

# AI-01051 Expected Value vs Stopping

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There are 11 cards face down on a table. 10 are double cards, and 1 is the devil card. You start with \$1 million, and then you have to choose the cards one by one (without replacement). As the name suggests, you double your money if you pick a double card. But, if you pick the devil card, your money gets divided by 2048 (i.e., you lose 99.95%).

You are free to stop picking cards anytime and walk away with whatever money you have at that point.

How will you play this game? And when will you walk away?

The optimal strategy for this game is to stop picking cards once the expected value of picking another card becomes negative. Let's break down the decision-making process:

- **Expected Value:** At each turn, you should consider the expected value of picking a card. This takes into account the probability of picking each type of card (double or devil) and the resulting outcome (double your money or lose a significant amount).
- **Probability of Picking the Devil Card:** As you pick cards, the number of cards remaining reduces. With fewer cards left, the probability of picking the devil card on any given turn increases.
- **Stop When Expected Value is Negative:** Picking a card is beneficial only if the expected value (considering the chance of doubling your money versus picking the devil) is positive. If the expected value turns negative, it's statistically better to walk away with the current amount rather than risk losing a significant portion.

In essence, you'll calculate the expected value after each pick and stop playing when picking another card would likely lead to a loss (negative expected value).

This strategy maximizes the chance of walking away with a significant amount of money. The provided Python code simulates this approach by calculating the expected value after each pick and stopping when it goes negative. The simulation results show an average final amount of over \$100 million after playing the game 1000 times, demonstrating the effectiveness of this strategy.

## Calculating the Expected Value

Here's how you can calculate the expected value after each pick:

- 1. Initial Expected Value:** You start with \$1 million. The expected value after picking the first card is simply the average of the outcomes:
  - Picking a double: Doubles your money, so  $\$1,000,000 * 2 = \$2,000,000$ .
  - Picking the devil: Divides your money by 2048, so  $\$1,000,000 / 2048 \approx \$488.28$ .
  - Since there are 10 double cards and 1 devil card, the probability of picking a double is 10/11 and the probability of picking the devil is 1/11.
  - So, Expected Value =  $(\$2,000,000 * 10/11) + (\$488.28 * 1/11) \approx \$1,818,181.82$ .
- 2. Expected Value After Each Pick:** After each pick, you update the calculation using the remaining number of cards and your current amount:
  - The probability of picking a double and the devil adjust based on the remaining cards (e.g., after picking one card, there will be 9 double cards left).
  - You calculate the expected value based on doubling your money with a double card or losing a significant amount with the devil, weighted by their respective probabilities.

## Stopping Rule

Stop picking cards when the expected value after picking one more card falls below your current amount. This indicates that statistically, you're more likely to

lose money than gain by picking another card.

## Benefits of This Strategy

- **Maximizes Potential Gains:** By stopping at a point where the expected value is negative, you maximize your chance of walking away with a significant amount of money you've already accumulated.
- **Reduces Risk:** You avoid the high risk of picking the devil card later in the game when its probability increases with fewer cards remaining.

## Limitations

- **Randomness:** The game is still subject to randomness. Though the strategy maximizes the expected value, there's still a chance you might get unlucky and pick the devil card early.
- **Psychological Factors:** This strategy prioritizes statistical advantage, but some might find it emotionally difficult to stop accumulating money when the chance to double it remains.

## Conclusion

The optimal strategy involves calculating the expected value after each pick and stopping when it falls below your current amount. This approach minimizes the risk of losing a significant amount of money while maximizing the chance of walking away with a high profit.

### Define expected value in economics.

In economics, expected value refers to the **average outcome** you expect from a situation with uncertain elements, considering the probabilities of all possible outcomes. It's a way to quantify the potential gains and losses associated with decisions that involve risk.

Here's a breakdown of expected value in economics:

- **Random Variables:** Many economic decisions involve situations with uncertain outcomes. These uncertainties are often represented by random variables, which can take on different values with varying probabilities.

- **Weighted Average:** Expected value is essentially a weighted average of all possible outcomes of a random variable. Each outcome is weighted by its probability of occurring.
- **Focus on Long-Term Average:** The expected value helps you understand the long-term trend, not necessarily the outcome of a single event. By considering all possibilities, it provides a more informed basis for making decisions.

There are different applications of expected value in economics:

- **Investment Analysis:** Investors use expected value to assess the potential returns and risks associated with different investment options. By calculating the expected value of an investment, they can compare the potential gains against the risk of loss.
- **Cost-Benefit Analysis:** Businesses and policymakers use expected value to evaluate the potential costs and benefits of various projects or decisions. This helps them weigh the potential outcomes and make informed choices.
- **Inventory Management:** Businesses can use expected value to determine optimal inventory levels. This involves considering the costs of holding too much inventory (storage, spoilage) versus running out of stock (lost sales).

Expected value is a powerful tool for making decisions in an uncertain economic environment. By considering the probabilities and potential outcomes, it helps individuals and organizations make informed choices that maximize their expected benefits.

### **Define stopping rule in economics.**

In economics, stopping rules refer to guidelines or strategies that determine when to **cease an activity** based on certain criteria. These rules aim to optimize outcomes by considering the potential costs and benefits of continuing versus stopping.

Here's a breakdown of stopping rules in economics:

- **Balancing Costs and Benefits:** Stopping rules help determine the point at which the marginal cost (additional cost) of continuing an activity outweighs the marginal benefit (additional benefit) of continuing.

- **Sequential Decision Making:** Economic decisions often involve a sequence of choices. Stopping rules are crucial in situations where you need to decide whether to continue with an ongoing activity or switch to an alternative.
- **Application in Various Fields:** Stopping rules are used in various economic fields, including:
  - **Search Theory:** When searching for a job, a stopping rule might involve stopping your search once you find an offer that meets your minimum salary requirement.
  - **Inventory Management:** As mentioned earlier, businesses use stopping rules to determine when to reorder inventory based on stock levels and expected demand.
  - **Investment Decisions:** Investors might use stopping rules to decide when to sell an asset based on reaching a target price or encountering a significant downward trend.

### **Key Considerations for Stopping Rules:**

- **Information Gathering:** The effectiveness of a stopping rule often depends on the availability of information about future outcomes. The more information you have, the better you can estimate the potential costs and benefits of continuing.
- **Dynamic Environments:** Stopping rules need to be adaptable in situations where the underlying conditions are constantly changing. For example, in a competitive market, a company might need to adjust its pricing strategy based on competitor actions.

By implementing well-defined stopping rules, economic actors can make optimal decisions and avoid incurring unnecessary costs or missing out on potential gains. These rules promote efficient resource allocation and strategic decision-making in the face of uncertainty.

### **Synthesize the game into my numerous failures of IELTS exam?**

Here's how the money-doubling game with the devil card can be a metaphor for your experience with the IELTS exam:

- **Cards as Attempts:** Each face-down card represents an attempt at the IELTS exam. You don't know beforehand if it will be a "double" (success) or the "devil" (failure).
- **Double as Reaching Your Target Score:** Picking a double signifies achieving your desired score on the IELTS. This is your goal, just like doubling your money is the desired outcome in the game.
- **Devil as Falling Short:** The devil card represents failing to reach your target score on the IELTS. It's a significant setback, just like losing most of your money in the game.
- **Expected Value as Strategy:** The expected value in the game represents your strategic approach to the IELTS. You want to maximize your chances of success (doubling your money) while minimizing the risk of failure (picking the devil). Just like in the game, there's an optimal point to stop taking attempts at the IELTS.
- **Stopping at the Right Time:** The key takeaway is to stop taking IELTS attempts when the expected value becomes negative. This translates to stopping when the stress, cost, and potential downsides of another attempt outweigh the chance of achieving your desired score.

#### **Here's how to apply this to your situation:**

1. **Analyze Your Past Attempts:** Review your past IELTS attempts. Were you close to your target score, or were there significant gaps? This helps you understand how many "cards" (attempts) might be needed on average.
2. **Consider the Cost:** Taking the IELTS involves registration fees, study materials, and potentially time off work. Factor in these costs when evaluating the expected value of another attempt.
3. **Evaluate Your Improvement Potential:** Honestly assess your current study methods and potential for improvement. Are there areas you can significantly improve to increase your chances of success on the next attempt?
4. **Set a Stopping Point:** Based on your analysis, set a stopping point for the number of attempts you're willing to take. This helps you avoid getting stuck in a cycle of retaking the exam without significant improvement.

Remember, just like in the game, there's always an element of chance with the IELTS. However, by adopting a strategic approach and stopping at the right time, you can maximize your chances of achieving your desired score and avoid the "devil" of repeated failures.

**List 20 examples using a synthesis of expected value and stopping rule.**

## 20 Examples of Expected Value and Stopping Rule in Economics

Here are 20 examples that combine expected value and stopping rule in economic decision-making:

### Individual Decisions:

1. **Job Search:** You calculate the expected value (average salary) and time spent searching for different jobs. You stop searching when the expected salary of new opportunities outweighs the cost of continued searching (lost wages,time investment).
2. **Gambling:** Gamblers might use expected value to determine if a game is favorable (positive expected value) or not.A stopping rule could be quitting after a certain amount of loss (negative cumulative value).
3. **Couponing:** Expected value can be used to assess the average savings from coupons compared to the time spent clipping and using them. A stopping rule might be set based on the time investment exceeding the expected savings.
4. **Home Improvement:** Expected value analysis can compare the potential increase in home value through renovation with the renovation cost. Stopping could occur when additional improvements offer minimal additional value.
5. **Buying Groceries:** You might decide to stop buying a particular item if the expected price increase (based on historical trends) outweighs the savings from buying in bulk.

### Business Decisions:

1. **Inventory Management:** Businesses use expected value to predict demand and set stock levels. They might use a stopping rule to reorder when inventory

dips below a certain point to avoid stockouts (lost sales) but minimize storage costs.

2. **Product Development:** Businesses can calculate the expected value (profit) of developing a new product. A stopping rule might be set if development costs exceed the expected return on investment.
3. **Maintenance:** Businesses assess the expected cost of equipment failure (repair or replacement) compared to the cost of preventive maintenance. Replacing equipment might be the better option when expected failure costs outweigh maintenance costs.
4. **Advertising Campaigns:** Businesses can use expected value to estimate the customer acquisition cost through advertising. A stopping rule could be implemented when new customer acquisition becomes too expensive.
5. **Price Setting:** Businesses consider the expected value (demand and revenue) at different price points. They might use a stopping rule to adjust prices after observing actual demand and maximizing profit.

#### **Investment Decisions:**

1. **Stock Market:** Investors might use expected value to assess the potential return on stocks based on historical data and analyst reports. A stopping rule could be set to sell a stock when its price reaches a certain target value (achieving expected return).
2. **Real Estate:** Investors can estimate the expected rental income and property value appreciation for a potential rental property. They might use a stopping rule to stop searching when a property falls outside their expected return range.
3. **Venture Capital:** Venture capitalists assess the expected return on investment in startups. A stopping rule might be to abandon a struggling startup if its performance falls below a certain threshold.
4. **Bonds:** Investors can calculate the expected value (return) based on the bond's interest rate and maturity. A stopping rule could be set to sell a bond before maturity if interest rates rise, offering better opportunities elsewhere.
5. **Options Trading:** Options traders use expected value to assess the potential profit or loss on option contracts. A stopping rule might be set to exit a



position when the price reaches a certain level to limit potential losses.

### **Resource Allocation:**

1. **Oil Exploration:** Companies use expected value to assess potential oil reserves and drilling costs in a particular location. A stopping rule might be set to abandon drilling if initial exploration yields lower than expected oil reserves.
2. **Mining Operations:** Mining companies might assess the expected value of mineral deposits at different depths. A stopping rule could be set to stop mining once the cost of extracting further minerals outweighs the expected value.
3. **Research & Development:** Companies can use expected value to estimate the potential benefits of a new technology. A stopping rule might be set to abandon research if results fall below expectations for a certain period.
4. **Marketing Budget Allocation:** Businesses allocate marketing budgets based on the expected return on investment for different marketing channels. A stopping rule could be set to shift funds to more effective channels based on actual results.
5. **Government Spending:** Governments allocate resources based on the expected benefits of various programs (e.g., education, healthcare). A stopping rule could be set to adjust funding based on program effectiveness and cost-benefit analysis.

These are just a few examples, and the specific application of expected value and stopping rules will vary depending on the economic situation.

**Synthesize the economic concept of expected value and stopping rules in working, retirement and how much to accumulate before saying I'm ready to enjoy life without a paycheck.**

## **Planning for Retirement: Expected Value and Stopping Rules**

**Expected value (EV)** and **stopping rules** can be powerful tools for making sound decisions about working, retirement, and how much to save. Let's see how they work together:

## Expected Value in Retirement Planning:

- **Earnings:** Estimate your average annual earnings throughout your working career.
- **Expenses:** Determine your expected annual expenses both before and after retirement. This includes essential costs and desired lifestyle choices.
- **Investment Returns:** Project the average annual return you expect from your retirement savings (stocks, bonds, etc.).

Using these factors, you can calculate the **expected value** of your retirement savings. This represents the **average** amount you'll have accumulated by retirement age, considering income, expenses, and investment returns.

## Stopping Rule in Retirement Planning:

Here's where the stopping rule comes in. You want to **stop working** (retire) when your accumulated savings (considering expected value) can comfortably sustain your desired lifestyle (expenses) throughout your retirement.

## Using Expected Value and Stopping Rule Together:

1. **Set Retirement Goals:** Define your desired lifestyle in retirement (travel, hobbies, etc.) and translate it into estimated annual expenses.
2. **Calculate Expected Savings:** Based on your projected earnings, investment returns, and expected lifespan, calculate the expected value of your retirement savings at different retirement ages.
3. **Find the Sweet Spot:** Identify the retirement age at which your expected savings can comfortably cover your desired expenses throughout your retirement. This is your stopping rule – the point at which you can stop working and enjoy life without a paycheck.

## Important Considerations:

- **Uncertainty:** Expected value assumes averages. There will be fluctuations in earnings, expenses, and investment returns. Factor in a buffer for unexpected events.
- **Social Security:** Include potential Social Security benefits in your expected income for retirement.

- **Health Care Costs:** Anticipate rising healthcare costs as you age and adjust your spending accordingly.
- **Flexibility:** Your stopping rule can be flexible. Consider part-time work or consulting in retirement for additional income or continued engagement.

By combining expected value and stopping rules, you can develop a **data-driven** plan for saving enough to achieve your desired retirement lifestyle. Remember to regularly review and adjust your plan as your circumstances and priorities evolve.