DATA609 HW11

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1. verify that the given function pair is a solution to the first-order system.

$$d_b = \frac{-v_0^2}{2k} + \frac{v_0^2}{k} = \frac{v_0^2}{2k} \qquad (11.29)$$

Figure 1

$\frac{dx}{dt} = \frac{d(-e^t)}{dt} = -e^t = -y$

$\frac{dy}{dt} = \frac{d(e^t)}{dt} = e^t = -x$

Therefore, it is a solution to the first-order system.

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6. Find and classify the rest points of the given autonomous system.

$$\textbf{1.}\ \ x = -e^t, \quad y = e^t$$

$$\frac{dx}{dt} = -y, \quad \frac{dy}{dt} = -x$$

Figure 2

$\frac{dx}{dt} = f(x, \quad y) = -(y-1) = 0$

Then y = 1

$\frac{dy}{dt} = g(x, \quad y) = x - 2 = 0$

Then x = 2 Therefore, the rest points is (2, 1)

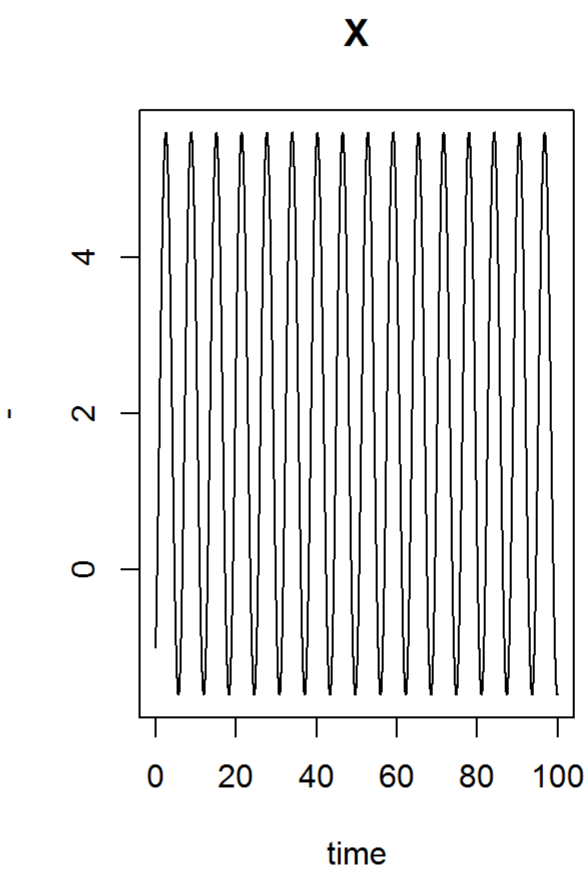
```
#Reference:http://www.di.fc.ul.pt/~jpn/r/odes/index.html
#install.packages("deSolve")
library(deSolve)
```

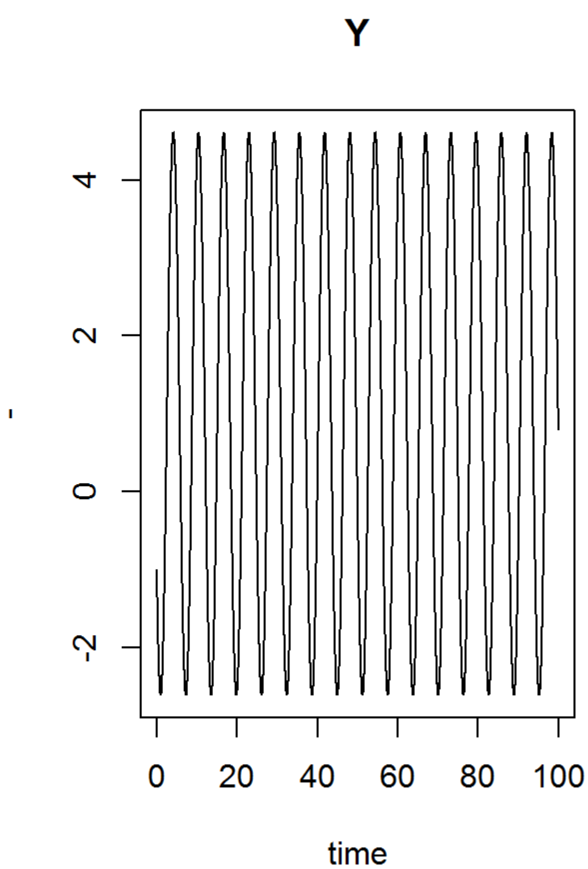
```
## Warning: package 'deSolve' was built under R version 3.3.3
```

```
parameters    <- c(a = 1, b = 1)
initial.state <- c(X = -1, Y = -1)

trajectory <-function(t, state, parameters) {
  with(as.list(c(state, parameters)),{
    dX <- 1 - b * Y
    dY <- a * X - 2

    list(c(dX, dY))
  })
}
times <- seq(0, 100, by = 0.01)
out <- ode(y = initial.state, times = times, func = trajectory, parms = parameters)
head(out)
```

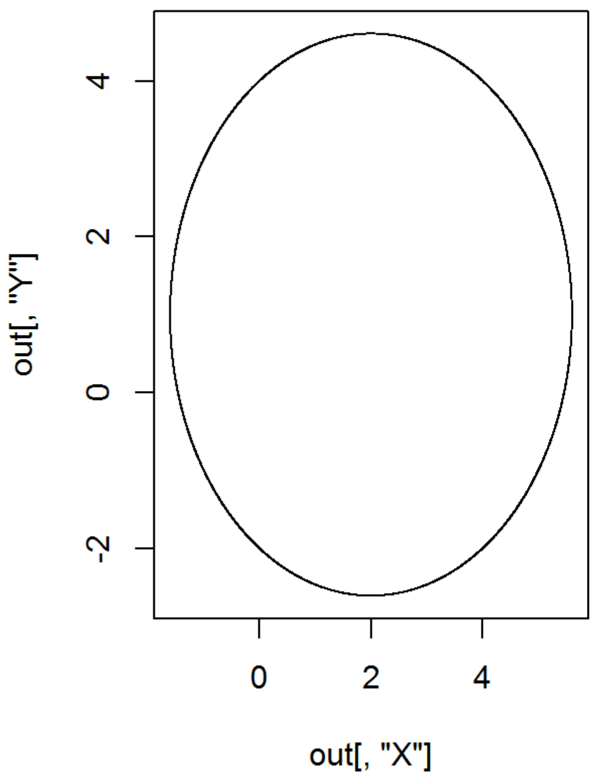
```
##      time          X         Y
## [1,] 0.00 -1.0000000 -1.000000
## [2,] 0.01 -0.9798505 -1.029899
## [3,] 0.02 -0.9594030 -1.059596
## [4,] 0.03 -0.9386596 -1.089086
## [5,] 0.04 -0.9176221 -1.118368
## [6,] 0.05 -0.8962929 -1.147437
```

```
plot(out, xlab = "time", ylab = "-")
```





```
plot(out[, "X"], out[, "Y"], pch = ".")
```



According to the trajectory of x and y, this is a periodic motion. Therefore, the rest point in this case is unstable.

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1. Apply the first and second derivative tests to the function f(y) = y^a / e^by to show that y = a / b is a unique critical point that yields the relative maximum f(a/b). Show also that f (y) approaches zero as y tends to infinity.

$f\,(y) = y^a/e^{by}$

$f'\,(y) = (ay^{a-1}e^{by} - y^a be^{by})/e^{2by}$

$f'\,(y) = (ay^{a-1} - by^a)/e^{by}$

$f'\,(y) = y^{a-1}(a - by)/e^{by} = 0$

Therefore, y = 0 or y = a/b we will obtain critical points.

$f''\,(y) = ((a-1)ay^{a-2} - aby^{a-1})e^{by} - (ay^{a-1} - by^a)be^{by}/e^{2by}$

$f''\,(y) = (y^{a-2}(a^2 - a - aby - aby + b^2y^2))/e^{by}$

When y = a/b

$f''\,(y) = (\frac{a}{b})^{a-2}(-a))/e^a$

$f''\,(y) = (\frac{a^{a-2}(-a)}{b^{a-2}e^a})$

$f''(y) = (\frac{-a^{a-1}}{b^{a-2}e^{a}})$

When f'‘is negative, we have relative maximum, when f'‘is positive we will have relative minimum. If b is positive and a is also positive, then f'' will be negative. The f(a/b) will achieve relative maximum.

$\lim_{y\to\infty} f(y) = \lim_{y\to\infty} y^{a}/e^{by}$

The numerator of this equation is always positive, therefore if b is a positive number the denominator which is an exponentiation is going to increase much faster as y approached infinity. The end result will approach zero. If b is a negative number, then the denominator will approached zero much faster, then the value f(y) is going to approach infinity.