DATA609 HW1

Bin Lin

2017-9-6

10. Your grandparents have an annuity. The value of the annuity increases each month by an automatic deposit of 1% interest on the previous month's balance.Your grandparents withdraw $1000 at the beginning of each month for living expenses. Currently, they have $50,000 in the annuity. Model the annuity with a dynamical system. Will the annuity run out of money? When? Hint: What value will an have when the annuity is depleted?

$$a_{n+1} = 1.01a_n - 1000 \; a_0 = 50000$$

```
annuity <- 50000
interest <- 0.01
withdrawl <- 1000
month <- 0

while(annuity > 0)
{
  annuity <- (1 + interest) * annuity - withdrawl
  month = month + 1
}
print("When the annuity run out of money:")
```

```
## [1] "When the annuity run out of money:"
```

```
month - 1
```

```
## [1] 69
```

```
print("What value is the annuity when it is depleted:")
```

```
## [1] "What value is the annuity when it is depleted:"
```

```
(annuity + 1000) / 1.01
```

```
## [1] 655.2788
```

9. The data in the accompanying table show the speed n (in increments of 5 mph) of an automobile and the associated distance an in feet required to stop it once the brakes are applied. For instance, n = 6 (representing 6 * 5 D 30 mph) requires a stopping distance of a6 = 47 ft.

a. Calculate and plot the change delta an versus n. Does the graph reasonably approximate a linear relationship?

Yes. The graph reasonable approximate a linear relationshiop.

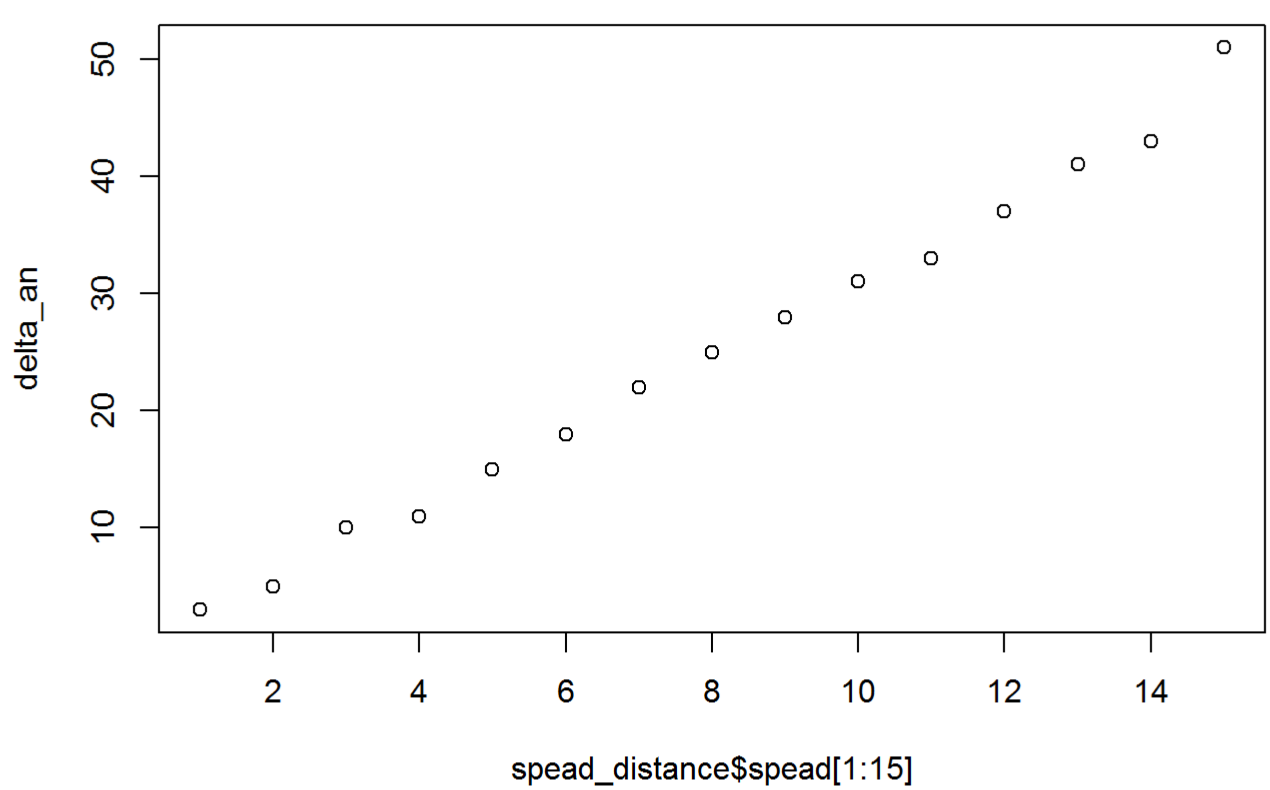
```
spead_distance <- data.frame(spead = 1:16, distance = c(3, 6, 11, 21, 32, 47, 65, 87, 112, 140,
171, 204, 241, 282, 325, 376))

delta_an <- c()
for (i in 1:15)
{
  delta_an[i] <- spead_distance$distance[i+1] - spead_distance$distance[i]
}

print("The value of the change delta is as follows:")
```

```
## [1] "The value of the change delta is as follows:"
```

```
delta_an
```

```
##  [1]  3  5 10 11 15 18 22 25 28 31 33 37 41 43 51
```

```
plot(spead_distance$spead[1:15], delta_an)
```



b. Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

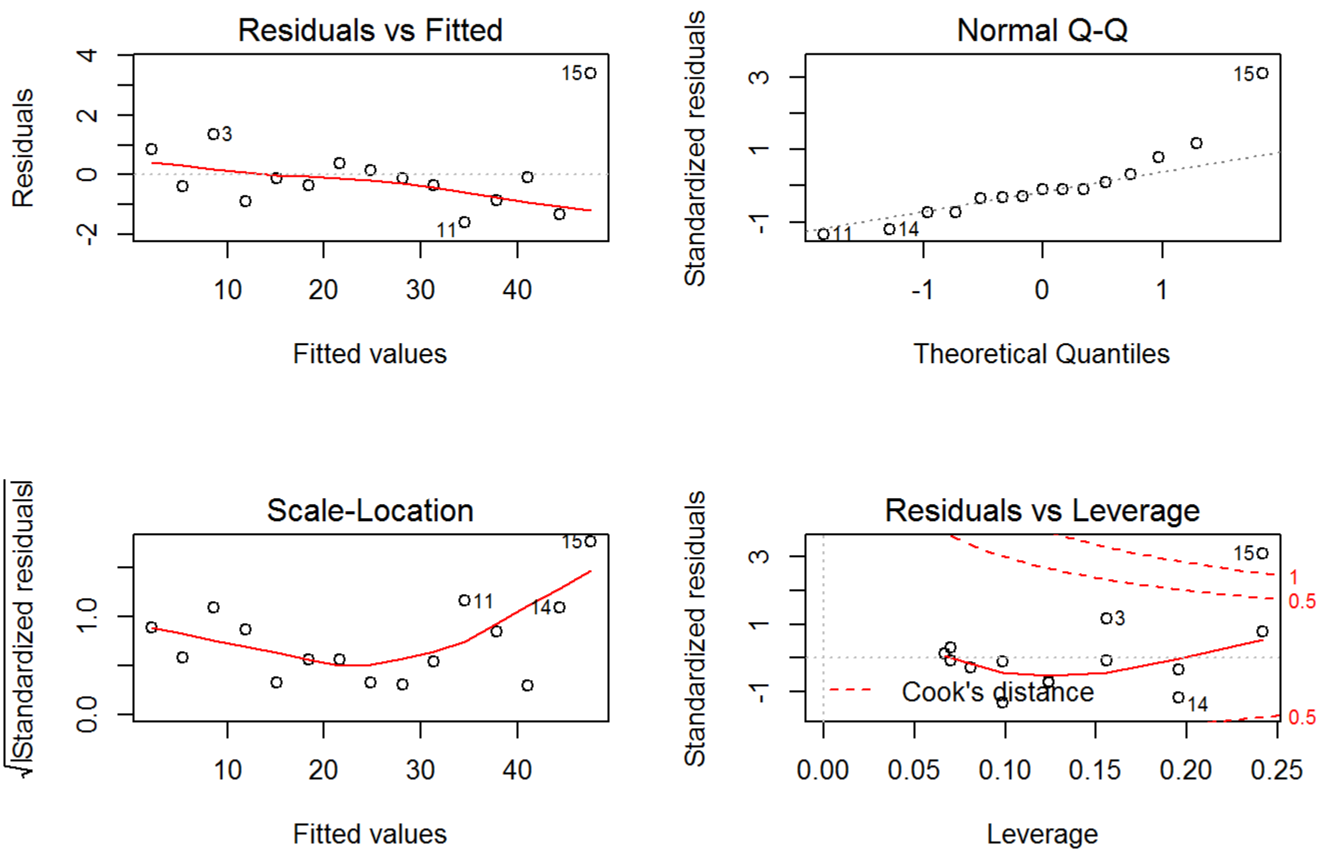
$$\Delta a_n = 3.24643 * n - 1.10476$$

According to the plot of the residual (first figure), we can tell that the residuals of the linear model appear to have no pattern. No bend or peak have noticed, therefore, the model is appropriate to explain the relationship between spead and the change in distance.

```
m1 <- lm(delta_an ~ spead_distance$spead[1:15])
summary(m1)
```

```
##
## Call:
## lm(formula = delta_an ~ spead_distance$spead[1:15])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.6060 -0.6202 -0.1274  0.2565  3.4083
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                -1.10476    0.68614   -1.61    0.131
## spead_distance$spead[1:15]  3.24643    0.07547   43.02  2.1e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.263 on 13 degrees of freedom
## Multiple R-squared:  0.993,  Adjusted R-squared:  0.9925
## F-statistic:  1851 on 1 and 13 DF,  p-value: 2.095e-15
```

```
par(mfrow = c(2, 2))
plot(m1)
```



13. Consider the spreading of a rumor through a company of 1000 employees, all working in the same building.We assume that the spreading of a rumor is similar to the spreading of a contagious disease (see Example 3, Section 1.2) in that the number of people hearing the rumor each day is proportional to the product of the number who have heard the rumor previously and the number who have not heard the rumor. This is given by $r_{n+1} = r_n + kr_n(1000 - r_n)$ where k is a parameter that depends on how fast the rumor spreads and n is the number of days. Assume k D 0:001 and further assume that four people initially have heard the rumor. How soon will all 1000 employees have heard the rumor?

After 11 days, all 1000 employees will have heard the rumor

```
r <- 4
k <- 0.001
day <- 0

while (ceiling(r) < 1000)
{
  r <- r + 0.001 * r * (1000 - r)
  day = day + 1
}

day
```

```
## [1] 11
```

```
r
```

```
## [1] 999.7277
```