magma : A Library of Universal Catalan Bijections

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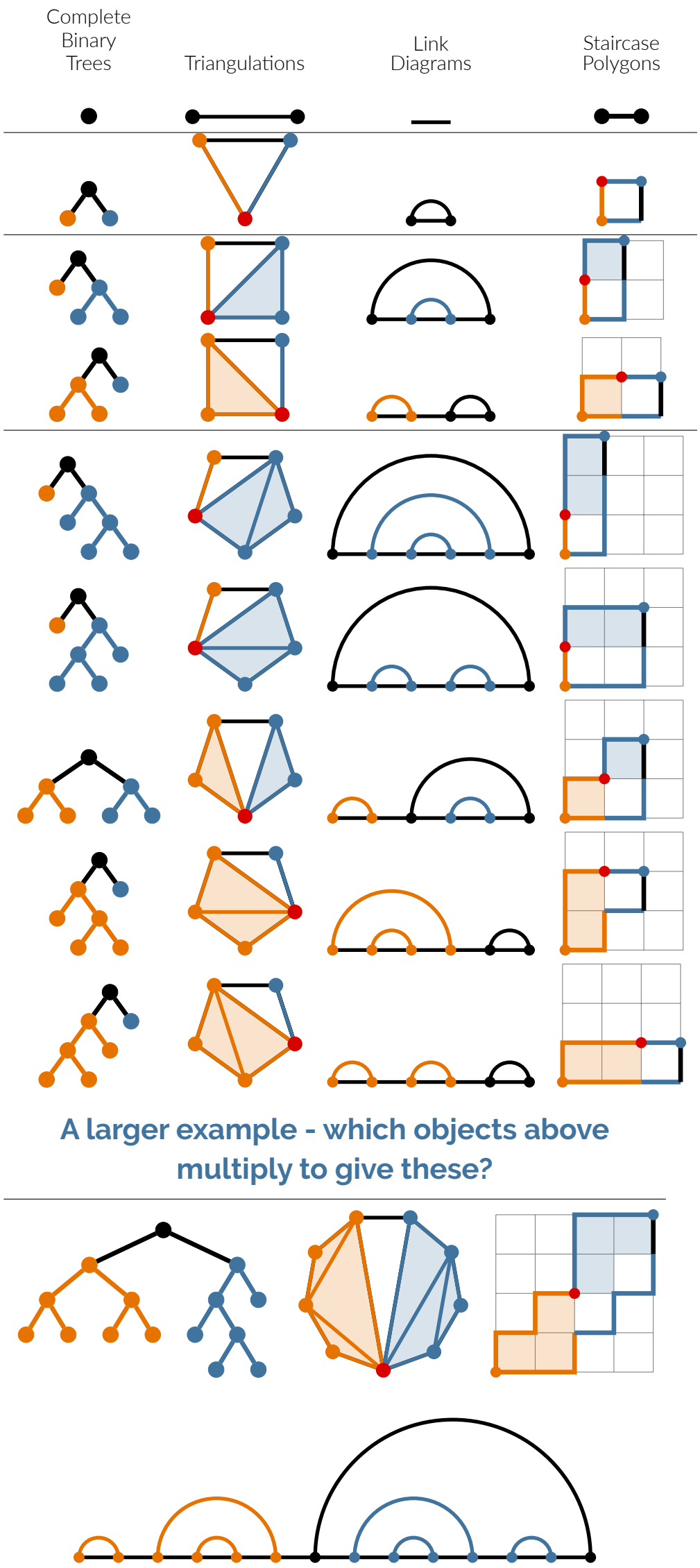

A larger example - which objects above multiply to give these?

$$1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862 \ldots$$

The Catalan Numbers are an integer sequence (A000108) counted by any of 214 combinatorial families described in R. Stanley's *Catalan Numbers*. We would like to understand properties of these families via bijections to other Catalan families, however, it would be infeasible to design $214 \times 213 = 4582$ direct bijections, and composing bijections lacks canonicity.

Through R. Brak's insight that any Catalan family is just a Free Magma on one generator, we reduce the problem of finding bijections between pairs of families, to finding a unique way of multiplying and factorising objects within a family (just once for each family!).

`magma`

`magma` is an open-source library/catalogue of Catalan Families, which provides a universal bijection function between any two Catalan Families. Given two Catalan Families, `A`, `B`, we obtain the bijection recursively:

```
bij(x) :: A -> B
  if x == A.generator:
    return B.generator
  else:
    first, second := A.factorise(x)
    return B.multiply( bij(first), bij(second) )
```

Therefore, anyone with their own Catalan Family can implement a valid `multiply` and `factorise` function and immediately obtain a bijective function to/from any other Catalan Family.

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