

# 物理实验数学中心

Physics Expeiment Center



## Introduction to college physics experiments

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**NJUPT** 

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## Classrooms of experiments

- ♦ 0. Introduction, J2-532
- ♦ 1. Spectrometer, J2-532
- ♦ 2. Michelson Interferometer, J2-534
- ♦ 3. Oscilloscope, J2-418
- ♦ 4. Double Bridge, J2-417
- ♦ 5. Wheatstone Bridge, J2-416
- ♦ 6. Dielectric constant measurement, J2-429
- ♦ 7. Torsional Pendulum, J2-427

https://github.com/bliseu/phylab

# Part 1

Rules of this course

## 1. Preparations

#### **Key-points:**

- > Reference books, Handouts
- >Goals, methods, principles
- >Experimental conditions
- > Preparation reports

#### **Preparation report includes:**

- 1. Title;
- 2. Your name, student ID, date;
- 3. Experimental goals;
- 4. Experimental principles;
- 5. Experimental apparatus (models, specifications);
- 6. Data Tables

### 2. Experiments

#### At the beginning of the class, I will:

- >Check your preparation reports of this lesson;
- >Ask for your previous experiment reports;
- >Show the experimental principles and the operation.

#### During the experiments, you should:

- > Follow the operating rules;
- >Observe experimental phenomena;
- > Record the data.

- Experimental data and phenomenon;

  Record: date, time, place, temperature, pressure, instrument serial number, title, specifications, original data and phenomenon...
- Requirement: smooth writing, plotting (when necessary), data, results.
- Good habit: process experimental data and submit reports in time.

## 3. Contents of experimental reports:

- 1、Title
- 2. Goals
- 3. Instruments
- 4. Principles
- 5. Contents & steps
- 6. Data processing
- 7. Discussions and analysis

**Before class** 

after class

## Part 2

Measurement error and instrumental error range in physics experiments

#### 1. Measurement data & unit

data = value+unit

e.g.: 175.0 cm



The international system of units
(SI): meter (m), kilogram
(kg), second (s), ampere (A), Kelvin (K), mole (mol), candela (cd)

### 2. Measurement classification

direct passive

e.g.

The cylinder's density

$$\rho = \frac{4m}{\pi D^2 h}$$

3. Significant digit and more...

- (1). Significant digit: total digit number from first non-zero digit to the last one.
- 4.60cm #4.600cm
- (2). Significant digit's nature
  - a). Significant digit tells the precision of measuring instrument

#### length measurement with different precision

```
steel ruler: L=46.0mm, \Delta_{\chi}=0.1mm, E_r=0.13%  
Vernier calipers: L=46.00mm, \Delta_{\chi}=0.02mm, E_r=0.026%  
micrometer: L=46.000mm, \Delta_{\chi}=0.004mm, E_r=0.006%
```

# b) Significant digit have nothing to do with decimal point's location and unit

e.g. 4.60 cm 0.0460m 46.0mm

measurement results

0.0125m=1.25cm 1.0900cm≠ 1.09cm

8.88m=8880mm?

### (3). Scientific notation

Scientific notation: a×10<sup>n</sup> (单位)

$$8.88m=8.88 \times 10^{3}$$
mm  
 $80.30g=0.0803$ kg?= $80300$ mg?  
 $80.30g=8.030 \times 10^{1}$ g= $8.030 \times 10^{4}$ mg  
 $=8.030 \times 10^{-2}$ kg

#### (4). rules for rounding

- •If the last digit of a number is less than 5, drop it;
- •If it is more than 5, the number to be rounded add 1;
- •If it is 5, make the last digit be even.

e.g.: if 
$$m=3.065$$
kg,  $u_{cm}=0.042$ kg

$$m=(3.06\pm0.05)$$
kg

e.g.

7.146 rounded to two digits	7.1
0.086 rounded to one digits	0.09
2.4352 rounded to three digits	2.44
17.415 rounded to four digits	17.42
17.425 rounded to four digits	17.42

### (5). algorithms

#### a) addition and subtraction

$$A + B + C = 14.7 8 + 0.0047 - 1.503$$
$$= 13.2877 = 13.28$$

Shortest decimal

#### b) Multiplication and division

e.g.

$$A \times B \times C = 24.56\overline{8} \times 3.4\overline{5} \times 128.\overline{4}$$
  
=  $10883.13264 = 1.09 \times 10^4$ 

Shortest digit

## (6), the rule to read data(direct)

instrumental error or minimum graduation value

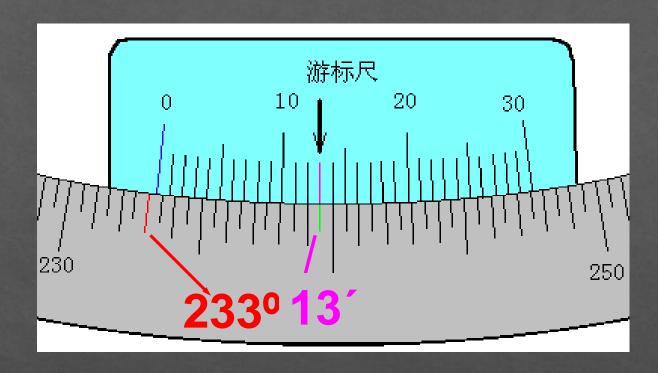
a: ruler

2cm or 20mm?



2.00cm or 20.0mm

#### b: angular vernier



e.g. vernier calipers

#### c: reading of digital meter



reading:23.9°C

## 4. error and uncertainty

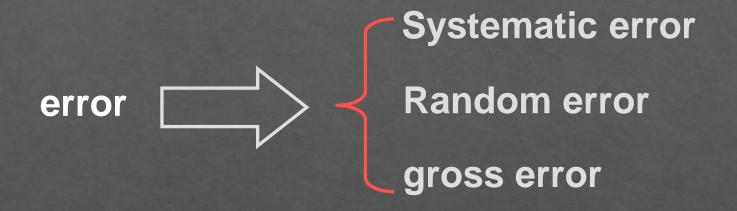
**Error and true value** 

true value: a

measured value: x

error:  $\varepsilon$   $\xi = \chi - \chi$ 

#### Classification of error



**Systematic error** 

## uncertainty

**★**uncertainty: range of fluctuation

#### the uncertainty u

- 1. A uncertainty:  $u_A$
- 2. B uncertainty:  $u_B$
- 3. Synthetic uncertainty:

$$u = \sqrt{u_{\rm B}^2 + u_{\rm A}^2}$$

#### A uncertainty

$$u_A = \frac{st}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{(n-1)} \cdot \frac{t}{\sqrt{n}}}$$

t distribution factor, n measurement times, probability P=0.95:

n	2	3	4	5	6	7	8	9	10
t	12.71	4.30	3.18	2.78	2.57	2.45	2.36	2.31	2.26
$\frac{t}{\sqrt{n}}$	8.99	2.48	1.59	1.24	1.05	0.93	0.84	0.77	0.72

#### **B** uncertainty

instrument error ,Limit error

instrument error : Δ

instrument error =Level × range

#### **B** uncertainty

$$u_{\rm B} = \Delta_{ins}$$

#### **Synthetic uncertainty**

$$u = \sqrt{u_B^2 + u_A^2}$$

#### measurement result:

$$N = N_M \pm u$$
 (unit)

*U*=Synthetic uncertainty,

 $N_{M}$ =best estimation of measurement data

#### Single measurement, Multiple/Repeated measurement

1. result of single direct measurement

single direct measurement

estimated true value: result of single measurement

$$u = u_B = \Delta_{ik}$$

$$N = N_m \pm \Delta$$
 (unit)

 $N_m$  have same decimal as  $\triangle$ 

#### 2 result of repeated measurement

$$N = \overline{N} \pm u$$
 (unit)

estimated true value: uncertainty:

$$u = \sqrt{u_A^2 + u_B^2}$$

$$\mu_{A} = \sqrt{\frac{\sum (N_{i} - \overline{N})^{2}}{(n-1)}} \cdot \frac{t}{\sqrt{n}} \qquad \mu_{B} = \Delta_{\emptyset}$$

# **Example 1:** Vernier caliper(division:50), cylinder diameter, 10 times.

times	1	2	3	4	5	6	7	8	9	10
d <sub>i</sub> (cm)	19.78	19.80	19.70	19.78	19.74	19.76	19.72	19.68	19.80	19.72

$$\bar{d} = \frac{\sum_{i=1}^{10} d_i}{10} = 19.75mm$$

$$u_{A} = S_{\overline{d}} \cdot \frac{t}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{10} (d_{i} - \overline{d})^{2}}{9}} \cdot 0.72 = 0.0108mm$$

$$u_B = \Delta_{fx} = 0.02 m m$$

#### uncertainty

$$u = \sqrt{u_A^2 + \Delta_{1/2}^2} = \sqrt{0.0108^2 + 0.02^2}$$

 $\approx 0.0227 \, mm \approx 0.023 \, mm$ 

#### diameter

$$d = \bar{d} \pm u_d = 19.750 \pm 0.023 (m m)$$

Same

decimal

## relative uncertainty

$$L_1$$
=(170.0±0.3) (cm)  $E_{r1}$ =0.18%  $L_2$ =(17.0 ±0.3) (cm)  $E_{r1}$ =1.8% relative uncertainty, $E_r$ 

$$E_r = \frac{u}{\overline{N}} \times 100\%$$

## (3) percentage error

$$E_0 = \frac{\left|N - N_0\right|}{N_0} \times 100\%$$

$$E_r=1.54\%$$
  $E_r=1.6\%$ 

$$E_r=3.82\%$$
  $\longrightarrow$   $E_r=3.9\%$ 

$$E_0 = 5.04\% \longrightarrow E_0 = 6\%$$

#### **Indirect measurement**

if, 
$$N = f(x, y, z...)$$

x, y, z.... direct measurement

if  $\overline{x}$ ,  $\overline{y}$ ,  $\overline{z}$  ... best estimates of direct measurement

best estimates of indirect measurement

$$\overline{N} = f(\overline{x}, \overline{y}, \overline{z} \cdots )$$

## **Estimate of uncertainty (indirect measurement)**

## N uncertainty

$$u_N = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 u_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 u_y^2 + \cdots}$$

### N relative uncertainty

$$E_r = \frac{u_N}{N} = \sqrt{\left(\frac{\partial \ln f}{\partial x}\right)^2 u_x^2 + \left(\frac{\partial \ln f}{\partial y}\right)^2 u_y^2 + \cdots}$$

#### Result of indirect measurement

$$N = \overline{N} + \mu_N$$

$$\overline{N} = f(\overline{x}, \overline{y}, \cdots)$$

## **★** calculation processing (uncertainty)

- 1. direct measurement  $(x, y, \dots)$  uncertainty  $(u_x, u_y, \dots)$
- 2. function  $N = f(x, y, \dots)$

**N** total derivative 
$$dN = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \cdots$$

3. N uncertainty  $u_N$ 

$$u_N = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 u_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 u_y^2 + \cdots}$$

- **★** calculation processing (relative uncertainty)
- 1. direct measurement uncertainty  $u_x$ ,  $u_y$ , ...;
- **2.** function  $N = f(x, y, \dots)$

logarithm  $N = \ln f(x, y, \dots)$ 

In *N*total derivative 
$$\frac{dN}{N} = \frac{\partial \ln f}{\partial x} dx + \frac{\partial \ln f}{\partial y} dy + \cdots$$

3. relative uncertainty of N

$$E_r = \frac{u_N}{N} = \sqrt{\left(\frac{\partial \ln f}{\partial x}\right)^2 u_x^2 + \left(\frac{\partial \ln f}{\partial y}\right)^2 u_y^2 + \cdots}$$

## **Example 2:** Using pendulum to measure

acceleration of gravity:  $g=4\pi^2l/T^2$ ,

 $T=2.000 \pm 0.002s, l=100.0 \pm 0.1cm$ , try to write a

gravitational acceleration g expression.

answer:

$$g = \frac{4\pi^2 l}{T^2}, \ln g = \ln 4\pi^2 + \ln l - 2 \ln T$$

$$\frac{\partial \ln g}{\partial l} = \frac{1}{l}, \frac{\partial \ln g}{\partial T} = -\frac{2}{T}$$

$$E_r = \frac{u_g}{g} = \sqrt{\left(\frac{\partial \ln g}{\partial l}\right)^2 u_l^2 + \left(\frac{\partial \ln g}{\partial T}\right)^2 u_T^2}$$

Thus: 
$$E_r = \sqrt{(\frac{u_l}{l})^2 + 4(\frac{u_T}{T})^2} = \sqrt{(\frac{0.1}{100})^2 + 4(\frac{0.002}{2.000})^2}$$
  
 $= 2.24 \times 10^{-3} = 0.23 \%$   
 $g = \frac{4\pi^2 l}{T^2} = \frac{4 \times 3.1416^2 \times 100.0}{2.000^2} = 987.0(cm/s^2)$ 

$$u_{cg} = E_r \cdot g = 2.24 \times 10^{-3} = 2.3 (cm / s^2)$$

Round to the nearest tenth, carry only!

acceleration of gravity 
$$g=(987.0\pm 2.3)$$
 (cm/s<sup>2</sup>)

align

# Part 3 Methods of data processing

## 1. Tabulation

Notes:

- a. Table design: reasonable, straightforward, complete.
- b. Title bar: physical quantity's name, mark,unit.
- c. Data: effective number.

- d. instruction and parameter (table name, measuring instrument specifications, environment condition).
- e. main->original data, important intermediate result.

#### Table Data of wire diameter measurement

Instrument: micrometer caliper, Level: 1, range:  $0 \sim 25$ mm, error:  $\pm 0.004$ mm,initio data:-0.005mm

Order	data (mm)	diameter D <sub>I</sub> (mm)	$D_i - \overline{D}$ (mm)	$(D_i - \overline{D})^2 (\times 10^{-8} mm^2)$
1	0.280	0.285	0.0022	484
2	0.278	0.283	0.0002	4
3	0.275	0.280	-0.0028	784
4	0.284	0.289	0.0062	3844
5	0.272	0.277	-0.0052	2704
6	0.278	0.283	0.0002	4
Average		0.282 8		$\sum (D_i - \overline{D})^2 = 7824$

$$S_{\overline{D}} = 1.7 \times 10^{-3} mm, \sigma_{\chi\chi} = \frac{\Delta_{\chi\chi}}{\sqrt{3}} = \frac{0.004 mm}{\sqrt{3}} = 2.3 \times 10^{-3} mm,$$

$$u_{cD} = \sqrt{S_{\overline{D}}^2 + {\sigma_{\chi\chi}}^2} = 2.9 \times 10^{-3} mm = 0.003 mm,$$

$$D = \overline{D} \pm u_{cD} = (0.283 \pm 0.003) mm$$

## 2. Graphical method

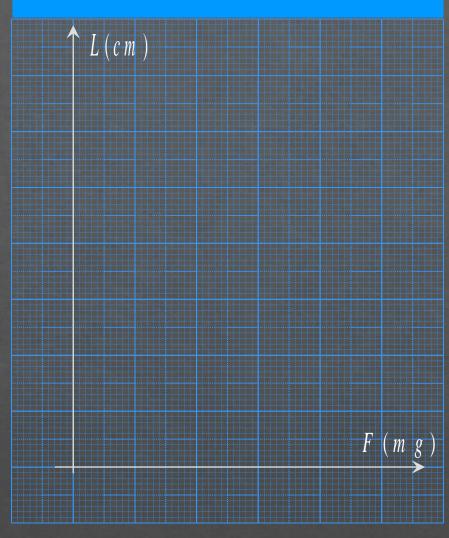
**≻**Numbers, Lines

**►** Mapping rules

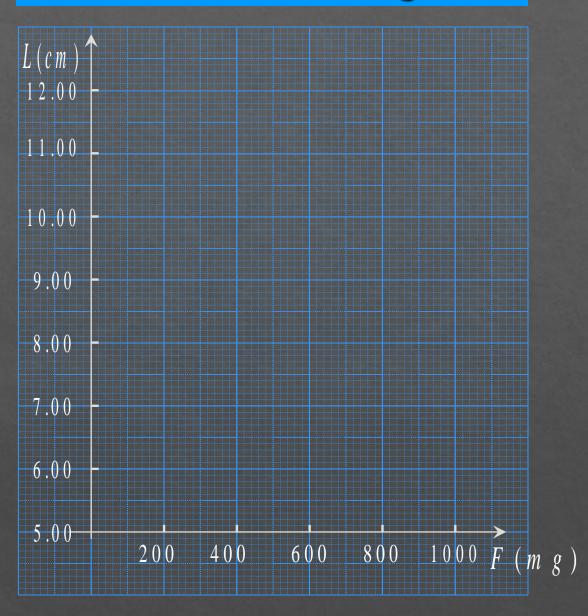
## Mapping rules

- 1. Plotting From Data table
- 2. Using coordinate paper

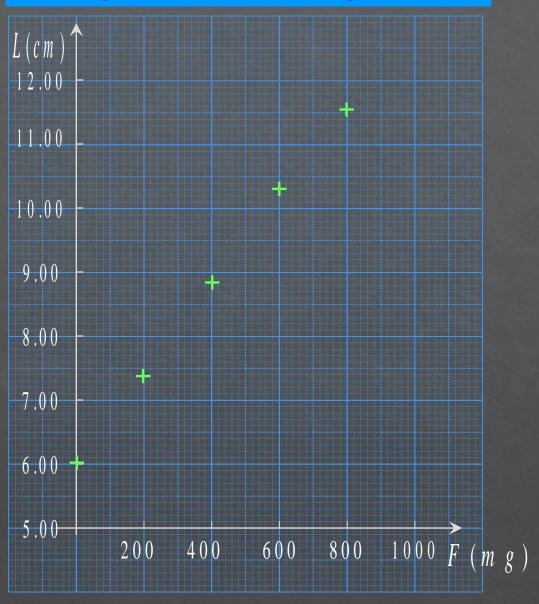
## 3. Coordinates



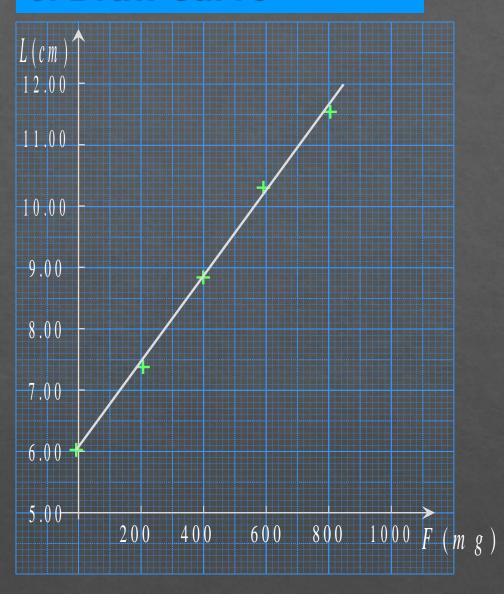
## 4.Coordinate indexing



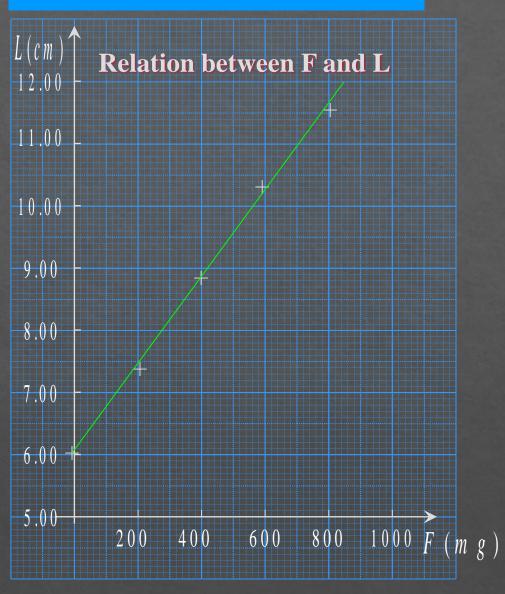
## 5. Experimental data points



## 6. Draw curve



## 7. Introductions



## **Graphical method**

1) Coordinate values

$$(x_1, y_1)$$
,  $(x_2, y_2)$ .

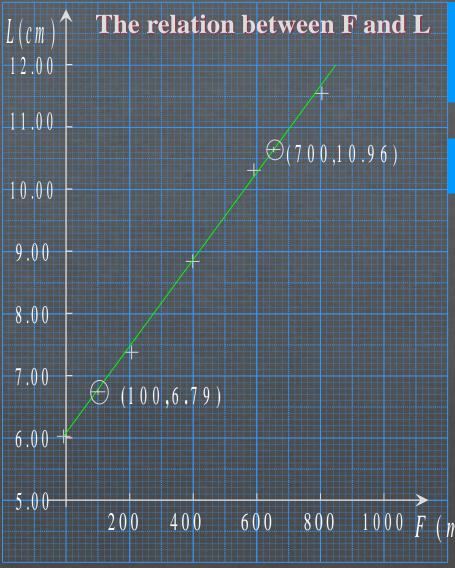
equation of

$$y = a + b x$$

straight line:

2) slope b: 
$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

3) intercept 
$$a: a = \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$$



•Mark 2 points on the line.

Note the distance

$$K = \frac{(700 - 100) \times 9.794 \times 10^{-6}}{(10.96 - 6.79) \times 10^{-2}}$$
$$= 0.141N / m$$

### 3. Method of successive minus

- method of successive minus:
  - experimental data->table
    verify law of variation of data.
  - Make full use of data, reduce measurement error.

➤ Verify the *linear* relationship of L<sub>i</sub> and F。

$$\Delta \overline{L} = \frac{1}{7} [(L_1 - L_0) + (L_2 - L_1) + \dots + (L_7 - L_6)]$$

$$= \frac{1}{7} [L_7 - L_0]$$

Intermediate data × beginning and end ○.

If we divide data into  $(L_7, L_6, L_5, L_4)$  and  $(L_3, L_2, L_1, L_0)$   $\circ$ 

$$\Delta \overline{L} = \frac{1}{4} [(L_7 - L_3) + (L_6 - L_2) \cdots + (L_4 - L_0)]$$

Make full use of data, Keep all the merits of Multi-times measurement.

## Summary

- 1. Rules of this course;
- 2. Significant digit, rules for rounding, algorithm, reading
- 3. Uncertainty calculation, rules for rounding

 $\mathbf{U}_{\mathbf{A}}$ ,  $\mathbf{U}_{\mathbf{B}}$ ,  $\mathbf{U}$ 

**Indirect measurement** 

#### 4. Measurement results:

$$N = N_M \pm u_c$$
 (unit)

 $u_c$ : Synthetic uncertainty,  $N_M$ : best estimation of measurement data

## 5. Data processing methods:

tabulation method, graphical method, method of successive minus

## END