

# A Naive Bayes Framework for Estimating the Competency of Institutions Applied to the United States Congress

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## 1 Introduction

In this note I introduce a Naive Bayes framework for estimating the competency of institutions in representing the interests of the public, for the simple case of institutions which make binary decisions informed exclusively by independently voting representatives of the public. Making the assumption that the competence of a representative in representing the public interest is a function of the public approval and disapproval percentages of the representative and the assumption that representatives represent party interests, I then use Gallup's Party Images polling and CSPAN voting records to visualize the competence of the chambers of the United States Congress in representing the public interest over successive six-month periods from 1993 to the present.

## 2 Naive Bayes Framework for Estimating the Competency of an Institution

An institution can be said to act in the public interest when making a decision if it makes the decision most likely to be in the public interest given the available evidence. The competency of an institution in representing the public interest can be estimated by the agreement between the decisions actually made by the institution and the decisions which were most likely to be in the public interest given the available evidence. An institution which always made the decision most likely to be in the public interest given the available evidence would have a competence of 1, and an institution which never made the decision most likely to be in the public interest given the available evidence would have a competence of 0.

If the available evidence is an observed voting profile of a body of representatives, each voting Yea or Nay in regards to taking an action, then the competency of an institution is the agreement between the actions actually taken by the institution and the actions most likely to be in the public interest given the observed representative voting profiles. Therefore, if I define a method for estimating the decisions most likely to be in the public interest given the observed voting profiles of a body of representatives, then I can use the observed voting profiles of a time period to measure the agreement between the institutional and "epistemic" decisions over the time period, and therefore the competency of the institution in representing the public interest during the time period.

In order to define a method for estimating the decision most likely to be correct given the observed voting profile of a body of representatives as evidence, and therefore be able to estimate the competency of institutions informed by voting representatives, this section first goes over how Bayes theorem can be applied in a binary setting to estimate the probability an option is correct given a set of available evidence, then reviews how an observed voting profile of a body of representatives can be used as evidence.

### 2.1 Bayes Theorem for Binary Decisions

Say a decision must be made between two options, option 0 and option 1. One of the options is "correct" - call it option  $X$ . The respective probabilities of option 0 and option 1 being correct given the available evidence  $v$  are the respective likelihoods of observing the the evidence  $v$  given option 0 and option 1 being correct, relative to total likelihood of observing the evidence  $v$ .

$$P(X = 0|v) = \frac{P(v|X = 0)P(X = 0)}{P(v|X = 0)P(X = 0) + P(v|X = 1)P(X = 1)}$$
$$P(X = 1|v) = \frac{P(v|X = 1)P(X = 1)}{P(v|X = 0)P(X = 0) + P(v|X = 1)P(X = 1)}$$

Where  $P(v|X = 0)P(X = 0)$  is the probability of observing the evidence  $v$  given the correct option being 0 multiplied by the prior probability the correct option is 0 and  $P(v|X = 1)P(X = 1)$  is similarly the probability of observing the evidence  $v$  given the correct option being 1 multiplied by the prior probability the correct option is 1.

Prior probabilities are probabilities calculated using the evidence available prior to the availability of the evidence currently being considered. If no evidence has been gathered prior to  $v$ , then no option can be favored by any evidence, and the prior probability of each option must be the same. In the binary case, both options would have a prior probability of  $\frac{1}{2}$ .

In general, the probability option  $x$  is the correct option given observed evidence  $v$  is

$$P(X = x|v) = \frac{P(v|X = x)P(X = x)}{\sum_{x=0}^1 P(v|X = x)P(X = x)}$$

Where the sum is over all possibilities for the correct option. The value of  $x$  for which  $P(X = x|v)$  is maximized is the option most likely to be correct given the available evidence  $v$ .

### 2.2 Voting as Producing Evidence

Assume a body of  $N$  voters is to produce the evidence meant to inform a decision as to whether or not to take an action. Calling taking the action option 1 and not taking the action option 0, the probability that the  $x$ th option is the correct decision given an observed voting profile  $\vec{v} = [v_1, v_2, \dots, v_N]$  as evidence is

$$P(X = x|\vec{v}) = \frac{P(\vec{v}|X = x)}{\sum_{x=0}^1 P(\vec{v}|X = x)}$$

Where  $P(\vec{v}|X = 0)$  is the probability of observing the voting profile  $\vec{v}$  given the correct decision being option 0,  $P(\vec{v}|X = 1)$  is the probability of observing the voting profile  $\vec{v}$  given the correct decision being option 1, and the prior probabilities have been factored out because the votes are assumed to be the only available evidence.

If the voters are naively assumed to vote independently conditional on the correct decision such that  $P(v_i = u|v_j = w, X = x) = P(v_i = u|X = x)P(v_j = w|X = x)\forall(i, j, x, u, v), i \neq j$ , and if the  $i$ th voter is assumed have competence  $p_i$  such that  $P(X = 1|v_i = 1) = P(X = 0|v_i = 0) = p_i$  and  $P(X = 0|v_i = 1) = P(X = 1|v_i = 0) = 1 - p_i$ , then the probability of observing the voting profile  $\vec{v}$  given the correct choice being the  $x$ th option is

$$P(\vec{v}|X = x) = \prod_{i=1}^N (x - p_i - v_i)(-1)^{v_i}$$

The probability that the  $x$ th option is the correct decision given the available evidence  $\vec{v}$  then simplifies to

$$P(X = x|\vec{v}) = \frac{1}{1 + \prod_{i=1}^N \left( \frac{1-p_i-v_i}{p_i-v_i} \right)^{-x}}$$

For a given vector of competence parameters  $\vec{p}$ , the option for which this equation takes the largest value is the option most likely to be the correct decision given the observed voting profile  $\vec{v}$  as evidence.

### 3 Application to the United States Congress

The chambers of the United States Congress take institutional action through majority rule. For each action considered by a chamber, each representative of the chamber votes Yea or Nay; if a majority (or supermajority) of the representatives vote Yea, then the proposed action is taken; if not, then the proposed action is not taken.

Congressional voting profiles and the resulting institutional decisions are available on CSPAN's website [1] [2] for each roll-call vote since 1989. If I assume vectors of representative competencies for each vote, then I can use the congressional voting profiles to estimate the corresponding epistemic decisions, and I can compare those epistemic decisions to the institutional decisions in order to estimate the competence of the chambers of Congress in representing the public interest during that time period.

It is, of course, not possible to know the actual values of the parameters. It is not possible to know exactly how competent a representative is in representing the public interest. In lieu of having the resources necessary to determine reasonable estimates empirically, the best I can do is make heuristically reasonable assumptions as to what the parameters might be approximated by, then interpret the resulting estimations as approximations particular to the assumptions.

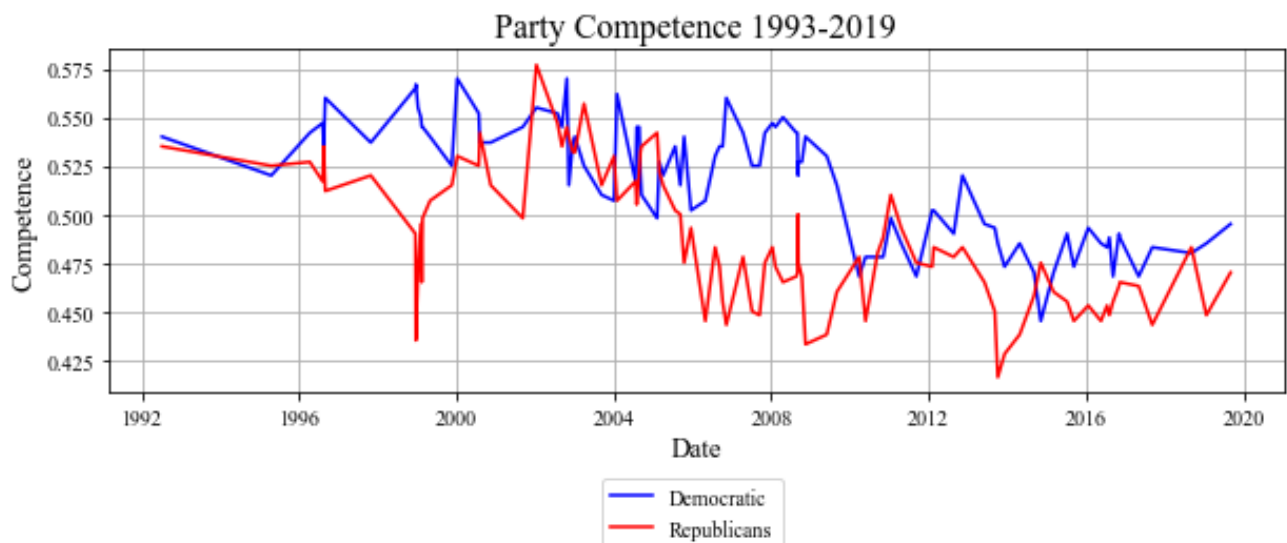
This section first goes over the parameter choices I made and how they might be improved upon, then uses the chosen parameters to estimate the competencies of the chambers of the U.S. Congress over successive six-month periods from 1993 to the present.

#### 3.1 Voter Competence Assumption

I assume that each voter is a party representative, and that the competence of a party representative at the time of a vote is the logistic of the difference of the party approval and disapproval percentages reported nearest to the time of the vote.

The heuristic reasoning leading me to assume that individual representatives are party representatives is that the public frequently elects representatives based on their party affiliation instead of their individual characteristics; and that representatives frequently vote along party lines instead of according to individual convictions.

The heuristic reasoning leading me to assume that the competence of party representatives is dependent on party approval and disapproval is that a party which is more approved than disapproved might be so because they are more likely than not to vote in the public interest, and visa versa; and that the probability a party votes in the public interest would increase diminishingly as the difference between its approval and disapproval ratings increases, and similarly decrease as the same difference decreases. The logistic of the difference between approval and disapproval fits this heuristic reasoning.



This figure displays the party competences for the Democratic and Republican parties from 1993 to the present, calculated using the Gallup Party Images series of polls [3]. These are the values which I take as individual voter competencies while producing the institution-epistemic agreement plots displayed in the next section.

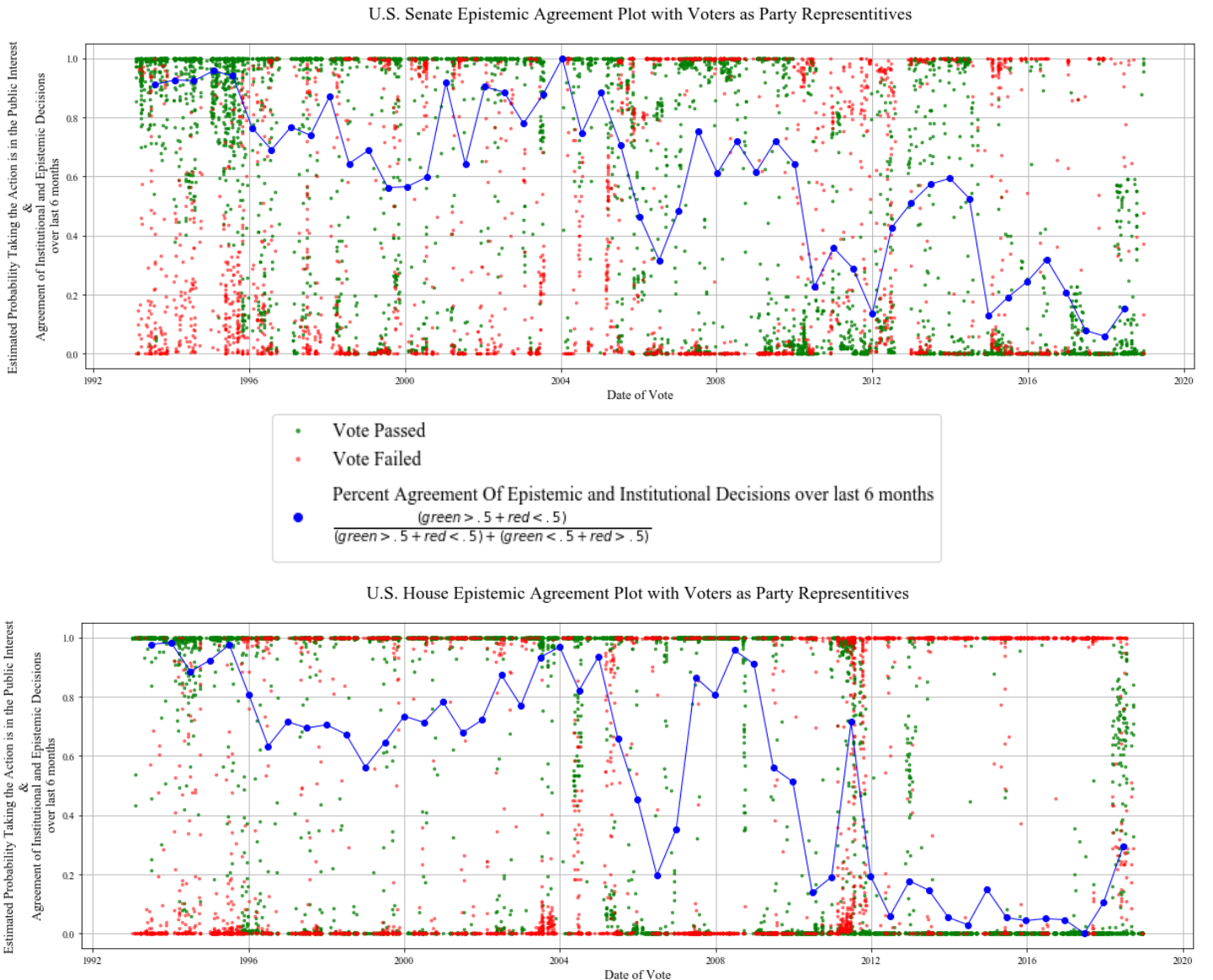
For example, Gallup polls concluding on September 15th 2019 reported the Democratic party as having approval and disapproval ratings of .48 and .50, respectively, and similarly the Republican party as having approval and disapproval ratings of .43 and .55. Consequently, for dates closer to September 15th 2019 than to any other date polls concluded, I take the competencies of Democratic and Republican representatives,  $p_D$  and  $p_R$ , to be

$$p_D = \frac{1}{1 + e^{-(.48-.50)}} = .495$$

$$p_R = \frac{1}{1 + e^{-(.43-.55)}} = .470$$

These competence parameter assumptions could be improved by fitting a trend line to the poll results, and taking the competencies from the exact date on the fit line instead of from the date of the nearest concluded poll. The assumptions could be further improved by conducting and aggregating a number of national polls asking how much each individual representative is trusted to act in the public interest. Preferably the polls would be taken at regular intervals and according to a regular standard. This would allow representatives to be treated as individuals instead of as party representatives, and for the probabilities of the public to directly factor into a standardized measure of the competence of institutions in representing the public interest.

### 3.2 Institution-Epistemic Agreement over Time



The blue lines display the estimated competencies of the chambers of the U.S. Congress in representing the public interest over successive six month periods from 1993 to the present. Each green/red dot represents an action voted on by a chamber of congress; its color indicates whether or not the action was taken, and its vertical position is the estimated probability taking it is in the public interest. By my assumed definition of institutional competence, a perfectly competent institution would have all green dots above .5 vertically and all red dots below .5 vertically, indicating that it took all actions which were most likely to be in the public interest and did not take all actions which were not most likely to be in the public interest.

The competency of U.S congressional institutions appears to have decayed over the last generation. The decay reflects the transition of our political parties from trustworthy to untrustworthy in the eyes of the public; once the parties crossed the competence threshold of .5, they became more likely to vote against the public interest than with the public interest, by definition.

Across the threshold, individual representatives are less reliable than a coin flip; a coin with Yea on one side and Nay on the other is more likely to vote in the public interest than a representative. In fact, each vote by a representative for an option becomes evidence against it, if the representative has a less than a 50% chance of voting in the public interest.

Because both political parties presently have a competence below .5, unanimous decisions are estimated as having an approximately 0% chance of being in the public interest. For party line votes, because the Republican party is presently less competent than the Democratic party, the Democratic choice is seen as being more likely than the Republican choice to be in the public interest; the Republican votes provide more evidence for the Democratic choice than the Democratic votes do for the Republican choice.

Because congressional voting data only started to be digitized in 1989 and party approval data only started to be measured in 1993, there is no convenient way for me to compare the trajectory observed here to earlier trends.

### References

[1] <https://www.c-span.org/congress/votes/?chamber=house>  
 [2] <https://www.c-span.org/congress/votes/?chamber=senate>  
 [3] [news.gallup.com/poll/24655/party-images.aspx](https://news.gallup.com/poll/24655/party-images.aspx)