DATA 605 Assignment 14

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2017-5-12

This week, we'll work out some Taylor Series expansions of popular functions.

$f\left(x\right)=\frac{1}{\left(1-x\right)}, x \in (-1,1)$

$f\left(x\right)=e^{x}$

$f\left(x\right)=\ln\left(1+x\right), x \in (-1,1)$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as a R-Markdown document.

Taylor Series Approximation is used to represent functions as an infinite sum of polynomial terms that are calculated using a function's derivatives evaluated at a single point. Taylor's Theorem states that any function that is infinitely differentiable can be represented as a polynomial of the following form:

$f\left(x\right)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^{n}=f\left(a\right)+f^{(1)}\left(a\right)\left(x-a\right)+\frac{f^{(2)}}{2!}(x-a)^{2}+\ldots\ldots$

Function One

$f\left(x\right)=\frac{1}{\left(1-x\right)}, x \in (-1,1)$

$f\left(x\right)=\frac{1}{\left(1-a\right)}+\frac{1}{\left(1-a\right)^{2}}\left(x-a\right)+\frac{\frac{2}{\left(1-a\right)^{3}}}{2!}\left(x-a\right)^{2}+\frac{\frac{6}{\left(1-x\right)^{4}}}{3!}\left(x-a\right)^{3}\ldots.$

$=\frac{1}{\left(1-a\right)}+\frac{\left(x-a\right)}{\left(1-a\right)^{2}}+\frac{\left(x-a\right)^{2}}{\left(1-a\right)^{3}}+\frac{\left(x-a\right)^{3}}{\left(1-a\right)^{4}}\ldots\ldots$

When a = 0,

$=1^{2}+x+x^{2}+x^{3}\ldots\ldots$

$=\sum_{n=0}^{\infty}x^{n}, \quad x\in(-1,1)$

Function Two

$f\left(x\right)=e^{x}$

$f\left(x\right)=e^{a}+e^{a}\left(x-a\right)+\frac{e^{a}}{2!}\left(x-a\right)^{2}+\frac{e^{a}}{3!}\left(x-a\right)^{3}\ldots\ldots$

when a = 0,

$=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}\ldots\ldots$

$=\sum_{n=0}^{\infty}\frac{x^{n}}{n!}$

Function Three

$f\left(x\right)=\ln\left(1+x\right), x \in (-1,1)$

$$f(x) = \ln(1+a) + \frac{1}{(1+a)}(x-a) - \frac{\frac{1}{(1+a)^2}}{2!}(x-a)^2 + \frac{\frac{2}{(1+a)^3}}{3!}(x-a)^3 \ldots\ldots$$

When a = 0,

$$= \ln(1) + x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\ldots$$

$$= \quad \sum_{n=0}^{\infty} (-1)^{(n+1)} \frac{x^n}{n}, \quad x \in (-1, 1)$$