PPP funds initially reduce defaults on impact date, but these defaults are simply delayed, rather than prevented. Pricing effects, such as the tighter debt price schedule, push against the benefits of the stimulus and undermine PPP's ability to reduce defaults. Some of the additional defaults under PPP during the recession's first year may be pushed forward in time; starting in the second year, the default rate under PPP is consistently lower than the default rate absent stimulus.

PPP is highly effective at shoring up ordinary firms' aggregate capital. Despite the contractionary conditions, the stimulus boosts ordinary firms' investment upon impact date, leading to a 0.6% rise in their aggregate capital. Their aggregate capital subsequently falls to 0.4% below steady state, as expectations of future lockdown shocks and general equilibrium effects weigh down target capital levels. This decline is short-lived; ordinary firms' aggregate capital overshoots its steady state level 8 quarters after impact date. PPP also generates a sustained increase in financial savings. In level terms, the increase in financial savings exceeds the rise in ordinary firms' aggregate capital, by a factor of 4.4 heading into the quarter after impact date. The rise in ordinary firms' capital and savings arises largely through the stimulus funds directly; aggregate debt is relatively flat in the PPP scenario. In part, this reflects PPP's ability to reduce financing constraints through increased assets. However, borrowing will also be constrained by the increased term premium generated by PPP. On impact date, the prepayment rate falls to 0.7%, less than half the steady state level.

In contrast with the PPP scenario, there is a substantial rise in aggregate debt under QE. 3.8% of firms choose to prepay upon impact date, as risk-free debt becomes less costly than in the steady state. Additionally, as in the no-stimulus case, many debt-free firms choose to borrow, taking advantage of the favorable debt prices and increased precautionary savings motivations. Consequently, there is a substantial rise in aggregate financial savings, which peak 11.6% above their steady state level. As a large share of the increase in debt comes from firms that are not immediately financially constrained, the effects of QE on ordinary firms' aggregate capital are more muted. Under QE, the trough of ordinary firms' aggregate capital is 2.1% below its steady state level, shaving off just 0.14 ppt of the decline observed under the no-stimulus baseline. However, QE does generate a faster recovery–aggregate capital across ordinary firms makes up half its peak losses two quarters sooner under QE than in the no-stimulus scenario.

6.3 Aggregate Flows over the Pandemic Recession

Figure 7 tracks the responses in aggregate output, consumption, investment, and labor across the first 20 quarters of the recession. Overall, the effects of QE and PPP tend to be relatively small. For example, over the first year of the recession, ordinary firms' output was 6.8% below its steady state level in the no-stimulus baseline. QE shaves off just 0.1 percentage point of this decline. PPP has largely effects on aggregate output, reducing the decline by 0.7 percentage point. By the second year of the recession, output in the baseline has returned to its steady state level. With QE, output over the second year is 0.1% higher than its steady state level, and under PPP its 0.7% higher. The boost from PPP relative to the no-stimulus baseline start to diminish starting in the third year after impact date. In contrast, the gains from QE increase for seven years, but still remain modest. In the seventh year after impact date, QE boosts output by 0.14% relative to the baseline, compared to a 0.41% gain under PPP.

The trends across total investment and labor qualitatively match those of aggregate output. In both cases, QE provides a slight boost relative to the no-stimulus baseline, while PPP provides more support. Given that PPP was intended to support small business payrolls, I'll note that the gains in labor generated by PPP are larger than those observed over output. Over the first year of the recession, hours worked under PPP fall 6.3%, whereas in the baseline hours worked fell 0.8 ppt more. In contrast, QE reduces the decline in employment by about 0.1 ppt over the first recession year. PPP and QE again boost employment by an additional 0.1 and 0.7 percentage points over the second year of the recession, relative to the baseline. Hours worked in the baseline are 1.0% above their steady state level in the second year. Under this scenario, PPP offset 23% of the decline in hours worked at eligible firms over the first year of the recession, bringing the fall up from 7.0% to 5.4%. However, since these ordinary firms are smaller, the savings in terms of overall hours worked is 11%.

The model generates declines in investment far larger than those found in the data. Conversely, the falls in consumption are far milder in the model than in the data. Expanding the set of shocks could potentially help bring the model closer to the data. For example, I could introduce a fall in households' utility from consumption (illustrating, for example, reduced benefits of going to restaurants during the pandemic). Holding aggregate resources constant, the reduction in consumption caused by that demand shock would necessitate an increase in aggregate investment. Despite the lack of a close quantitative match to the data, I believe the model still provides useful insights into the relative effects of QE and PPP. QE has very negligible effects on aggregate consumption; the slightly stronger output growth under QE is allocated as capital. This helps propel the longer-term recovery, but does not provide much relief to the household during the crisis. In contrast, PPP generates a steeper trend in aggregate consumption, falling more upon impact date but rising more quickly after the first year of the recession. This is because PPP more successfully boosts aggregate capital than QE: investment falls less during the first year of the recession, allowing firms to subsequently produce more.

6.4 Simultaneous Implementation of the Paycheck Protection Program & Quantitative Easing

In practice, PPP and QE were implemented simultaneously during the COVID-19 recession. Table 6 summarizes how the economy responds to pandemic when both policies are deployed, relative to the no-stimulus baseline. For comparison, I also run the same comparisons for the PPP- and QE-only scenarios. For discussion purposes, I refer to the scenario in which PPP and QE are implemented simultaneously as the full-policy scenario.

The full-policy scenario only modestly boosts aggregate flows, relative to the PPP-only scenario. This is consistent with the modest gains generated by the QE-only scenario. The benefits of the full-policy scenario are roughly equal to the sum of the PPP- and QE-only scenarios. For example, over the first year of the recession, under the full-policy scenario output is 0.81% higher, relative to the no-stimulus baseline. This is 0.06 ppt more than when PPP is implemented independently, effectively the same as the 0.05% boost generated by QE on its own. That the combined force of the full-policy scenario is roughly the same as the sum of its parts indicates that the two policies do not substantially amplify or diminish one another's real economic effects.

In contrast, the simultaneous implementation of PPP and QE does alter trends over financial outcomes. In particular, while on its own PPP increased the default rate over the first year of the recession, when PPP is implemented alongside QE the default rate is lower than in the no-stimulus baseline. Moreover, the declines in defaults under the full-policy scenario are sustained, indicating that firms' defaults are being prevented, not simply delayed. This highlights that the rising term premium indirectly generated by PPP, and the tightening it imposes across the debt price schedule, is responsible for the higher initial default rate under the PPP-only scenario. When QE reverses this term premium, the PPP funds are better able to prevent defaults. Lastly, the full-stimulus scenario leads to much more substantive increases in financial savings, boosting it by 9.5% relative to the no-stimulus baseline. This rise is much larger than the combined increase in savings across the PPP- and QE-only scenarios. In part, this is driven by a larger increase in debt under the PPP-only scenario. This is another instance in which the higher term premium of the PPP-only scenario is evident. On its own, PPP generates a smaller rise in debt than the no-stimulus baseline, but when it is implemented alongside QE aggregate debt rises more.

The full-policy scenario underscores the benefits of simultaneously implementing PPP and QE. PPP helps minimize the economic downturn and speed up the recovery. QE is more effective at targeting the financial economy, especially at reducing the default rate. PPP and QE target different channels: PPP funds reduce liquidity constraints, while QE eases borrowing constraints. Consequently, the two policies' effects do not substantially interact with one another in the aggregate. This allows the economy to enjoy the best of both worlds: a stronger aggregate recovery and fewer defaults.

7 Channels of Policy Transmission

7.1 Paycheck Protection Program & General Equilibrium Effects

Section 6 highlighted various general equilibrium effects, and how these vary across the three recession scenarios. To quantify the net impact of these effects, I consider a counterfactual scenario in which prices follow their trajectories in the no-stimulus baseline, but firms receive PPP funds on impact date. This allows the direct effects of PPP to be disentangled from the indirect effects the programs generates through equilibrium forces. Since QE operates through prices, that type of stimulus does not lend itself as well to this sort of counterfactual exercise.

Figure 6 compares the aggregate performance of ordinary firms under this counterfactual exercise to when PPP is given in general equilibrium and when no stimulus is given. Under the partial equilibrium counterfactual, ordinary firms increase their capital by 2.9% in the aggregate, 2.2 percentage points more than what PPP generates under general equilibrium effects. In contrast, the rise in financial savings in the counterfactual is initially smaller than in the full solution with PPP. This illustrates the degree to which aggregate price movements lower target capital levels, encouraging firms to allocate funds into financial savings rather than physical capital.

The counterfactual without general equilibrium effects substantially reverses defaults. This In the counterfactual in which prices do not respond to PPP, there are fewer defaults. The default rate over the first year of the recession is 1.8%, compared to 2.3% under PPP in general equilibrium and 2.2% in the no-stimulus baseline. By the second year of the recession, the default rate has fallen below its steady state level in the counterfactual. The counterfactual reduces the default rate more than when PPP is implemented alongside QE in general equilibrium (Table 6). This is because, as Figure 2 showed, PPP pushes up the risk-free rate under general equilibrium. Equivalently, households and firms are more impatient under PPP than under the baseline scenario. Without this increase in patience, there is less incentive for firms to gamble and allocate funds towards present dividends, instead of risky future returns.

Table 7 summarizes how aggregate flows in the counterfactual compare to when no stimulus is given and when PPP is given in general equilibrium. Absent general equilibrium effects, PPP would reduce the loss in output by an additional 1.1 ppt, over and above the 0.7 ppt saved by PPP in general equilibrium. This suggests that, in terms of output, general equilibrium forces absorb 64% of the benefits of the stimulus. A similar result holds for employment. The results for aggregate investment are even more striking. Under general equilibrium, PPP shores up investment over the first recession year by 5.2 ppt, relative to the no-stimulus scenario, but annual investment is still down 33.7% from the steady state. In contrast, in the absence of general equilibrium effects, PPP would actually drive investment 4.6% above its steady level, despite the severe recession. Aggregate flows are boosted in the counterfactual scenario both by ordinary firms, which enjoy an increase in capital over all (Figure 7), and by super firms, which also have higher discount factors under baseline prices and therefore retain more capital (Figure 5).

Over all, the model points to PPP having very strong general equilibrium effects. These effects, particularly the downward pressure on firms' discounting and the larger term premium, lower capital investment, leading to more funds being allocated towards financial savings instead. Additionally, these effects reduce PPP's ability to prevent firm defaults. Lastly, in the absence of general equilibrium effects, PPP would boost employment and output over the first year of the recession by another 1 percentage point of the steady state level.

7.2 Maturity Frictions & Stimulus Programs

Since Myers (1977), issues related to debt overhang have been a focus of the economics literature. Maturing debt allows firms to recommit to their obligations to lenders, reducing time-inconsistency issues. When debt and investment choices are made jointly, endogenously priced debt disciplines firm decisions, improving long-term prospects. Additionally, more productive firms can benefit from maturity by being able to borrow on more favorable terms, allowing them to expand more. Since productivity is persistent, on average these firms enjoy long-run benefits thanks to their increased size.

At the same time, firms with maturing debt must divert resources away from productive uses in order to repay their obligations. During a downturn, this "cash-on-hand" channel can be especially severe, as firm revenue is unexpectedly low. Additionally, firms that need to take on more borrowing face a worsening "debt-pricing" channel, as lenders tighten borrowing conditions in response to rising default risk. As highlighted previously, the debt-pricing channel tightens even more in settings with long-term debt, as lenders charge a term premium. In particular, while some firms may not experience rising default risk in a recession, the term premium affects all firms.

While the literature has identified a negative relationship between debt maturity and capital investment during recessions (e.g., Almeida et al, 2011), it is hard to decompose the source of this negative relationship through empirical analyses. In this quantitative model, however, since maturity shocks are exogenous, identification is far more straight forward. Moreover, the two policies examined, QE and PPP, cleanly target one channel more than the other. For example, PPP increases firms' cash-on-hand, reducing the negative effects of debt payment. In contrast, QE does not directly affect cash-on-hand, but by eliminating the term premium, it eases the debt-pricing channel. Is either policy effective at easing the constraints on investment driven by the maturity of long-term debt?

Table 8 addresses this question by examining a panel of 1,000,000 firms randomly drawn from the equilibrium distribution of firms. I simulate this panel over the first year of the recession across each of the four main scenarios: no-stimulus, PPP-only, QE-only, and PPP and QE simultaneously implemented ("full policy"). By using the same random sample of firms across each setting, I ensure that the only differences in the simulations are driven by the differences in economic conditions endogenously generate by the model equilibria. For comparison purposes, the first data column of Table 8 replicates the specification from Table 4.

Absent stimulus, the COVID-19 recession increases the maturity friction by about 0.5 ppt, and dampens the gains from prepayment by 0.2 ppt. This is qualitatively consistent with the empirical analysis of the Global Financial Crisis (e.g., Almeida et al, 2011). While the recession scenario does not have a financial crisis, such as the one experienced in 2008, between the term premium and rising credit risk, there is still a substantial degree of tightening in financing conditions.

Stimulus measures generate milder maturity frictions during the recession. PPP lowers the maturity-shock effect by 0.2 ppt lower, relative to the point estimate from the no-stimulus baseline. QE is more effective, lowering the point estimate by 0.4 ppt, just 0.1 ppt above the point estimate under the steady state. This indicates that, during the recession, the larger maturity friction is largely driven through the debt-pricing channel. The full-stimulus scenario further corroborates this: implementing stimulus funds in addition to QE has a negligible effect of the debt-maturity shock.

There are also substantial movements in the prepayment effect on annual investment. Under PPP, the prepayment choice is associated with another 1.3 ppt more growth than in the no-stimulus scenario. In contrast, the prepayment effect is 1.5 ppt lower when QE is implemented on its own. Selection plays a role here. Since the term premium is increased by the PPP-only scenario, only the firms that benefit the most from prepayment will make this choice then. In contrast, by eliminating the term premium, QE encourages more firms to prepay. The firms that are willing to pay the prepayment cost in either scenario tend to be those with the most to gain.

In sum, the COVID-19 recession increases the negative effects of debt maturity shocks on annual investment by 0.5 ppt, relative to the steady state. The debt maturity friction is eased by both PPP and by QE, though QE generates twice as large a reduction. Hence, in this environment tightening credit conditions is the principle channel through which the recession increases debt maturity's reduction on firm-level investment.

7.3 Stimulus & Firm-Level Investment

Using the same random sample of firms from Table 8, I now explore the effects of the stimulus on the firm level. Since I run each of the 1,000,000 randomly chosen firms through each of the recession scenarios, I can directly attribute differences across the scenarios to the overall effects of the stimulus. In Tables 9 and 10, I focus specifically on four-quarter capital growth over the first year of the recession. If a firm had an investment rate of 7.6% absent stimulus and an investment rate of 8.9% under PPP, then the overall effects of PPP pushed this firm's annual investment up 1.3 percentage point over the first year of the recession. I used the no-stimulus scenario as a baseline to assess how PPP and QE differentially affected firm-level investment. This provides insights on how effective the stimulus measures are at relaxing various financial constraints.

Table 9 presents some simple summary statistics on how the stimulus measures affect investment. In this table, I exclude firms without debt to focus solely on those currently facing financial frictions. Changes in investment are winsorized at the 1% level to reduce the effects of outliers on the regressions. On average, PPP has a larger effect on investment than QE, pushing the mean rate up 4.3 ppt, against a 0.4 ppt gain under quantitative easing. However, PPP generates extreme movements on both sides: the standard deviations of changes to firm investment is 5.8 ppts, more than twice the 2.1 ppt standard deviation generated by quantitative easing. The full-stimulus scenario increases annual investment at about 70% of firms, relative to the PPP-only scenario. The large reductions in growth at the bottom 10 percent of firms is mainly driven by general equilibrium effects on firms that are not presently financially constrained. As illustrated in Figure 5, the stimulus measures further reduce the investment levels of unconstrained firms.

Table 10 examines how the stimulus' effects varied with financial savings, size, leverage, and exogenous shocks driving debt maturity and lockdowns. I include both a dummy for positive savings as well as savings' share of assets since just a quarter of firms with positive debt also have positive savings. Both stimulus programs disproportionately affect firms with less savings as a share of assets. Similarly, all else equal, both forms of stimulus increase the investment of smaller firms more than larger firms. The size effect is weighed down further under PPP when a debt maturity shock is realized, whereas under QE and the full-policy scenario the debt maturity shock reduces the size effect. Financial savings and size are both standard proxies for reduced financial frictions. While the model abstracts from liquidity risk, firms will only hold savings once their capital stock is so large that the marginal return from the risk-free asset dominates the expected value of a marginal rise in capital. Similarly, a firm with more assets will more easily be able to obtain the unconstrained optimal level of capital.

PPP and QE both increase investment more at firms with higher leverage. For a firm with median leverage, these gains are equivalent to about a quarter of a standard deviation under PPP and the full-policy scenario, while QE on its own generates a gain of about 5% of a standard deviation. The leverage effect is strengthened by maturity shocks under QE and the full policy scenarios. For firms receiving the maturity shock in these settings, at the median leverage ratio investment is 60% and 35% higher with stimulus than in the no-stimulus baseline. In contrast, debt maturity does not significantly alter the leverage effect when PPP is implemented alone. The PPP-only scenario does strengthen investment gains across firms choosing to prepay, whereas the QE- and full-policy scenarios generate negative effects through this channel. This reflects the same selection forces noted in Table 8: only the firms that benefit the most from prepayment will choose to do so when PPP is implemented on its own, due to the tighter debt price schedule.

8 Conclusion

For firms in the US, COVID-19 is the rare example of an aggregate shock that could plausibly be out of the state space. In 2019, it would be hard to imagine that sectors and localities would be subject to rolling lockdowns or to guidelines that would prevent them from fully operating for weeks or months on end. It would be harder still for a firm to justify holding the level of savings necessary to survive such a crisis without support. In the US, as was true elsewhere, governments developed an array of novel programs to mitigate the pandemic recession's effects on firms.

I find that the Paycheck Protection Program can reduce the severity of a contraction, dampening the fall in capital and boosting output. However, the Program is less able to reduce defaults, in part because general equilibrium effects reverse the benefits of the funding. By reducing the term premium, quantitative easing does prevent many of the excess defaults caused by the recession. Moreover, quantitative easing relaxes the investment frictions associated with debt maturity. This relaxation lowers firms' debt overhang, by making payment more manageable. But since only a small fraction of firms have long-term debt maturing in any given period, the short-run effects of quantitative easing on aggregate flows are quite modest. Simultaneously implementing the Paycheck Protection Program and quantitative easing allows the economy to enjoy the best of both worlds. The aggregate economy recovers more quickly, and more defaults are prevented.

The underlying environment examined here captures several salient aspects of firms' financing and investment behavior in the United States. A natural way to extend on this would be to incorporate a financial sector with its own frictions. Such frictions would allow the model to also capture the feedback effects of increased default risk on aggregate lending conditions (e.g., Elenev, Landvoight, and Van Nieurwerburgh, 2020). Since it abstracts from these effects, the current model offers a lower bound on the benefits of stimulus. Including financial-sector frictions would also allow for quantitative easing to be implemented in a way more similar to what has occurred in practice, with the central bank purchasing assets from financial institutions. Studies on the effects of quantitative easing in the aftermath of the Global Financial Crisis highlight the importance of the banking channel (e.g., Chodorow-Reich, 2014). The current paper isolates the firm debt-demand channel. Accounting for this alongside the banking channel would allow for a more complete accounting of quantitative easing's effects on the economy.

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Table 1: Parameters (Annualized)

	Parameter	Symbol	Value
Externally Set	Discount factor	β	0.96
	Labor share of output	η	0.6
	Capital depreciation rate	δ	0.067
	Exit rate	ρ	0.085
	Coupon payment	c	$1/\beta - 1$
	Average debt maturity	λ	1/5
	Persistence of idiosyncratic shocks	ρ_ε	0.757
	Super Firm Share, Type 1	–	0.11%
	Super Firm Share, Type 2	–	0.18%
Internally Set	Leisure utility	B	2.1
	Capital share of output	α	0.275
	Variance of idiosyncratic shocks	σ_ε^2	0.075
	Operating expense	κ	0.12
	Entrant endowment rate	ι	0.025
	Capital recovery	χ	0.54
	Default cost	ν	0.0525
	Debt prepayment cost	γ	0.75%
	Super Firm Productivity, Type 1	–	1.826
	Super Firm Productivity, Type 2	–	1.566

Table 2: Steady State Moments (Annualized)

Moment	Data	Model
Capital/Output	2.28	2.17
Investment/Capital	0.069	0.069
Share of Time Worked	1/3	0.336
Default rate (%)	1.65	1.61
Share of Debt Prepaid (%)	2.3	4.7
Recovery rate (%)	54.4	76.6
Mean risk premium (%)	2.65	0.96
Mean Annual Capital Growth	0.122	0.118
St. Dev. Annual Capital Growth	0.337	0.363
Mean Debt/Assets	0.46	0.52
St. Dev. Debt/Assets	0.38	0.31
Employment Share, Firms < 1 year	0.03	0.03
Emp. Share, Firms $\in (1, 10)$ years	0.21	0.28

Firm-level model moments winsorized at 1% level

Capital data: Cooper & Haltiwanger (2006): LRD

Leverage data: Dinlersoz et al (2019): LOCUS private firms

Employment data: Ottonello & Winberry (2020): LBD

Table 3: Firm Distribution (%)
over Quarterly Employment Growth

	Data	Model
(-2, 0.2]	7.6	5.4
(-0.2,-0.05]	16.7	17.0
(-0.05, 0.05]	51.2	55.5
(0.05, 0.2]	16.9	7.7
(0.2, 2)	7.6	14.4

Growth calculated as $2 * (n_{t+1} - n_t)/(n_{t+1} + n_t)$

Data source: Davis et al. (2010)

Table 4: Debt Maturity & Annual Capital Growth (%) in the Steady State

			Metric:		
	Pooled	Base	Size: $\ln(k+a)$	Leverage	Savings: $a/(k+a)$
Debt Paid Back	-0.088	–	–	–	–
Maturity Shock	–	-1.002***	-1.422	-0.976	-1.225***
Metric x Maturity Shock	–	–	0.892	1.881	10.233***
Metric^2 x Maturity Shock	–	–	-0.317	-2.746	-14.021***
Firm Prepays	–	3.705***	19.469***	-4.642*	3.861***
Metric x Prepayment	–	–	-15.445***	-0.804	-122.813***
Metric^2 x Prepayment	–	–	1.899***	16.418***	123.025***
N. Obs.	445,579	445,579	445,579	445,579	445,579
R^2	0.265	0.265	0.266	0.266	0.265

Base group: firms with non-maturing debt. Firms with 0 debt excluded.

Continuous variables winsorized at 1% level.

Additional controls: gross assets, leverage, savings, productivity

$***: p < 0.01,\ **: p < 0.05,\ *: p < 0.1$

Table 5: Exogenous Productivity Shocks

Quarter	Labor Disutility Shock (%)	Lockdown Share	Lockdown Duration
Q1	2.5	1/3	1/3
Q2	1.875	1/5	1/3
Q3	1.25	1/10	1/6
Q4	0.625	1/10	1/6
Q5+	0	0	–

Table 6: Model Dynamics under Stimulus, relative to No-Stimulus Baseline (%)

Economy-Wide	Output PPP	Output QE	Output Both	Consumption PPP	Consumption QE	Consumption Both	Investment PPP	Investment QE	Investment Both	Employment PPP	Employment QE	Employment Both
Year 1	0.75	0.05	0.81	-0.12	-0.01	-0.13	8.44	0.21	8.77	0.87	0.07	0.94
Year 2	0.77	0.09	0.87	0.04	0.00	0.04	4.39	0.60	4.85	0.73	0.08	0.83
Year 3	0.67	0.10	0.77	0.14	0.01	0.15	3.14	0.67	3.56	0.54	0.08	0.62
Ordinary Firms	**Capital PPP**	**Capital QE**	**Capital Both**	**Default (ppt) PPP**	**Default (ppt) QE**	**Default (ppt) Both**	**Savings PPP**	**Savings QE**	**Savings Both**	**Debt PPP**	**Debt QE**	**Debt Both**
Year 1	1.89	0.12	2.01	0.12	-0.16	-0.12	3.85	3.83	9.47	-1.15	5.12	6.24
Year 2	1.92	0.20	2.14	-0.16	-0.12	-0.27	2.99	5.79	9.49	-1.43	7.71	7.12
Year 3	1.91	0.25	2.16	-0.25	-0.08	-0.32	3.28	5.84	9.56	-1.35	7.73	7.19

Aggregate capital, savings, and debt across ordinary firms measured at end of year.

Table 7: Aggregate Flows across First Recession Year
% Deviations from Steady State

	No Stimulus	PPP GE	PPP PE
Output	-6.8	-6.1	-5.0
Employment	-7.1	-6.3	-5.3
Investment	-38.9	-33.7	4.6

PPP GE: PPP stimulus given in general equilibrium.

PPP PE: Stimulus given; prices follow no-stimulus scenario.

Table 8: Debt Maturity's Effects on Four-Quarter Capital Growth, %

	Steady State	COVID-19 Recession: No Stimulus	COVID-19 Recession: PPP	COVID-19 Recession: QE	COVID-19 Recession: PPP & QE
Debt Matures	-1.002^{***}	-1.498^{***}	-1.317^{***}	-1.094^{***}	-1.088^{***}
Firm Prepays	3.705^{***}	3.492^{***}	4.809^{***}	1.970^{***}	2.476^{***}

Base group: firms with non-maturing debt. Firms with 0 debt excluded.

Additional controls: gross assets, leverage, savings, productivity

$***: p < 0.01,\ **: p < 0.05,\ *: p < 0.1$

Table 9: Summary Statistics
on Four-Quarter Capital Growth
relative to No-Stimulus Scenario (ppt)

	PPP	QE	PPP & QE
10th Percentile	-4.3	-0.4	-4.6
25th Percentile	-2.1	-0.1	-1.5
Median	6.3	0.0	6.6
75th Percentile	8.1	0.1	8.5
90th Percentile	9.9	0.6	10.7
Mean	4.3	0.4	4.7
Standard Deviation	5.8	2.1	6.2

Winsorized at 1% level. Firms with 0 debt excluded.
Firms examined at the recession impact date.

Table 10: Four-Quarter Capital Growth
relative to No-Stimulus Scenario (ppt)

	PPP	QE	PPP & QE
Dummy for Positive Savings	-0.734^{***}	-0.058***	-0.785^{***}
$a/(k+a)$	-1.640^{***}	0.092^{***}	-0.569^{***}
Size: $\ln(k+a)$	-3.343^{***}	-0.397^{***}	-4.365^{***}
Debt Maturity x Size	-0.206^{**}	0.428^{***}	0.286^{***}
Debt Prepayment x Size	0.094	-3.485^{***}	-3.905^{***}
Lockdown x Size	0.013	0.056^{***}	0.068^{*}
Leverage	2.951^{***}	0.240^{***}	2.845^{***}
Debt Maturity x Leverage	0.136	2.260^{***}	1.727^{***}
Debt Prepayment x Leverage	1.752^{***}	-4.162^{***}	-10.752^{***}
Lockdown x Leverage	-0.033	0.129^{**}	0.128
Debt Maturity	0.446^{*}	-1.544^{***}	-1.041^{***}
Debt Prepayment	-1.563^{**}	10.782^{***}	14.078^{***}
Lockdown	0.177	-0.152**	0.020

a: savings, k: physical capital. Base group: firms with non-maturing debt.
Firms with 0 debt excluded. Additional controls: productivity.
Growth & leverage winsorized at 1% level.
$***:\ p<0.01,\ **:\ p<0.05,\ *: p<0.1$

Figure 1: Capital Decision Rules for Firms with the Lowest Productivity and Non-Maturing Debt

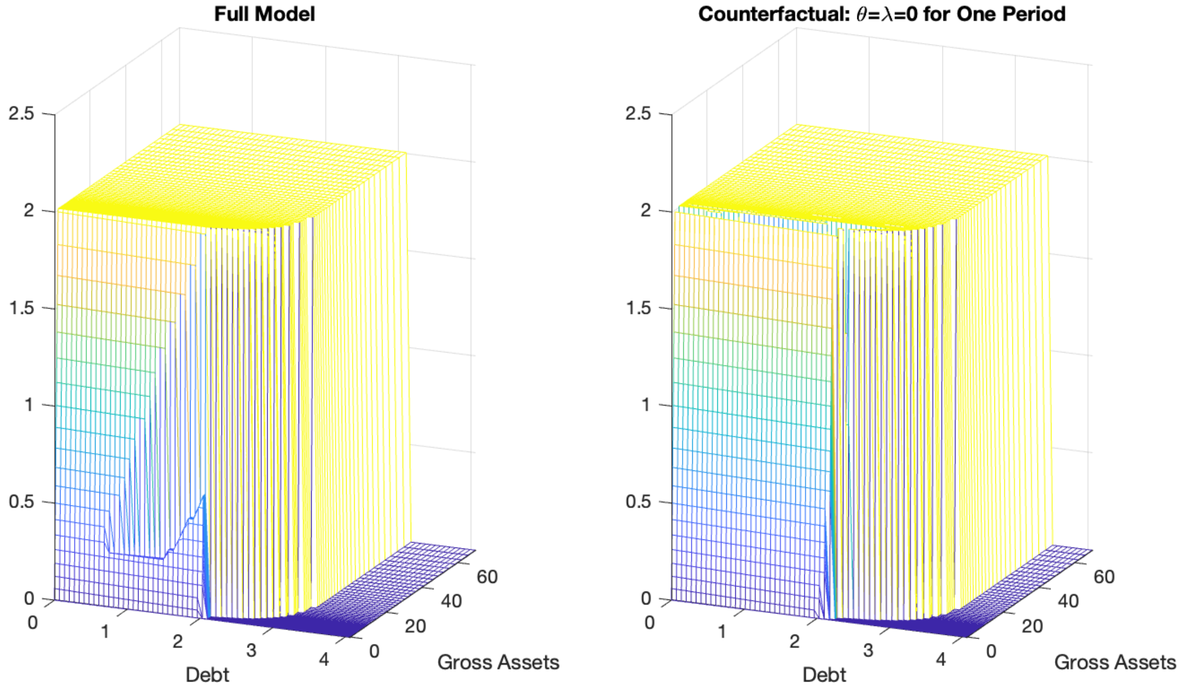
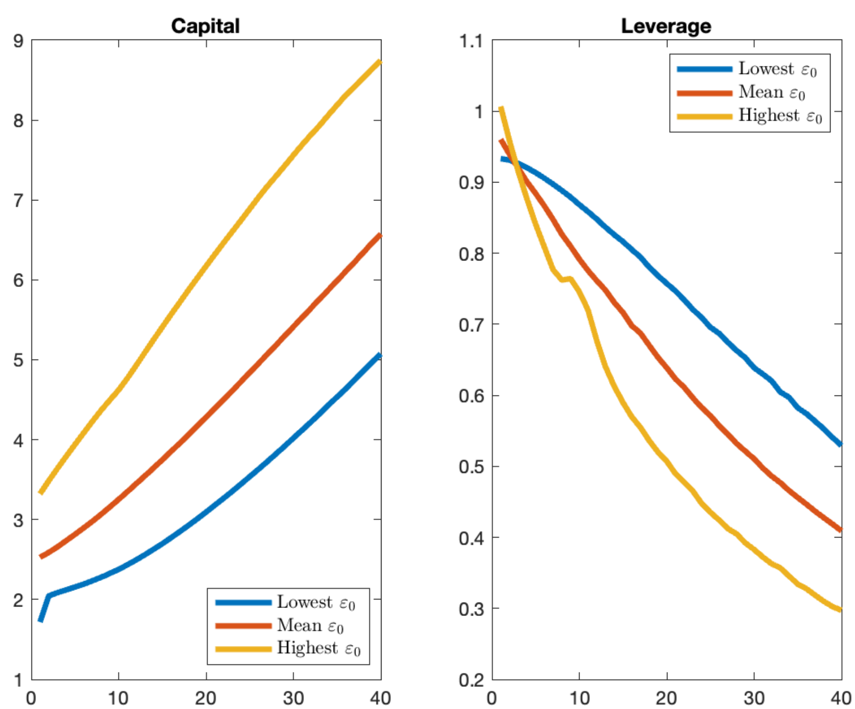
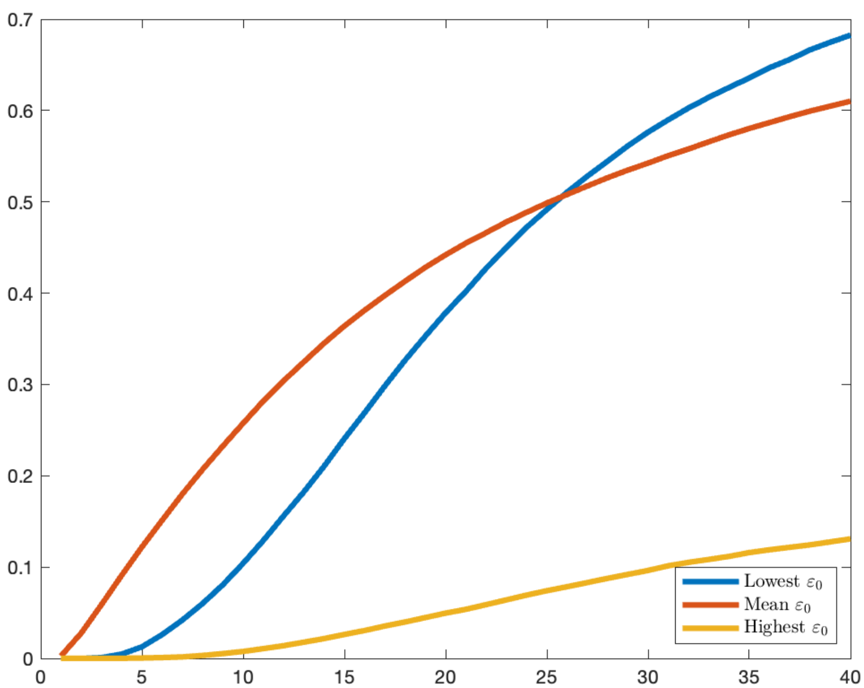


Figure 2: Firm Lifecycle Dynamics in Steady State, by Initial Productivity



Averages across continuing firms. x-axis: quarters since entry.

Figure 3: Cumulative Prepayment Probability in Steady State, by Initial Productivity



Averages across continuing firms. x-axis: quarters since entry.

Figure 4: Price Dynamics, Percentage Point Differences from Steady State

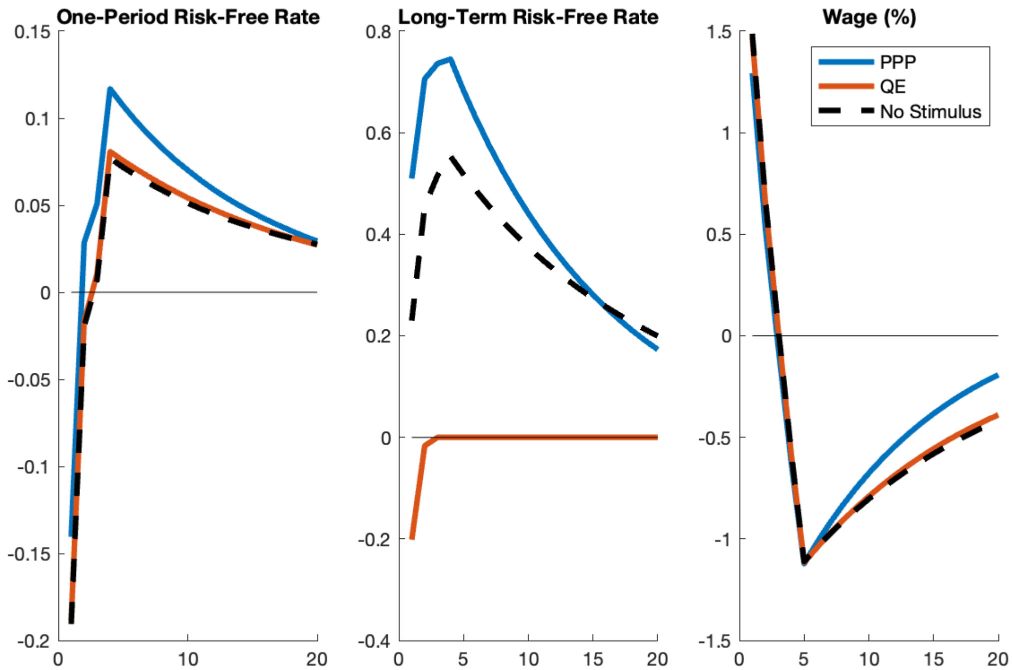


Figure 5: Super Firms' Capital Stock, Percent Deviation from Steady State

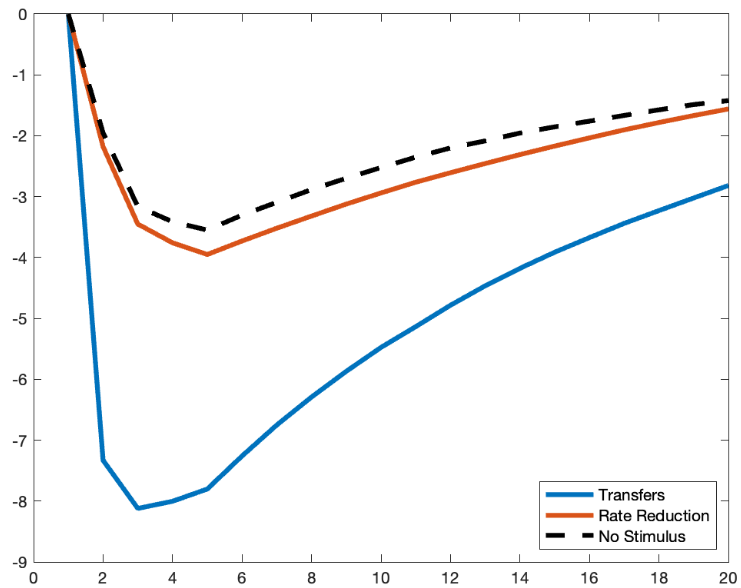


Figure 6: Ordinary Firms' Aggregate Stcks, Percent Deviations from Steady State Level

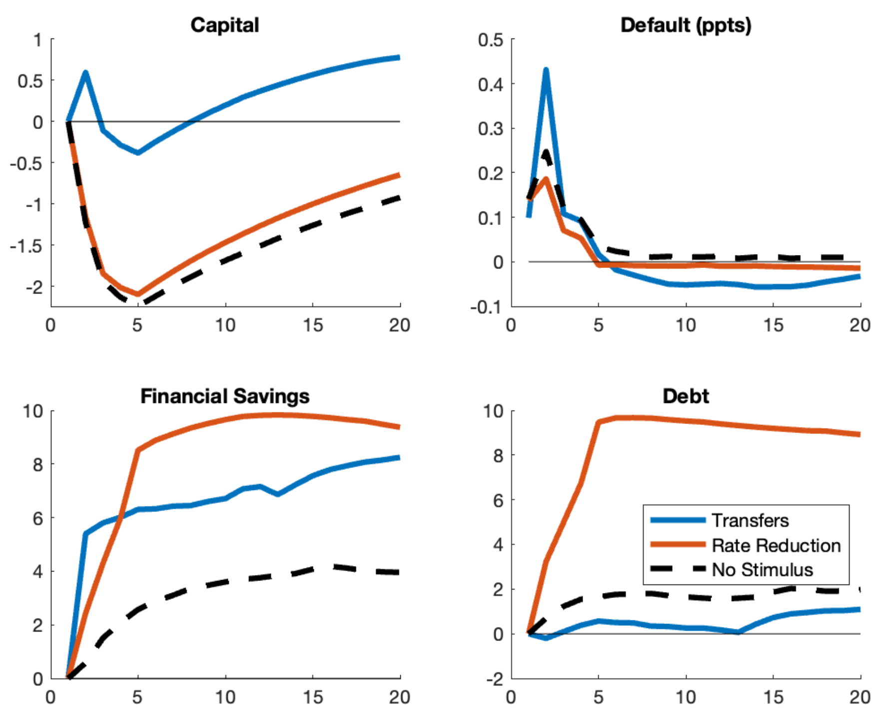


Figure 7: Aggregate Flows, Percent Deviations from Steady State Level

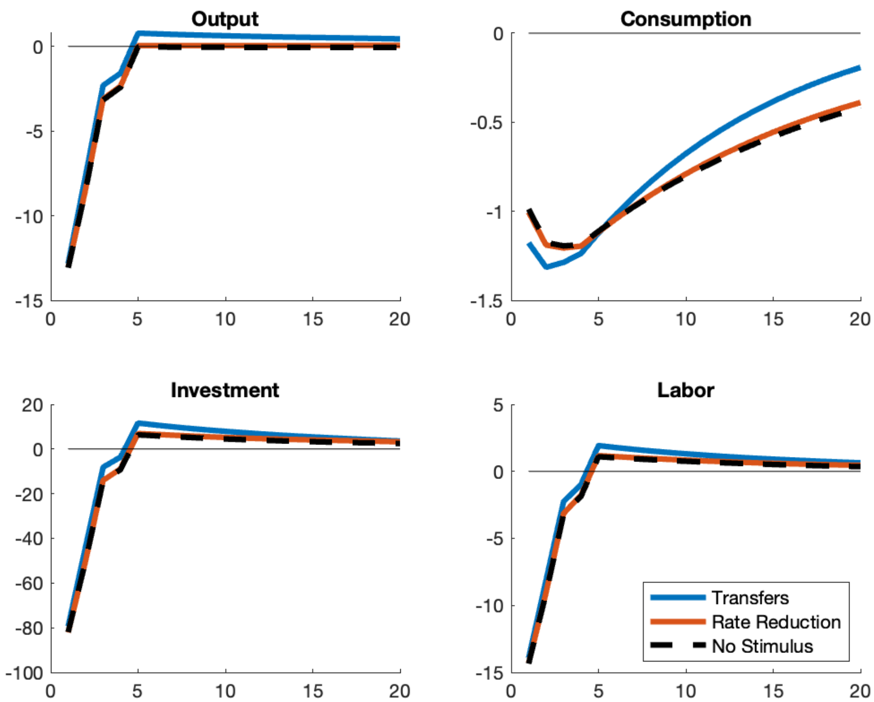
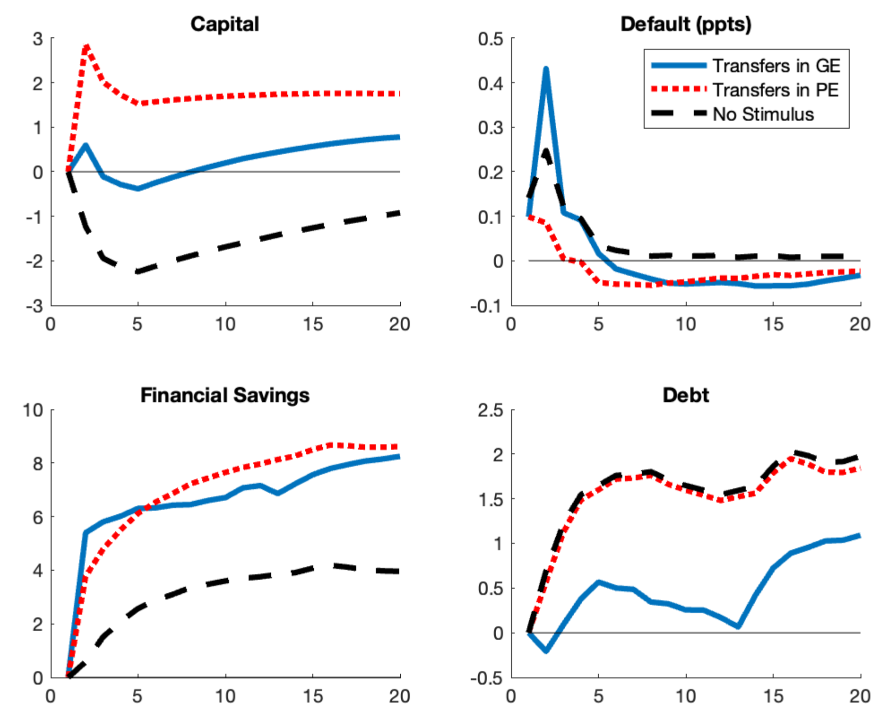


Figure 8: Ordinary Firms' Aggregate Stocks, Percent Deviations from Steady State Level



Appendix

Aggregation for Definition of Equilibrium

Here, I map the beginning-of-period distribution, μ, into the distributions that determine output and investment during the period, μ_Y and μ_I. μ_Y differs from μ due to the restructuring of defaulted firms, which leads to a loss of $(1-\chi)$ fraction of capital. The restructuring of defaulted firms also leads μ_I to differ from μ. In addition, μ_I incorporates new entrants, which replaced firms that received the exit shock and were not restructured.

For ease of notation, let $\mathbb{D}^e$ be an indicator for an exiting firm that lenders put through default. Recall that $\mathbb{D}^n$ and $\mathbb{D}^m$ are indicators for firms that endogenously default under non-maturing and maturing debt. The distributions μ and μ^Y are defined on the Borel algebra $\mathcal{A}$, generated by the open subsets of the product space $\mathcal{A} = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{E}$, where $\mathbb{E} \equiv \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$. For any $(A, \varepsilon_m) \in \mathcal{A}$, μ^Y, is measured by

$$\begin{aligned}
\mu^Y(A,\varepsilon_m) = (1-\lambda)(1-\rho) &\int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{D}_t^n(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m]) \\
+(1-\lambda)(1-\rho) &\int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^n(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\lambda(1-\rho) &\int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{I}_t^m(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\lambda(1-\rho) &\int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^m(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\rho &\int_{(k,a,b)|(k,a,b)\in A} (1-\mathbb{I}_t^e(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times\varepsilon_m]) \\
+\rho &\int_{(k,a,b)|(\chi k,a,0)\in A} \mathbb{I}_t^e(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times\varepsilon_m])
\end{aligned} \tag{15}$$

The state space for μ^I must also account for whether or not a firm's debt matures, as decision rules vary along this margin. I denote this Borel algebra as $\mathcal{A}^I$, which is generated by the open sets of the produce space $\mathcal{A}^I = 2 \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{E}$. Let 1 be an indicator variable for a firm whose debt matures, with 0 reflect a firm with non-maturing debt.

$$\mu_t^I(0, A, \varepsilon_m) = (1-\lambda)(1-\rho)\int_{(k,a,b)|(k,a,b)\in A}(1-\mathbb{D}_t^n(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m]) \tag{16}$$

$$\begin{aligned}\mu_t^I(1, A, \varepsilon_m) = (1-\lambda)(1-\rho)&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_t^n(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m])\\ +\lambda(1-\rho)&\int_{(k,a,b)|(k,a,b)\in A}(1-\mathbb{I}_t^m(k,a,b,\varepsilon_m))\mu(d[k\times a\times b\times \varepsilon_m])\\ +\lambda(1-\rho)&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_t^m(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m])\\ +\rho\mathbb{1}[(0,\iota a_e,0)\in A]\bar{\pi}_m^{\varepsilon}&\int(1-\mathbb{I}_t^e(k,a,b,\varepsilon))\mu(d[k\times a\times b\times \varepsilon])\\ +\rho&\int_{(k,a,b)|(\chi k,a,0)\in A}\mathbb{I}_e(k,a,b,\varepsilon_m)\mu(d[k\times a\times b\times \varepsilon_m])\end{aligned} \tag{17}$$

where $\mathbb{1}$ is an indicator variable for whether the entrants' state is an element of A and $\bar{\pi}^{\varepsilon}$ is the ergodic distribution over firms' idiosyncratic productivities. Entrants' financial assets are a fraction of firms' average physical capital:

$$a_e = \iota\int k\mu(d[k\times a\times b\times \varepsilon]) \tag{18}$$

The aggregate law of motion for the distribution of firms relies on μ_I as an intermediate distribution. For any $(A, \varepsilon_m)\in\mathcal{A}$, μ_{t+1} is measured by

$$\begin{aligned}\mu_{t+1}(A,\varepsilon_n) = &\int_{(k,a,b,\varepsilon_m)|(g_k^n(\cdot),g_a^n(\cdot),b)\in A}\pi_{m,n}^{\varepsilon}\mu_I(0,d[k\times a\times b\times \varepsilon_m])\\ +&\int_{(k,a,b,\varepsilon_m)|(g_k^m(\cdot),g_a^m(\cdot),g_b^m(\cdot))\in A}\pi_{m,n}^{\varepsilon}\mu_I(1,d[k\times a\times b\times \varepsilon_m])\end{aligned} \tag{19}$$

where the arguments for g_k, g_a, g_b, and $\mathbb{D}$ are (k,a,b,ε) and t subscripts are left off to reduce notation.