Computational Methods

UNIT-4

Assignment -4

Ques 1 Classify the following partial differential equation as hyperbolic, parabolic and elliptic :-

(i) $y\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$ Elliptic

(ii) $e^x u_{xx} + \cos y\, u_{xy} - u_{yy} = 0$ Parabolic

(iii) $u_{xx} + u_{yy} + u_x + \sin x\, u_y - u = x^2 + y^2$ Hyperbolic

Ques 2 Using Taylor's method, compute $y(0.2)$ to three decimal precision from $\frac{dy}{dx} = 1 - 2xy$ $y(0) = 0$

$$\frac{dy}{dx} = 1 - 2xy \qquad y(0) = 0$$

$$y(x) = y(x_0) + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x)$$

where $y(x_0) = 0$, $x_0 = 0$

$y'(x) = 1 - 2xy \quad = 1 - 2(0) = 1$

$y''(x) = -(2y + 2xy') = -(2(0) + 2(0)(1)) = 0$

$y'''(x) = -(2y' + 2xy'' + 2y') = -(2(1) + 2(0) + 2(1))$
$= -4$

$$y(0.2) = 0 + (0.2-0)(1 - 2x_0y_0) + \frac{(0.2-0)^2}{2!}(-2y_0 - 2xy'(0)) + \frac{(0.2)^3}{3!}y'''(0)$$

$$= 0 + 0.2(1) + \cancel{\frac{0.2(0.2)}{2}}(0) + \frac{0.2(0.2)(0.2)}{6}(-4)$$

$$= 0.2 + \left(\frac{-0.032}{6}\right)$$

$$= 0.2 + (-0.0053)$$

$\boxed{y(0.2) = 0.195}$ Ans.

Ques 3 Using Picard's Method of ~~...~~ successive approximation obtain a solution up to fifth approximation of equation

$$\frac{dy}{dx} = x + y \qquad y = 1, x = 0$$

Check your answer by finding exact particular solution

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\, dx \quad - (1)$$

Here, $x_0 = 0 \quad y_0 = 1 \qquad f(x, y) = x + y$

Put $n = 1$ in (1)

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$= 1 + \int_0^x (x + 1)\, dx$$

$$= 1 + \left[x + \frac{x^2}{2}\right]_0^x$$

$$\boxed{y_1 = 1 + x + \frac{x^2}{2}}$$

Put $n = 2$ in (1) $\quad y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\, dx$

$$= 1 + \int_0^x \left(x + 1 + x + \frac{x^2}{2}\right) dx$$

$$= 1 + \int_0^x \left(1 + 2x + \frac{x^2}{2}\right) dx$$

$$= 1 + \left[x + \frac{2x^2}{2} + \frac{x^3}{6}\right]_0^x$$

$$\boxed{y_2 = 1 + x + x^2 + \frac{x^3}{6}}$$

Put $n = 3$ in (1) $\quad y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\, dx$

$$= 1 + \int_0^x 2x + 1 + x^2 + \frac{x^3}{6}\, dx$$

$$\boxed{y_3 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}}$$

Put (1) ...

Put $n=4$ in (1)

$$y_4 = y_0 + \int_{x_0}^{x} f(x, y_3)\, dx$$

$$= 1 + \int_{0}^{x} \left(1 + 2x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}\right) dx$$

$$\boxed{y_4 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120}}$$

Put $n=5$ in (1)

$$y_5 = y_0 + \int_{x_0}^{x} f(x, y_4)\, dx$$

$$y_5 = 1 + \int_{x_0}^{x} 2x + 1 + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120}\, dx$$

$$\boxed{y_5 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{720}} \quad \text{— (II)}$$

Exact Solution:

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} - y = x$$

$$IF = e^{\int -dx} = e^{-x}$$

$$y(IF) = \int Q(IF)\, dx + c$$

$$ye^{-x} = \int xe^{-x}\, dx + c$$

$$ye^{-x} = \left[x\int e^{-x} dx - \int \left[\frac{d}{dx}(x) \int e^{-x}\right] dx\right]$$

$$ye^{-x} = -xe^{-x} - \int -e^{-x}\, dx$$

$$ye^{-x} = -xe^{-x} - e^{-x} + c$$

$$\boxed{y = -x - 1 + ce^{x}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} - - -$$

$$2e^x = 2 + 2x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + - - -$$

$$2e^x - 2 - 2x = x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} - - - \quad \text{— (III)}$$

Put (III) in (II)

$$y_5 = 1 + \cancel{x} + (2e^x - 2 - 2x) \Rightarrow -1 - x + 2e^x$$

$$\boxed{y_5 = 2e^x - x - 1}$$ approximate.

Ans.

$\cancel{y_5} =$

Ques 4 Consider the initial value Problem

$$\frac{dx}{dt} = 2 + (x - t - 1)^2, \quad x(1) = 2$$

Using Runge - kutta method of 4th order find $x(1.5)$ taking $h = 0.5$

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = hf(x_0, t_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, t_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, t_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, t_0 + k_3)$$

$$f(x, t) = 2 + (x - t - 1)^2 \qquad x_0 = 2 \quad t_0 = 1$$

$$k_1 = 0.5\left(2 + (2 - 1 - 1)^2\right) = 0.5(2) = 1$$

$$k_2 = 0.5\left(2 + (2.25 - 1.5 - 1)\right) = 0.875$$

$$k_3 = 0.5\left(2 + (2.25 - 1.43 - 1)\right) = 0.906$$

$$k_4 = 0.5\left(2 + (2.5 + 2.906)\right) = 0.797$$

$$\Rightarrow y_1 = 2 + \frac{1}{6}\left[1 + 2(0.875 + 0.906) + 0.797\right]$$

$$\boxed{x_1 = 2.893} \qquad t_1 = t_0 + h = 1 + 0.5 = 1.5$$

$$\boxed{x(1.5) = 2.893}$$

Ans.

Ques 5 Solve differential Equation

$$\frac{dy}{dx} = x + y \qquad y(0) = 1$$

for $y(0.1)$ & $y(0.2)$ by using modified euler's method correct to four decimal places.

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

where $y^*_{n+1} = y_n + h f(x_n, y_n)$

$f(x, y) = x + y \qquad x_0 = 0 \qquad y_0 = 1 \qquad h = 0.1$

$$y^*_1 = y_0 + h f(x_0, y_0) = 1 + 0.1[1 + 0] = 1.1$$

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y^*_1)\right]$$

$x_1 = x_0 + h$
$0 + 0.1$
$= 0.1$

$$= 1 + \frac{0.1}{2}\left[1 + [0.1 + 1.1]\right]$$

$$= 1 + 0.05[1 + 1.2]$$

$$= 1 + 0.05[2.2]$$

$$\boxed{y_1 = 1.11 = y(0.1)}$$

Ans.

$$y^*_2 = y_1 + h f(x_1, y_1) \Rightarrow 1.11 + 0.1[0.1 + 1.11]$$

$$= 1.231$$

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y^*_2)\right]$$

$$= 1.11 + \frac{0.1}{2}\left[0.1 + 1.11 + 0.2 + 1.231\right]$$

$$\boxed{y_2 = 1.24205 = y(0.2)}$$

Ans.

Ques7 Solve the initial value problem

$$\frac{dy}{dx} = 1+xy^2 \quad y(0)=1$$ find $y(0.4)$ by Milne's method,

when it is given that

x	0.1	0.2	0.3
y	1.105	1.223	1.355

$x_0 = 0 \quad y_0 = 1 \quad f_1 = 1+x_1y_1^2 = 1+(0.1)(1.105)^2 = 1.1221$

$h = 0.1 \quad f_2 = 1+x_2y_2^2 = 1+(0.2)(1.223)^2 = 1.2991$

$f_3 = 1+x_3y_3^2 = 1+(0.3)(1.355)^2 = 1.5508$

Milne Predictor

$$y_4 = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

$$= 1 + \frac{4(0.1)}{3}\left[2(1.1221) - 1.2991 + 2(1.5508)\right]$$

$$= 1.53956$$

$$f_4 = 1+x_4y_4^2 = 1+(0.4)(1.534)^2 = 1.947$$

Milne Corrector

$$y_4 = y_2 + \frac{h}{3}\left(f_2 + 4f_3 + f_4\right)$$

$$= 1.223 + \frac{0.1}{3}\left[1.2991 + 4(1.5508) + 1.947\right]$$

$$y_4 = 1.53797$$

$$f_4 = 1+x_4y_4^2 = 1+(0.4)(1.53797)^2$$

$$= 1.947$$

f_4

$$\boxed{y_4 = 1.53 = y(0.4)}$$

Ans.