Mohammad Abdullah

20100144

Q2 part 3

We suppose that our greedy algorithm is not optimal i.e. MV stops at more stations than necessary. Let the sequence of Fuel Stations of our greedy approach be L1,L2,….Lx. Since this is not optimal, an optimal sequence of stops must contain less stops. Let P1,P2,...,Py be such an optimal solution, where y < x. Let k be the largest index for which: L1,L2,...,Lk = P1,P2,...,Pk. Consider the stop k+1. We know that Lk+1 is not equal to Pk+1. Since our greedy approach selected the k+1 fuel station as the farthest away from Lk which is equal to Pk but within n Kilometers of Lk, Lk+1 must be closer to Lk than Lk+1, since if Pk+1 were even farther away from Lk MV would have run out of fuel, and hence the optimal solution wouldn't even be a solution. Hence, we can replace Pk+1 with Lk+1 in the optimal solution, without affecting neither the size nor the correctness of the optimal solution. Now, we repeat the same exchange procedure for each subsequent stop from k+2 to y, transforming the optimal solution into P1,P2...,Pk,Lk+1,...,Ly = L1,...,Lk,Lk+1,...,Ly. Remember that y < x. Since the optimal solution is a solution, it means that stop Py is within n Kilometers from Destination H, and hence you wouldn't need to stop for fuel anymore. But by our exchange argument above Ly is either equal to Py or even closer to Destination than Py is. Hence, our greedy algorithm would have stopped after Ly without producing the additional stops Ly+1,...,Lx. This contradicts our assumption that y < x, and this in turn contradicts our original assumption that our greedy method is not optimal. Hence, our greedy method is in fact optimal as we wanted to prove.