This note explores the effect of varying wealth inequality on expected GDP growth rate in a simplified economy, under the assumption that consumer behavior is motivated by the local distribution of available consumption opportunities. First, I introduce a theoretical framework for estimating expected GDP growth rate given a distribution of wealth and a distribution of available consumption opportunities. Second, I apply the framework to estimate how expected GDP growth rate varies with wealth inequality and distribution of opportunity.

Consider a simple economy with no taxes or subsidies, composed of M units of wealth distributed among N economic agents. Taking the value-added perspective, the increase of the GDP of the economy over a time period δt - the GDP growth rate - can be written as the total wealth added through productive processes by the agents during the time period δt.

$$\delta GDP = \sum_{i=1}^{N} p_i - c_i$$

Where p_i is the wealth produced by the productive processes completed during δt by economic agent i and c_i is the wealth consumed by the same agent in the the same processes. Assuming a constant return on investment r such that $p_i = (1+r)c_i$, the growth rate can be written as

$$\delta GDP = \sum_{i=1}^{N} rc_i$$

Where the product rc_i is the surplus value produced by economic agent i during δt. Consequently the expected GDP growth rate can be written as

$$E[\delta GDP] = r\sum_{i=1}^{N} E[c_i]$$

Assuming that the amount of wealth consumed by an economic agent during time period δt is a random variable with a probability distribution characterized by the agent's wealth, such that the expected amounts consumed by agents of identical wealth are identical; and that an economic agent can only consume as much wealth as it has, expected GDP growth rate can be written as

$$E[\delta GDP] = r\sum_{w=0}^{M} N(w)\sum_{c=0}^{w} cP(c,w)$$

Where $N(w)$ is the number of economic agents having wealth w, the inner sum is over all possible amounts of wealth an economic agent of wealth w can consume, and $P(c,w)$ is the probability that an economic agent with wealth w will consume amount of wealth c.

Assuming that the probability an agent with wealth w consumes an amount c is the number of opportunities to consume amount c relative to the total number of consumption opportunities available to an agent with wealth w, $P(c,w)$ can be written as

$$P(c,w) = \frac{O(c)}{\sum_{c=0}^{w} O(c)} = \frac{O(c)}{O_a(w)}$$

Where $O(c)$ is the number of opportunities to consume amount of wealth c, and $O_a(w)$ is the total number of consumption opportunities available to an economic agent of wealth w. The expected GDP growth rate is then

$$E[\delta GDP] = r\sum_{w=0}^{M} \frac{N(w)}{O_a(w)}\sum_{c=0}^{w} cO(c)$$

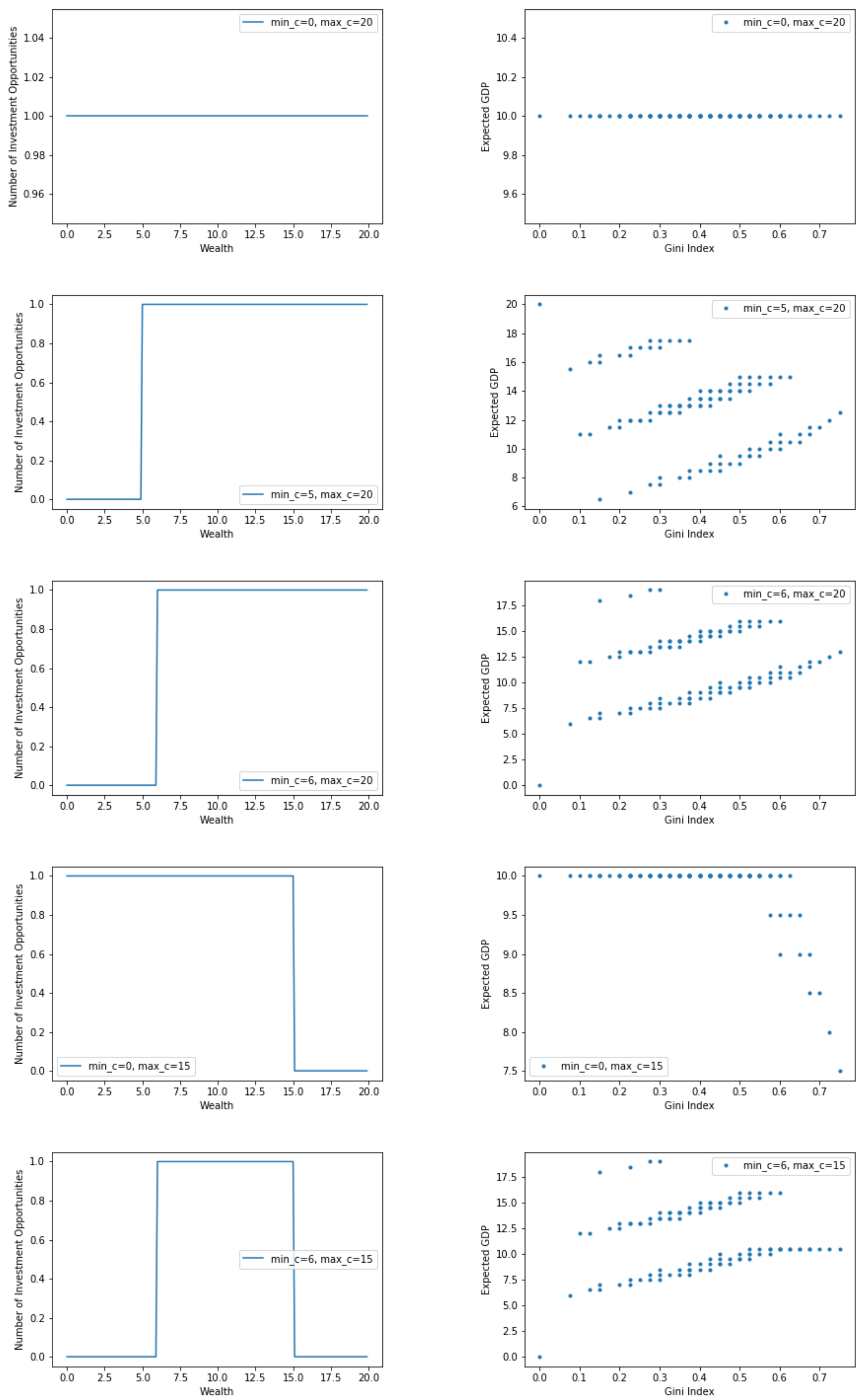
Given a wealth distribution $N(w)$ and an opportunity distribution $O(c)$, the expected GDP growth rate can be easily computed.

In order to estimate how changing wealth inequality tends to change expected GDP growth rate, I plotted Gini index against expected GDP growth rate for all possible $N = 4, M = 20$ wealth distributions for uniform, sinusoidal, exponential growth, and exponential decay type opportunity distributions with various minimum and maximum allowable consumption amounts.

Imposing minimum and maximum consumption amounts is an imperfect attempt to account for the barriers to entry to the capital market and the limitations to the amount of capital which can be marketed, respectively. Meaning, the imposition accounts for the fact that if someone has only enough wealth to pay for food and rent, then they wont be investing; and the fact that there are, for example, no ways to use 120 trillion dollars (roughly the net worth of the US) at once.

Uniform Opportunity Distribution

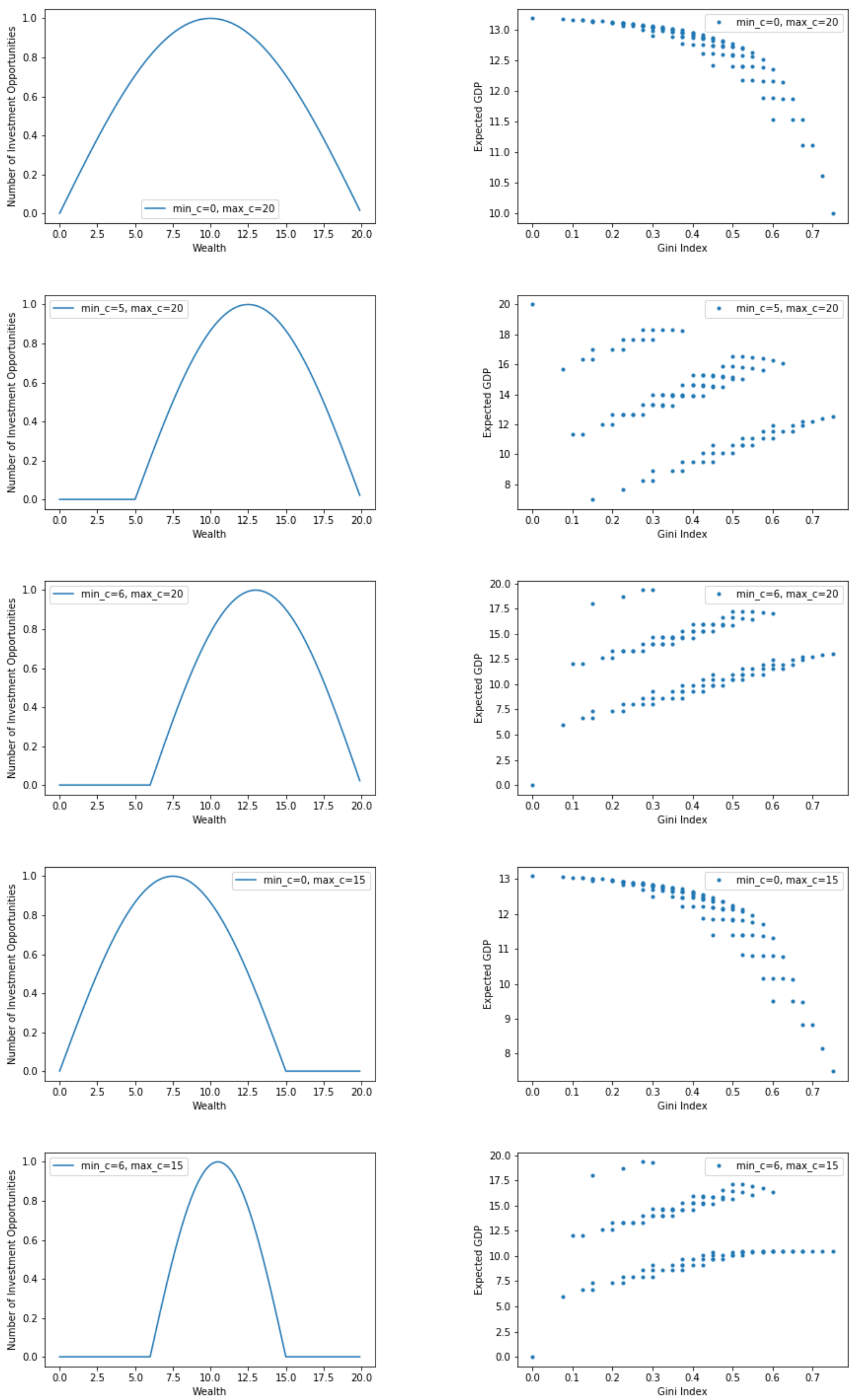
$$O(c) = \begin{cases} 1 & c < max_c, c > min_c \\ 0 & otherwise \end{cases}$$



If there is no minimum or maximum investment, then all wealth distributions have equal growth. If there is a minimum investment, but small enough that each agent can have enough to invest, then equality maximizes growth; and decreasing the number of agents with enough to invest tends to decrease growth, but, within a given number of agents having enough to invest, increasing inequality tends to increase growth. If there is a minimum investment large enough that not every agent can have enough to invest, then equality produces no growth, and maximum growth comes from maximizing inequality after maximizing the number of agents having enough to invest. If there is a maximum investment, then increasing inequality past a critical range tends to decrease growth. If there is a minimum and maximum investment, then the effect on growth is a combination of the effects of applying a minimum and a maximum investment individually.

Sinusoidal Opportunity Distribution

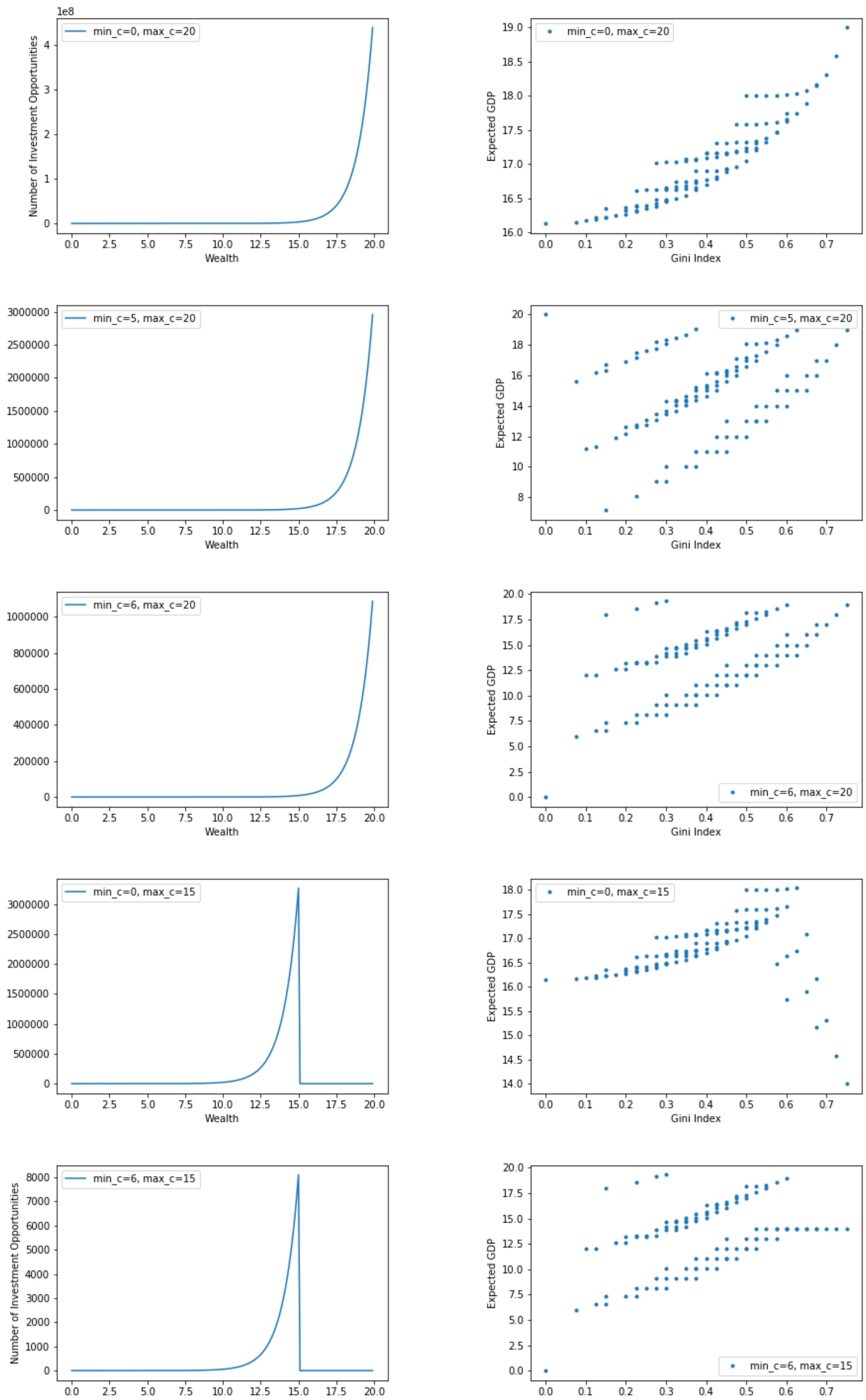
$$O(c) = \begin{cases} sin(\frac{\pi(c-min_c)}{(max_c-min_c)}) & c < max_c, c > min_c \\ 0 & otherwise \end{cases}$$



If there is no minimum or maximum investment amount, then equality maximizes growth, and increasing inequality slowly decreases growth until, after a critical inequality range, increasing inequality rapidly decreases growth. The decrease occurs because in the sinusoidal opportunity case as an agent's wealth increases the likelihood it invests all its wealth decreases, by virtue of the assumed wealth-investment relation. Adding minimum and maximum investment amounts has the same general effect as the uniform case.

Exponential Growth Opportunity Distribution

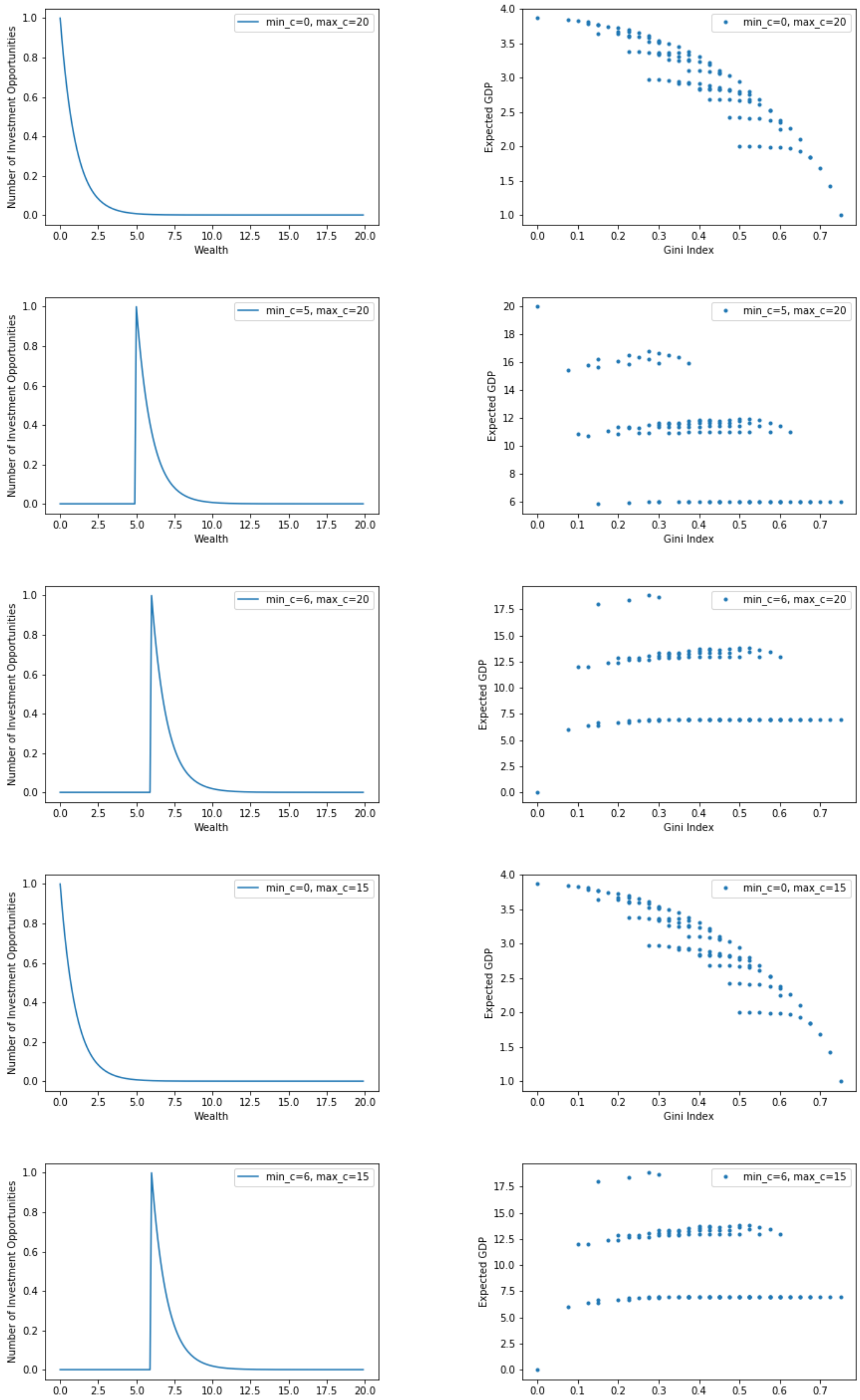
$$O(c) = \begin{cases} e^{c-min_c} & c < max_c, c > min_c \\ 0 & otherwise \end{cases}$$



If there is no minimum or maximum investment, then maximizing inequality maximizes expected GDP growth. That maximizing inequality maximizes growth is different from the uniform and sinusoidal cases, where minimizing inequality maximizes growth. The difference appears because in the exponential opportunity case as an agent's wealth increases the likelihood it invests all its wealth increases, by virtue of the assumed wealth-investment relation. Adding minimum and maximum investment amounts has the same general effect as the uniform case.

Exponential Decay Opportunity Distribution

$$O(c) = \begin{cases} e^{-(c-min_i)} & c < max_c, c > min_c \\ 0 & otherwise \end{cases}$$



If there is no minimum or maximum investment, then minimizing inequality maximizes expected GDP growth. Adding minimum and maximum investment amounts has the same general effect as the uniform case.