

Frankl's Conjecture of Family of Sets closed under Union

Let F be a Family of Sets such that:

$$e, g \in F \Rightarrow e \cup g \in F.$$

Frankl's Conjecture is that there is at least one Element of those Sets that is in at least 50% of all Sets in F .

My claim is: Frankl is right and there can be more than one Element with that Feature. And I can point out which more quickly than just counting. In A Power Set all of Elements of the Sets of that Powerset have that feature.

Proof:

1.) There is a Subset of F which i want to call $S(F)$. It contains the equally smallest of all Sets of F .

$$h, f \in F \wedge h \in S(F) \wedge f \notin S(F) \Rightarrow |h| < |f|$$

$S(F)$ can't be empty.

2.) Elements of F have exactly one of the following Relations between each other:

$$x, y, z \in F. x \neq y. y \neq z. z \neq x.$$

R1.) $x \cup y = y$ I call that absorbtion, y absorbs x .

R2.) $x \cup y = x$ Still absorbtion.

R3.) $x \cup y = z$ The Union is a bit larger than x or y . The Set x and the Set y depend on the existenz of z in F . Because F is closed under union. Let's Phrase it a Emmision. The Sets x an y emit z .

If $x \in S(F)$ than absorbtion R2.) can't happen because x is a Among the Smallest Sets of F .

3.) Lets say $x \in S(F)$ and let's define two new Subsets of Sets. I want to call them $E(x)$ and $A(x)$. They are defined as follows:

$$l \in A(x) \Rightarrow x \cup l = l$$

$A(x)$ is just the Set of Absorbers of x . Each element in $A(x)$ has 100% content of x .

$$l \in E(x) \Rightarrow x \cup l = k. k \neq l \wedge k \neq x.$$

$E(x)$ is just the Set of Emmitters when united with x . The Result of such a union is k . And the Result is itself Element of $A(x)$! Also Elements of E might contain elements of x . But never 100%.

$$k \in A(x)$$

So for each Element in $E(x)$ there is a Element in $A(x)$. But not the other way around.

$$E(x) \cup A(x) = F$$

$$|A(x)| \geq |E(x)|$$

Now Remember that each element of $A(x)$ has a 100% of the elements of x . Q.E.D. Wrong!!!!!!

It Turns out that even if $S(F)$ is just Singletons x ist not neccessarly abundand. Which is explained here: <https://mathoverflow.net/a/254065/34341>