

Cartesian Product of a finite Sets with them Selfs

Lets assume S is a set of Cardinality 2. Further label the two Elements of the Set (a) and (b). Multiply that Set n-times with it self (Cartesian Product). Let's say that is equal to raise that Set S by a Power of n. We get Set T where each Member of T is made of a Tuples of length n and each Tuples consists only of 'a' and/or 'b' Entries. T has 2^n Tuples. All Variations possible are in that Set.

$$\overbrace{(a; a; b; a \dots)}^{\text{Tuple with } n \text{ Entries}} \in \{a, b\}^n = T_n \quad |T_n| = |\{a, b\}^n| = |S|^n$$

It gets interesting when u replace a and b with ciphers (0 and 1). If so each Tuple can be interpreted as a (binary) Number. For ease of imagination let's say the Tuple (1;0;0;1....) corresponds with the Number 1001.... So we can pair every n-digit(or less) binary and natural Number with one Tuple in $T(n)$. When we make a shift Operation to every and each Tuple by shifting (n minus m) digits - coming from the left – behind the hole Number demarcation sign (the point). If m is equal to 1 all digits are behind the Point. Except the first. This way we can interpret each Tuple as a Number between 0 and 2. If we make n bigger the Interval where the Numbers lay don't change. Because the Tuples grow from the Right. Instead we capture more Numbers between 0 and 2. Of course m is still equal with 1 and operating. Then we consider n equal to $|\mathbb{N}| (= \aleph_0)$. Now we have **all** Numbers between 0 and 2. That Means $|\mathbb{R}| = |T| = 2^{|\mathbb{N}|}$. As we know the 2 is there because our Set S has two Elements. But of course that is not a must. We got no obligation to bind this to binary number encoding. As u may see this means $|\mathbb{R}| = b^{|\mathbb{N}|}$ where b is an arbitrary Natural Number larger then 1.