

Frankl's Conjecture of Family of Sets closed under Union

Let F be a Family of Sets such that:

$$e, g \in F \Rightarrow e \cup g \in F.$$

Frankl's Conjecture is that there is at least one Element of those Sets that is in at least 50% of all Sets in F.

My claim is: Frankl is right and there can be more than one Element with that Feature. And I can point out which more quickly than just counting. In A Power Set all of Elements of the Sets of that Powerset have that feature.

Proof:

1.) There is a Subset of F which i want to call S(F). It contains the equally smallest of all Sets of F.

$$h, f \in F \wedge h \in S(F) \wedge f \notin S(F) \Rightarrow |h| < |f|$$

S(F) can't be empty.

2.) Elements of F have exactly one of the following Relations between each other:

$$x, y, z \in F. x \neq y. y \neq z. z \neq x.$$

R1.) $x \cup y = y$ I call that absorbtion, y absorbs x.

R2.) $x \cup y = x$ Still absorbtion.

R3.) $x \cup y = z$ The Union is a bit larger than x or y. The Set x and the Set y depend on the existenz of z in F. Because F is closed under union. Let's Phrase it a Emmision. The Sets x an y emit z.

If $x \in S(F)$ than absorbtion R2.) can't happen because x is a Among the Smallest Sets of F.

3.) Lets say $x \in S(F)$ and let's define two new Subsets of Sets. I want to call them E(x) and A(x). They are defined as follows:

$$l \in A(x) \Rightarrow x \cup l = l$$

A(x) is just the Set of Absorbers of x. Each element in A(x) has 100% content of x.

$$l \in E(x) \Rightarrow x \cup l = k. k \neq l \wedge k \neq x.$$

E(x) is just the Set of Emmitters when united with x. The Result of such a union is k. And the Result is itself Element of A(x)! Also Elements of E might contain elements of x. But never 100%.

$$k \in A(x)$$

So for each Element in $E(x)$ there is a Element in $A(x)$. But not the other way around.

$$E(x) \cup A(x) = F$$

$$|A(x)| \geq |E(x)|$$

Now Remember that each element of $A(x)$ has a 100% of the elements of x . Q.E.D. Wrong!!!!!!

It Turns out that even if $S(F)$ is just Singletons x is not necessarily abundant. Which is explained here: <https://mathoverflow.net/a/254065/34341>

But i think that is also Wrong. I don't understand the mathoverflow.net post. I think if $S(F)$ is just singletons i'm right. I will post more here soon.

I Posted the first draw to to Gower's Weblog. (<https://gowers.wordpress.com/>) as Anymous. And an earlier Version of this here on Github (<https://github.com/blitzDing/Frankle.git>).

And David Bevan. Answerd to me the Following:

There may be two emitters of x whose union with x is the same. For example, if $x = \{1, 2\}$, then the emitters $\{1, 3\}$ and $\{2, 3\}$ give the same union with x , namely $\{1, 2, 3\}$.

So it cannot be deduced that $|A(x)| \geq |E(x)|$.

See <https://mathoverflow.net/a/254065/34341> for a specific example in which no element of a smallest set is abundant. It's good to read and understand existing results like this.

Best wishes,

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He is right if x is not a Singleton i believe. I thought about it and here is what i came up with. Let's assume $S(F)$ are not Singletons.

Oh! That's a problem...