

DATA609 HW10

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3. The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson, "On the Growth of the Sheep Population in Tasmania," Trans. R. Soc. S. Australia 62(1938): 342-346).

t (year)	1814	1824	1834	1844	1854	1864
$P(t)$	125	275	830	1200	1750	1650

Figure 1

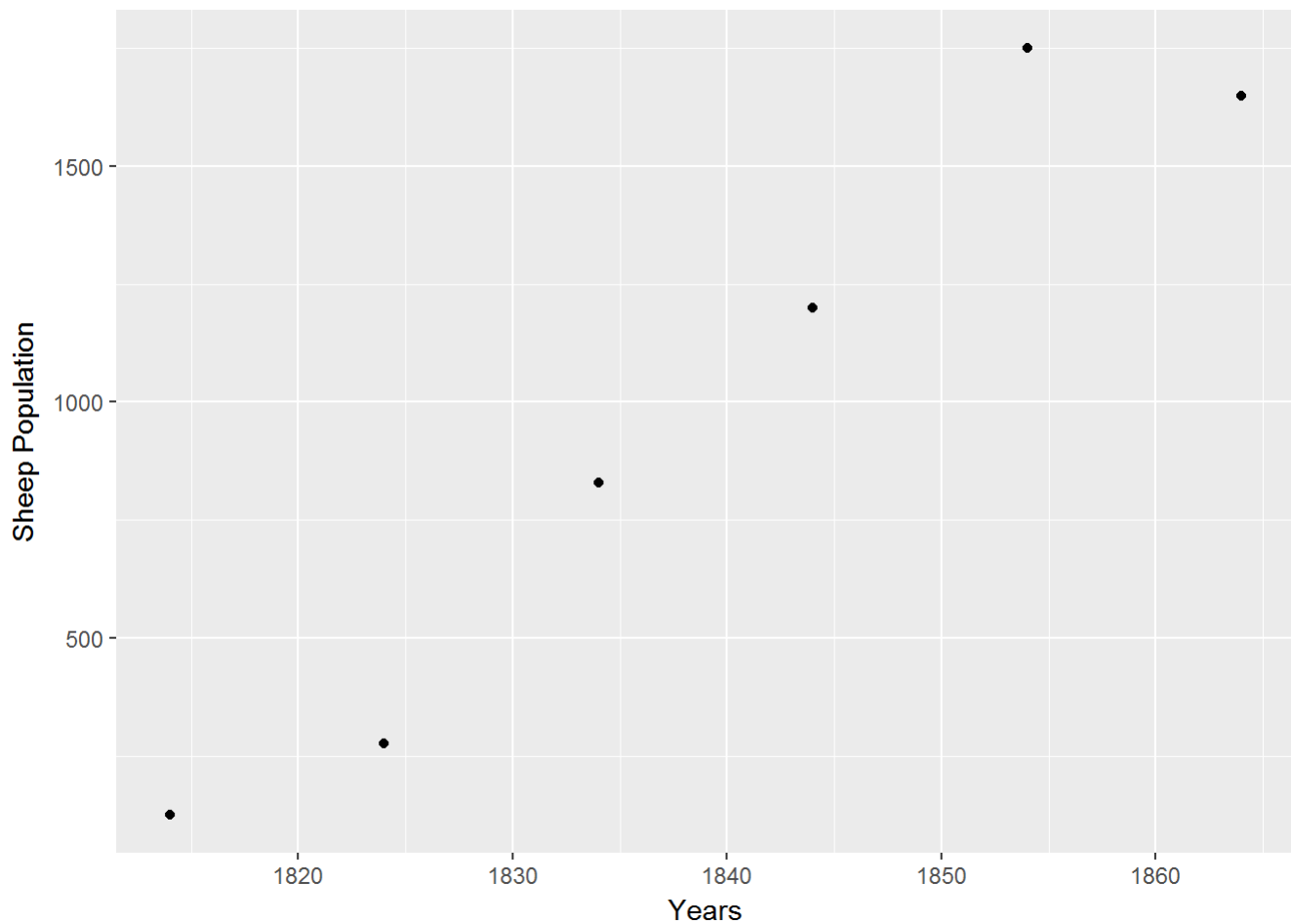
- a. Make an estimate of M by graphing $P(t)$

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.3.3
```

```
t <- c(1814, 1824, 1834, 1844, 1854, 1864)
Pt <- c(125, 275, 830, 1200, 1750, 1650)
df <- data.frame(t, Pt)
```

```
ggplot(data = df, aes(x = t, y = Pt)) + geom_point() + xlab("Years") + ylab("Sheep Population")
```

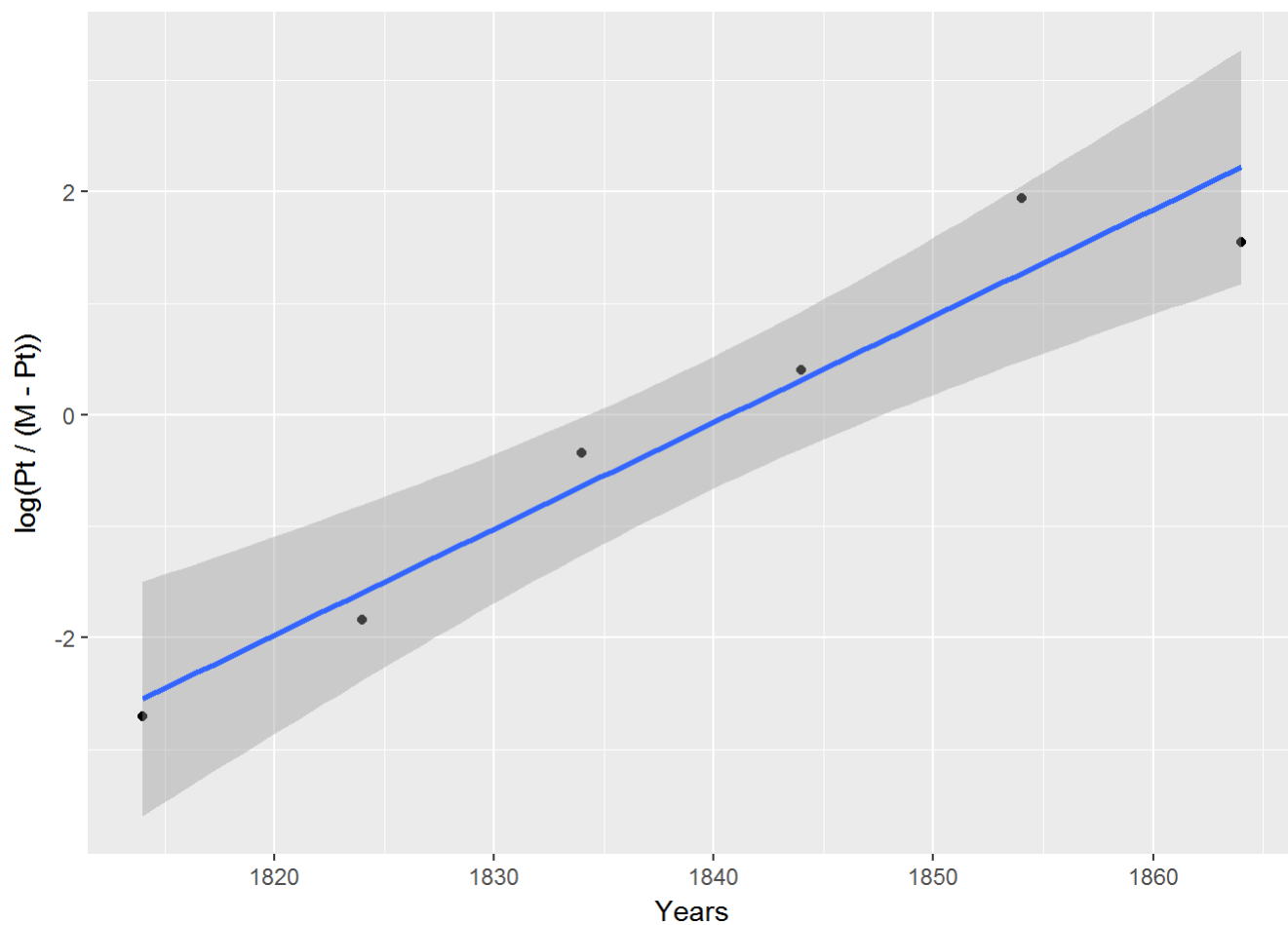


From this graph, I will estimate M , which is maximum population, to be approximately 2000.

b. Plot $\ln[P / (M - P)]$ against t . If a logistic curve seems reasonable, estimate rM and t^* .

According to the following graph, the logistic regression follows approximately linear relationship. Therefore, logistic curve is reasonable.

```
M <- 2000
ln_PMP <- log(Pt / (M - Pt))
df <- cbind(df, ln_PMP)
ggplot(data = df, aes(x = t, y = ln_PMP)) + geom_point() + stat_smooth(method = "lm") + xlab("Years") + ylab("log(Pt / (M - Pt))")
```



```
m <- lm(ln_PMP ~ df$t)
coefficients(m)
```

```
## (Intercept)      df$t
## -175.59646688  0.09539543
```

```
rM <- 0.09539543
C <- 1814 * rM - 175.59646688
t_star <- (- C / rM) + 1814
```

```
#Slope
rM
```

```
## [1] 0.09539543
```

```
#Constant
C
```

```
## [1] -2.549157
```

#Time when population P reaches half the M .
t_star

[1] 1840.722

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6. Suggest other phenomena for which the model described in the text might be used.

Patient blood glucose level has to keep in certain range. Glucose that is higher than a certain value is called hyperglycemia. Long term consequences of hyperglycemia may involve damages to the kidney, nerves, retina, and vascular. However, if the blood glucose is lower than a level, the situation is called hypoglycemia. Patient might experience seizures, coma, even death. Many factors will affect the sugar level inside the body. For instance, the food that people eat, family histories, physical activities et cetera. If patient take medications, then medications will be another factor that can influence blood glucose level.

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1.

a. Using the estimate that $db = 0.054v^2$, where 0.054 has dimension $\text{ft}\cdot\text{hr}^2/\text{mi}^2$, show that the constant k in Equation (11.29) has the value $19.9 \text{ ft}/\text{sec}^2$.

$$d_b = \frac{-v_0^2}{2k} + \frac{v_0^2}{k} = \frac{v_0^2}{2k} \quad (11.29)$$

Figure 1

$$d_b = \frac{V_0^2}{2K}$$

$$d_b = 0.054 V^2 = 0.0251 \cancel{\text{ft}^2} V^2 = \frac{V^2}{2K}$$

$$0.054 \frac{\text{ft} \cdot \text{hr}^2}{\text{mile}^2} \times \frac{3600^2 \text{s}^2}{\text{hr}^2} \times \frac{\text{mile}^2}{5280^2 \text{ft}^2}$$

$$= \frac{0.054 \cancel{\text{ft}} \times 3600^2 \text{s}^2}{5280^2 \text{ft}}$$

$$= 0.0251 \text{ft}^{-1} \text{s}^2$$

$$0.0251 \text{ft}^{-1} \text{s}^2 = \frac{1}{2K}$$

$$K = 19.92 \text{ft/s}^2$$

Figure 1

b. Using the data in Table 4.4, plot d_b in ft versus $v^2/2$ in ft^2/sec^2 to estimate $1/k$ directly.

```
speed <- c(20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80)
distance <- c(20, 28, 40.5, 52.5, 72, 92.5, 118, 148.5, 182, 220.5, 266, 318, 376)
```

```
speed <- speed * 5280 / 3600
speed
```

```
## [1] 29.33333 36.66667 44.00000 51.33333 58.66667 66.00000 73.33333
## [8] 80.66667 88.00000 95.33333 102.66667 110.00000 117.33333
```

```
v_square_half <- speed ^ 2 / 2
df1 <- data.frame(v_square_half, distance)
ggplot(data = df1, aes(x = v_square_half, y = distance)) + geom_point() + stat_smooth(method = "lm") + xlab("v^2/2") + ylab("Braking Distance")
```

