

AI-01038 Exponent vs Compound Interest

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What is an exponent? How does it relate to compound interest concept? Explain and delve.

An exponent is a mathematical notation that signifies how many times a number (base) needs to be multiplied by itself. For instance, 2^4 represents 2 multiplied by itself four times ($2 \times 2 \times 2 \times 2$).

The concept of exponents is fundamental in understanding compound interest. Compound interest refers to the interest earned on both the initial principal amount and the accumulated interest from previous periods. Let's say you invest \$100 at an annual interest rate of 5%. After one year, you'll earn \$5 in interest, bringing your total balance to \$105. In the second year, the 5% interest is calculated not just on the initial \$100 but also on the \$5 earned earlier. So, you earn \$5.25 in the second year, and your total balance becomes \$110.25.

This repetitive calculation of interest on interest can be effectively represented using exponents. If you invest \$P at an annual interest rate of R for n years, the total amount (A) you'll have at the end can be expressed as:

$$A = P(1 + R/100)^n$$

Here, P is the principal, R is the interest rate, and n is the number of years. The term $(1 + R/100)$ represents the factor by which your investment grows each year, and it's raised to the power of n to account for the repeated compounding of interest.

The power of compound interest becomes even more evident when considering longer timeframes. With longer periods, the effect of compounding becomes more substantial. Even a small increase in the interest rate or investment period can significantly impact the final amount. This is why Albert Einstein famously called compound interest the "eighth wonder of the world."

Derive the formula

Absolutely! Here's the derivation of the compound interest formula:

We can imagine compound interest accruing in stages over time. Let's say you invest an initial principal amount (P) at an annual interest rate (R). After one year, you'll earn interest (I) on the principal amount:

$$I = P * (R/100)$$

This interest is then added to the principal to get the amount (A) at the end of the first year:

$$A_1 = P + I = P + (P * R/100) = P (1 + R/100)$$

Now, in the second year, the interest is calculated not just on the original principal (P) but also on the interest earned in the first year (I). This effectively means the interest is earned on the total amount (A_1) from the previous year.

$$I_2 = A_1 * (R/100) = P(1 + R/100) * (R/100)$$

We can substitute A_1 from the previous step:

$$I_2 = P (1 + R/100) * (R/100) = P * (1 + R/100)^2$$

Following this pattern, the total amount (A) after n years can be expressed as:

$$A = P (1 + R/100)^n$$

This formula captures the essence of compound interest, where interest is earned on both the initial principal and the accumulated interest from previous periods.