DATA609 HW13

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4. Discuss how you might go about validating the nuclear arms race model. What data would you collect? Is it possible to obtain the data?

Some important data values we need to collect, as shown in the following:

1: the number of missiles possessed by countries that are involved in the nuclear arm races (probably in an annual basis)

2: Any anti missile weapons that can increase the survival of missiles in that country.

3: Mobile Launching Pads, underground basis, and multiple warheads et cetera.

It is possible to collect the data that are revealed by the government.The data that is available in the public tend to underestimate the real arm forces. The Cold War period is the perfect time we can study the arm races. Because there is no real war, each country can use either friendly strategy and enemy strategy rather than trying to produce weapons as soon and as much as possible.

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1. Build a numerical solution to Equations (15.8).

$y_{n+1} = 120 + \frac{1}{2}x_n$

$x_{n+1} = 60 + \frac{1}{3}y_n$

x0=100

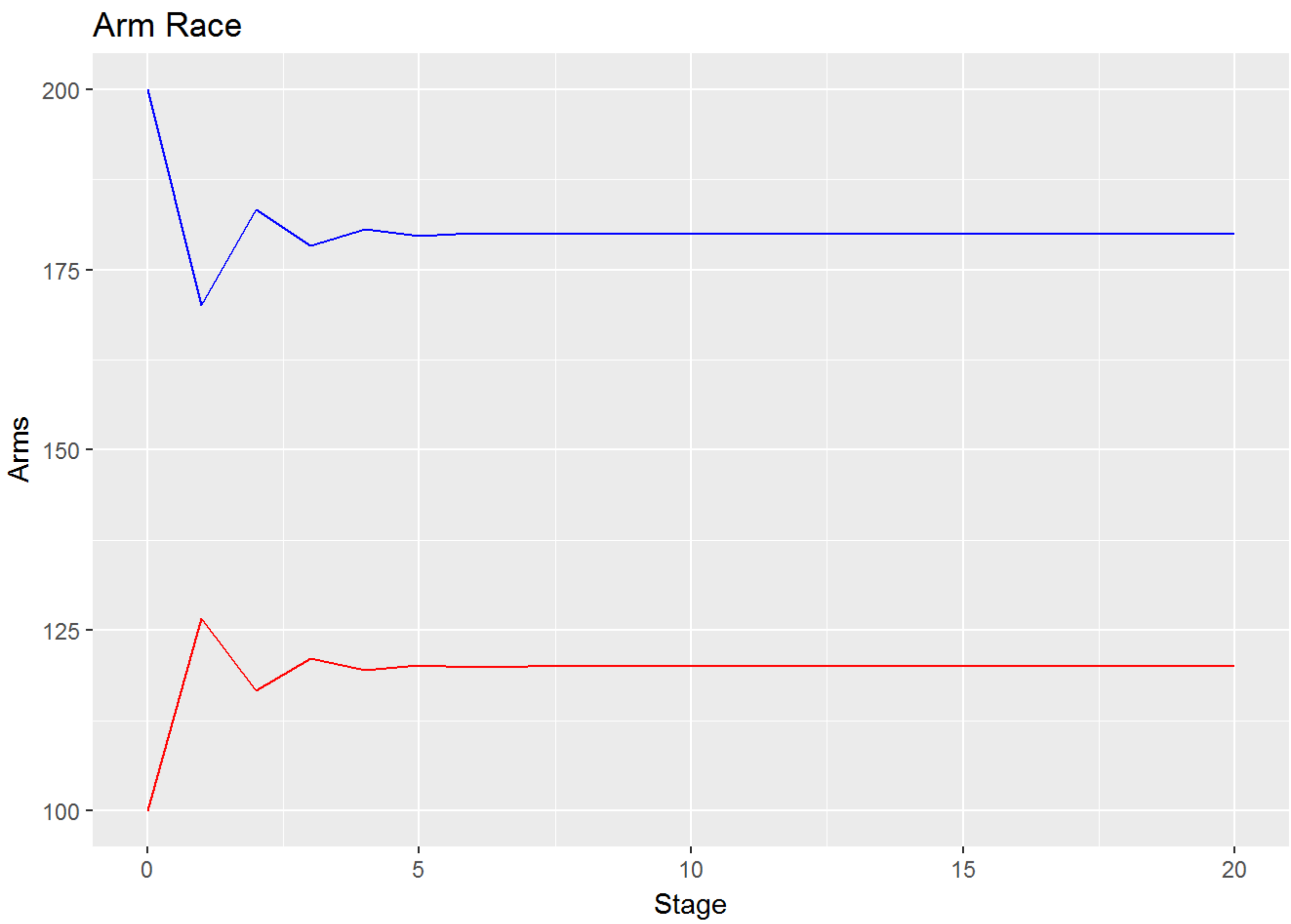
y0=200

a. Graph your results.

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.3.3
```

```
n <- 0
x <- 100
y <- 200
iteration <- 20

df <- data.frame(n, x, y)

for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 2 * x_temp
  x <- 60 + 1 / 3  * y_temp
  df <- rbind(df, c(n, x, y))
}
ggplot(df, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y), color = "b
lue") + labs(title = "Arm Race", x = "Stage", y = "Arms")
```



b. Is an equilibrium value reached?

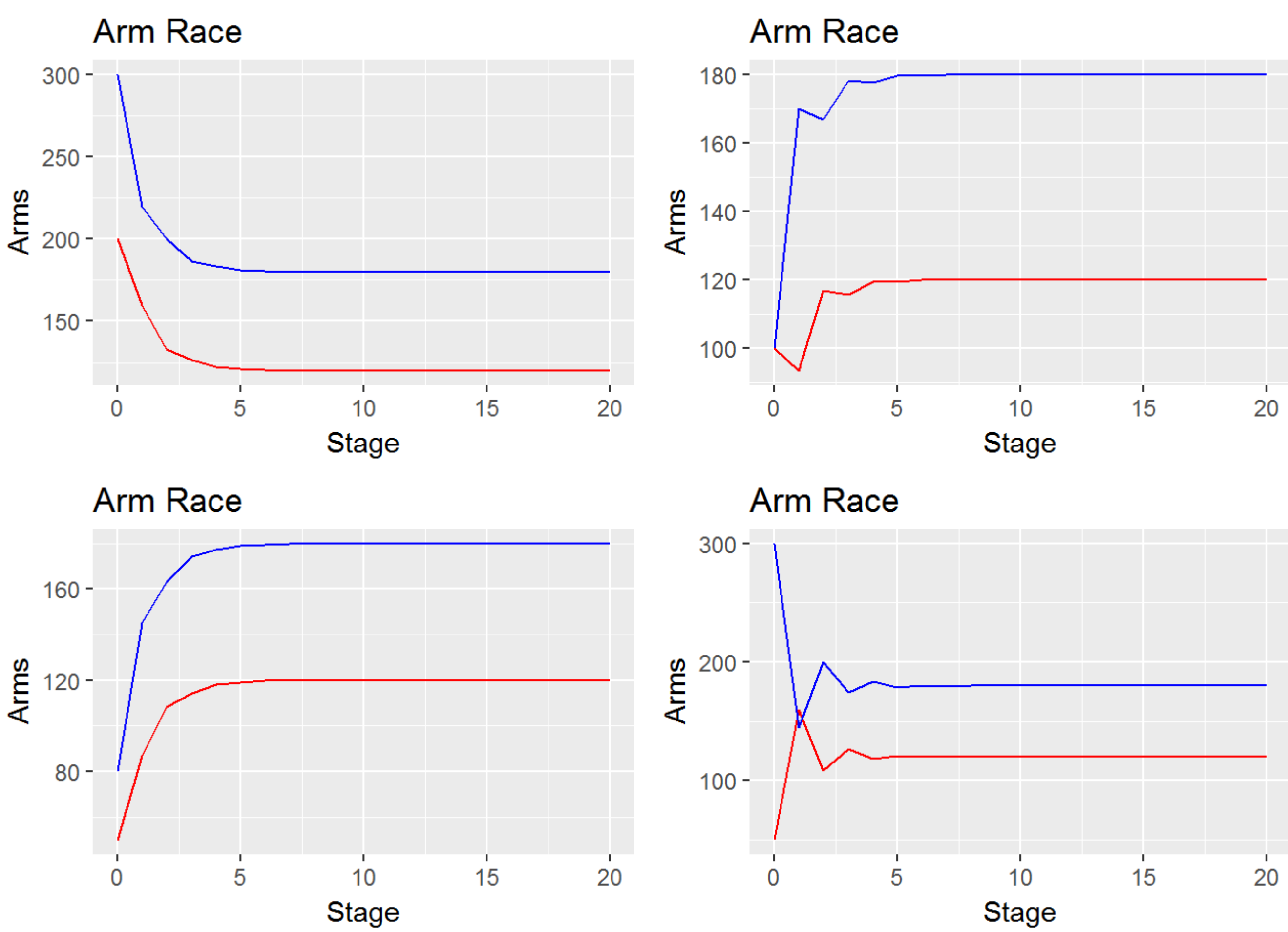
Yes, the equilibrium value for x and y is 120 and 180 respectively.

c. Try other starting values. Do you think the equilibrium value is stable?

The equilibrium value is stable.

```r
n <- 0
x <- 200
y <- 300
iteration <- 20
df1 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 2 * x_temp
  x <- 60 + 1 / 3  * y_temp
  df1 <- rbind(df1, c(n, x, y))
}


n <- 0
x <- 100
y <- 100
iteration <- 20
df2 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 2 * x_temp
  x <- 60 + 1 / 3  * y_temp
  df2 <- rbind(df2, c(n, x, y))
}


n <- 0
x <- 50
y <- 80
iteration <- 20

df3 <- data.frame(n, x, y)

for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 2 * x_temp
  x <- 60 + 1 / 3  * y_temp
  df3 <- rbind(df3, c(n, x, y))
}


n <- 0
x <- 50
y <- 300
```

```
iteration <- 20
df4 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 2 * x_temp
  x <- 60 + 1 / 3  * y_temp
  df4 <- rbind(df4, c(n, x, y))
}



plot1 <- ggplot(df1, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")
plot2 <- ggplot(df2, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")
plot3 <- ggplot(df3, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")
plot4 <- ggplot(df4, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")

library(gridExtra)
grid.arrange(plot1, plot2, plot3, plot4, ncol=2, nrow = 2)
```

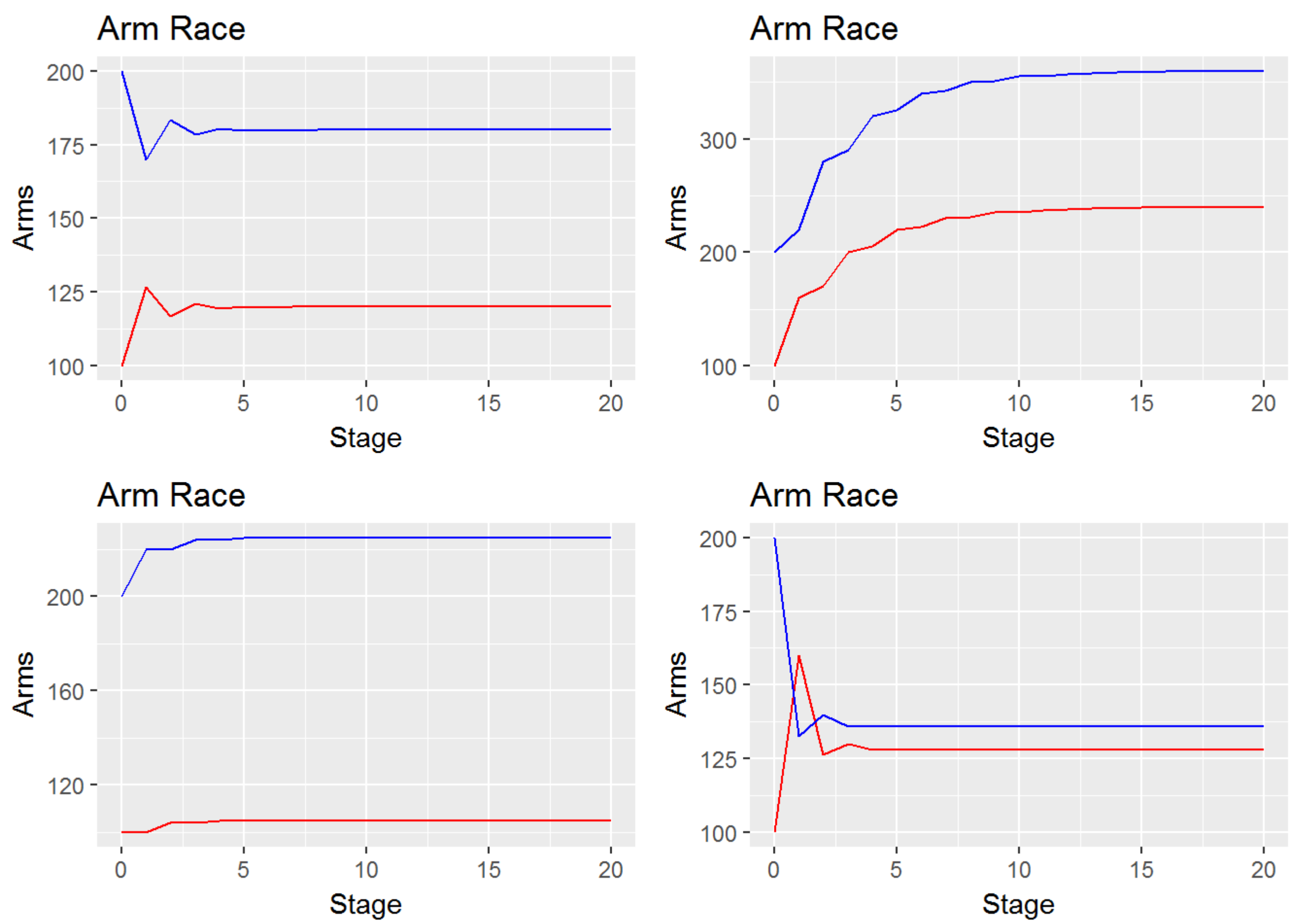


d. Explore other values for the survival coefficients of Countries X and Y . Describe your results.

The survival coefficients of countries X and Y can greatly affect the equilibrium values

```
n <- 0
x <- 100
y <- 200
iteration <- 20
df5 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 3 * x_temp
  x <- 60 + 1 / 5  * y_temp
  df5 <- rbind(df5, c(n, x, y))
}
plot5 <- ggplot(df, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y), c
olor = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")


n <- 0
x <- 100
y <- 200
iteration <- 20
df6 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 * x_temp
  x <- 60 + 1 / 2  * y_temp
  df6 <- rbind(df6, c(n, x, y))
}
plot6 <- ggplot(df6, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")


n <- 0
x <- 100
y <- 200
iteration <- 20
df7 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 * x_temp
  x <- 60 + 1 / 5  * y_temp
  df7 <- rbind(df7, c(n, x, y))
}
plot7 <- ggplot(df7, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")
```

```
n <- 0
x <- 100
y <- 200
iteration <- 20
df8 <- data.frame(n, x, y)
for (i in 1:iteration)
{
  x_temp <- x
  y_temp <- y
  n <- i
  y <- 120 + 1 / 8 * x_temp
  x <- 60 + 1 / 2  * y_temp
  df8 <- rbind(df8, c(n, x, y))
}
plot8 <- ggplot(df8, aes(x = n)) + geom_line(aes(y = x), color = "red") + geom_line(aes(y = y),
color = "blue") + labs(title = "Arm Race", x = "Stage", y = "Arms")

grid.arrange(plot5, plot6, plot7, plot8, ncol=2, nrow = 2)
```



Page B-21 4. Verify the result that the marginal revenue of the q + 1st unit equals the price of that unit minus the loss in revenue on previous units resulting from price reduction.

$$MR_{q+1} = TR_q^{'} = P_q + P_q^{'} * q$$

$$P_q^{'} = \frac{P_{q+1} - P_q}{1} = P_{q+1} - P_q$$

Therefore:

$$MR_{q+1} = P_q + (P_{q+1} - P_q) * q$$

Since P(q+1) < P(q), then

$$MR_{q+1} = P_q - (P_q - P_{q+1}) * q$$