

The Eyring Equation and Precision

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The Eyring equation is a classic of physical chemistry.¹ The free energy of activation for a reaction can be determined and, if a series of rate constant values are available at different temperatures, we can determine the enthalpy and the entropy of activation. These parameters are very helpful in interpreting reaction mechanism. But can these results be trusted? How could you estimate their reliability? Unless you are calculating the standard deviations from your plot you may be unaware of a low quality relationship.

Introduction

The Eyring equation was developed almost 100 years ago^{2,3} by Henry Eyring,⁴ Michael Polanyi⁵ and Meredith Evans.⁶

The Eyring equation was analytically derived from statistical thermodynamics and transition state theory. Unlike the Arrhenius equation, which was just a guess based on empirical observations, the Eyring equation provides useful information. The equation is expressed as...

$$k = \kappa \frac{k_B}{h} T \cdot e^{\frac{-\Delta G^\ddagger}{RT}}$$

...where κ is the transmission coefficient (assumed to be unity, but that's not always the case), R is the gas constant, k_B is the Boltzmann constant and h is the Planck constant. k is the observed rate constant and T is the absolute temperature.

The free energy term is an expression of enthalpy and entropy...

$$\Delta G^\ddagger = \Delta H^\ddagger - T\Delta S^\ddagger$$

...and so the Eyring equation can be expressed as...

$$k = \kappa \frac{k_B}{h} T \cdot e^{\frac{-\Delta H^\ddagger}{RT}} e^{\frac{\Delta S^\ddagger}{R}}$$

This can be presented in a linear form...

$$\ln \frac{k}{T} = \frac{-\Delta H^\ddagger}{R} \frac{1}{T} + \frac{\Delta S^\ddagger}{R} + \kappa \frac{k_B}{h}$$

The famous Eyring plot is a plot of $\ln k/T$ vs. $1/T$. The slope will be the value of $-\Delta H^\ddagger/R$ and the intercept will be the sum $\Delta S^\ddagger/R + \kappa k_B/h$. Assuming that ΔH^\ddagger and ΔS^\ddagger do not change with temperature in the range being studied, the plot should produce a straight line.

This document was produced using the L^AT_EX typesetting language with the Tufte-handout document class. Chemical diagrams were created in *ChemDoodle* Plots were typeset using *Matplotlib* and *Python* tools. Diagrams were created and edited using *Affinity Designer*.

¹ Read chapter 7 of "Modern Physical Organic Chemistry" by Anslyn & Dougherty, *University Science Books*, 2006 or any textbook that discussed chemical kinetics.

² "The activated complex in chemical reactions." H. Eyring, *J. Chem. Phys.*, 1935, 3, 107-115, <https://doi.org/10.1063/1.1749604>

³ "Some applications of the transition state method to the calculation of reaction velocities, especially in solution." M.G. Evans, M. Polanyi, *Trans. Faraday Soc.*, 1935, 31, 875-894, <https://doi.org/10.1039/TF9353100875>

⁴ Eyring was born in Mexico in 1901 and his family fled to the USA in 1912 during the Mexican revolution. Eyring was a Mormon. His father practiced polygamy and was married to the two great-aunts of former US presidential candidate Mitt Romney at the same time. Eyring was awarded just about every honour in chemistry except the Nobel prize.

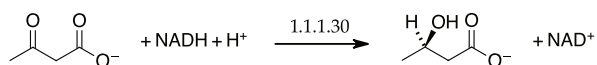
⁵ Polanyi fled Germany in 1933 and was appointed a professor of physical chemistry at the University of Manchester. He was a deep thinker and was appointed to a chair in social science at U. Manchester in 1948. He passionately argued that human nature was greater than the sum of its chemical parts. Two of his students, Eugene Wigner (a nuclear physicist who helped draft the famous "Einstein letter" that led to the creation of the atomic bomb) and Melvin Calvin (a leader in the elucidation of carbon fixation in photosynthesis) won Nobel prizes. His son, John Polanyi, is a professor of chemistry at U. Toronto and won the Nobel prize in 1986.

⁶ It wasn't a team effort; it was a race. All three proposed the equation around the same time in 1935. Evans was a student of Eyring and a colleague of Polanyi. Perhaps he was the link?

An Example

To demonstrate the Eyring plot, we must first obtain some experimental data. Here we will present some results from an enzyme kinetics experiment.⁷ the authors determine the Michaelis-Menten k_{cat} value for a reaction over a series of temperatures. As long as the chemical change is the rate-determining-step and not some other step (such as substrate binding or product release)⁸ then the Eyring equation will apply.⁹

The authors reported rate/temperature data for the reduction of the β -ketoacids 3-oxobutanoate and 3-oxopentanoate by NADH in the presence of *hydroxybutyrate dehydrogenase* (EC 1.1.1.30) from *Acinetobacter baumannii*.
0.12345(9)



The rate of reaction was followed by measuring the change in NADH concentration using UV-vis spectrometry. This is a precise method and should give repeatable results.

The Data

Below are two sets of data from the publication (always check the supplementary material for experimental details and data tables not presented in the publication.)

Temp. (/ K)	k_{cat} / s^{-1}			
	3-oxobutanoate		3-oxopentanoate	
283	3.4	± 0.1	0.21	± 0.01
288	5.3	± 0.2	0.28	± 0.01
293	7.6	± 0.2	0.46	± 0.02
298	11.7	± 0.3	0.60	± 0.03
303	15.2	± 0.1	1.16	± 0.02
308	21.3	± 0.9	1.47	± 0.03
313	27.8	± 0.9	2.07	± 0.09
318	39	± 3	2.7	± 0.1
323	52	± 4	4.1	± 0.2
325	61	± 2		
328	69	± 3	6.1	± 0.4
330	79	± 7	6.6	± 0.9

⁷ "Linear Eyring Plots Conceal a Change in the Rate-Limiting Step in an Enzyme Reaction", Teresa F. G. Machado, Tracey M. Gloster, and Rafael G. da Silva, *Biochemistry*, 2018, 57, 6757–6761. <https://doi.org/10.1021/acs.biochem.8b01099>.

⁸ It is an investigation into this very possibility that is the focus of this paper.

⁹ The Eyring equation requires a reaction that is a "single step." Multistep reactions with high-energy intermediates will also work as long as there is no significant concentration of the intermediate present during the reaction. If a significant portion of the reactant is converted to an intermediate that then converts to product in a r.d.s., such as in enzyme burst kinetics, the Eyring equation would not be appropriate. If one could determine the separate rate constants, then the Eyring equation could be successfully applied to each step.

Table 1: Michaelis-Menten turnover number, k_{cat} , vs. temperature for reaction of *hydroxybutyrate dehydrogenase* with 3-oxobutanoate and 3-oxopentanoate.

Data taken from tables S-1 and S-3 of the supplementary material for the the paper.⁷

Your Challenge

Complete the following activities.

- Construct an Eyring plot using the data above and perform a linear curve fit. Then complete the following activities.
 - Perform the curve fit with and without consideration of the estimated experimental errors. When using the experimental error, Express them using error bars on the plots. How did you propagate the error from the rate data through to the $\ln k_{cat}/T$ term?
 - Report the slope and intercept, their respective standard deviation and the squared Pearson correlation constant (r^2) for each substrate. Why are these parameters slightly different when we do or do not include the experimental errors for each point?
 - How you think the errors were determined in these data sets?
 - Calculate the values for ΔH^\ddagger and ΔS^\ddagger with error-propagation.^{10,11} Comment on the precision of each parameter. The r^2 value was great; why is the precision so poor? Compare your results with the results of the authors. Comment on any differences.
 - One of the parameters had lower precision than the other. Why does the design of the Eyring plot result in this poorer precision? Plot the Eyring plot with the x -axis starting from the origin (zero). Does that image provide any insight?
 - Using these activation parameters, calculate the predicted value of k_{cat} for each of the two cases. Using error-propagation, determine the estimated standard deviation for your results. How was the precision for this value?
 - Most Eyring plots in the literature use four or five data points. Select any four data points from the sets above and repeat the above exercises. What happened to your precision when using less data?
- Linearizing mathematical models for the convenience of using linear regression analysis is very 1980's.¹² Perhaps we could try fitting the data directly to k_{cat} vs. T using non-linear curve fitting? Repeat the exercises described above. How is the precision now?
- Document your methods for creating the plots, performing the curve fits and performing calculations with correct error-propagation. Explain how your method of documentation provides an unambiguous description of your math and methods. Does your documentation provide a tool that can be easily reused by yourself or your lab-mates in analyzing other sets of rate data at different temperatures?
- Where in this process did Microsoft Excel become useless to you?

¹⁰ Error propagation for the Eyring equation is expressed using the following two relationships:

$$\sigma_{\Delta H^\ddagger} = \sqrt{2 \frac{R^2 T_{max}^2 T_{min}^2}{(T_{max} - T_{min})^2} \left(\frac{\sigma_{k_{cat}}}{k_{cat}} \right)^2}$$

$$\sigma_{\Delta S^\ddagger} = \sqrt{\frac{R^2 (T_{max}^2 + T_{min}^2)}{(T_{max} - T_{min})^2} \left(\frac{\sigma_{k_{cat}}}{k_{cat}} \right)^2}$$

¹¹ Error propagation formula adapted from "A Static Agostic α -CH \cdots M Interaction Observable by NMR Spectroscopy: Synthesis of the Chromium(II) Alkyl $[\text{Cr}_2(\text{CH}_2\text{SiMe}_3)_6]^{2-}$ and Its Conversion to the Unusual 'Windowpane' Bis(metallacycle) Complex $[\text{Cr}(\chi^2\text{-C, C}' - \text{CH}_2\text{SiMe}_2\text{CH}_2)_2]^{2-}$." P.M. Morse et al., *Organometallics*, **1994**, *13*, 1646-1655. <https://doi.org/10.1021/om00017a023>

¹² Without a doubt, the '80's was the greatest decade – except for the computers. I miss my Commodore 64, but my current computer is one million times more powerful in every metric: 64 kilobytes vs. 16 gigabytes; no storage (or whatever an audio cassette tape could hold) vs. one terabyte; 4528 transistors vs. 16 million transistors; 1.5 MHz vs. 3.2 GHz; dial-up bulletin-board services using 54 kb/s modems vs. the internet at 300 mb/s over optical fibre.

We have enormous computing power at our fingertips now. Why do we stick with methods meant for graph paper?