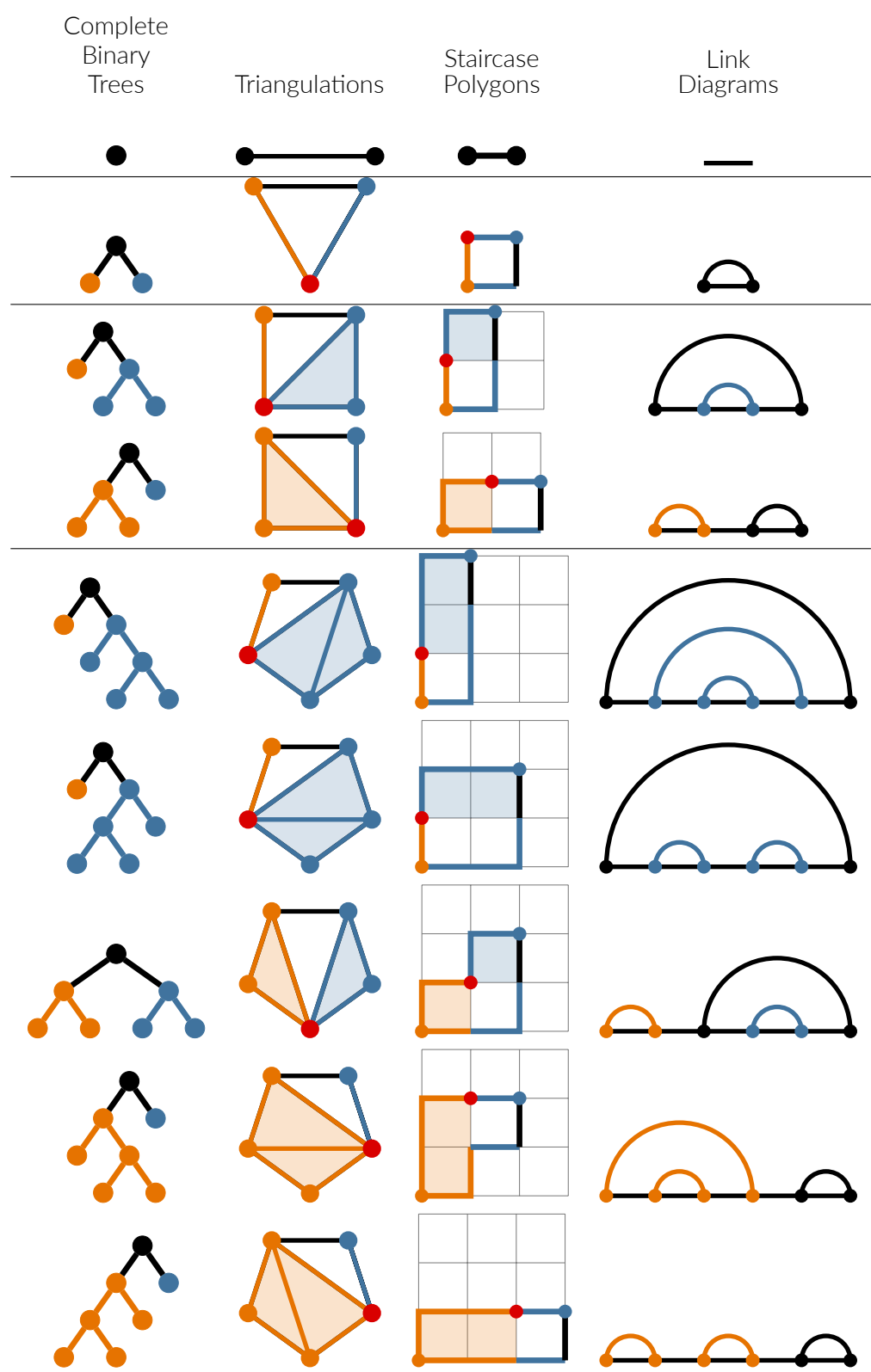
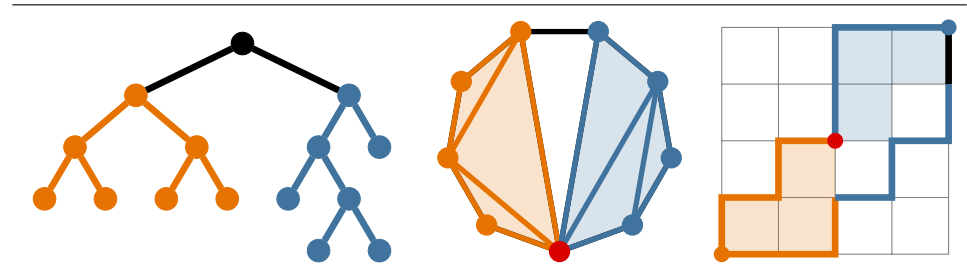


# magma : A Library of Universal Catalan Bijections

Billy Price, Supervisor: Richard Brak



A larger example - what does the corresponding link diagram look like?



1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862 . . .

The Catalan Numbers are an integer sequence (A000108) counted by any of 214 combinatorial families described in R. Stanley's *Catalan Numbers*. We would like to understand properties of these families via bijections to other Catalan families, however, it would be infeasible to design  $214 \times 213 = 4582$  direct bijections, and composing bijections lacks canonicity.

Through R. Brak's insight that any Catalan family is just a Free Magma on one generator, we reduce the problem of finding bijections between pairs of families, to finding a unique way of multiplying and factorising objects within a family (just once for each family!).

## magma

**magma** is an open-source library/catalogue of Catalan Families, which provides a universal bijection function between any two Catalan Families. Given two Catalan Families, **A**, **B**, we obtain the bijection, **bij** :: **A** -> **B** recursively:

```
def bij(x):
    if x == A.generator:
        return B.generator
    else:
        first, second := A.factorise(x)
        return B.multiply( bij(first), bij(second) )
```

Therefore, anyone with their own Catalan Family can implement a valid **multiply** and **factorise** function and immediately obtain a bijective function to/from any other Catalan Family.

## Representing Families

A central challenge to this project was deciding on the best way to represent each family's objects in the code, especially as many Catalan Families are geometric in nature. As is this re